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# Improving throughput in multihop networks using node mobility

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February 2014

# Contents

This study is a copy paste of [10].

# 1 Introduction

Gupta and Kumar [7] studied the capacity of wireless networks consisting of randomly located nodes which are immobile. They showed that if each source node has a randomly chosen destination node, the useful network capacity is of order $C\sqrt{n/\log n}$ where $n$ is the number of nodes and $C$ is the nominal capacity of each node. However, if the nodes are mobile and follow i.i.d. ergodic motions in a square area, Grossglauser and Tse [6] showed that the network capacity can rise to $O(nC)$[1] by using the mobility of the nodes. In this case, a source node relays its packet to a random mobile relay node which transmits this packet to its destination node only when they come close together, *i.e.*, at a distance of order $1/\sqrt{n}$. Therefore, the time it takes to deliver a packet to its destination would be of order $\sqrt{n}L/v$ where $v$ is the average speed of the nodes and $L$ is the length of the fixed square area where nodes are deployed. In contrast, in Gupta and Kumar's result [7], the packet delivery delay tends to be negligible, although the network capacity drops by a factor of $\sqrt{n \log n}$.

In this study, we aim to maximize the capacity of mobile networks while keeping the mean packet delivery delay bounded with an increasing number of nodes. To relay packets towards their destinations, mobile nodes use our proposed georouting strategy, called the *Constrained Relative Bearing* (CRB) scheme. We show that, in a random walk mobility model, this strategy achieves a network capacity of order $\frac{n}{\log n \log\log n}C$ with a time to delivery of order $L/v$. Our main contribution is summarized in Table 1. Note that in random walk mobility models, nodes have free space motion and move in straight lines with constant speed. This mobility model is a subclass of the free space motion mobility model. Therefore, we can also extend our result to mobility models where the average free space distance $\ell$ is non zero.

Let us consider an example of an urban area network in a fixed square area of length $L$ with the number of nodes $n = 10^6$, nominal bandwidth $C = 100$ kbps and delay per store-and-forward operation of 1 ms. The average packet delivery delay for Gupta and Kumar's case would be around one second but with a network capacity of 10 Mbps. In the case of Grossglauser and Tse, the network capacity would increase to about 100 Gbps but if the straight

[1] We recall the following notation: (i) $f(n) = O(g(n))$ means that there exists a constant $c$ and an integer $N$ such that $f(n) \leq cg(n)$ for $n > N$. (ii) $f(n) = \Theta(g(n))$ means that there exist two constants $c_1$ and $c_2$ and an integer $N$ such that $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for $n > N$.

| | *Network Capacity* | *Delivery Delay* |
|---|---|---|
| Gupta & Kumar [7] | $O\left(\sqrt{\frac{n}{\log n}}\right)$ | negligible |
| Grossglauser & Tse [6] | $O(n)$ | $O\left(\frac{\sqrt{n}}{v}\right)$ |
| *Our work* | $O\left(\frac{n}{\log n \log\log n}\right)$ | $O\left(\frac{1}{v}\right)$ |

Table 1: Network Capacity vs. Delivery Delay Trade-off.

line crossing time $L/v$ is about one hour (*e.g.*, with cars as mobile nodes), the time to delivery would be around one month. However, our model which uses the mobility of nodes along with the proposed CRB scheme, to relay packets to their destinations, would lead to a network capacity of 10 Gbps with a time to delivery of about one hour.

This study is organized as follows. We first summarize some important related work and results in Section 2. We discuss the models of our network and CRB scheme in Sections 3 and 4 respectively. The analysis of capacity and delay can be found in Section 5 and we confirm this analysis using simulations in Section 6. We also discuss possible extensions of our work in Section 7 and concluding remarks can be found in Section 8.

# 2 Related Studies

The main difference between the models proposed by Gupta and Kumar [7] and Grossglauser and Tse [6] is that in the former case, nodes are static and packets are transmitted between nodes like "hot potatoes", while in the latter case, nodes are mobile and relays are allowed to carry buffered packets while they move. Both strategies are based on the following model: if $p_n$ is the transmission rate of each node, *i.e.*, the proportion of time each node is active and transmitting, the radius of efficient transmission is given by $r_n \sim L\sqrt{\frac{\kappa}{np_n}}$ when $n$ approaches infinity for some constant $\kappa > 0$ which depends on the protocol, interference model, etc.

In the context of [7], the number of relays a packet has to traverse to reach its destination is $h_n = O(1/r_n)$. Consequently, $np_nC$ must be divided by $h_n$ to get the useful capacity: $np_nC/h_n = O(C\sqrt{p_n n})$. In order to ensure connectivity in the network, so that every source is able to communicate with its randomly chosen destination, $p_n$ must satisfy the limit $p_n \leq O(1/\log n)$. This leads to Gupta and Kumar's maximum capacity of $O(C\sqrt{n/\log n})$ with "hot potatoes" routing.

In contrast, in the context of [6], the network does not need to be connected since the packets are mostly carried in the buffer of a mobile relay. Therefore there is no limit on $p_n$ other than the requirement that it must be smaller than some $\varepsilon < 1$ that depends on the protocol and some other physical parameters. Thus $r_n$ is $O(1/\sqrt{n})$. In Grossglauser and Tse's model, the source transmits the packet to the closest mobile relay or keeps it until it finds one. This mobile relay delivers the packet to the destination when it comes within range of the destination node. Such a packet delivery requires a transmission phase which also includes retries and acknowledgements so that the packet delivery can be eventually guaranteed.

The proposed model of [6] requires a GPS-like positioning system and the knowledge of the effective range $r_n$. The estimate of $r_n$ could be achieved via a periodic beaconing from every node, where each beacon contains the position coordinates of the node, so that a node knows the typical distance for a successful reception. However, the relay cannot rely on beaconing in order to detect when it is in the reception range of the destination. The reason is that a node stays in the reception range of another node for a short time period of order $r_n/v = 1/\sqrt{n}$ and this cannot be detected via a periodic beaconing with bounded frequency since $p_n = O(1)$ (the frequency of periodic beaconing should be $O(\sqrt{n})$). We may also assume that the destination node is fixed and its Cartesian coordinates are known by the mobile relay. Otherwise, if the destination node is mobile, there would be a requirement for this node to track its new coordinates and disseminate this information in the wireless network as in [15, 20].

It is also interesting to note that Diggavi, Grossglauser, and Tse [3] showed that a constant throughput per source-destination pair is feasible even with a more restricted mobility model. Franceschetti et al. [4] proved that there is no gap between the capacity of randomly located and arbitrarily located nodes. Throughput and delay trade-offs have appeared in [5, 19] where the delay of multi-hop routing is reduced by increasing the coverage radius of each transmission, at the expense of reducing the number of simultaneous transmissions the network can support. We will show that, in our work, we have a delay of $O(1/v)$ and throughput per source-destination pair of $O(\frac{1}{\log n \log\log n})$. If we take the notation of $\sqrt{a(n)}$ in [5, 19] to measure the average distance traveled toward the destination between two consecutive emissions of the same packet, then we will show that our scheme yields $\sqrt{a(n)} = \Theta(1/\log n)$. If we compare with the result of [5,19], we should have a throughput of $\Theta(\frac{1}{\log n\sqrt{n \log n}})$ but our scheme delivers a higher throughput by a factor greater than $\sqrt{n}$. In fact, if $\ell$ is the average free space distance of the random walk, then our scheme yields $\sqrt{a(n)} = \Theta\left(\frac{1}{\frac{1}{\ell}+\log n}\right)$. The apparent contradiction comes from the fact that the authors in [5, 19] consider a mobility model based on Brownian motion. This corresponds to having $\ell = 0$ and, in this case, our scheme would be equivalent to the "hot potatoes" routing of [7] with $\sqrt{a(n)} = \Theta(r_n)$. Let us point out that the Brownian motion mobility is an important yet worst case model and it is not realistic for real world situations such as urban area mobile networks. In the section devoted to generalizations, we extend our result to fit a more general mobility model where mobile nodes follow fractal trajectories with $\ell = \ell_n = \Theta(1/\log n)$ and the throughput of our scheme remains $\Theta(\frac{1}{\log n \log\log n})$.

On the practical side, many protocols have been proposed for wireless multi-hop networks. These protocols may be classified in topology-based and position-based protocols. Topology-based protocols [2,8,18] need to maintain information on routes potentially or currently in use, so they do not work effectively in environments with a high frequency of topology changes. For this reason, there has been an increasing interest in position-based routing protocols. In these protocols, a node needs to know its own position, the one-hop neighbors' positions, and the destination node's position. These protocols do not need control packets to maintain link states or to update routing tables. Examples of such protocols can be found in [1, 9, 12–14, 16, 17, 21]. In contrast to our work, they do not analyze the trade-off between the capacity and the delay of the network under these protocols and their scaling properties.

# 3 Network and Mobility Settings

We consider a network of $n$ mobile nodes with their initial positions uniformly distributed over the network area. Each mobile node transmits packets to a randomly chosen fixed node, called its destination node, which is also randomly located in the network area. We assume that mobile nodes are aware of their own Cartesian coordinates, *e.g.*, by using GPS or from the initial position, a mobile node could use the knowledge of its motion vector to compute its Cartesian coordinates at any given time.

Initially we consider that only mobile nodes participate in the relay process to deliver packets to its destination node. The case where the fixed nodes may also participate in the relay process is discussed in Section 7. A mobile node should be aware of the Cartesian coordinates of the destination node of a packet it carries. Indeed it can be assumed that this information is included in all packets or is relayed with the packets. Hence our model only requires that a source or relay node is aware of the cartesian coordinates of the destination node, which is assumed to be fixed. Note that if the destination node is mobile, a mechanism to disseminate its updated cartesian coordinates in the network can be used, *e.g.*, [15,20]. However, this is outside the scope of this paper as we particularly focus here on the throughput-delay tradeoff.

With the available information, a mobile relay can determine:

- its heading vector, which is the motion vector when its speed is non zero,
- its bearing vector, which is the vector between its position and the position of a packet's destination; and,
- the relative bearing angle, *i.e.*, the absolute angle between its heading and bearing vectors.

In the example of Fig. 1, node $A$ is carrying a packet for node $D$. This figure also shows the heading vector of mobile node $A$ and its bearing vector and relative bearing angle for destination node $D$. Note that a mobile relay may carry packets for multiple destinations but can easily determine the bearing vector and relative bearing angle for each destination node.

# 4 Model of the CRB Scheme

In this section, will present the parameters and specifications of the model of our georouting scheme.

## 4.1 Parameters

We define the parameters $\theta_c$, called the carry angle, and $\theta_e$, called the emission angle. Each mobile node carries a packet to its destination node as long as its relative bearing angle, $\theta$, is smaller than $\theta_c$ which is strictly smaller than $\pi/2$. When this condition is not satisfied, the packet is transmitted to the next relay.

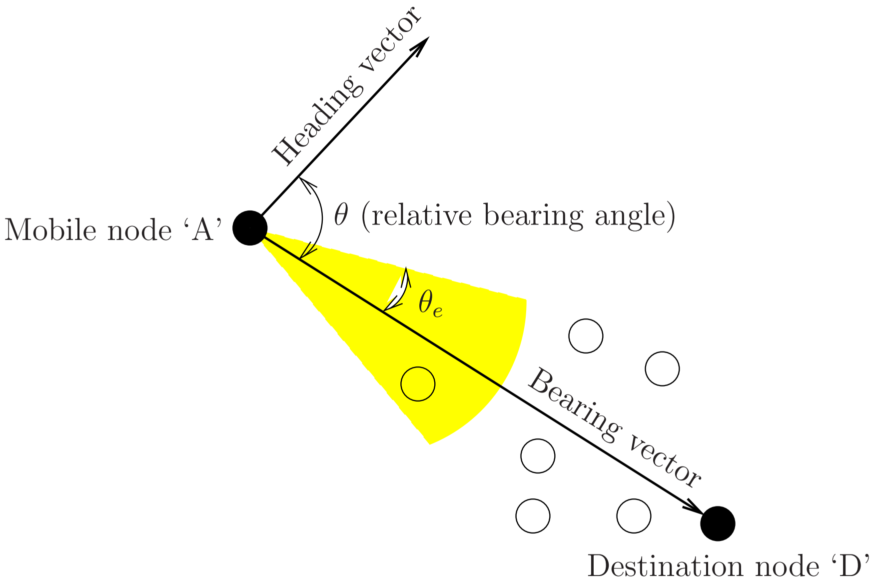


Figure 1: Figurative representation of our model. Unfilled circles represent the potential mobile relays for a packet transmitted by node $A$ for node $D$.

## 4.2 Model Specification with Radio Range Awareness

In the following description, we initially assume that each node is aware of the effective range of transmission $r_n$. This means that there is a periodic beaconing that allows this estimate to be made. In Section 4.3, we investigate how to specify our model without an estimate of the effective range $r_n$.

Let us assume that node $A$ is carrying a packet for node $D$. The velocity of node $A$ is denoted by $\mathbf{v}(A)$.

- If node $A$ is within range of node $D$, it transmits the packet to $D$ directly otherwise,
- if the relative bearing angle is smaller than $\theta_c$, node $A$ continues to carry the packet; otherwise,
- node $A$ transmits the packet to a random neighbor mobile node inside the cone of angle $\theta_e$, with the bearing vector as the axis, and then takes no further interest in the packet.

In order to better understand the model of our georouting scheme, let us consider the example in Fig. 1. We assume that node $A$ is out of range of node $D$ and, because of that, it cannot deliver the packet directly. However, if $\theta < \theta_c$, node $A$ will continue to carry the packet for node $D$. Otherwise, it transmits the packet to one of the random mobile relays, represented by unfilled circles in the figure.

### 4.2.1 Transmission procedure

To transmit the packet towards another mobile node, node $A$ proceeds as follows:

- it first transmits a *Call-to-Receive* packet containing the positions of nodes $A$ and $D$;
- a random mobile node $B$ which receives this *Call-to-Receive* packet can compute the angle $(AB, AD)$. If this angle is smaller than $\theta_e$, it replies with an *Accept-to-Receive* packet containing an identifier of node $B$;
- node $A$ sends the packet to the first mobile node which replied with an *Accept-to-Receive* packet.

The first node which sends its *Accept-to-Receive* packet notifies the other receivers of the *Call-to-Receive* packet, to cancel their transmissions of *Accept-to-Receive* packets. There may be more than one (but finite) transmissions of *Accept-to-Receive* packets in case two or more receivers are at a distance greater than $r_n$ from each other.

Note that this procedure does not need any beaconing or periodic transmission of hello packets. The back-off time of nodes, transmitting their *Accept-to-Receive* packet, can also be tuned in order to favor the distance or displacement towards $D$, depending on any additional optional specifications.

## 4.3 Model Specification without Radio Range Awareness

Estimating of $r_n$ would require that the nodes employ a periodic beaconing mechanism. If such a mechanism is not available or desirable, the CRB scheme relies on the signal to interference plus noise ratio (SINR) to transmit packets to their destinations or random mobile relays. In other words, a mobile node can relay a packet to its destination node or another mobile node only if the SINR at the receiver is above a given threshold.

Note that in this case, the specification of the transmission procedure is also modified so that it terminates when the final destination receives the packet. To transmit the packet towards its destination node or another mobile node, node $A$ proceeds as follows:

- it first transmits a *Call-to-Receive* packet containing the positions of nodes $A$ and $D$;

- if node $D$ receives this packet, it responds immediately with an *Accept-to-Receive* packet with the highest priority. Node $A$, on receiving this packet, relays the packet to node $D$; otherwise,

- the procedure of selecting a random mobile node, as the next relay, is similar to the procedure described in Section 4.2.1. A random mobile node $B$, which receives the *Call-to-Receive* packet, computes the angle $(AB, AD)$. If this angle is smaller than $\theta_e$, it responds with an *Accept-to-Receive* packet;

- node $A$ relays the packet to the first mobile node which sent its *Accept-to-Receive* packet successfully. The first node which transmits its *Accept-to-Receive* packet also makes the other receivers cancel their transmissions of *Accept-to-Receive* packets.

# 5 Performance analysis

In our georouting scheme, the average transmission rate of each mobile node is $p_n = O(1/\log\log n)$. We will also show that the number of transmissions per packet is $O(\log n)$ and this would lead to a useful network capacity $O(C\frac{n}{\log n \log\log n})$. Note that our scheme could lead to packet loss because of transmission failure but we show that the probability of this packet loss is the inverse power of $\log n$ and it tends to zero as $n$ approaches infinity.

We assume that the network area is a square area and without loss of generality we assume that it is a square *unit* area. The mobile nodes move according to i.i.d. random walk: from a uniformly distributed initial position, the nodes move in a straight line with a certain speed and randomly change direction. The speed is randomly distributed in an interval $[v_{\min}, v_{\max}]$ with $v_{\min} > 0$. To simplify the analysis, we assume that $v_{\min} = v_{\max} = v$. We also assume that each node changes its direction with a Poisson point process of rate $\tau$. When a mobile node hits the border of the network, it simply bounces back off like a billiard ball. This leads to the *isotropic property* (Jacquet et al. [11]): at any given time the mobile nodes are uniformly distributed in the square and move in uniformly selected directions independently of their position.

The radius of efficient transmission $r_n$ is derived from the value of $p_n$ and is given by $r_n = \sqrt{\beta\frac{\log\log n}{\pi n}}$ , for some $\beta > 0$. Therefore, the average number of neighbors of an arbitrary node at an arbitrary time is $\beta \log\log n$. In order to keep the average cumulated load finite, the nodes have an average transmission rate of $p_n = \frac{1}{\beta\log\log n}$. Therefore, the actual density of simultaneous transmitters is $\frac{n}{\beta \log \log n}$.

## 5.1 Methodology

The parameters of interest are the following:

- The delay $D_n(r)$ of delivering a packet to the destination when the packet is generated in a mobile node at distance $r$ from its destination node.
- The average number of times $F_n(r)$ the packet changes relay before reaching its destination when it has been generated in a mobile node at distance $r$ from its destination node.

In order to demonstrate the actual performance of our proposed CRB scheme, we aim to derive an upper-bound on the parameters $D_n(r)$ and $F_n(r)$. In the next two sub-sections, we assume w.l.o.g. that there is always a relay node, to receive the packet, in the emission cone (as the node density and angle, $\theta_e$, are sufficiently large) when a relay change must occur.

## 5.2 Delivery Delay

We ignore the queueing delays which can become apparent when several packets could be in competition in the same relay to be transmitted at the same time.

**Theorem 1.**

$$D_n(r) \leq \frac{r}{v\cos(\theta_c)} + \frac{F_n(r)}{v}\left(t_f v - \frac{r_n}{\cos(\theta_c)}\right) , \tag{1}$$

*where $t_f$ is the store and forward delay quantity.*

*Proof.* We can write $D_n(r)$ as

$$D_n(r) \leq F_n(r)t_f + \frac{r - F_n(r)r_n}{v\cos(\theta_c)} ,$$

where $F_n(r)t_f$ is the average total delay of store and forward operations and $\frac{r-F_n(r)r_n}{v\cos(\theta_c)}$ is an upper bound of the delay when mobile relays carry a packet at constant speed $v$ with a relative bearing angle always smaller than $\theta_c$. Reordering this equation leads to (1). □

**Remark 1.** *If $r_n$ is greater than $t_f v$, the second factor in* (1) *becomes negative which gives the upper bound of $D_n(r) \leq \frac{r}{v\cos(\theta_c)}$ . However, if $r_n$ is less than $t_f v$, $D_n(r)$ will also include an $O(\log n)$ factor because of $F_n(r)$.*

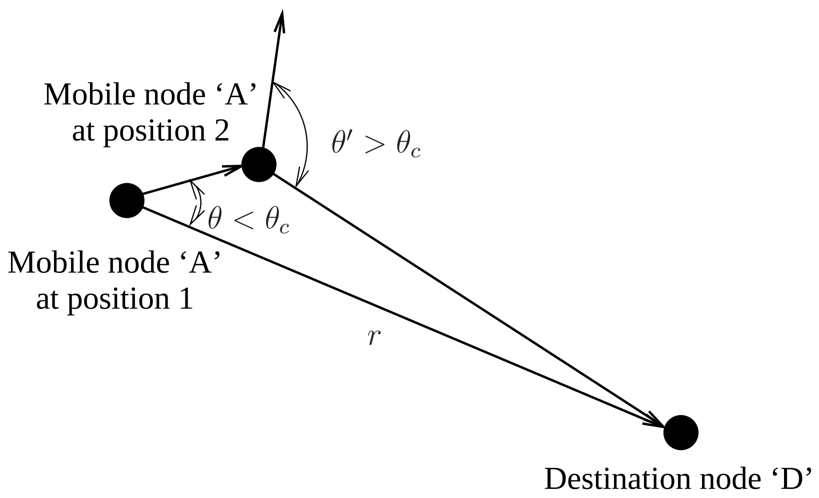


Figure 2: Figurative description of relay change due to turn. At position 1, $\theta < \theta_c$ and node $A$ carries the packet for node $D$. At position 2, node $A$ changes its heading vector and must transmit the packet.

## 5.3 Number of Relay Changes

There are two events that trigger relay changes.

1. Relay change due to turn, *i.e.*, the mobile node, carrying the packet, changes its heading vector such that the relative bearing angle becomes greater than $\theta_c$.
2. Relay change due to pass over, *i.e.*, the mobile node keeps its trajectory and the relative bearing angle becomes greater than $\theta_c$.

Let us consider a packet generated at distance $r$ from its destination. Let $F_n^t(r)$ be the average number of relay changes due to turn. Equivalently, let $F_n^p(r)$ be the average number of relay changes due to pass over. Therefore, we have $F_n(r) = F_n^t(r) + F_n^p(r)$ and we expect that the main contribution of $O(\log n)$ in $F_n(r)$ will come from $F_n^p(r)$.

### 5.3.1 Number of Relay Changes Due to Turn

We prove the following theorem:

**Theorem 2.** *We have the bound:*

$$F_n^t(r) \leq \frac{\pi - \theta_c}{\theta_c} \frac{\tau}{v \cos(\theta_c)} r \ .$$

*Proof.* We consider the case in Fig. 2 and assume that a mobile node is carrying a packet to its destination located at distance $r$. The node changes

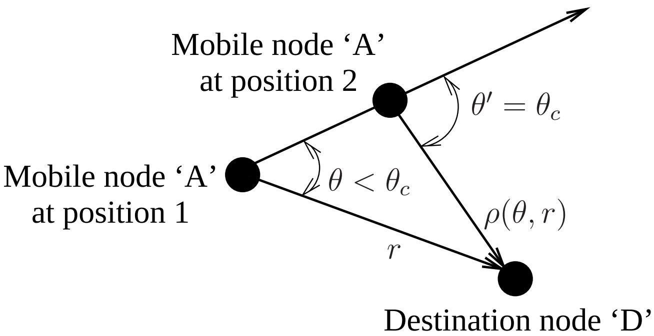


Figure 3: Figurative description of relay change due to pass over. At position 1, $\theta < \theta_c$ and node $A$ carries the packet for node $D$. At position 2, node $A$ has the same heading vector but $\theta' = \theta_c$ and it must transmit the packet.

its direction with Poisson rate $\tau$. When the node changes its direction, it may keep a direction that stays within angle $\theta_c$ with the bearing vector and this will not trigger a relay change. This occurs with probability $\frac{\theta_c}{\pi}$. Otherwise, the packet must change relay. But the new relay may have a relative bearing angle greater than $\theta_c$ which would result in an immediate new relay change. Therefore, at each direction change, there is an average of $\frac{\pi-\theta_c}{\theta_c}$ relays. Multiplied by $D_n(r)$ this gives our upper-bound of $F_n^t(r)$.

Note that we have not considered the turn due to bounces on the borders of the square map. But it is easy to see via straightforward geometric considerations that they cannot actually generate a relay change. □

#### 5.3.2 Number of Relay Changes Due to Pass Over

We prove the following theorem:

**Theorem 3.** *We have the bound:*

$$F_n^p(r) \leq \frac{\pi \tan(\theta_c)}{\theta_c^2} \log\left(\frac{r}{r_n}\right) .$$

*Proof.* Here we consider the case of Fig. 3. We assume that a mobile node at distance $r$, from its destination, has a relative bearing angle equal to $\theta$. If it keeps its trajectory (*i.e.,* does not turn), it will need to transmit to a new relay when it passes over the destination, *i.e.*, when it arrives at a distance of $\rho(\theta, r) = \frac{\sin(\theta)}{\sin(\theta_c)} r$ from the destination. The function of $\theta$ $\rho(\theta, r)$ is bijective from $[0, \theta_c]$ to $[0, r]$. For $x \in [0, r]$ let $\rho^{-1}(x, r)$ be its inverse.

Assume that $r$ is the distance to the destination when the relay receives the packet or just after a turn. Thus the angle $\theta$ is uniformly distributed on

$[0, \theta_c]$, *i.e.*, with a constant probability density $\frac{1}{\theta_c}$. The probability density of the pass over event at $x < r$ (assuming no direction change) is therefore

$$\frac{1}{\theta_c}\frac{\partial}{\partial x}\rho^{-1}(x,r) = \frac{\sin(\theta_c)}{\theta_c \cos(\rho^{-1}(x,r))r} = \frac{\tan(\rho^{-1}(x,r))}{\theta_c}\frac{1}{x} \; .$$

Since $\rho^{-1}(x,r) \leq \theta_c$, the point process where the packet would need a relay change due to pass over is upper bounded by a Poisson point process on the interval $[r_n, r]$ and of intensity equal to $\frac{\tan(\theta_c)}{\theta_c}\frac{1}{x}$ for $x \in [r_n, r]$.

A relay change due to pass over corresponds to an average of $\frac{\pi}{\theta_c}$ relays which is higher than the average number of relays in the case of relay change due to turn. The reason for this higher number is the fact that when relay change due to turn, a mobile relay may still move in a direction that stays within the carry angle $\theta_c$ and continue to carry the packet. Neglecting the decrement of distance during each transmission phase

$$F_n^p(r) = \int_{r_n}^{r} \frac{\pi \tan(\theta_c)}{\theta_c^2}\frac{dx}{x} = \frac{\pi \tan(\theta_c)}{\theta_c^2} \log\left(\frac{r}{r_n}\right) \; .$$

□

We have thus

$$F_n(r) \leq \frac{\pi - \theta_c}{\theta_c}\frac{\tau}{\cos(\theta_c)v} r + \frac{\pi \tan(\theta_c)}{\theta_c^2} \log\left(\frac{r}{r_n}\right) \; .$$

Therefore we have a main contribution of $O(\log n)$ relay changes that comes from $\log(1/r_n)$. The result holds because we assume that there is always a receiver in each relay change. In the next sub-section we remove this condition to establish a result with high probability.

## 5.4 Number of Relay Changes With High Probability of Success

In the previous subsection we assumed that there is always a receiving relay in the emission cone at each relay change and that the relay change is always successful. The case with failed relay change would introduce additional complications. For example one could use fixed relays if a packet cannot be delivered to a mobile relay. However, to simplify the present contribution, we will show that with high probability, *i.e.*, with probability approaching 1 when $n$ approaches infinity, every relay change succeeds.

**Theorem 4.** *With high probability on arbitrary packets, all relay changes succeed for this packet and on average number* $F_n(r)$ *and the delay is* $D_n(r)$.

*Proof.* We use a modified stochastic system to cope with failed relay changes. The modification is the following: when there is no relay in the emission cone during a relay change a *decoy* mobile relay is created in the emission cone that will receive the packet. Each decoy relay is used only for one packet and disappears after use. Notice that the modified system is *not* a practical scheme in a practical network. The analysis in the previous section still holds and in particular $F_n(r)$ is now the average unconditional number of relay changes (including those via decoy relays) for any packet starting at distance $r$ from the destination.

Let $P_n(r)$ be the probability that a packet starting at distance $r$ has a failed relay change. The probability that a relay change fails is equal to $(1-\theta_e r_n^2)^{n-1} \sim e^{-n\theta_e^2 r_n^2} = (\log n)^{-\beta\frac{\theta_e}{\pi}}$. Therefore the average number of failed relay changes $E_n(r) \leq F_n(r)(\log n)^{-\beta\frac{\theta_e}{\pi}}$ which tends to zero when $\beta\frac{\theta_e}{\pi} > 1$, since $F_n(r) = O(\log n)$. The final result comes since $P_n(r) \leq E_n(r)$. □

# 6 Simulations

We performed simulations with CRB georouting scheme in two contexts:

1. a simplified context where the network is modeled under the unit disk model;
2. a realistic context where the network operates under slotted ALOHA and a realistic SINR interference model is considered. The simulations of the CRB scheme are stressed to the point that the motion timings are not so large compared to slot times.

## 6.1 Under Disk Graph Model

In this section, we consider a network of $n$ mobile nodes. We assume that all the nodes have the same radio range given by

$$r_n = \sqrt{\frac{\beta_0 \log\log n}{\pi n}} .$$

Each mobile node moves according to an i.i.d. random walk mobility model, *i.e.*, it starts from a uniformly distributed initial position, moves in a straight line with a constant speed and uniformly selected direction and bounces back off on the borders of the square area (like billiard balls).

In the next section (Section 6.2), we will further explore the effect of interference on the simulations, but for the moment we only consider a source mobile node and its randomly located destination node which is fixed. We adopt the disk graph model of interference, *i.e.*, two nodes are connected or they can exchange information if the distance between them is smaller than a certain threshold (called radio range), otherwise, they are disconnected. A mobile node relays the packet only if the relative bearing angle, *i.e.*, the absolute angle made by the heading vector and the bearing vector, becomes greater than $\theta_c$. Otherwise, it continues to carry the packet.

### 6.1.1 Simulation parameters and assumptions

The aim of our simulations is to verify the scaling behavior of average delay and number of hops per packet with increasing numbers of nodes in the network. Therefore, the number of mobile nodes, $n$, in the network varies from 10000 to two million nodes. The values of the the other parameters, which remain constant, and do not impact on the scaling behavior are listed as follows.

*(i)* The parameters of the CRB scheme, $\theta_c$ and $\theta_e$, are taken to be $\pi/6$.
*(ii)* The speed of all mobile nodes is constant, *i.e.*, 0.005 unit distance per slot.
*(iii)* All mobile nodes change their direction according to a Poisson point process with mean equal to 10 slots.
*(iv)* The value of the constant factor $\beta_0$ is assumed to be equal to 40.

### 6.1.2 Results

We have evaluated the following parameters.
*(i)* Average delay per packet.
*(ii)* Average number of hops per packet.

We considered the Monte Carlo Method with 100 simulations. The delay of a packet is computed from the time its processing started at its source mobile node until it reaches its destination node. Figure 4 shows the average delay per packet with an increasing number of nodes. We notice that as $n$ increases, the average delay per packet appears to approach a constant upper bound which can be computed from (1). Figure 5 shows the average number of hops per packet with increasing values of $n$.

## 6.2 With slotted ALOHA under the SINR interference model

In this section, we will present the simulations of the CRB georouting scheme with a transmission model which does not rely on the estimate of $r_n$ and is based on the required minimal SINR threshold.

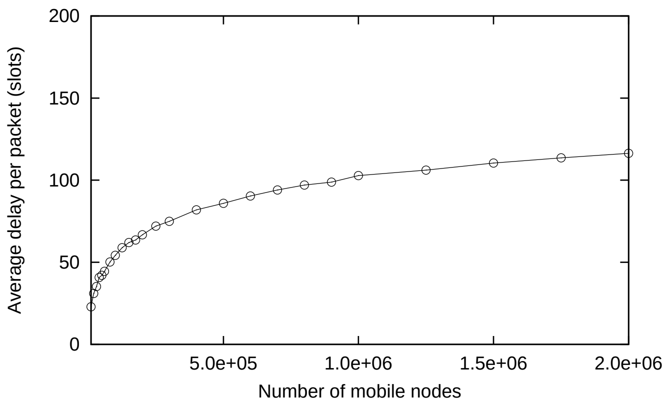


Figure 4: Average delay per packet.

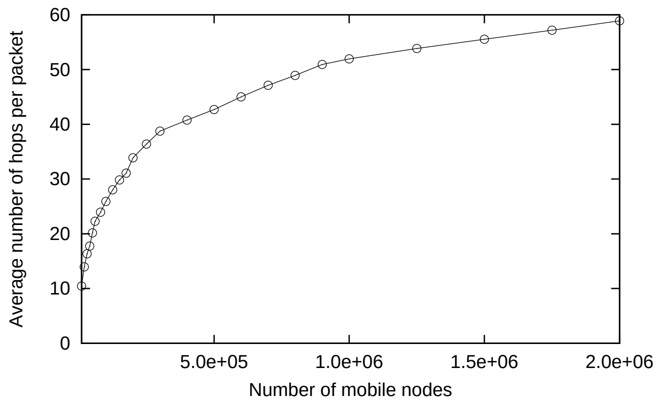


Figure 5: Average number of hops per packet.

### 6.2.1 Transmission model

Our transmission model is as follows. Let $P_i$ be the transmission power of node $i$ and $\gamma_{ij}$ be the channel gain from node $i$ to node $j$ such that the received power at node $j$ is $P_i\gamma_{ij}$. The transmission from node $i$ to node $j$ is successful only if the following condition is satisfied

$$\frac{P_i\gamma_{ij}}{N_0 + \sum_{k\neq i} P_k\gamma_{kj}} > K \ ,$$

where $K$ is the desired minimum SINR threshold for successfully receiving the packet at the destination and $N_0$ is the background noise power. For the moment, we ignore multi-path fading or shadowing effects and assume that the channel gain from node $i$ to node $j$ is given by $\gamma_{ij} = |z_i - z_j|^{-\alpha}$ , where $\alpha > 2$ is the attenuation coefficient and $z_i$ is the location of node $i$.

### 6.2.2 Simulations under the SINR interference model

For the theoretical analysis in Section 5, we have assumed that the effective range of successful transmission is

$$r_n = \sqrt{\beta \frac{\log \log n}{\pi n}},$$

which requires that the mobile nodes have an average transmission rate of $p_n = \beta / \log \log n$. In other words, if the mobile nodes emit packets at the given average rate, the average distance of successful transmission under the SINR interference model is of $O(r_n)$ and the results from the theoretical analysis are also applicable.

We assume that time is slotted and mobile nodes determine their relative bearing angles at the beginning of a slot. We also assume that all the nodes are synchronized and simultaneous transmitters in each slot emit a *Call-to-Receive* packet at the beginning of the slot. Moreover, we also assume that fixed nodes do not emit any packet except, maybe, an *Accept-to-Receive* packet in response to a transmission by a mobile node.

In our simulation environment, $n$ mobile nodes start from a uniformly distributed initial position and move independently in straight lines and in randomly selected directions. They also change their direction randomly at a rate which is a Poisson point process. Each mobile node sends packets towards a unique destination (fixed) node, and all the destination nodes are also uniformly distributed in the network area.

In order to keep the load in the network finite, the packet generation rate at a node, $\rho_n$, should be of $O(p_n/X_n)$ where $X_n$ is the expected number of transmissions per packet. From the theoretical analysis, we know that $X_n = O(\log(\frac{n}{\beta_2})) + c$ , where $c$ is a constant if $\theta_c$ is non-varying. In our simulations under the SINR model, we assume that the knowledge of $r_n$ is not available and mobile nodes use the minimal SINR threshold to successfully receive a packet. We also assume that each mobile node generates packets, destined for its unique fixed destination node, at a uniform rate given by

$$\rho_n = \frac{1}{\beta_1 \log(\frac{n}{\beta_2}) \log \log n} \ , \tag{2}$$

for some $\beta_1 > 0$ and $\beta_2 > 0$. We have not taken into account the value of constant $c$ and have observed that the simulation results are asymptotically correct because, with $n$ increasing, the value of $c$ should be insignificant compared to the $O(\log(n/\beta_2))$ factor.

### 6.2.3 Simulation parameters and assumptions

The aim of our simulations is to verify the scaling properties of network capacity, delay and number of transmissions per packet with increasing number of nodes in the network. The number of mobile nodes, $n$, in the network varies from 250 nodes to 100,000 nodes. All the nodes use uniform unit nominal transmission power and the background noise power $N_0$ is assumed to be negligible. The values of the other parameters are listed as follows.

*(i)* The parameters of the CRB scheme, $\theta_c$ and $\theta_e$, are taken to be $\pi/6$.

*(ii)* The speed of all mobile nodes is constant, *i.e.*, 0.01 unit distance per slot.

*(iii)* All mobile nodes change their direction independently and randomly according to a Poisson point process with mean equal to 10 slots.

*(iv)* The values of the constant factors $\beta_1$ and $\beta_2$ are assumed to be equal to 500 and 1 respectively.

*(v)* The SINR threshold, $K$, is assumed equal to 1.

*(vi)* Attenuation coefficient, $\alpha$, is assumed to be equal to 2.5.

In our simulations, we make the following assumptions.

*(i)* Each mobile node generates an infinite number of packets, at rate $\rho_n$, for its respective destination node.

*(ii)* A mobile node may carry, in its buffer, its own packets as well as the packets relayed from other mobile nodes. Therefore, it may have more than one packet in its buffer which it must transmit because their respective relative bearing angles are greater than $\theta_c$. In such a case, it first transmits the packet which is furthest from its destination.

### 6.2.4 Results

We have examined the following parameters.

*(i)* Throughput capacity per node, $\lambda_n$.

*(ii)* Average number of hops, $h_n$, and transmission attempts, $t_n$, per packet.

*(iii)* Average delay per packet.

The throughput capacity per node, $\lambda_n$, is the average number of packets arriving at their destinations per slot per mobile node. With $n$ increasing, throughput capacity per node should follow the following relation

$$\lambda_n = \frac{\eta}{\beta_1 \log(\frac{n}{\beta_2}) \log\log n} \ , \tag{3}$$

for some $0 < \eta < 1$ which depends on $K, \alpha$ and protocol parameters. Note that the values of these constants do not affect the asymptotic behavior of $\lambda_n$ which is also observed in our simulation results.

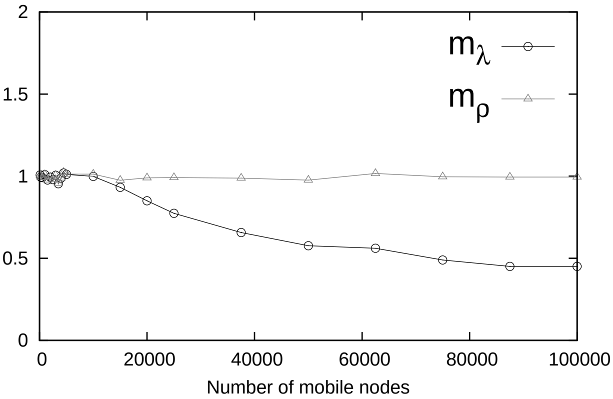


Figure 6: Verification of network throughput capacity with plots of $m_\lambda$ and $m_\rho$.

In order to verify the asymptotic character of the simulated packet generation rate and throughput capacity, we have analyzed the parameters $m_\rho$ and $m_\lambda$ which are given by

$$
\begin{aligned}
m_\rho &= \rho_n \left( \beta_1 \log \left( \frac{n}{\beta_2} \right) \log\log(n) \right) \ , \\
m_\lambda &= \lambda_n \left( \beta_1 \log \left( \frac{n}{\beta_2} \right) \log\log(n) \right) .
\end{aligned}
$$

From the definition of $\rho_n$ in (2), the value of $m_\rho$ should be constant at 1 whereas, with $n$ increasing, the value of $m_\lambda$ should converge to the constant $\eta$. From Figure 6, the value of $\eta$ is found to be approximately equal to 0.45. Figure 7 shows the simulated and the theoretical packet generation rate, $n\rho_n$, and the throughput capacity, $n\lambda_n$, in the network. The theoretical values of $n\rho_n$ and $n\lambda_n$ are computed from (2) and (3).

Figure 8 shows the average number of hops, $h_n$, and transmission attempts, $t_n$, per packet. The value of $t_n$ is slightly higher than the value of $h_n$ because of the possibility that a successful receiver may not be found in each transmission phase, *i.e.*, in the cone of transmission formed with $\theta_e$. With $n$ increasing, $h_n$ and $t_n$ are expected to grow in $O(\log(n/\beta_2))$. To verify this behaviour in simulation results, we examine the parameters $m_h$ and $m_t$ given by

$$
\begin{aligned}
m_h &= h_n \frac{1}{\log(\frac{n}{\beta_2})} \ , \\
m_t &= t_n \frac{1}{\log(\frac{n}{\beta_2})} .
\end{aligned}
$$

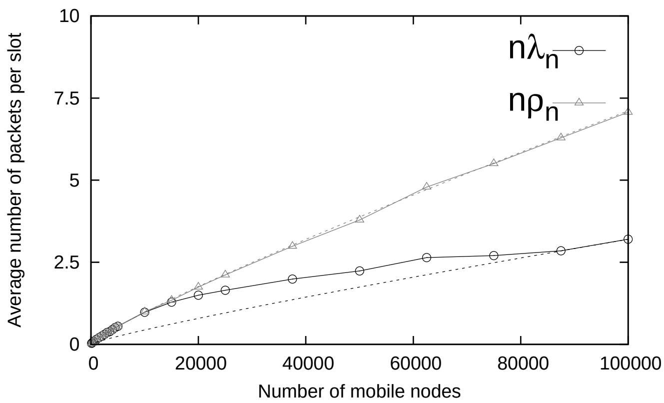


Figure 7: Simulated (solid lines) and theoretical (dotted lines) network throughput capacity, $n\lambda_n$, and network packet generation rate, $n\rho_n$.

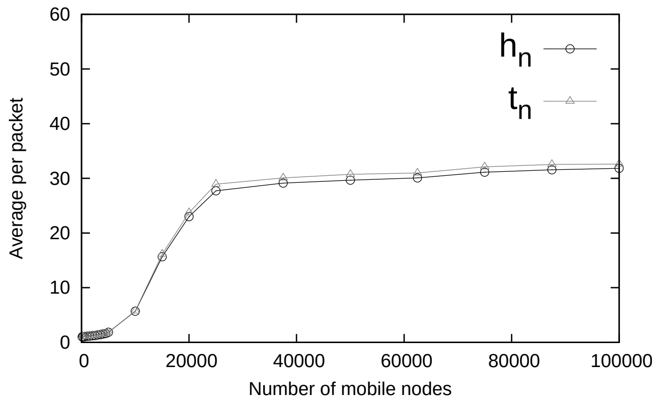


Figure 8: Average number of hops, $h_n$, and transmission attempts, $t_n$, per packet.

If the values of $h_n$ and $t_n$ are in $O(\log(n/\beta_2))$, the values of $m_h$ and $m_t$ should approach a constant value, which is the case in Figure 9.

The delay of a packet is computed from the time its processing starts at its source mobile node until the time it arrives at its destination node. Figure 10 shows the average delay per packet. As the number of mobile nodes increases, the average delay appears to approach a constant value.

It can be observed that when $n$ is small, the average number of hops per packet is almost of $O(1)$ which also means that the average delay per packet is of $O(1)$ and the network throughput capacity is of $O(\eta n)$. In simulation results however, it is bounded by the network packet generation rate which is of $O(\frac{n}{\log n \log\log n})$. This can be observed in Figures 7, 8 and 10. The reason is that when $n$ is small, the number of simultaneous transmissions

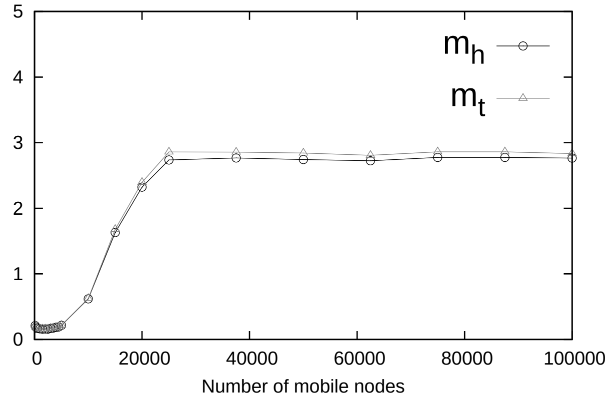


Figure 9: Verification of number of hops and transmission attempts with plots of $m_h$ and $m_t$.

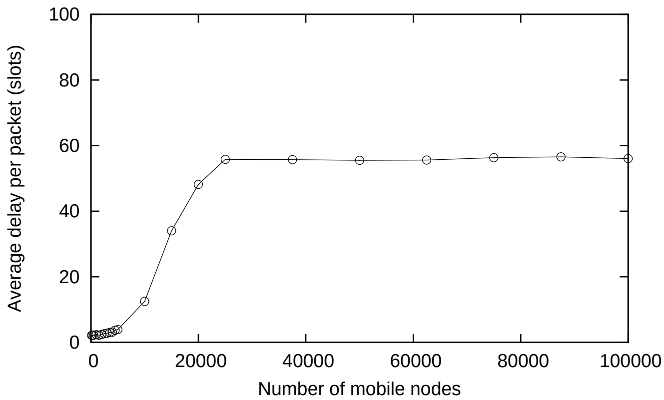


Figure 10: Average delay per packet.

in the network is also small and packets can be delivered by the mobile nodes, directly to their destination nodes, in $O(1)$ hops. As $n$ increases, the number of simultaneous transmitters increases and consequently the effective transmission range of each transmitter shrinks. Therefore, the dominant factor in the number of transmissions per packet comes from the fact that a mobile relay has to be close to the destination to deliver a packet. According to our theoretical analysis, $h_n$ and $t_n$ grow in $O(\log n)$ which is also observed in the simulation results. The simulations also show that, asymptotically, network throughput capacity is of $O(\frac{n}{\log n \log\log n})$ and the average delay per packet is of $O(1/v)$ which complies with our theoretical analysis.

# 7 Extensions and general mobility models

In our discussion, we primarily focussed on the capacity-delay tradeoff and thus for the sake of clarity initially assumed that the fixed nodes can only receive packets destined for them. We could also consider a slight variation in the specification of the model of the CRB scheme such that the fixed nodes also participate in the routing of packets to their destination nodes. For example, during a transmission phase, if a packet cannot be transmitted to its destination node or relayed to a random mobile neighbor in the cone of transmission, it can be relayed to a fixed node. This fixed node should emit this packet immediately to its destination node or to any mobile relay in the neighborhood. Note that this will also help increase the connectivity of the network.

The condition about i.i.d. random walks can be relaxed and the result about the expected number of relay changes will still remain valid. In other words, the i.i.d. random walk model can be seen as a worst case compared to realistic mobility models. If the mobile relays move like cars in an urban area, then we can expect that their mobility model will significantly depart from the random walk. Indeed cars move toward physical destinations and in their journey on the streets toward their destination, their heading after each turn is positively correlated with the heading before the turn. This implies that the probability that a relay change is needed after a turn is smaller than it would be under a random walk model, where headings before and after a turn are not correlated. Furthermore on a street, the headings are positively correlated (consider Manhattan one-way streets) and in this case a relay change due to a pass over will have more chance to arrive at a relay with a good heading (one half instead of $\theta_c/\pi$). Again this would lead to fewer relay changes due to pass over.

The result still holds if we assume that the turn rate $\tau$ depends on $n$ and $\tau = \tau_n = O(\log n)$. In this case, the mobility model would fit even better for the realistic mobility of an urban area. Indeed, the trajectories of cars should be fractal or self-similar, showing more frequent turns when cars are close to their physical destination (different than packet destination) or when leaving their parking lot. In this case, the overall turn rate tends to be in $O(\log n)$ with a coefficient depending on Hurst's parameter of the trajectory. This would lead to the same estimate of $O(\log n)$ relay changes per packet.

Figure 11 illustrates a self-similar trajectory in an urban area. It shows a two-dimensional trajectory (upper half) and its traveled distance (lower half). The successive turns are indicated by $T_1, \ldots, T_7$. The trajectory after any turn $T_i$ looks like a reduced copy of the original trajectory. The CRB scheme may need some adaptation to cope with some unusual street configurations,

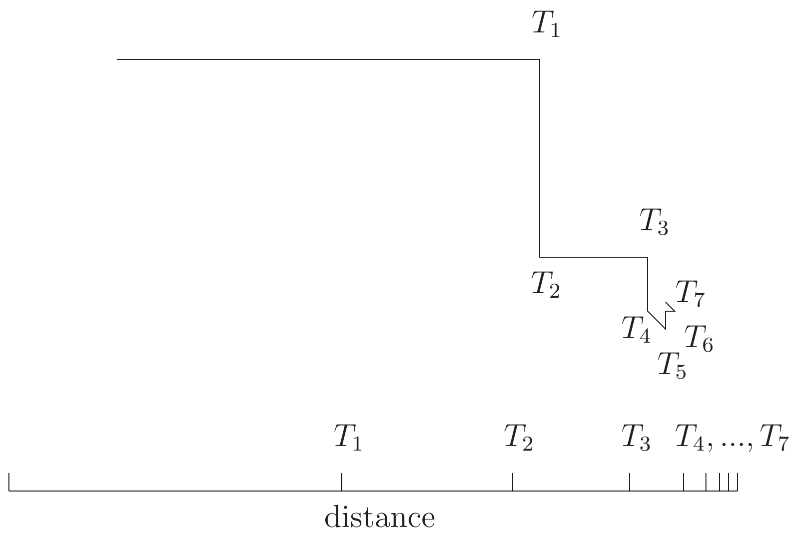


Figure 11: Illustration of self-similar trajectories in urban areas.

*e.g.*, to replace the Cartesian distance with the Manhattan distance on the street map.

# 8 Conclusions

We have examined asymptotic capacity and delay in mobile networks with a georouting scheme, called CRB, for communication between source and destination nodes. Our results show that CRB allows the network capacity of $O(\frac{n}{\log n \log \log n})$ with a packet delivery delay of $O(1)$ and transmissions per packet of $O(\log n)$ to be achieved. It is noticeable that this scheme does not require any sophisticated overhead for implementation. However, in this case, the mobile nodes must be aware of their position via a GPS system, for example.

We have shown the asymptotic performance via analytical analysis under a unit disk graph model with random i.i.d. walks. The analytical results have been confirmed by simulations and in particular under ALOHA with the SINR model. We have seen that the performance of CRB can be maintained even with non i.i.d. random walks, the latter being a worst case scenario. However, this result would require that the mobile nodes maintain the same heading for $O(1/\log n)$ time. A next step would be to analyze the performance of this scheme on real traffic traces in urban areas.

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