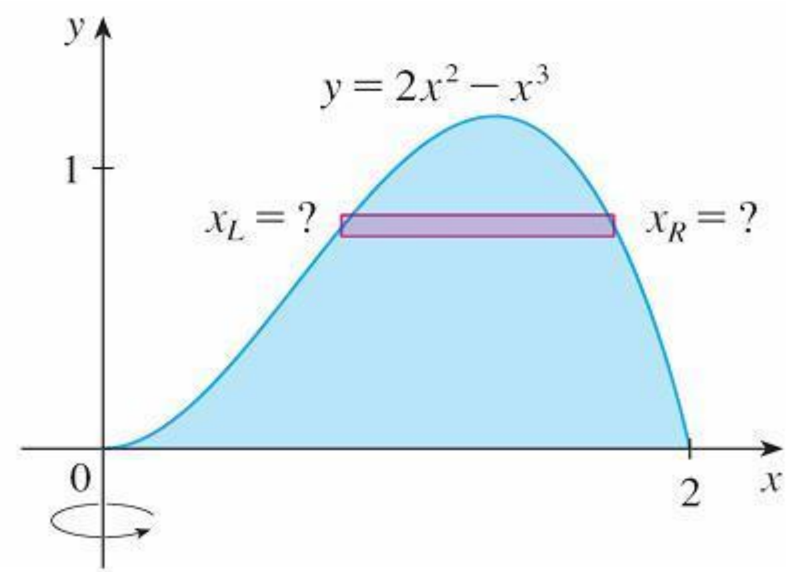
6.3 Volumes by Cylindrical Shells

Volumes by Cylindrical Shells

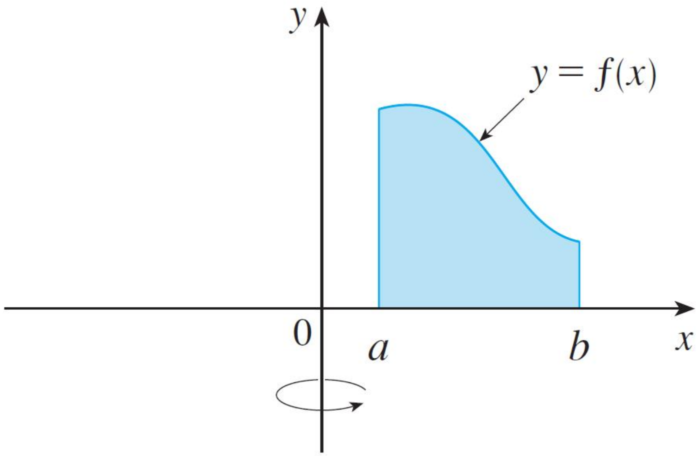
Let's find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

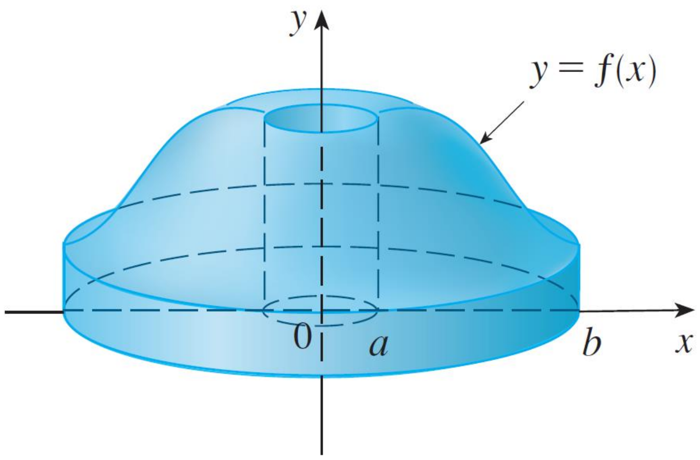


To compute the inner radius and the outer radius of the washer, we have to solve the cubic equation $y = 2x^2 - x^3$ for x in terms of y; Hard!

Volumes by Cylindrical Shells

Let S be the solid obtained by rotating about the *y*-axis the region bounded by $y = f(x)$ [where $f(x) \geq 0$], $y = 0$, $x = a$, and $x = b$, where $b > a \geq 0$.



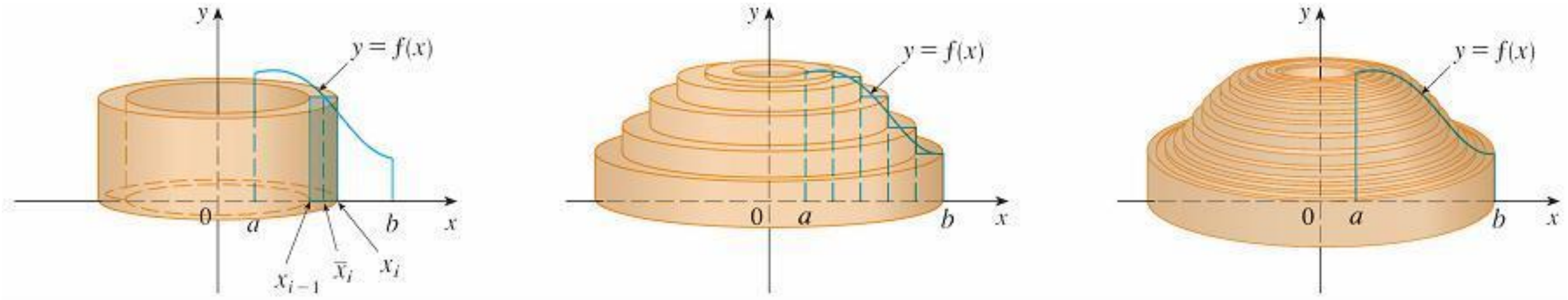


We divide the interval $[a, b]$ into *n* subintervals $[x_{i-1}, x_i]$ of equal width Δx and let $\overline{x}_i$ be the midpoint of the *i*th subinterval.

Volumes by Cylindrical Shells

If the rectangle with base $[x_{i-1}, x_i]$ and height $f(\bar{x}_i)$ is rotated about the y-axis, then the result is a cylindrical shell with average radius $\bar{x}_i$, height $f(\bar{x}_i)$, and thickness Δx its volume is

$$V_i = (2\pi\bar{x}_i)[f(\bar{x}_i)]\,\Delta x$$



$$\lim_{n\to\infty}\sum_{i=1}^{n} 2\pi\bar{x}_i f(\bar{x}_i)\,\Delta x = \int_a^b 2\pi x f(x)\,dx$$

Example

Find the volume of the solid obtained by rotating about the *y*-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

Solution:

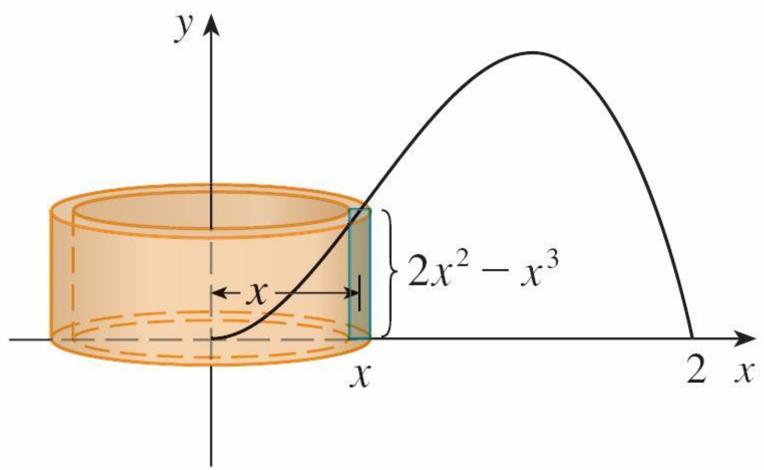


Figure 6