Class Work 13

Sols.

Names:

Eliminate the parameter to find a Cartesian equation of the curves:

$x = e^t,\ y = e^{-2t}$

$x^2 = e^{2t} = \frac{1}{y}$

$\boxed{yx^2 = 1}$

Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

$x = t\cos(t), y = t\sin(t)\ ;\ \ t = \pi.$

$\frac{dy/dt}{dx/dt} = \frac{\sin t + t\cos t}{\cos t - t\sin t}$

$dy/dt \,\big|_{t=\pi} = -\pi$

$\frac{dx}{dt}\Big|_{t=\pi} = -1$

slope $= \pi$

$x(\pi) = -\pi$

$y(\pi) = 0$

an equation of the tangent line at $(-\pi, 0)$ with slope π : $\boxed{y = \pi(x+\pi)}$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{d\,x^2}$ when $x = t^2 + 1$ and $y = e^t - 1$. For which values of t is the curve concave upwards?

$t \neq 0$

$\boxed{\frac{dy}{dx} = \frac{e^t}{2t}}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left(\frac{e^t}{2t}\right)}{2t} = \boxed{\frac{2te^t - 2e^t}{(2t)^3}}$ $t \neq 0$

$\frac{d^2y}{dx^2} > 0$ for $\boxed{x > 1 \text{ or } x < 0}$

Find the exact area of the surface obtained by rotating the curve $x = 1 + 2y^2,\ \ 1 \le x \le 2$, about the x-axis.

$2\pi \int y\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = 2\pi \int y\sqrt{1+16y^2}\, dy = \frac{2\pi}{32} \cdot \frac{2}{3} \left(1+16y^2\right)^{3/2} +$

$\left[\frac{\pi}{24}\left(1+16y^2\right)^{3/2}\right]_0^1 = \frac{\pi}{24}\left[17\sqrt{17} - 1\right]$