

7.2

Trigonometric Integrals

Trigonometric Integrals

Strategy for Evaluating $\int \sin^m x \cos^n x \, dx$

- (a) If the power of cosine is odd ($n = 2k + 1$), save one cosine factor and use $\cos^2 x = 1 - \sin^2 x$ to express the remaining factors in terms of sine:

$$\begin{aligned}\int \sin^m x \cos^{2k+1} x \, dx &= \int \sin^m x (\cos^2 x)^k \cos x \, dx \\ &= \int \sin^m x (1 - \sin^2 x)^k \cos x \, dx\end{aligned}$$

Then substitute $u = \sin x$.

- (b) If the power of sine is odd ($m = 2k + 1$), save one sine factor and use $\sin^2 x = 1 - \cos^2 x$ to express the remaining factors in terms of cosine:

$$\begin{aligned}\int \sin^{2k+1} x \cos^n x \, dx &= \int (\sin^2 x)^k \cos^n x \sin x \, dx \\ &= \int (1 - \cos^2 x)^k \cos^n x \sin x \, dx\end{aligned}$$

Then substitute $u = \cos x$. [Note that if the powers of both sine and cosine are odd, either (a) or (b) can be used.]

- (c) If the powers of both sine and cosine are even, use the half-angle identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

It is sometimes helpful to use the identity

$$\sin x \cos x = \frac{1}{2} \sin 2x$$

Trigonometric Integrals

Use a similar strategy to evaluate integrals of the form

$$\int \tan^m x \sec^n x \, dx.$$

Trigonometric Integrals

Strategy for Evaluating $\int \tan^m x \sec^n x \, dx$

- (a) If the power of secant is even ($n = 2k, k \geq 2$), save a factor of $\sec^2 x$ and use $\sec^2 x = 1 + \tan^2 x$ to express the remaining factors in terms of $\tan x$:

$$\begin{aligned}\int \tan^m x \sec^{2k} x \, dx &= \int \tan^m x (\sec^2 x)^{k-1} \sec^2 x \, dx \\ &= \int \tan^m x (1 + \tan^2 x)^{k-1} \sec^2 x \, dx\end{aligned}$$

Then substitute $u = \tan x$.

- (b) If the power of tangent is odd ($m = 2k + 1$), save a factor of $\sec x \tan x$ and use $\tan^2 x = \sec^2 x - 1$ to express the remaining factors in terms of $\sec x$:

$$\begin{aligned}\int \tan^{2k+1} x \sec^n x \, dx &= \int (\tan^2 x)^k \sec^{n-1} x \sec x \tan x \, dx \\ &= \int (\sec^2 x - 1)^k \sec^{n-1} x \sec x \tan x \, dx\end{aligned}$$

Then substitute $u = \sec x$.

Trigonometric Integrals

$$\int \tan x \, dx = \ln |\sec x| + C$$

1

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

Example

Find $\int \tan^3 x \, dx$.

Trigonometric Integrals

2 To evaluate the integrals (a) $\int \sin mx \cos nx \, dx$, (b) $\int \sin mx \sin nx \, dx$, or (c) $\int \cos mx \cos nx \, dx$, use the corresponding identity:

$$(a) \quad \sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$$

$$(b) \quad \sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$(c) \quad \cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

Example 9

Evaluate $\int \sin 4x \cos 5x \, dx$.