REVIEW

Topics and sample problems

- Integration by substitution:

Use the substitution method to solve the following indefinite integrals:

$\int \sqrt{5t+3}\, dt$; $\int \frac{2t}{t^2+7}\, dt$; $\int x^2 e^{x^3+7}\, dx$; $\int \frac{dy}{y\sqrt{\ln y}}$

- Integration by parts:

Evaluate the integrals

$\int x \cos x\, dx$; $\int t^3 \ln(t)\, dt$; $\int_0^{\pi} t^2 \sin t\, dt$; $\int x \sec^2 x\, dx$

- Areas between curves:

Find the area of the region bounded by the given curves

- $y = 12 - x^2$, $y = x^2 - 6$
- $y^4 = x$, $y = \sqrt{2-x}$, $y = 0$

- Volumes by the disc/washer methods:

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line

a) $xy - 1 = 0$, $y = 0$, $x = 1$, $x = 2$, $x \geq 0$; about $x = -1$.

b) $y = e^x$, $y = 0$, $x = -1$, $x = 1$; about the x-axis.

- Volumes by the shell method:

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

a) $y = x - x^2$, $y = 0$; about $x = 3$.

b) $y = x$, $y = x^2$; about the y-axis.

- Improper integrals: Convergence/divergence:

Determine whether the integrals are convergent or divergent. Explain your answers:

a) $\int_1^{\infty} \frac{1}{x}\, dx$; $\int_{-\infty}^{0} x e^x\, dx$; $\int_2^5 \frac{1}{\sqrt{x-2}}\, dx$; $\int_e^{\infty} \frac{1}{x(\ln x)^2}\, dx$

- Parametric equations. Tangent lines. Arc lengths:

1. Eliminate the parameter to find a Cartesian equation of the given curves

a) $x = t^2 - 3$, $y = t + 2$, $-3 \leq t \leq 3$; b) $x = \sqrt{t}$, $y = 1 - t$.

2. Find dy/dx and d^2y/dx^2. For which values of t is the curve concave upward? a) $x = e^t$, $y = te^{-t}$; b) $x = t - \ln t$, $y = t + \ln t$.

3. Find an equation of the tangent line to $x = e^t \sin \pi t$, $y = e^{2t}$ at $t = 0$.

- Polar coordinates. Area of a region described in polar coordinates:

1. Identify the curves. a) $r^2 = 5$ b) $\theta = \pi/3$; c) $r^2 \cos 2\theta = 1$
2. Find the points on the curve $r = 3\cos\theta$, where the tangent line is horizontal.
3. Find the area of the region bounded by $r = e^{-\theta/4}$, $\pi/2 \le \theta \le \pi$

- Sequences:

Find the limit of each sequence or explain why it does not exist.

a) $a_n = \frac{2n}{3n+1}$; b) $a_n = 2 + \frac{(-1)^n}{n}$; c) $a_n = 1 + \frac{10^n}{9^n}$; d) $a_n = \frac{3\sqrt{n}}{\sqrt{n}+1}$

- Series (divergence, limit comparison, integral and alternating tests):

Determine whether the series converges or diverges

$\sum_{n=1}^{\infty} \frac{1}{n^3+8}$; $\sum_{n=2}^{\infty} \frac{9^n}{1+10^n}$; $\sum_{1}^{\infty} \frac{1}{\sqrt{n^2+1}}$; $\sum_{1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+2}}$; $\sum_{1}^{\infty} \frac{n^{100}\,100^n}{n!}$

- Power series. Interval and radius of convergence:

1. Find the radius of convergence and interval of convergence of the series.

$\sum_{1}^{\infty} (-1)^n n x^n$; $\sum_{1}^{\infty} \frac{(-1)^{n-1} x^n}{n\,5^n}$; $\sum_{1}^{\infty} \frac{\sqrt{n}}{8^n} (x+6)^n$; $\sum_{1}^{\infty} n!\,(2x-1)^n$

- Taylor and Maclaurin series

1. Find the Maclaurin series for

$f(x) = (1-x)^{-2}$; $f(x) = \cos x$; $f(x) = \sinh x$

$f(x) = e^{-2x}$; $f(x) = x\cos x$

Find the associated radius of convergence.