7.5 Strategy for Integration

Strategy for Integration

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} \quad (n \neq -1)$	**2.** $\int \dfrac{1}{x}\,dx = \ln\lvert x\rvert$
3. $\int e^x\,dx = e^x$	**4.** $\int b^x\,dx = \dfrac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$	**6.** $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$	**8.** $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$	**10.** $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert$	**12.** $\int \csc x\,dx = \ln\lvert \csc x - \cot x\rvert$
13. $\int \tan x\,dx = \ln\lvert \sec x\rvert$	**14.** $\int \cot x\,dx = \ln\lvert \sin x\rvert$
15. $\int \sinh x\,dx = \cosh x$	**16.** $\int \cosh x\,dx = \sinh x$
17. $\int \dfrac{dx}{x^2 + a^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$	**18.** $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right), \quad a > 0$
***19.** $\int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a}\ln\left\lvert \dfrac{x-a}{x+a}\right\rvert$	***20.** $\int \dfrac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert$

Strategy for Integration

1. Simplify the Integrand if Possible

Sometimes the use of algebraic manipulation or trigonometric identities will simplify the integrand and make the method of integration obvious. Here are some examples:

$$\int \sqrt{x}\,(1+\sqrt{x})\,dx = \int (\sqrt{x}+x)\,dx$$

$$\int \frac{\tan\theta}{\sec^2\theta}\,d\theta = \int \frac{\sin\theta}{\cos\theta}\cos^2\theta\,d\theta$$

$$= \int \sin\theta\cos\theta\,d\theta = \tfrac{1}{2}\int \sin 2\theta\,d\theta$$

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2. Look for an Obvious Substitution

Try to find some function $u = g(x)$ in the integrand whose differential $du = g'(x)\, dx$ also occurs, apart from a constant factor. For instance, in the integral

$$\int \frac{x}{x^2 - 1}\, dx$$

we notice that if $u = x^2 – 1$, then $du = 2x\, dx$.

Strategy for Integration

3. Classify the Integrand According to Its Form

If Steps 1 and 2 have not led to the solution, then we take a look at the form of the integrand $f(x)$.

(a) *Trigonometric functions.* If $f(x)$ is a product of powers of sin x and cos x, of tan x and sec x, or of cot x and csc x, then we use the substitutions.

(b) *Rational functions.* If f is a rational function, we use the procedure involving partial fractions.

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(c) *Integration by parts.* If $f(x)$ is a product of a power of x (or a polynomial) and a transcendental function (such as a trigonometric, exponential, or logarithmic function), then we try integration by parts, choosing u and dv.

(d) *Radicals.* Particular kinds of substitutions are recommended when certain radicals appear.

(i) If $\sqrt{\pm x^2 \pm a^2}$ occurs, we use a trigonometric substitution.

(ii) If $\sqrt[n]{ax + b}$ occurs, we use the rationalizing substitution $u = \sqrt[n]{ax + b}$. More generally, this sometimes works for $\sqrt[n]{g(x)}$.

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$$\int \frac{dx}{1 - \cos x} = \int \frac{1}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x}\, dx = \int \frac{1 + \cos x}{1 - \cos^2 x}\, dx$$

$$= \int \frac{1 + \cos x}{\sin^2 x}\, dx = \int \left(\csc^2 x + \frac{\cos x}{\sin^2 x} \right) dx$$

Can We Integrate All Continuous Functions?

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The same can be said of the following integrals:

$$\int \frac{e^x}{x}\,dx \qquad \int \sin(x^2)\,dx \qquad \int \cos(e^x)\,dx$$

$$\int \sqrt{x^3 + 1}\,dx \qquad \int \frac{1}{\ln x}\,dx \qquad \int \frac{\sin x}{x}\,dx$$

In fact, the majority of elementary functions don’t have elementary antiderivatives.