CW 19

Sols

Names:

Find the radius of convergence and interval of convergence of the series.

1. $\sum_{n=1}^{\infty} \frac{(-1)^n 4^n x^n}{\sqrt{n}}$

Ratio Test $\lim_n \frac{4^{n+1}|x|^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{4^n|x|^n} =$

$= 4|x| < 1$ the $\sum$ series conv. absol.

$4|x| > 1$ diverges.

$|x| = \frac{1}{4}$?

Radius of convergence $\boxed{R = \frac{1}{4}}$

$x = \frac{1}{4}$ $\sum (-1)^n \frac{1}{\sqrt{n}}$ converges

$x = -\frac{1}{4}$ $\sum \frac{1}{\sqrt{n}}$ diverges interval of convergence $\left(-\frac{1}{4}, \frac{1}{4}\right]$

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2. $\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$ $\lim_n \frac{|x|^{2n+2}}{(n+1)!} \cdot \frac{n!}{|x|^{2n}} = \lim_n \frac{|x|^2}{n+1} = 0$

absolutely cv.

$R = \infty$ interval of convergence $(-\infty, \infty)$

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3. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2+1}$ Ratio test $\lim_n \frac{|x-2|^{n+1}}{(n+1)^2+1} \cdot \frac{n^2+1}{|x-2|^n} = |x-2|$

Radius of convergence $R = 1$

$|x-2| = 1$ $\sum \frac{1}{n^2+1}$ converges.

Interval of convergence $[1, 3]$

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4. $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n} = \sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n \left(x - \frac{1}{2}\right)^n$

Root test $\lim \sqrt[n]{\frac{|2x-1|^n}{5^n}} = \frac{2}{5}\left|x - \frac{1}{2}\right|$

Radius of convergence $\boxed{R = 5/2}$ | interval of convergence $\left(\frac{1}{2} - \frac{5}{2}, \frac{1}{2} + \frac{5}{2}\right) = (-2, 3)$