

Solve the following problems.

WK #1

1. Find the derivative of each of the following functions:

$$g(x) = \int_0^x \sqrt{t+t^5} dt; \quad g'(x) = \boxed{x+x^5}$$

$$h(x) = \int_x^0 \sqrt{t+t^3} dt; \quad h'(x) = \boxed{-\sqrt{x+x^3}}$$

$$f(x) = \int_0^{\sin(x)} \sqrt{t+t^3} dt; \quad f'(x) = \boxed{\sqrt{\sin x + (\sin x)^3} \cdot \cos x}$$

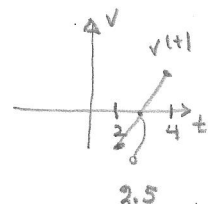
2. The velocity function (meters per second) for a particle moving along a line is

$$v(t) = 2t - 5, \quad 2 \leq t \leq 4. \text{ Find}$$

- (a) the displacement and the distance traveled by the particle in the given interval.

displacement $\int_2^4 (2t-5) dt = \left[t^2 - 5t \right]_2^4 = (16-20) - (4-10) = \boxed{2}$

distance traveled $\int_2^4 |2t-5| dt = \int_2^{2.5} (5-2t) dt + \int_{2.5}^4 (2t-5) dt = \frac{1}{2} \cdot 1 \cdot \frac{1}{2} + \frac{3}{2} \cdot 3 \cdot \frac{1}{2} = \frac{10}{4} = \boxed{\frac{5}{2}}$



- (b) the displacement and the distance traveled by the particle when t varies from 2 to 3.

displacement $\int_2^3 (2t-5) dt = \left[t^2 - 5t \right]_2^3 = (9-15) - (4-10) = -6+6 = \boxed{0}$

distance traveled $\int_2^3 |2t-5| dt = \int_2^{2.5} (5-2t) dt + \int_{2.5}^3 (2t-5) dt = \frac{1}{2} \cdot 1 \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot 1 = \boxed{\frac{1}{2}}$

3. Use the substitution method to solve the following indefinite integrals:

a. $\int \sqrt{5t+3} dt$

let $u = 5t+3$. Then $du = 5dt$

$$\int \sqrt{5t+3} dt = \frac{1}{5} \int u^{1/2} du = \frac{1}{5} \cdot \frac{2}{3} u^{3/2} + C = \boxed{\frac{2}{15} (5t+3)^{3/2} + C}$$

b. $\int \frac{2t}{t^2+5} dt$

$u = t^2+5 \quad du = 2t dt$

$$\int \frac{2t}{t^2+5} dt = \int \frac{1}{u} du = \ln|u| + C = \boxed{\ln(t^2+5) + C}$$

c. $\int x^2 e^{x^3+5} dx$

$u = x^3+5 \quad du = 3x^2 dx$

$$\int x^2 e^{x^3+5} dx = \frac{1}{3} \int e^u du = \frac{1}{3} e^u + C = \boxed{\frac{1}{3} e^{x^3+5} + C}$$