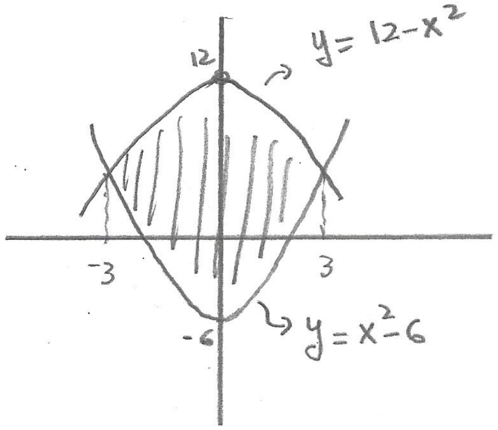
Calculus II - CW#3

Sols

Names:

1. Sketch the region enclosed by the given curves and find its area. Show your work.

$y = 12 - x^2$, $y - x^2 + 6 = 0$.



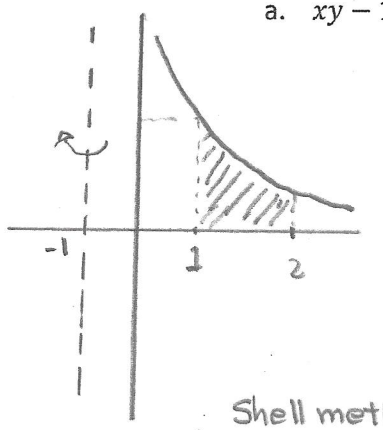
$$\text{Area} = \int_{-3}^{3} \left[(12-x^2)-(x^2-6)\right] dx =$$

$$= 2\int_0^3 \left[18-2x^2\right] dx = 2\left[18x - \frac{2}{3}x^3\right]_0^3 =$$

$$= 2\left[18\cdot 3 - \frac{2}{3}\cdot 3^3\right] = \boxed{72}$$

2. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Show your work.

a. $xy - 1 = 0, y = 0, x = 1, x = 2,\ x \geq 0$; about the x=-1.



$$\text{Volume} := \int_0^1 A(y)\, dy = \int_0^{1/2} \pi\left[3^2-2^2\right] dy + \int_{1/2}^{1} \pi\left[\left(\frac{1}{y}+1\right)^2 - 2^2\right] dy$$

$$= \frac{5\pi}{2} + \int_{1/2}^{1} \pi\left[\frac{1}{y^2} + \frac{2}{y} - 3\right] dy = \frac{5\pi}{2} + \pi\left[-\frac{1}{y} + 2\ln y - 3y\right]_{1/2}^{1}$$

$$= \boxed{2\pi + 2\pi \ln 2} = 2\pi(1+\ln 2)$$

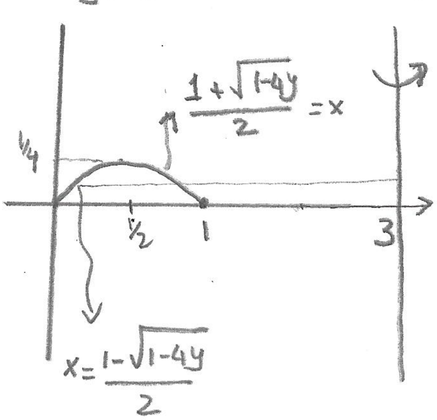
Shell method

$$2\pi\int_1^2 (x+1)\cdot\frac{1}{x}\, dx = 2\pi\left[x + \ln x\right]_1^2 = 2\pi\left[2+\ln 2 - 1\right] =$$

$$= \boxed{2\pi\left[1+\ln 2\right]}$$

b. $y = x - x^2, y = 0$; about x=3.

$x^2 - x + y = 0$



$$\text{Volume} = \pi\int_0^{1/4} \left[\left(3 - \frac{1-\sqrt{1-4y}}{2}\right)^2 - \left(3 - \frac{1+\sqrt{1-4y}}{2}\right)^2\right] dy$$

$$= \pi\int_0^{1/4} 5\sqrt{1-4y}\, dy = -\frac{5\pi}{4}\left[\frac{(1-4y)^{3/2}}{3/2}\right]_0^{1/4} =$$

$$= \boxed{\frac{5\pi}{6}}$$

Shell method

$$\text{Volume} = 2\pi\int_0^1 (3-x)(x-x^2)\, dx =$$

$$= \boxed{\frac{5\pi}{6}}$$