CW 20

Sols

Names

Find a power series representation for the function and determine the radius of convergence

1. $f(x) = \ln(1-x)$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots = \sum_{n=0}^{\infty} x^n \qquad |x|<1$$

$$-\ln(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} \cdots$$

$$\boxed{\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}} \qquad R=1$$

2. $f(x) = \dfrac{x}{2-x} = \dfrac{x/2}{1-x/2}$

$$\frac{1}{1-\frac{x}{2}} = \sum_{n=0}^{\infty} \frac{x^n}{2^n} \qquad \frac{x/2}{1-x/2} = \boxed{\sum_{n=0}^{\infty} \frac{x^{n+1}}{2^{n+1}}} \qquad \boxed{R=2}$$

$|x|<2$

3. $f(x) = \dfrac{1+x}{(1-x)^2} = \dfrac{x+1}{[2-(x+1)]^2} = \dfrac{1}{2}\,\dfrac{\frac{x+1}{2}}{\left[1-\frac{x+1}{2}\right]^2}$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad |x|<1$$

$$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1}$$

$$\frac{1}{\left(1-\frac{x+1}{2}\right)^2} = \sum_{n=1}^{\infty} n\frac{(x+1)^{n-1}}{2^{n-1}}$$

$$\frac{1}{2}\cdot\frac{x+1}{2}\Big/\left(1-\frac{x+1}{2}\right)^2 = \boxed{\frac{1}{2}\sum_{n=1}^{\infty} n\frac{(x+1)^n}{2^{n+2}}} \qquad \boxed{R=2}$$

4. $f(x) = \dfrac{x^2}{1+x^2}$

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} \qquad |x|<1$$

$$\frac{x^2}{1+x^2} = \boxed{\sum_{n=0}^{\infty} (-1)^n x^{2(n+1)}} \qquad \boxed{R=1}$$