CW # 18

Sols

Names:

1. Use the ratio test to determine whether the series is convergent or divergent. Show your work.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{(n+1)\,4^{2n+1}}$

Ratio Test

$$\lim_{n} \frac{\dfrac{10^{n+1}}{(n+2)\,4^{2n+3}}}{\dfrac{10^n}{(n+1)\,4^{2n+1}}} = \lim_{n} \frac{10\,(n+1)}{4^2\,(n+2)} = \frac{5}{8} < 1$$

Convergent

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b. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi/3)}{n!}$

$$\left| \frac{\cos \frac{n\pi}{3}}{n!} \right| \leq \frac{1}{n!}$$

Ratio Test implies $\sum \frac{1}{n!}$ converges.

$$\lim_{n} \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} = \lim_{n} \frac{1}{n+1} = 0 < 1$$

Hence $\displaystyle\sum_{n} \frac{\cos(n\pi/3)}{n!}$ is absolutely convergent.

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c. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

Ratio test

$$\lim_{n} \frac{\dfrac{(2n+2)!}{(n+1)!^2}}{\dfrac{2n}{n!^2}} = \lim \frac{(2n+2)(2n+1)}{(n+1)^2}$$

$= 4 > 1$

divergent

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2. Use the root test to determine whether each series is convergent or divergent.

a. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

$$\lim_{n} \frac{n^2+1}{2n^2+1} = \frac{1}{2} < 1$$

Convergent

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b. $\displaystyle\sum_{n=1}^{\infty} (\arctan n)^n$

$$\lim_{n} \arctan(n) = \pi/2 > 1$$

divergent