Class Work 12

Names: Sols.

Find the exact area of the surface obtained by rotating the curve about the x-axis.

$y = x^3,\ 0 \le x \le 5$

Surface area $= 2\pi \int_0^5 x^3 \sqrt{1+9x^4}\,dx = \frac{2\pi}{36} \int_1^{1+9\cdot 5^4} u^{1/2}\,du$

Let $u = 1+9x^4$

$du = 36x^3\,dx$

$= \frac{\pi}{18} \cdot \frac{2}{3} \left[u^{3/2} \right]_1^{1+9\cdot 5^4} =$

$= \boxed{\frac{\pi}{27}\left((1+9\cdot 5^4)^{3/2} - 1 \right)}$

Find the exact area of the surface obtained by rotating the curve about the x-axis.

$$y = \frac{x^3}{6} + \frac{1}{2x}, \quad \frac{1}{2} \le x \le 1.$$

Area $= 2\pi \int_{1/2}^{1} y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$= 2\pi \int_{1/2}^{1} \left(\frac{x^3}{6} + \frac{1}{2x}\right) \sqrt{1+\left(\frac{x^2}{2} - \frac{1}{2x^2}\right)^2}\,dx =$

$= 2\pi \int_{1/2}^{1} \left(\frac{x^3}{6} + \frac{1}{2x}\right) \sqrt{1+\frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}}\,dx =$

$= 2\pi \int_{1/2}^{1} \left(\frac{x^3}{6} + \frac{1}{2x}\right) \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2}\,dx = 2\pi \int_{1/2}^{1} \left(\frac{x^3}{6} + \frac{1}{2x}\right)\left(\frac{x^2}{2} + \frac{1}{2x^2}\right) dx =$

$= 2\pi \int_{1/2}^{1} \left(\frac{x^5}{12} + \frac{x}{4} + \frac{x}{12} + \frac{1}{4x^3}\right) dx = 2\pi \left[\frac{x^6}{72} + \frac{x^2}{6} - \frac{1}{8x^2} \right]_{1/2}^{1} =$

$= \boxed{2\pi \left(\frac{1}{72} + \frac{1}{6} - \frac{1}{8} - \frac{1}{(64)(72)} - \frac{1}{24} + \frac{1}{2} \right)}$