ClassWork 14

Sols

Names:

At what point(s) on the curve $x = 3t^2 + 1,\ y = t^3 - 1$ does the tangent line has slope ½?

$$\frac{dy}{dx} = \frac{3t^2}{6t} = \frac{1}{2} \quad \therefore\ t = 1$$

$t \neq 0$

The tangent line to the given curve at (4, 0) has slope ½.

Find the area enclosed by the x-axis and the curve $x = t^3 + 1$, $y = 2t - t^2$.

$$\int_0^2 (2t - t^2)\, 3t^2\, dt = \int_0^2 (6t^3 - 3t^4)\, dt = \left[\frac{3}{2}t^4 - \frac{3}{5}t^5\right]_0^2$$

$$= 24 - \frac{3}{5}\cdot 32 = \frac{120 - 96}{5} = \boxed{\frac{24}{5}}$$

Set up an integral that represents the length of the curve given by $x = t + \sqrt{t}, y = t - \sqrt{t},\ 0 \le t \le \pi$.

$$\int_0^{\pi} \sqrt{\left(1 + \frac{1}{2\sqrt{t}}\right)^2 + \left(1 - \frac{1}{2\sqrt{t}}\right)^2}\ dt$$

Find the exact length of the curve $x = e^t - t, y = 4\, e^{\frac{t}{2}}, 0 \le t \le 2$.

$$\int_0^2 \sqrt{(e^t - 1)^2 + \left(2e^{t/2}\right)^2}\ dt = \int_0^2 (e^t + 1)\, dt = \left[e^t + t\right]_0^2 = e^2 + 1$$

Write the points $(2, 3\pi/2)$ and $(-1, -\pi/6)$ in Cartesian coordinates.

$$\left(2\cos\frac{3\pi}{2},\ 2\sin\frac{3\pi}{2}\right) = (0, -2)$$

$$\left(-1\cos\left(-\frac{\pi}{6}\right),\ -1\sin\left(-\frac{\pi}{6}\right)\right) = \left(-\frac{\sqrt{3}}{2},\ \frac{1}{2}\right)$$