Class Work 9

Names: Sols

Evaluate the integrals

$\int \frac{y}{(y+4)(y-1)^2}\,dy$. Decompose the given function into partial fractions.

$$\frac{y}{(y+4)(y-1)^2} = \frac{A}{y+4} + \frac{B}{y-1} + \frac{C}{(y-1)^2} \; ; \quad y = A(y-1)^2 + B(y-1)(y+4) + C(y+4)$$

Let $y=1$ $\boxed{C = \frac{1}{5}}$ $\quad y=0 \quad 0 = A-4B+4C$

$y=-4$ $\boxed{A = -\frac{4}{25}}$ $\quad \boxed{B = \frac{4}{25}}$

$$\int \frac{y}{(y+4)(y-1)^2}\,dy = -\frac{4}{25}\int \frac{dy}{y+4} + \frac{4}{25}\int \frac{dy}{y-1} + \frac{1}{5}\int \frac{dy}{(y-1)^2} =$$

$$= -\frac{4}{25}\ln|y+4| + \frac{4}{25}\ln|y-1| - \frac{1}{5}\frac{1}{y-1} + C$$

$\int \frac{1}{x^3\sqrt{x^2-1}}\,dx$. Use a trigonometric substitution.

$x = \sec t \qquad \cos t = \frac{1}{x} \qquad t = \cos^{-1}\left(\frac{1}{x}\right) \qquad \sin t = \sqrt{1-\frac{1}{x^2}}$

$dx = \sec t \tan t \, dt$

$$\int \frac{\sec t \cancel{\tan t}\, dt}{\sec^3 t \cancel{\tan t}} = \int \cos^2 t\, dt = \int \left(\frac{1}{2} + \frac{1}{2}\cos 2t\right) dt = \frac{1}{2}t + \frac{1}{4}\sin 2t + C$$

$$= \boxed{\frac{1}{2}\cos^{-1}\left(\frac{1}{x}\right) + \frac{1}{2}\cdot\frac{1}{x}\sqrt{1-\frac{1}{x^2}} + C}$$

$$\int \frac{\ln(x)}{x\sqrt{1+(\ln(x))^2}}\,dx = \frac{1}{2}\int u^{-\frac{1}{2}}\,du = u^{1/2} + c$$

$$= \boxed{\sqrt{1+(\ln(x))^2} + C}$$

$u = 1 + (\ln(x))^2$

$du = 2\ln(x)\cdot\frac{1}{x}\,dx$