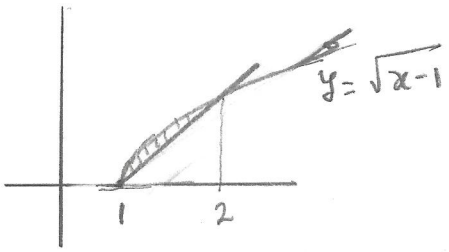
Calculus II — 4

Sols

Names:

1. Sketch the region enclosed by the given curves and find its area. Show your work.

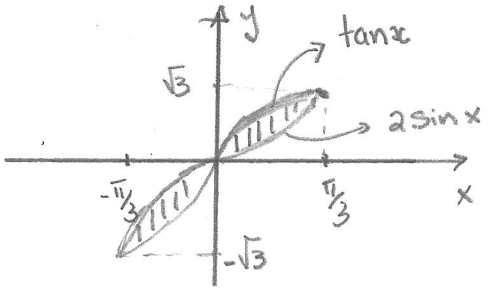
a. $y = \sqrt{x-1}$, $x - y = 1$.



$$\int_1^2 \left[\sqrt{x-1} - (x-1)\right] dx = \int_0^1 (\sqrt{u} - u)\, du$$

$$= \frac{2}{3} - \frac{1}{2} = \boxed{\frac{1}{6}}$$

b. $y = \tan(x)$, $y = 2\sin(x)$, $-\pi/3 \le x \le \pi/3$.



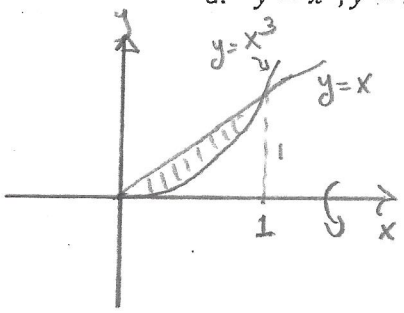
area: $$2\int_0^{\pi/3} (\tan x - 2\sin x)\, dx =$$

$$2\left[-\ln(\cos x) + 2\cos x\right]_0^{\pi/3} = 2\left[+\ln 2\right]$$

$$= \boxed{2\ln 2}$$

2. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Show your work.

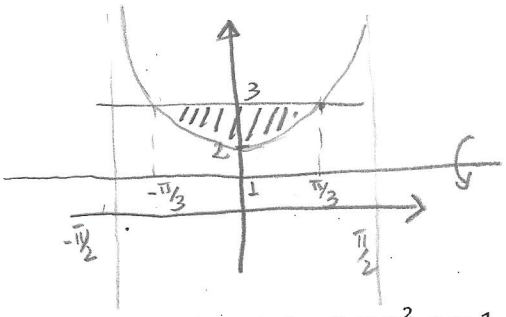
a. $y = x^3, y = x, x \ge 0$; about the x-axis.



Volume: $$\pi \int_0^1 (x^2 - x^6)\, dx =$$

$$= \pi\left[\frac{1}{3} - \frac{1}{7}\right] = \boxed{\frac{4}{21}\pi}$$

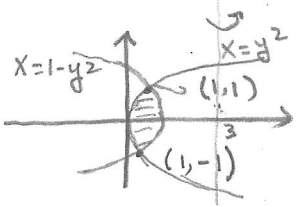
b. $y = 1 + \sec(x), y = 3$; about y=1. $-\frac{\pi}{3} \le x \le \pi/3$



$$2\pi \int_0^{\pi/3} \left[2^2 - \sec^2(x)\right] dx = 2\pi\left[4\cdot\frac{\pi}{3} - \tan\frac{\pi}{3} + \underbrace{\tan 0}_{0}\right]$$

$$= \boxed{2\pi\left[\frac{4}{3}\pi - \sqrt{3}\right]}$$

c. $x = y^2, x = 1 - y^2$; about x=3.



Volume $$2\pi \int_0^1 \left[(3-y^2)^2 - (3-1+y^2)^2\right] dy = 2\pi\left[5 - \frac{10}{3}\right] = \boxed{\frac{10\pi}{3}}$$