Class Work –7

Sols

Name:

Find:

1. $\int \sin^3 x \, dx$ $= \int \sin x \, (1-\cos^2 x) \, dx = \int (u^2-1)\, du = \frac{1}{3}u^3 - u + C$

$u = \cos x$ $\quad du = -\sin x \, dx$ $\quad \therefore \boxed{\int \sin^3 x \, dx = \frac{1}{3}\cos^3 x - \cos x + C}$

2. $\int t \cos^5(t^2)\, dt.$ $= \frac{1}{2}\int \cos^5 u \, du = \frac{1}{2}\int (1-\sin^2 u)^2 \cos u \, du =$

$\boxed{\begin{array}{l} u = t^2 \\ du = 2t\,dt \end{array}}$

$z = \sin u$
$dz = \cos u \, du$

$= \frac{1}{2}\int (1-z^2)^2 \, dz =$

$= \frac{1}{2}\left[z - \frac{2}{3}z^3 + \frac{1}{5}z^5\right] + C$

$\boxed{\int t \cos^5(t^2)\, dt = \frac{1}{2}\left[\sin(t^2) - \frac{2}{3}\sin^3(t^2) + \frac{1}{5}\sin^5(t^2)\right] + C}$

3. $\int (\tan^2(x) + \tan^4(x))\, dx\,.$

$\int \tan^2 x \cdot \sec^2 x \, dx = \int u^2 \, du = \frac{1}{3}u^3 + C = \boxed{\frac{1}{3}(\tan x)^3 + C}$

$u = \tan x$

4. $\int x \sec(x) \tan(x)\, dx.$

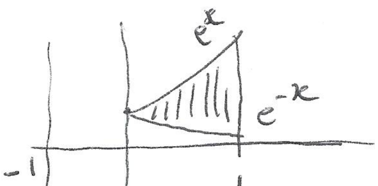
Integration by parts. $\int u\,dv = uv - \int v\,du = x \sec x - \int \sec x \, dx$

$u = x$ $\quad du = dx$

$= \boxed{x \sec x - \ln|\sec x + \tan x| + C}$

$dv = \sec x \tan x \, dx$ $\quad v = \sec x$

5. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves $y = e^x,\ y = e^{-x},\ x = 1$ about $x = -1$.



$2\pi \int_0^1 (x+1)\,[e^x - e^{-x}]\, dx$ $\quad$ [Shell method]

$\int_0^1 (x+1)(e^x - e^{-x})\, dx = \left[(x+1)(e^x - e^{-x})\right]_0^1 - \int_0^1 (e^x + e^{-x})\, dx = 2(e - e^{-1}) - \left[e^x - e^{-x}\right]_0^1$

$= e - e^{-1}$

$u = x+1$ $\quad du = dx$

$dv = (e^x - e^{-x})\, dx$ $\quad v = e^x + e^{-x}$

Ans: $\boxed{2\pi(e - e^{-1})}$