

5.5

The Substitution Rule

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4 The Substitution Rule If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I , then

$$\int f(g(x)) g'(x) dx = \int f(u) du$$

Definite Integrals

6 The Substitution Rule for Definite Integrals If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

Example

Evaluate. $\int_0^4 \sqrt{2x + 1} \, dx$

Symmetry

7 Integrals of Symmetric Functions Suppose f is continuous on $[-a, a]$.

(a) If f is even [$f(-x) = f(x)$], then $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$.

(b) If f is odd [$f(-x) = -f(x)$], then $\int_{-a}^a f(x) dx = 0$.

$$\int_{-1}^1 \frac{\tan x}{1 + x^2 + x^4} dx = 0$$