ClassWork ~~15~~ 17

Names: **Sols**

1. Determine whether each of the following series is convergent if divergent. If you decide the series is convergent find its sum. Otherwise, explain why.

a. $\sum_{k=1}^{\infty} \frac{k(k+7)}{(k+2)^2}$ diverges because

$$\lim_{k} \frac{k(k+7)}{(k+2)^2} = 1$$

b. $\frac{1}{10} + \frac{1}{16} + \frac{1}{22} + \frac{1}{28} + \cdots = \sum_{n=1}^{\infty} \frac{1}{6n+4}$ divergent

$$\lim_{n} \frac{\frac{1}{n}}{\frac{1}{6n+4}} = 6$$

Since $\sum_{1}^{\infty} \frac{1}{n}$ diverges the Limit Comparison test implies that $\sum_{1}^{\infty} \frac{1}{6n+4}$ diverges

2. Find the values of p for which the series $\sum_{k=2}^{\infty} \frac{4}{k\,(\ln k)^p}$ is convergent. Explain your answer.

$$\int_2^{\infty} \frac{4}{x(\ln x)^p}\,dx \underset{p \neq 1}{=} \lim_{M\to\infty} \left[\frac{4(\ln x)^{-p+1}}{-p+1}\right]_2^M = \lim_{M\to\infty} \left[\frac{4(\ln M)^{1-p}}{1-p} - \frac{4(\ln 2)^{1-p}}{1-p}\right]$$

$p>1$ the limit exists $\therefore \sum_{k=2}^{\infty} \frac{4}{k(\ln k)^p}$ converges

$p<1$ diverges

$$p=1 \quad \int_2^{\infty} \frac{4}{x\ln x}\,dx = \lim_{M\to\infty} [4\ln(\ln M) - 4\ln(\ln 2)] = \infty \quad . \quad \sum_{2}^{\infty} \frac{4}{k\ln k} \text{ diverges}$$

$\boxed{\text{The given series converges for } p>1}$

3. Show that the curve $x = 5\cos(t)$, $y = 2\sin(t)\cos(t)$ has two tangents at (0, 0) and find their equations.

$5\cos t = 0$ $\quad t = \frac{\pi}{2}$ and $\frac{3\pi}{2}$

$\sin 2t = 0$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos 2t}{-5\sin t}$$

tangent lines:

$$m\Big|_{t=\pi/2} = \frac{-2}{-5} = \frac{2}{5} \qquad \boxed{y = \frac{2}{5}x}$$

$$m\Big|_{t=\frac{3\pi}{2}} = -\frac{2}{5} \qquad \boxed{y = -\frac{2}{5}x}$$