Class Assignment 18

Sols

Names:

MATH 1920, section 003, Fall 2016

Personal Study Plan - 11.9: Representations of Functions as Power Series

Fernanda Botelho
Instructor: Fernanda Botelho
University of Memphis

Quiz Results

1. Question: SCalcET8 11.9.003.MI.

Find a power series representation for the function. (Center your power series representation at x = 0.)

$$f(x) = \frac{1}{7+x} = \frac{1}{7}\,\frac{1}{1-\left(-\frac{x}{7}\right)} = \frac{1}{7}\sum_{n=0}^{\infty}\left(-\frac{x}{7}\right)^n$$

$$f(x) = \sum_{n=0}^{\infty}\left(\boxed{(-1)^n \frac{x^n}{7^{n+1}}}\right)$$

Determine the interval of convergence. (Enter your answer using interval notation.)

$\boxed{(-7, 7)}$

2. Question: SCalcET8 11.9.005.MI.

Find a power series representation for the function. (Give your power series representation centered at x = 0.)

$$f(x) = \frac{2}{7-x} = \frac{2}{7}\,\frac{1}{1-\frac{x}{7}} = \frac{2}{7}\sum_{n=0}^{\infty}\frac{x^n}{7^n}$$

$$f(x) = \sum_{n=0}^{\infty}\left(\boxed{\frac{2x^n}{7^{n+1}}}\right)$$

Determine the interval of convergence. (Enter your answer using interval notation.)

$\boxed{(-7, 7)}$

3. Question: SCalcET8 11.9.013.

(a) Use differentiation to find a power series representation for

$$f(x) = \frac{1}{(6+x)^2}. = \frac{1}{6^2} \frac{1}{\left(1+\frac{x}{6}\right)^2}$$

$$f(x) = \sum_{n=0}^{\infty} \left((-1)^n (n+1) \frac{x^n}{6^{n+2}} \right)$$

What is the radius of convergence, R?

$R =$ 6

(b) Use part (a) to find a power series for

$$f(x) = \frac{1}{(6+x)^3}. = \frac{1}{6^3} \frac{1}{\left(1+\frac{x}{6}\right)^3}$$

$$f(x) = \sum_{n=0}^{\infty} \left((-1)^n \frac{(n+2)(n+1)}{2} \frac{x^n}{6^{n+3}} \right)$$

What is the radius of convergence, R?

$R =$ 6

(c) Use part (b) to find a power series for

$$f(x) = \frac{x^2}{(6+x)^3}.$$

$$f(x) = \sum_{n=2}^{\infty} \left((-1)^n \frac{n(n-1)}{2} \frac{x^n}{6^{n+1}} \right)$$

What is the radius of convergence, R?

$R =$ 6

$$\frac{1}{(1+x)^2} = \sum_{n=0}^{\infty} (-1)^n (n+1) x^n$$

$|x| < 1$

$$\frac{-2}{(1+x)^3} = \sum_{n=1}^{\infty} (-1)^n (n+1) n x^{n-1}$$

$$\frac{1}{(1+x)^3} = \sum_{n=0}^{\infty} (-1)^n \frac{(n+2)(n+1)}{2} x^n$$

$$\frac{1}{(1+x)^3} = \sum_{n=2}^{\infty} (-1)^n \frac{n(n-1)}{2} x^{n-2}$$

4. Question: SCalcET8 11.9.015.MI.

Find a power series representation for the function. (Give your power series representation centered at x = 0.)

$f(x) = \ln(5 - x)$

$$f(x) = \ln(5) - \sum_{n=0}^{\infty} \left(\frac{x^{n+1}}{(n+1)5^{n+1}} \right) = \ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n 5^n}$$

Determine the radius of convergence, R.

$R =$ 5

$$\ln\left[1 - \frac{x}{5}\right]$$

$$\left(\ln(1-x) = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} \right)$$

5. Question: SCalcET8 11.9.025.MI.

Evaluate the indefinite integral as a power series.

$$\int \frac{t}{1-t^5}\,dt$$

$$C + \sum_{n=0}^{\infty} \left(\frac{1}{5n+2} t^{5n+2} \right)$$

What is the radius of convergence R?

$R =$ 1

$$\frac{t}{1-t^5} = \sum_{n=0}^{\infty} t^{5n+1}$$