Class Work 10

Sols

Name:

Evaluate the integrals. Decompose each given function into partial fractions

1. $\int \frac{x}{x^2-1}\,dx = \frac{1}{2}\int\left(\frac{1}{x-1}+\frac{1}{x+1}\right)dx = \boxed{\frac{1}{2}\ln|x-1|+\frac{1}{2}\ln|x+1|+C}$

$\frac{x}{x^2-1} = \frac{A}{x-1}+\frac{B}{x+1}$ $\quad A(x+1)+B(x-1)=x$

$A=\frac{1}{2}=B$

2. $\int \frac{x^3}{x^2-1}\,dx = \int\left(x+\frac{x}{x^2-1}\right)dx = \frac{x^2}{2}+\frac{1}{2}\ln|x^2-1|+C$

3. $\int \frac{1}{x^2-x-6}\,dx$; $\frac{1}{(x-3)(x+2)} = \frac{A}{x-3}+\frac{B}{x+2}$

$1 = A(x+2)+B(x-3)$

$A=\frac{1}{5}$; $B=-\frac{1}{5}$

$\int \frac{1}{x^2-x-6}\,dx = \int \frac{\frac{1}{5}}{x-3}\,dx - \frac{1}{5}\int\frac{1}{x+2}\,dx$

$= \boxed{\frac{1}{5}\ln|x-3| - \frac{1}{5}\ln|x+2|+C}$

4. $\int \frac{x^2}{x^2-6-x}\,dx =$ ↳ $= \boxed{\frac{x^2}{2}+\frac{9}{5}\ln|x-3| - \frac{4}{5}\ln|x+2|+C}$

$\frac{x^2}{x^2-x-6} = 1+\frac{x+6}{x^2-x-6} = 1+\frac{A}{x-3}+\frac{B}{x+2} = 1+\frac{9}{5}\frac{1}{x-3}-\frac{4}{5}\frac{1}{x+2}$

$A(x+2)+B(x-3) = x+6 \rightarrow -5B=4 \quad 5A=9$

5. $\int \frac{x^3}{x^2+1}\,dx = \int\left(x-\frac{x}{x^2+1}\right)dx = \frac{x^2}{2}-\frac{1}{2}\ln|x^2+1|+C$

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