Effect of hyperbolic heat transfer on metal

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Abstract

Hyperbolic heat conduction equation has received considerable interest because of its wide applicability. In this project hyperbolic heat transfer though a solid rod subjected to periodic heat flux is found using an explicit numerical method and the results are compared with the result from commercially available COMSOL software.

1. INTRODUCTION

The use of heat sources such as laser and microwaves with extremely short duration or very high frequency has found numerous applications in many different areas of mathematical physics, applied sciences and engineering. In such situations, the classical Fourier's heat conduction theory becomes inaccurate and the non-Fourier effect becomes more reliable in describing the diffusion process and predicting the temperature distribution. The classical Fourier's law of heat conduction assumes that the phase lag between the heat flux and temperature gradient is zero. By this assumption, Fourier heat conduction leads to parabolic governing equations which theoretically show infinite propagation speed of thermal signals in the material but in 1994 the experiments done by Peshkov demonstrated the finite speed of heat propagation. In order to eliminate the paradox of infinite speed of heat wave and to develop a model for heat conduction with finite speed, Cattaneo and Vernotte in 1985 independently suggested a modified heat-flux model with thermal relaxation time. Here heat flow does not start instantaneously, but raises gradually with a relaxation time τ with respect to the temperature gradient.

2. MATHEMATICAL MODELLING

In this work one dimensional heat transfer through an iron rod of length 1 metre subjected to periodic heat flux with amplitude q_0 and frequency w for a small interval of time is considered. The situation is illustrated schematically in Fig. 1.

2.1 Governing equations

From Cattaneo and Vernotte (CV) model the hyperbolic heat

$$a\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} + \tau\frac{\partial^2 T}{\partial t^2}$$

transfer equation is

(1)

Here a is thermal diffusivity and τ is the relaxation time This hyperbolic heat equation is now considered subject to the following boundary and initial conditions

$$-k\frac{\partial T(0,t))}{\partial x} = \tau\frac{\partial q(0,t)}{\partial t} + q(0,t) \quad (2)$$

$$\frac{\partial T(1,t)}{\partial x} = 0 \quad (3)$$

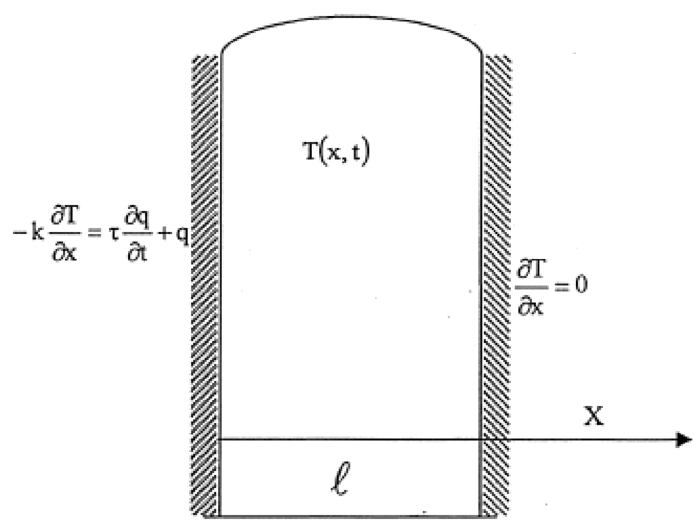

Fig 1 A schematic representation of geometry and coordinates.

$$T(x,0) = 0\,,\ \frac{\partial T(x,0)}{\partial x} = 0 \tag{4}$$

Where q (0, t) = 500 + q_0 coswt and q(x, 0) = 0 here q_0 = 5e4 w/m^2 and w = 1000Hz. The propagation speed V = $\sqrt{a\tau}/l$ where l is length of the rod.

3. METHDOLOGY

Hyperbolic heat equation is solved using MATLAB code in which the equation is discretised using explicit finite difference method and the domain is divided into 300 elements. The increment in time is set to 0.0001 for stability. We assumed the relaxation time for iron is 0.15e-15 and the thermal diffusivity as 2.1923e-5 m^2/s. Fourier heat transfer for the same problem was also considered. To verify the results obtained from code Comsol multi physics software with mathematical PDE interface was used

4. RESULTS AND DISCUSSION

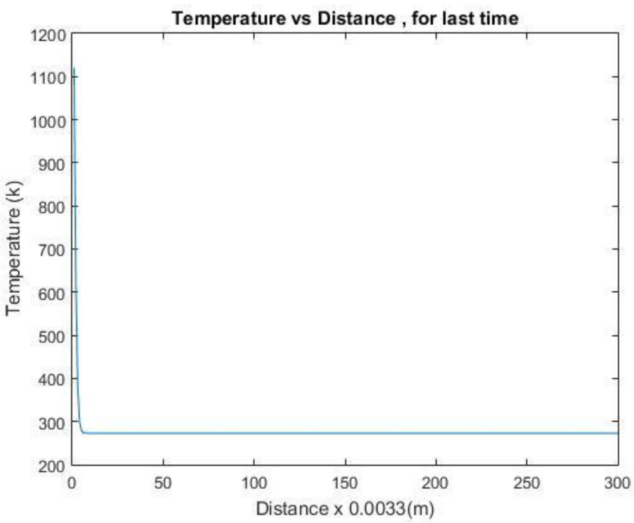

Fig2 Temperature vs distance graph obtained from code

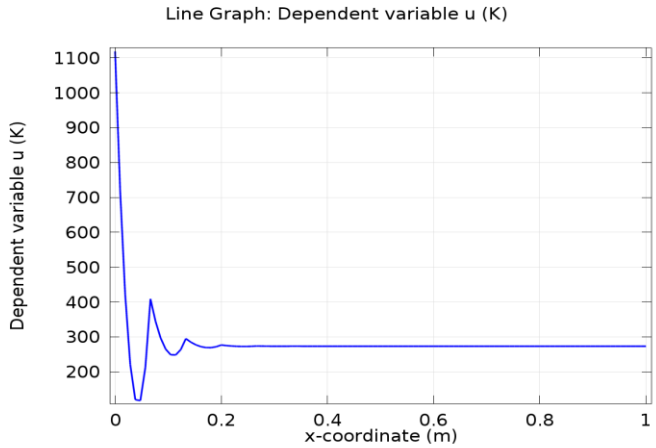

Fig3 Temperature vs distance graph obtained from COMSOL

In figure 2 and 3 temperature at different positions for the given time 0.5 seconds is shown the data from both methods shows similar trend but the graph obtained from the COMSOL shows little fluctuations at 10cm but in the graph obtained from MATLAB code shows an average value instead of fluctuation. This is can be avoided if we use very small time step in the code.

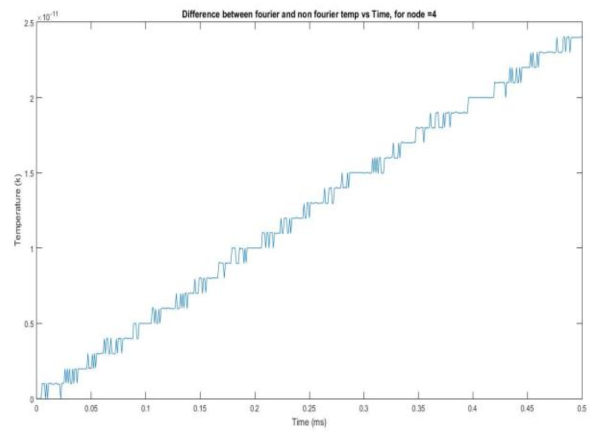

Fig4 Difference between Fourier and Hyperbolic heat transfer

It is clear from the graph that there is residue of order 10e-11 is present between Fourier and Non fourier heat transfer models.

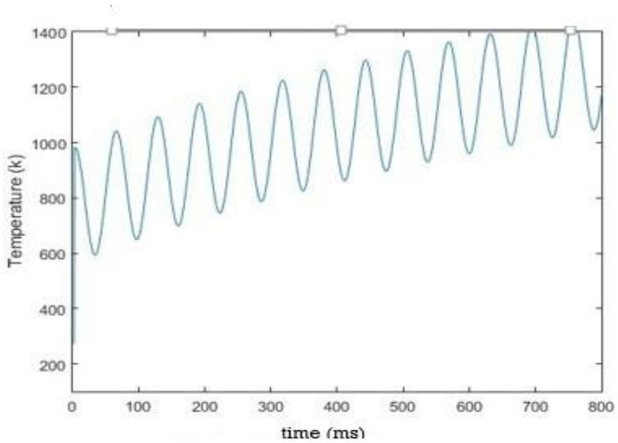

Fig5 Left Boundary condition

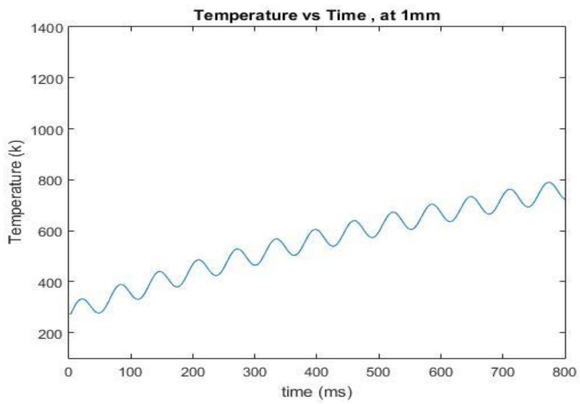

Fig6 Temperature at 1mm distance showing the figures.

5. CONCLUSIONS

The results obtained from both code and COMSOL shows similar trend. Both figures 5 and 6 shows the effect of periodic heat flux. The Fourier and Non Fourier heat transfer shows a residue of the order 10e-11 this because of the time lag. Also the propagation speed of thermal wave is 3.3923e+07 m/s for Fourier heat transfer it is the speed of light.

6. REFERENCES

1. Peterson, G.P_ Sobhan, C.B-Microscale and Nanoscale Heat Transfer _ Fundamentals and Engineering Applications-CRC Press (2008)
2. H. Jackson, C.I. Walker, Thermal conductivity, second sound and phonon±phonon interactions in NaF, Phys.
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Observations and Calculations

TABLE 1
(Cold Run)

Sl.No	Drag Force F (N)	Static Pressure Ps (mm of H_2O)	Total Pressure P (mm of H_2O)	Blower Total Pressure Pb (mm of H_2O)	Temperature of Intake Air Ti (^oC)	Mach Number
1	0.27	2	16	24	41.8	0 .0439
2	0.43	4	26	39	45.7	0.055
3	1.42	13	71	95	46.3	0.089
4	2.56	37	161	194	44.8	0.13
5	3.93	59	241	283	43.9	0.157
6	5.26	76	299	353	42.7	0.174

TABLE 2
(Hot Run)

Sl.No	Pitot Static Tube Position From Inlet X (cm)	Thrust F (N)	Static Pressure Ps (mm of H_2O)	Total Pressure P (mm of H_2O)	Fuel Temperature T_f (^oC)	Exhaust Temperature T_e (^oC)
1	+5	2.3	12	36	38.2	820
2	0	2.6	18	36	37.6	846
3	-5	2.5	21	36	37.7	843
4	-10	2.44	19	36	37.6	874
5	-15	2.64	11	32	37.6	900
6	-20	2.72	13	25	37.7	920
7	-25	2.68	12	19	37.5	925.3
8	-30	2.79	9	17	37.6	937.8
9	-35	2.57	7	16	37.5	921.3
10	-40	2.59	4	16	37.6	932.9

Sample Calculations

(a) Cold run (set 2)

Static Pressure, Ps

= 4 mm of water + Patm
=.004×1000×9.81 + Patm
= 39.2 + 101325 Pa
= 101364.24 Pa

Total Pressure, P

= 36 mm of water + Patm
= 255.06 + 101325 Pa
= 101580.06 Pa

Using equation(1)

Intake MachNumber

$= \sqrt{5 \times ((101580.06/101364.24)^{.2857} - 1)}$

$= 0.055$

(b)Hot run (set 2)

Ambient

Static Pressure, Ps

= 18 mm of water + Patm

= 176.58 + 101325 Pa

= 101501.58 Pa

Total Pressure, P0a

= 36 mm of water + Patm

= 353.16 + 101325 Pa

= 101678.16 Pa

Inlet Mach Number, Ma

$= \sqrt{5 \times ((101678.16/101501.58)^{.2857} - 1)}$

= 0.0498

Temperature at inlet, Ti

= 320.6 K

Stagnation temperature, Toa

= 320.68 K

Density at inlet side, ρa

= Pa/(RTa)

= 1.103 Kg/m^3

Inlet Velocity, Va

$= \sqrt{(\gamma RT a)} \times Ma$

= 17.87 m/s