SSSP in DAGs

Initialization: $\forall v \{d[v] \leftarrow \infty \;\; p[v] \leftarrow \phi\}$ $d[s] \leftarrow 0$.

1. Perform top-sort of vertices
2. Relax all o/g edges from ~~vertices~~ each vertex in the order obtained from 1.
3. T.C. $O(n+m)$.

Correctness Let the shortest path from s to x be $(x_0, x_1, x_2, \ldots x_{k-1}, x_k)$ $x_0 = s$ and $x_k = x$.

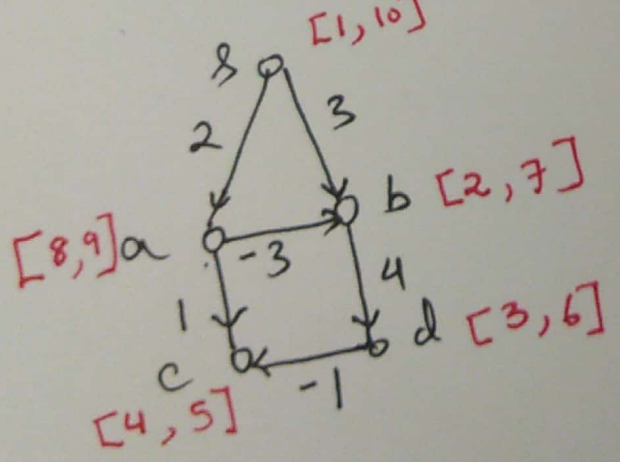
In top-sort $\Rightarrow$ edge (x_0, x_1) precedes (x_1, x_2) etc

x_0 precedes x_1 when you relax them.

That is, $\int$ in any given path edges are relaxed from L to R $\therefore$ By path relaxation property

$d[x_k] = \delta[x_k]$.

c finishes first at $t=5$ etc.



We relax (s,a) and (s,b).

$\downarrow d[a]$ $\quad d[b] \quad d[c] \quad d[d]$

$[\,2 \quad 3 \quad \infty \quad \infty\,]$

We relax (a,b) (a,c)

$[\,2 \quad -1 \quad 3 \quad \infty\,]$

We relax (b,d)

$[\,2 \quad -1 \quad 3 \quad 3\,]$

We relax (d,c)

$[\,2 \quad -1 \quad 2 \quad 3\,]$

No more o/g edges

A cut is a partition of V into V_1 and $V - V_1 = V_2$.

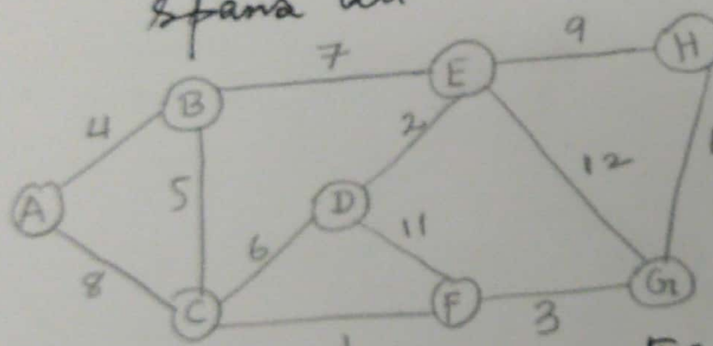
An edge crosses the cut if its end points are in different sets. i.e. $(V_1 ; V_2)$.

A light edge is an edge with min weight, a light edge with end points in (V_1, V_2) or (...) is a light edge that crosses the cut.

MST Problem: Undirected connected graph (simple) with edge weights +ve, 0 or -ve.

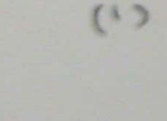
Tree: Connected subgraph of G that is acyclic.

MST: " " " " and spans all vertices.



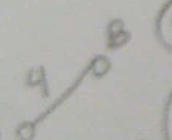
(i) Prim's starting from vertex A (You can start anywhere).

The cut is always (Included vertices, Remaining vertices). Light edge is AB with $w = 4$.



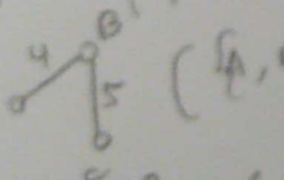
({A}, {B, C, D, E, F, G, H}) BC $w = 5$

({A, B} {C, D, E, F, G, H}) CF $w = 1$

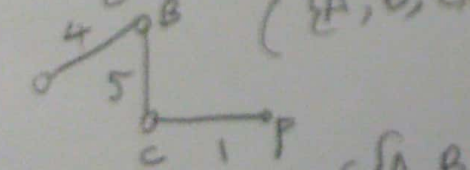


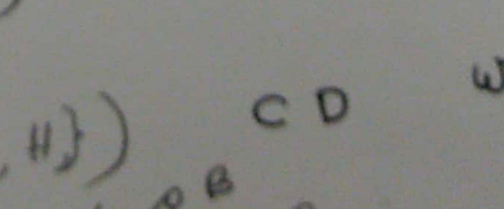
({A, B, C} {D E F G H}) FG $w = 3$

({A, B, C, F}, {D, E, G, H}) CD $w = 6$

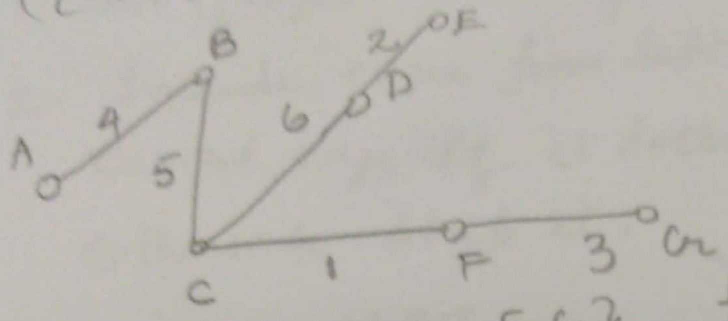


({A, B, C, F, G}, {D, E, H})

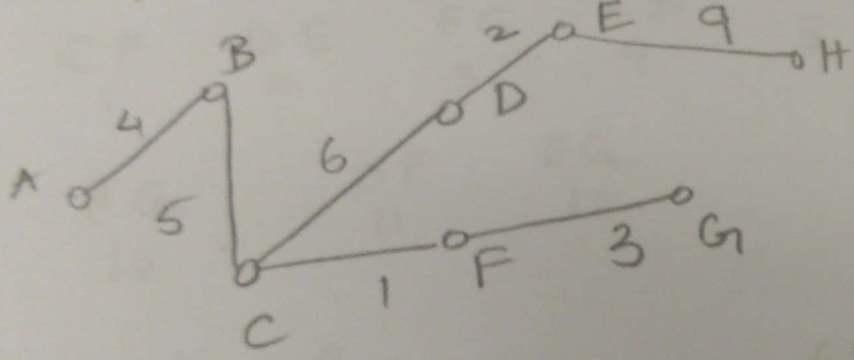




({A, B, C, D, G, F}, {E, H}) LE : DE w=2



({A, B, C, D, E, F, G}, {H}) LE : EH w=9



Cost 30

Kruskal's algorithm:

1. Sort edges ascending on edge weights $O(m \lg m)$
2. Include edges from left. I.e. Include the current edge if it doesn't create a cycle. Otherwise skip it.
3. O/P MST.

CF 1, DE 2, FG 3, AB 4, BC 5, CD 6, BE 7, AC 8, EH 9

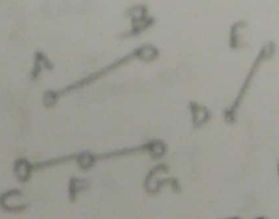
HG 10, DF 11, EG 12

<u>Components</u>

1. CF → Representative C (least id.)
2. CF, DE → rep D.
3. CF → Rep C, DE

   ↖ G

   Size <u>CF</u> = Size <u>FG</u>

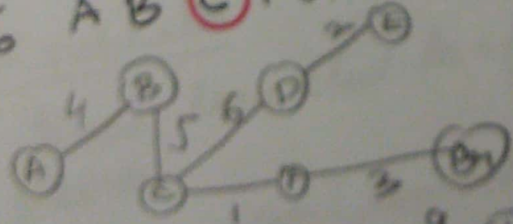


union by rank.

4. CFG, DE, AB

   Size CFG > Size AB

5. <u>ABCFG</u> DE ~~ABG~~
6. A B C F G DE



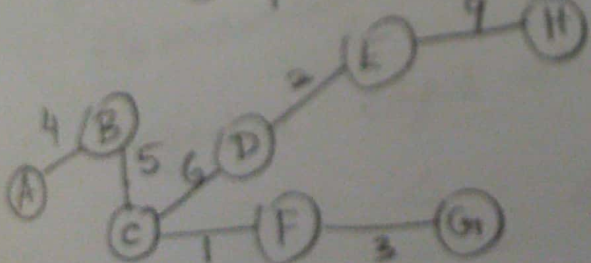
BE → Rejected

find (B) = find (E)

AC → Rejected

<u>EH → OK</u>

7 A B C D E F G H



DF → Rejected

EG → Rejected

<u>Cost</u>: 30

The total complexity of union & find $O(m \log m)$. ∴ T.C. $O(m \log m)$

Assume that $A \subseteq$ Some MST.

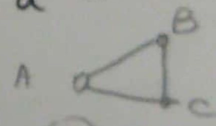
Generic MST

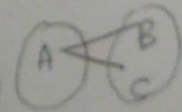
```
A ← φ
While A is a subset of some spanning tree
    Find (u, v) that is safe for A
    A ← A ∪ {(u, v)}
Return A.
```

Safe: If $A \subseteq$ Some MST and (u,v) is safe for A
then $A \cup \{(u,v)\} \subseteq$ Some MST.

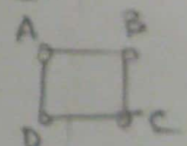
Th: Let $A \subseteq T^*$ a MST in (G, w).
weighted graph.
Let $(S, V-S=\bar{S})$ be a cut that respects A.
Let (u,v) be a light edge that crosses $(S, \bar{S})$.
Then (u,v) is safe for A.

Double crossing Lemma: If a cycle crosses a cut then
it does so even number of times.



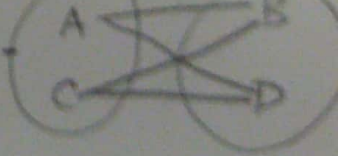


In Prim's cut is $(E_T, E \setminus E_T)$
E_T: Edges in subtree formed so far

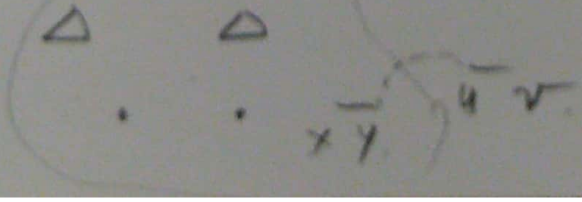


Kruskals

Some connected components are on one side
~~Cother~~
Remaining connected components one the other side



Rejected edges are not considered.

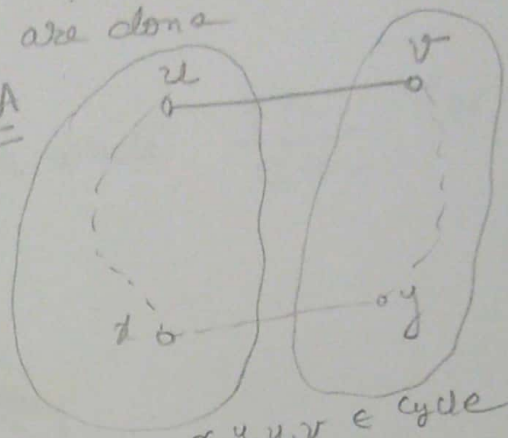


Proof of generic theorem

Let (u,v) be a light edge that crosses V_1, V_2 $(V_2 = V - V_1)$
$V_1 \neq \Phi$, $V_2 \neq \Phi$. Let T^* be MST of the weighted graph (G, w).

Case (i) if $(u,v) \in T^*$ then we are done

Fig A



$x, y, u, v \in$ cycle

Case (ii) $(u,v) \notin T^*$.

$\because$ T^* spans $V \Rightarrow$ Some edge that crosses the cut $\in T^*$.

Say, this edge is x, y.

Furthermore T^* spans u and v

$\therefore \{(u,v)\} \cup T^*$ creates a single cycle. By double crossing lemma this cycle C crosses (V_1, V_2) even number of times.

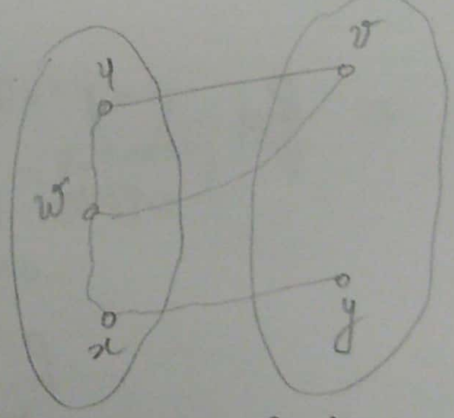
Among all these edges (u,v) is a light edge. $\therefore$

Fig A $T^* \cup \{(u,v)\} \setminus \{(x,y)\}$ is a S.T. whose cost $\leq T^*$.

(u,v) could have been chosen earlier.

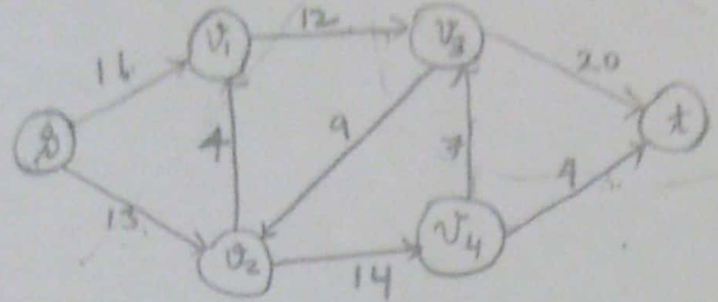
$\therefore$

Fig B $T^* \cup \{(u,v)\} \setminus \{(w,y)\}$ is also a M.S.T.

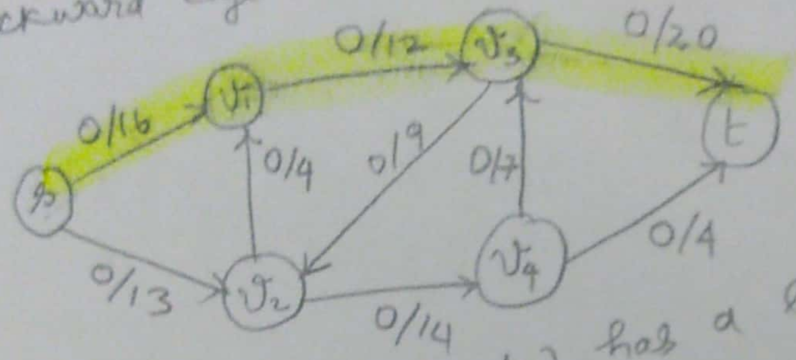


$x, y \notin$ Cycle

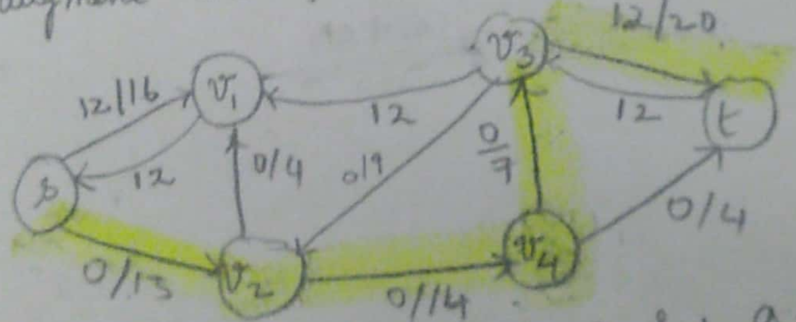
The other cases are symmetric.



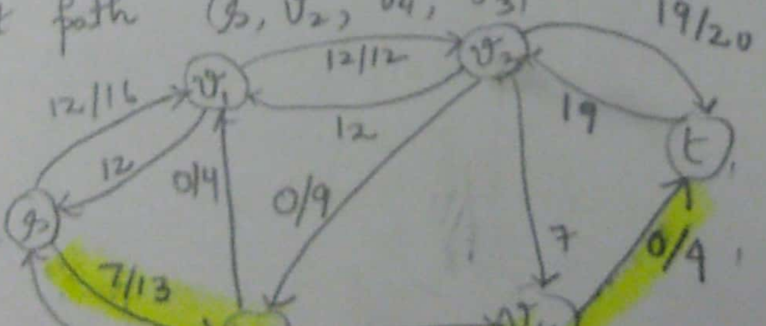
Convention: For forward edge we write $\frac{f(e)}{c(e)}$ for backward edge we write $f(e)$



The s-t path (s, v_1, v_3, t) has a capacity of 12
We augment the flow



The s-t path (s, v_2, v_4, v_3, t) has a capacity of 7



The s-t path (s, v_2, v_4, t) has a capacity of 4