Optimal Double Route Search in a Telecommunication Network: A Telecontrol Network Application

Juan Ramón Feijoo, Jesús Cid-Sueiro, *Senior Member, IEEE*, Juan Antonio García-López, Patricia Conde, and José Luis Rojo-Álvarez, *Member, IEEE*

***Abstract*—High availability communication networks with very low failure rates are often designed by using physical diversity, i.e., the traffic between a given pair of nodes is routed by using several physically disjoint paths. The selection of the pair of routes that maximizes the connectivity of a node is not an easy problem, because such connectivity cannot be expressed as an additive function of the availability of links and nodes in the path pairs. Previous algorithms for searching the optimal route use additive costs, which can be a loose assumption either when high failure rates can be locally present, or when fully disjoints paths do not exist. In this paper, we construct a Bayesian network to encode the probabilistic relation between the connectivity of a node and the availability of nodes and links. The problem of selecting an optimal pair of routes becomes equivalent to optimizing the structure of the Bayesian network. By introducing appropriate approximations on the double route availability equations, we propose a new algorithm, which outperforms other classical methods when there are sporadic high error rate elements in the network. Simulations and an application example in an electric transport telecontrol network show the performance of the method when compared to standard search.**

***Index Terms*—Bayesian networks, network optimization, telecommunication networks, telecontrol.**

ACRONYM

RFE	Remote Front End Concentrator.
SVM	Support Vector Machine.
REE	Red Eléctrica de España.
TC	Telecontrol Circuit.
SDH	Synchronous Digital Hierarchy.
OGW	Optical Ground Wire.
VSAT	Very Small Aperture Terminal.

NOTATION

$A_n(i)$	Availability of node i
$U_n(i)$	Unavailability of node i
$A_e(i)$	Availability of link i
$U_e(i)$	Unavailability of link i
$A_r(i)$	Availability of path i
$U_r(i)$	Unavailability of path i
$C(i)$	Connectivity of node i
$L(i)$	Connectivity loss of node i
n_i	Node in a Bayesian network
b_{ij}	Branch in a Bayesian network
e_{ij}	Availability of the link between nodes i and j
R_i	Path from a source node n_0 to destination node n_f as a sequence of nodes
$c(R)$	Cost associated to a path R in a network
$E(R)$	Expected loss increment of a path cost
$d(i,j)$	Distance from node n_i to node n_j
S_k	Set of paths in a network
N_c	Number of complete paths in S_k
λ_i	Length of path Q_i
n_R	Terminal node of R
$d(,n_f)$	Distance to the RFE node, n_f

Manuscript received November 30, 2008; revised September 11, 2009 and March 28, 2010; accepted August 02, 2010. Date of publication January 10, 2011; date of current version March 02, 2011. This work has been supported by research projects MOTTIBA from REE, TEC2007-68096-C02/TCM, and TEC2008-01348/TEC from Spanish Ministry of Science and Innovation. Associate Editor: S. Shieh.

J. R. Feijoo is with the Red Eléctrica de España, Madrid, Spain. He is also with the Depto. de Teoría de la Señal y Comunicaciones, Universidad Carlos III de Madrid, Madrid, Spain (e-mail: jfeijoo@ree.es; jfeijoo@tsc.uc3m.es).

J. Cid-Sueiro is with the Depto. de Teoría de la Señal y Comunicaciones, Universidad Carlos III de Madrid, Madrid, Spain (e-mail: jcid@tsc.uc3m.es).

J. A. García-López is with the Red Eléctrica de España, Madrid, Spain (e-mail: juangl@ree.es).

P. Conde is with the Depto. de Teoría de la Señal y Comunicaciones, Universidad de Vigo, 36310 Vigo, Spain (e-mail: pconde@gmail.com).

J. L. Rojo-Álvarez is with the Depto. de Teoría de la Señal y Comunicaciones, Universidad Rey Juan Carlos, Madrid, Spain (e-mail: joseluis.rojo@urjc.es).

Digital Object Identifier 10.1109/TR.2010.2103591

I. INTRODUCTION

NETWORK optimization under constraints is a general, classical problem, which has been widely studied in the literature. One of the main constraints to be considered for the design is often the network reliability, which in telecommunication networks has been estimated by analyzing different criteria, such as component types and redundancy [1], [2]. Other constraints, such as bandwidth, backbone topology, or delay, have also been taken into account [3], [4]. In other networks, like those conceived for electricity distribution, the growth follows the demand, and hence reliability is also one of the main constraints at their design stage [5]–[7]. In these telecommunication networks, a convenient way of increasing reliability is by using physical diversity, usually by establishing at least two alternate routes (between each node and the telecontrol station) with the minimum number of shared elements.

In this paper, network availability is measured in terms of the connectivity of its nodes, i.e., the average time fraction that each node has a functioning connection with the selected destination. Other issues, as the delay or the throughput of the available communication links are assumed to be less critical than the connectivity, which is a reasonable assumption in telecontrol networks, where the connection loss, which implies the inability to communicate network failures to control stations, is much more critical than the quality of the communication links.

The optimization of node connectivity involves three main steps: 1) estimating the availability of single network components (nodes and links); 2) estimating the connectivity of each node with the desired destination node, based on the availability of the nodes and links in the established routes; and 3) optimizing the availability of the whole network by selecting, for each node, the pair of alternate routes that maximizes its connectivity.

Because the availability and connectivity measures are usually unknown in advance, statistical and probabilistic methods must be used during the three steps of this process. Statistical learning models have been widely used to assist the network analysis and planning, focusing on different aspects, such as optical components? fault probabilities [8] or global availability [9]. Given that these models have to be trained with observed events and availability in a given network, different learning-from-samples methods have been used [10]–[14]. In [15], a fault prediction system was developed that used variables like temperature, weather forecast, cable types, and traffic intensities, so that when observations are introduced, fault probabilities can be updated. In [16], a Support Vector Machine (SVM) was used to model the availability of network elements in terms of their geographical and physical features, for the case of a high reliability communication network (specifically, an electric network), in which a low number of events are available.

The focus of this work is in the last two steps of the 3-step procedure described above. We use the formalism of Bayesian Networks to express the reliability of a Telecommunication network in terms of the estimated availability of its single components. After this, we design a new algorithm for searching the optimal routes. More specifically, we propose a method to find the optimal pair of routes in a telecommunication network from a source node to a destination node, in the sense of maximizing the node connectivity.

Bayesian Networks provide a systematic procedure to compute the connectivity of a node, avoiding the need to express it explicitly as a function of node and link availability. This makes the design of the network based on an exact connectivity metric feasible. Moreover, the model construction described in this paper is not restricted to double-path networks, thus making straightforward the extension of the double-path optimization discussed in this paper to the design of multiple-path telecontrol networks.

The algorithm focuses on the connectivity optimization. Algorithms for double-path search that optimize additive metrics on weighted graphs have been proposed in the literature, which are computationally efficient [17], [18]. However, the use of additive metrics is suboptimal, and may degrade the network availability. Because the optimization process must be carried out *offline*, during the network design, computational issues are not critical. In this paper, we focus on the optimization of the exact (non-additive) connectivity metric, at the expense of admissible increments of the computational cost.

The scheme of the paper is as follows. In the next section, after a related work summary, the problem statement and notation are introduced, and the background on related network optimization algorithms and Bayesian Networks is presented. Then, in Section III, the availability cost is approximated, and the Dijkstra algorithm is adapted for use in this setting. In Section IV, the new Bayesian Network based algorithm for connectivity search is presented. In Section V, results on simulations and on a case study in an electric network show the performance of our algorithms. Finally, in Section VI, conclusions are presented.

II. Background and Problem Statement

A. Related Work

Path search algorithms for network optimization have been the subject of research for years, and a number of algorithms for various conditions and constraints have been proposed, most of them dealing with additive metrics, e.g., the number of hops. A classical algorithm, proposed by Dijkstra [19], finds a single path minimizing an arbitrary cost measure in a graph, provided that the cost increment of adding a new node or link is independent of which nodes or links are already in the path.

A telecontrol network is a special telecommunication network that is used to operate high power electrical installations, such as substations and transformers. It consists of point to point circuits that connect the electrical control center (usually located in the electrical company headquarters) to the substations. Its application is to operate the electrical grid, by conveying the commands that open and close the breakers and the mechanical switches that terminate the electrical lines. It also conveys information like the electrical magnitudes, module, and phase of voltage and intensity.

The network optimization problem addressed in this work consists of selecting, for each node in the telecontrol network, the pair of paths maximizing the connectivity, which will be called the *most reliable pair*. The Dijkstra algorithm can be used to find the most reliable *single* route, but its extension to find the optimal pair of routes is not easy: in general, the optimal pair is not equal to the best path plus the second-best and, furthermore, the best single path may not be in the best pair. This is because, as we will see later, the connectivity of a pair of routes cannot be expressed as a linear function of the availability of the single nodes and links in the paths.

In highly reliable networks with similar components (i.e. when the availability of nodes and links range in the same order of magnitude), any disjoint pair of routes is more reliable than any pair of routes sharing some nodes or links. This suggests an alternative optimization criteria: among all pairs minimizing the total cost of the shared components, select the one minimizing the total cost of the non-shared components (i.e., the disjoint sub-paths). Several extensions of the Dijkstra

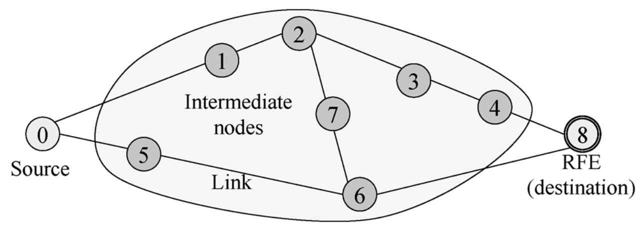

Fig. 1. Double path selection problem. There are four paths from source node 0 to the RFE destination node (number 8): 0-1-2-3-4-8, 0-5-6-8, 0-1-2-7-6-8, and 0-5-6-7-2-3-4-8. The goal is to select the pair of paths maximizing the connectivity of node 0. This must be done for every node in the network.

algorithm have been proposed for this purpose, like the Suurballe algorithm [17], or the algorithm proposed by Bhandari [18], which involves only two steps of the Dijkstra algorithm. However, when the reliability of the network components has a great diversity (i.e., the availability of the network components may differ in several orders of magnitude), the connectivity of a node cannot be approximated as a sum of the individual cost of each component. In that cases, the pair of routes obtained by classical algorithms can be highly suboptimal, which motivates the investigation of novel algorithms maximizing the connectivity of nodes.

Finally, p-cycles have received much attention in the literature [20], [21], for local recovery (adjacent to failure) with preconfigured protection cycles (virtual rings) formed in the spare capacity of the network. Nevertheless, the focus of work around p-cycles has been more devoted to scenarios with high degree of element failure and multiple failure survivability, which is beyond the scope of the present work.

B. Problem Statement and Notation

The objective of this work is to create algorithms to obtain the two routes that have the best joint availability in a telecommunication network with a semi-mesh configuration. These two routes connect a source node to a destination node, the last one being a Remote Front End (RFE) Concentrator (see Fig. 1), and they are alternate paths in a standby configuration for an important service whose failure condition can be modeled as a logical AND gate of the condition of a two path parallel circuit. The service keeps on working while at least one of the paths keeps on working, and the service comes down only if both paths come down.

A path is a concatenation of several links and nodes. One link or node down is necessary and sufficient for the path being down. One path can be down as a consequence of other causes different from the failure of a link, for instance, a power failure in the installation at a node.

For describing the algorithm, some definitions are needed.

- *Availability*: Time fraction of a link or a node in a functioning condition (from now, in steady state).
- *Unavailability*: Time fraction of a link or a node not in a functioning condition.
- *Connectivity*: Time fraction of a selected node connected to its corresponding RFE.
- *Connectivity loss*: Time fraction of a selected node not connected to its corresponding RFE.

Connectivity loss is given by the probability of a node being unable to connect to the RFE at a randomly chosen time. As far as availability and connectivity are defined in statistical terms, it will be possible to apply statistical models for yielding a probabilistic behavior. According to the notation introduced for availability, unavailability of node, link, and path, it is clear that $U_n(i) = 1 - A_n(i)$ for nodes, and similarly for links, paths, and connectivity and connectivity loss $(L(i) = 1 - C(i))$. Note that, in the definitions of path availability, the path includes the origin and destination nodes.

For comparison purposes, in this work we considered other optimization criteria from reliability, such as the number of hops, and the availability. Though hops or availability optimization can sometimes yield paths with shared elements with a low total availability, they are useful complementary tools for connectivity and disjoint hops optimization. The characteristics of these optimization methods, and their algorithmic implementations are next described.

1) Hop Optimization: The selection of the two paths starting from each node and reaching its RFE with a minimal number of hops can be made computationally efficient, and yields the same results as availability optimization when node and link availability are similar. Thus, it is an alternative to other optimization criteria when there are no reliable estimates of node and link availability. However, because it does not take into account if the selected paths share some components or not, it can be inadequate to achieve high global connectivity in the network. In any case, hop optimization is used as a reference to show the advantages of using availability estimates, and searching for disjoint paths during network optimization.

The algorithm implemented in this work (which is a simple extension of the Dijkstra algorithm) determines the two shortest paths from one node to another in a graph by pruning the whole set of paths. At the i-th iteration, the Dijkstra algorithm expands the selected set of paths (with i hops), by adding a new hop to any neighbor node. All possible neighbor nodes are used, so several paths may result from growing each selected path. At the pruning step, if several paths converge at the same intermediate node, then the best two paths are selected, and the others are eliminated. This step improves upon the Dijkstra algorithm, which takes only the best single path during the pruning step. In this way, the number of nodes to be explored in the next steps is substantially reduced. As an example, in a Pentium M 1.6 GHz computer, the search of the two best paths for each node in the telecontrol network analysed in Section V (465 nodes and 1164 links) can be made in 6.3 seconds.

2) Not Disjoint Availability Optimization: In this algorithm, the two paths with the highest availability are searched. Different from hop optimization, this method takes into account the availability of the nodes and the links, although it does not account for common nodes or links in the selected pair of paths.

The Dijkstra algorithm was initially conceived for obtaining the path with the lowest number of elements, or with minimal total cost, in networks where each link or node has an associated cost, and the total cost is the sum of the cost of all path components. Because of the probabilistic nature of the availability measure, path availability is not the sum of the availability of its components. However, a first order approximation, explained later, can be applied to express the unavailability of a path as the sum of the unavailability of its elements, and the same algorithm as in hop optimization can be used for considering associated costs given by unavailability values.

C. Bayesian Networks

For any set of random variables, a direct acyclic graph can be constructed as follows. Each random variable is represented by a node in the graph, and the directed links are chosen in such a way that the joint probability distribution of all the variables can be computed as the product of the conditional probability of each variable given its parents in the graph. When this correspondence between the graph structure and the joint probability is satisfied, the graphical model is said to be a Bayesian network [22], [23].

Bayesian network methods provide a systematic approach to probabilistic modeling. Once the Bayesian network is constructed (by identifying the graph structure and the conditional probability values of each node given its parents), efficient algorithms exist to compute arbitrary probabilities about the variables in the graph. General-purpose software applications are available for Bayesian network implementation.

For the purposes of optimizing the connectivity of nodes in a telecommunication network, the main advantage of Bayesian network methods is that they avoid deriving the explicit formulas for the connectivity of nodes as a function of link and node availability.

Fig. 2 shows the structure of a Bayesian network to compute the connectivity of node 0 for the network in Fig. 1, which is organized in four layers of binary variables, described as follows.

1 Nodes and links: Bayesian network node n_i is a binary variable taking value 1 if node i (of the communication network in Fig. 1) is available, and 0 otherwise. In a similar way, Bayesian network node e_{ij} accounts for the availability of the link between nodes i and j.
2 Branches: Bayesian network node b_{ij} is 1 if all nodes and links in the chain connecting node i to node j are available. Because all components of the branch are used for the same and only purpose (i.e., serving communications between nodes i and j), grouping them in a single variable is useful to reduce computations.
3 Paths: Each node in this layer accounts for the availability of one of the paths selected to connect node 0 with its RFE.
4 Connectivity: The last level refers to the connectivity of the selected node.

The Bayesian network links go from the lower to the higher layers in it, because any path or branch may operate iff each component is functioning properly. In this construction, there are two implicit assumptions: the availability of a node is s-independent of the availability of the other nodes, and the availability of a link is s-independent of the availability of the other links.

Besides a graphical model, the probability of each node associated to its parents in the graph must be specified. First layer probabilities are given by individual node and link availability. For the second and third layers, we assume a deterministic model such that any branch or path works iff all its components work. For instance, branch variable b_{06} in Fig. 2 takes value 1 iff $e_{05} = 1$, $n_5 = 1$, and $e_{56} = 1$; otherwise, $b_{06} = 0$. Thus, the corresponding conditional probabilities are 1 or 0 accordingly.

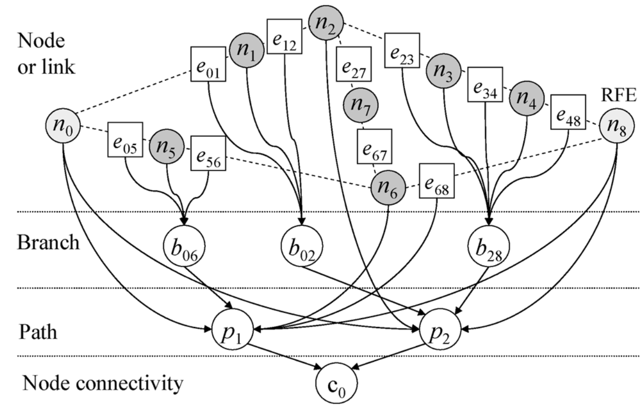

Fig. 2. Bayesian network model for computing the connectivity of node 0 in the communication network in Fig. 1. The model assumes that the paths used by node 0 to connect to its RFE are 0-1-2-3-4-8 and 0-5-6-8.

Path variables are considered in a similar way. Fourth layer variable c_0 represents the service state of node 0 ($c_0 = 1$ if the node is connected to its RFE, and 0 otherwise), and is also a deterministic function of the path variables, p_i. Though, in this work, the state of all variables is a deterministic function of variables in lower layers, more general failure conditions can be also introduced to represent failure causes other than links or nodes [24]. In that case, conditional probability values other than 0 or 1 can be considered.

After the Bayesian network is constructed, usual inference algorithms can be applied to compute the node connectivity, and the full potential of Bayesian network models can be used, including the possibility of employing soft or virtual evidence. Sometimes the state of a node or a link has not been observed, but we have some distribution over its possible values (for example, if we ignore its exact value, but we know its likelihood ratio). This knowledge can be introduced in the network, and the failure probabilities can be recomputed. Also, it is possible to learn rules and relationships among the observed data, and use this knowledge to improve prediction accuracy [9], [13].

Note that, though this paper is focused on the problem of searching a double path for each node, the extension of the Bayesian network model to telecontrol services with more than two paths is immediate, by just adding new path variables in the third layer.

The availability values of nodes and links required to apply the Bayesian network can be estimated from data. In [16], we applied statistical learning techniques based on Support Vector Machines (SVM) to predict the node and link availability based on sparse data of the historic behavior of the network. Fig. 3 shows the overall architecture of the evaluation system to be integrated in the target network model.

III. Computing the Connectivity of a Node

One important advantage of Bayesian network methods is that they avoid the need to obtain explicit expressions for the connectivity of a node as a function of node and link availability. However, in this section we obtain such expressions for the connectivity of a node with a double path because they will serve to show that, in general, it is not an additive function of the availability values.

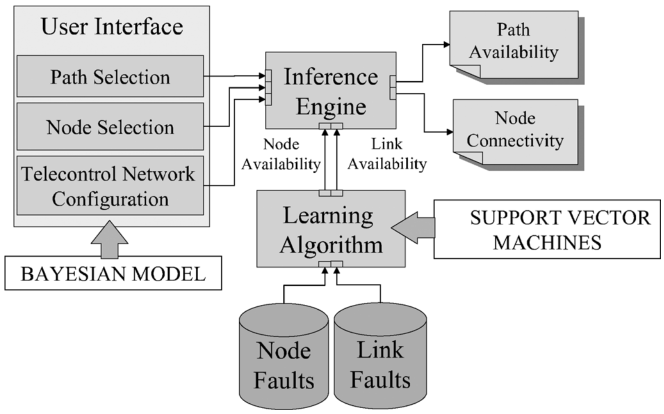

Fig. 3. Overall model architecture.

A. Path Availability

A path R from source node n_0 to destination node n_f is a sequence of nodes $\mathcal{N} = \{n_0, \dots, n_f\}$ that are connected by means of a link sequence $\mathcal{E} = \{e_{0,1}, \dots, e_{f-1,f}\}$, so that $e_{i,i+1}$ is a link between node n_i and node n_{i+1}, for every $i, 0 \leq i < f$.

A path is available if all of its nodes and links are available. Therefore,

$$A_r(R) = \prod_{j=0}^{f} A_n(n_j) \prod_{k=0}^{f-1} A_e(e_{k,k+1}). \tag{1}$$

Similarly,

$$U_r(R) = 1 - A_r(R) = 1 - \prod_{j=0}^{f} A_n(n_j) \prod_{k=0}^{f-1} A_e(e_{k,k+1}). \tag{2}$$

Usually, node and link unavailability are very low, and hence first order estimations are very precise. By neglecting in (2) every product with two or more unavailability, we can write

$$\begin{aligned} U_r(R) &= 1 - \prod_{j=0}^{f} (1 - U_n(n_j)) \prod_{k=0}^{f-1} (1 - U_e(e_{k,k+1})) \\ &\approx \sum_{j=0}^{f} U_n(n_j) + \sum_{k=0}^{f-1} U_e(e_{k,k+1}). \end{aligned} \tag{3}$$

This equation states an additive unavailability model, where the impact of adding a new node gives an unavailability increment that depends only on the unavailability of the added node or link. Additive models could actually be more accurate than the non-additive ones because of problems with finite precision terms in the form $1 - I$, when I can take values about 10^{-9}. Moreover, an additive model permits the use of efficient methods like the well-known Dijkstra algorithm [19] in order to search the most available path.

B. Node Connectivity

Node connectivity is the probability that at least one route from one individual node to its destination is working correctly, and its determination is somehow more complex. First, a node n_0 is connected to RFE c if the following three conditions are met.

1 Node n_0 is available.

2 Node c is available.

3 At least one of the telecontrol paths from n_0 to c is available.

We assume that each node has two telecontrol paths to its RFE, i.e., even though there may be many paths connecting each node with its RFE, we assume that only two of them can be used by the telecontrol network. If the telecontrol paths are fully disjoint (i.e. they do not share any node and any link), then node connectivity can be determined as follows.

Let $R_0, \dots, R_{K-1}$ be the telecontrol paths for node n_0. Likewise, let $\bar{R}_k$ be the nodes and links from R_k excluding the terminal nodes (i.e., n_0 and C). In this case, we can write

$$C(n_0) = A(n_0) A\left(\bigcup_{k=0}^{K-1} \bar{R}_k\right) A(C) \tag{4}$$

where the second factor represents the availability of the disjoint set of sub-paths. Taking this into account, and provided that terminal nodes are available, the connectivity of node n_0 is lost iff all paths $\bar{R}_k$ fail, and by assuming that node and link failures are s-independent, we can write

$$A\left(\bigcup_{k=0}^{K-1} \bar{R}_k\right) = 1 - \prod_{k=0}^{K-1} U_r(\bar{R}_k). \tag{5}$$

Let $\{n_{k,0}, \dots, n_{k,f_k}\}$, and $\{e_{k,1,2}, \dots, n_{k,f_{k-1},f_k}\}$ be the set of nodes, and links from path k (so that $n_{k,0} = n_0$ y $n_{k,f_k} = C$). In that case, using (2), we have

$$\begin{aligned} U_r(\bar{R}_k) &= 1 - A_r(\bar{R}_k) \\ &= 1 - \prod_{i=1}^{f_k - 1} A_n(n_{k,i}) \prod_{j=0}^{f_k - 1} A_e(e_{k,j,j+1}). \end{aligned} \tag{6}$$

Combining (4), (5), and (6), we obtain

$$\begin{aligned} C(n_0) &= A(n_0) A(C) \\ &\times \left(1 - \prod_{k=0}^{K-1} \left(1 - \prod_{i=1}^{f_{k-1}} A_n(n_{k,i}) \prod_{j=0}^{f_k - 1} A_e(e_{k,j,j+1})\right)\right). \end{aligned} \tag{7}$$

C. Shared Elements in Paths

The connectivity of a node with two non-disjoint paths can be computed by separating the contribution of shared and disjoints nodes and links. If R_0 and R_1 are the two alternative paths, then

$$\begin{aligned} C(n_0) = &\prod_{j \in \Phi_n}^{f} A_n(n_j) \prod_{k \in \Phi_e}^{f-1} A_e(e_{k,k+1}) \\ &\times \left(1 - \left(1 - \prod_{i \in \Phi_{n,0}} A_n(n_{0,i}) \prod_{j \in \Phi_{e,0}} A_e(e_{0,j,j+1})\right)\right. \\ &\left.\times \left(1 - \prod_{i \in \Phi_{n,1}} A_n(n_{1,i}) \prod_{j \in \Phi_{e,1}} A_e(e_{1,j,j+1})\right)\right) \end{aligned} \tag{8}$$

where Φ_n, and Φ_e are the sets of shared nodes, and links, respectively; and $\Phi_{n,i}$, and $\Phi_{e,i}$ are the sets of non-shared nodes, and links respectively, from path i, for $i = 0, 1$.

D. First Order Approximation

If we again replace availability products with first order approximations expressed in terms of their corresponding unavailability, we can express the connectivity loss, $L(n_0) = 1 - C(n_0)$, as

$$L(n_0) \approx \sum_{j\in\Phi_n}^{f} U_n(n_j) + \sum_{k\in\Phi_e}^{f-1} U_e(e_{k,k+1})) + \left(\sum_{i\in\Phi_{n,0}} U_n(n_{0,i}) + \sum_{j\in\Phi_{e,0}} U_e(e_{0,j,j+1})\right) \times \left(\sum_{i\in\Phi_{n,1}} U_n(n_{1,i}) + \sum_{j\in\Phi_{e,1}} U_e(e_{1,j,j+1})\right). \quad (9)$$

Note that terms corresponding to non-shared links appears multiplying, and therefore their contribution to the loss probability of the node is negligible (with respect to the contribution of the shared links). Hence it can be approximated as

$$L(n_0) \approx \sum_{j\in\Phi_n}^{f} U_n(n_j) + \sum_{k\in\Phi_e}^{f-1} U_e(e_{k,k+1}) \quad (10)$$

which depends only on shared elements.

Equations (9), and (10) show that, in general, the most reliable pair will have two physically-disjoint paths, because common links or nodes have a first-order influence in (10), indicating that a single fault in any common element could take down both paths, and the whole service. Nevertheless, the most reliable pair may have non disjoint paths for two main reasons:

- there is no alternative non-disjoint link or node; and
- the availability of disjoint paths is very low because the availability of alternative paths is low, and thus paths with some common links to keep high levels of availability may be more reliable.

IV. Network Optimization

A. Bhandari Algorithm

Equations (9), and (10) suggest an approximate procedure for searching the most reliable pair. Among those path pairs minimizing the failure rate of their common nodes and links (according to (10)), find the pair minimizing the product of the loss probabilities corresponding to the disjoint parts of each path (according to (9)). For computational efficiency, a further approximation can be made by replacing this product by a sum of loss probabilities of the disjoint parts. In such a case, an algorithm proposed by Bhandari [18], [25] can be applied. The proposed algorithm would consist of the following steps.

1 Set link, and node unavailability as the costs of each element in a path.
2 Apply the algorithm in [18], [25] to find the most disjoint routes with minimum overall cost.
3 Use (9), and (10) to determine the connectivity of a node.

The Bhandari algorithm involves two steps of the Dijkstra algorithm, and can be extended to the case where no disjoint paths exist by searching the minimization of the total cost. It is computationally efficient, but the replacement of a multiplicative cost in (9) by an additive cost may have undesired consequences. Fig. 4 shows a simple example of a network where the minimization of an additive cost may be suboptimal. Assume that the node unavailability are 0, and the link unavailability take the values x, y, and z (as shown in the figure), where $x < y \ll z$. The pair of routes obtained by connectivity optimization and the disjoint path optimization based on the Bhandari algorithm are different. While the connectivity loss of the source node for the optimal pair is close to $6yz$, the loss resulting from the Bhandari algorithm is around z^2. If z is large, the availability of the path pairs obtained by both algorithms is substantially different (the ratio, $6y/z$ can be arbitrarily slow). The differences between both algorithms are due to Connectivity Optimization working to concentrate the less reliable links in only one of the paths, hence achieving a better global availability.

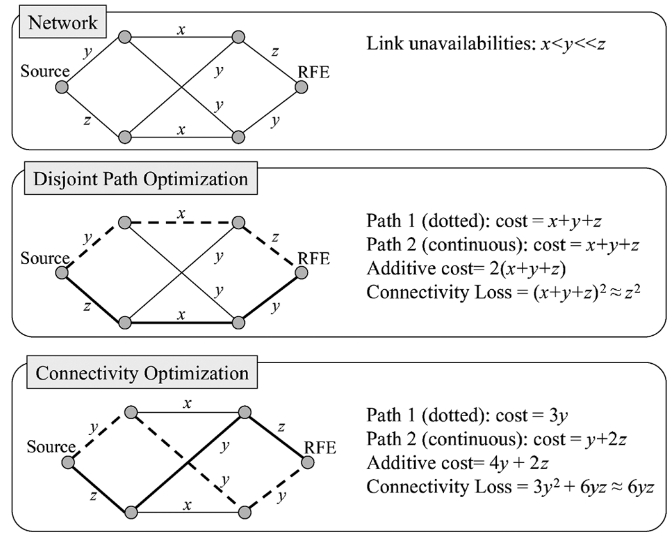

Fig. 4. Difference between connectivity optimization and the minimization of the sum of unavailabilities.

In the example in Fig. 4, the network has six nodes with zero unavailability, and link unavailability taking values x, y, and z such that $x < y \ll z$ (up). The pair of routes obtained by disjoint path optimization based on the Bhandari algorithm (center) is suboptimal: while the connectivity loss of the source node for the optimal pair (down) is close to $6yz$, the connectivity resulting from the Bhandari algorithm is around z^2. If z is large, the availability of the path pairs is substantially different. This example shows that, because the connectivity is a nonlinear function of the unavailability, the optimization of an additive cost (as the one used by the Bhandari algorithm) is suboptimal, and even in simple networks the connectivity loss of the pair of paths obtained by the Bhandari algorithm can be several orders of magnitude higher than the optimal value. The example also provides some evidence that the connectivity optimization could be especially efficient in networks where highly reliable links coexist with unreliable links. In heterogeneous networks, with a high degree of variability in the link availability, connectivity optimization could improve the connectivity of some nodes whose paths are not fully disjoint, or may have some links with low availability.

B. Proposed Algorithm

The new optimization algorithm proposed here minimizes the connectivity loss given by (9). Even efficiently implemented, the computational burden is higher than that of other approaches based on the Dijkstra algorithm, and in some cases it enforces the stopping of the optimization algorithm in suboptimal solutions. In our experiments, we will show the differences in time and loss probability resulting from both approaches. Our proposed algorithm obtains a *quasi* optimal solution in a reasonable time, which implies that this search obtains for almost all nodes the telecontrol pair providing maximal connectivity. The optimization process may stop when the probability of finding a better pair of paths is low enough.

The algorithm proceeds by exploring over a set of possible paths, which are progressively built by adding a new hop, and computing the availability of all the possible paths. The sequence of steps is as follows.

1 Initialization:
   a) Establish a Bayesian Network with the network elements, and its associated failure probabilities [24].
   b) Let n_0 be the starting node, and n_f be its RFE. Let $S_0 = \{n_0\}$ be the set of active paths.

2 While there are incomplete paths (i.e. paths that do not end in the RFE node) in S_k:
   a) Path selection: Select an incomplete path $R^* \in S_k$ to progress.
   b) Path extension: Compute all paths obtained by extending R^* from its terminal node with a single hop to a new node not in R^*.
   c) Path set extension: Obtain S'_k by replacing R^* in S_k by the extended paths.
   d) Evaluation: Compute the failure probability of all the new path pairs using the inference engine of the Bayesian Network (or, equivalently, (9))
   e) Pruning: Set S_{k+1} as the result of removing all suboptimal paths in S_k.
   f) Increase k by unity.

The path selection, and the pruning steps are next detailed.

C. Path Selection

The selection of the path to be extended is a key to get an efficient optimization algorithm. If a reliable path pair is obtained at iteration k, many pairs of active paths in S_k with lower availability can be eventually discarded, and subsequently, more paths can be removed from S_k, which reduces the exploration space, and the computational load. The proposed path selection method is based on computing a cost measure $c(R)$ for each exploration path, R, which evaluates the expected availability of the path pairs. The selected path is

$$R^* = \arg\min_{R} c(R) \tag{11}$$

The cost $c(R)$ is computed as follows. Let $\bar{R}$ be a complete path constructed by extending R (which is incomplete) with a virtual *error free* link (i.e. a link with zero unavailability), that connects the terminal node of R with the RFE node, n_f. For any complete path $Q_i \in S_k$, let $L(\bar{R}, Q_i)$ be the connectivity loss of the starting node, n_0, for the path pair $(\bar{R}, Q_i)$, and let us define the average connectivity loss as

$$\tilde{L}(R) = \frac{1}{N_c} \sum_{i=1}^{N_c} L(\bar{R}, Q_i) \tag{12}$$

Note that $\tilde{L}(R)$ cannot be used as a cost measure, because it ignores the expected increment of the cost caused by the true (non-virtual) links that remain to complete path R. For this reason, we compute the cost as

$$c(R) = \tilde{L}(R) + E(R) \tag{13}$$

The expected loss increment, $E(R)$, is estimated based on the average loss per link on the current complete pairs, as

$$E(R) = \frac{1}{N_c} \sum_{i=1}^{N_c} E(R, Q_i) \tag{14}$$

$$E(R, Q_i) = \frac{\tilde{L}(Q_i)}{\lambda_i} d(n_R, n_f) \tag{15}$$

To compute such distance, a classical result from Graph Theory was employed, which says that, if $\mathbf{A}$ is a graph matrix, with components a_{ij} (hence $a_{ij} = 1$ if there is a link from node i to node j, and $a_{ij} = 0$ otherwise, and $a_{ii} = 1$), then matrix $\mathbf{B}(k)$ is given by

$$\mathbf{B}(k) = \mathbf{A}^k \tag{16}$$

and it has components $b_{ij}(k)$ that are equal to the number of paths that reach node i from j in no more than k hops. Therefore, if $b_{ij}(k) = 0$, there is no path reaching node j in k hops. Therefore, the distance from node i to node j, defined as the shortest path length from i to j, can be computed as

$$d(i, j) = \min_{b_{ij}(k) > 0} \{k\} \tag{17}$$

which means that $d(i, j)$ is the smallest value of k in which $b_{ij}(k) > 0$.

D. Pruning

The optimization algorithm computes the availability of all pairs formed with the active paths in S_k. At the same time, as soon as a pair of complete paths (i.e., paths terminating at the RFE) is available, the algorithm keeps in memory the most reliable complete pair. Because the availability of an incomplete pair cannot increase when any of its paths is extended with new links, all pairs with lower availability than the currently most reliable are discarded. A path R is pruned from the path set S_k iff the availability of all possible pair containing R is lower than that of the currently most reliable pair.

E. Complexity

The optimization process stops when there are no incomplete paths in S_k. Depending on the network size, and the loss probability distributions, this can take a long time. In terms of complexity, the proposed algorithm is not competitive with respect to the Bhandari algorithm; while the computational complexity of the former grows exponentially with the number of nodes (for a bounded number of links per node), an efficient implementation of the Bhandari algorithm requires $O(N_c log(N_c))$ operations where N_c is the number of nodes. However, because the optimization of a telecontrol network has no severe time constraints, the processing times are not critical issues in this application. For the network with 465 nodes and 1164 links used in our experiments, the computation time of the Bhandari algorithm is about a few seconds, while the proposed algorithm takes several hours, which is feasible. Moreover, the algorithm tends to explore potentially reliable paths with preference, so that a premature stop of the optimization algorithm usually provides quasi-optimal path pairs.

V. Case Study: REE Telecontrol Network

Red Eléctrica de España (REE) runs the High Voltage grid on the Spanish mainland, ensuring the technical conditions to enable power to flow continuously from the power generators to the centers of consumption. One of the main issues in this electric grid operation is reliability of the telecommunications, and in consequence a high reliable telecommunication network has been built to cover the needs arising from the management of electric power transmission, and the operation of the electricity system. Telecontrol service in this high voltage transmission network uses telecommunication circuits, and it is always designed with two different physical paths, so that in case of one path being affected by a fault, the other will still be able to operate, and the telecontrol service will continue without any interruption. Each of the paths uses several consecutive links.

The theoretical model described in the preceding sections was applied to assess the availability of REE telecontrol service, so that an analytical estimation of the availability in the telecontrol service could be made, and accurate comparison of the global impact of different circuit configurations could be analysed and quantified. Each element in the telecommunication system (related to the telecontrol circuit) is modeled basing on constructive characteristics, and observed data, such as historical failure records. After loading the appropriate and available empirical data in the model, the impact of failures in the availability can be assessed.

In REE, a Telecontrol Circuit (TC) is an end-to-end V.24 circuit that links the Operation Central System (sited at REE Headquarters) with the Remote Telecontrol Units (sited at the electrical substations). The TC physically operates switches and bays in the electrical installation. There is an intermediate step, the RFE, which concentrates the telecommunication TC of a local area. These telecommunication circuits are implemented with modems whose analog line is carried through a SDH transmission network implemented over optical fiber network OGW cables. In this way, REE has developed a semi-mesh fiber network, as shown in Fig. 5. Sometimes it has been hardly possible to establish two completely different paths over the optical fiber infrastructure, and hence, the convenience of alternative links had to be chosen among carrier power lines, radio links, VSAT circuits, or leased lines [26].

A statistical learning approach, using Support Vector Machines, was previously made for modeling the availability of

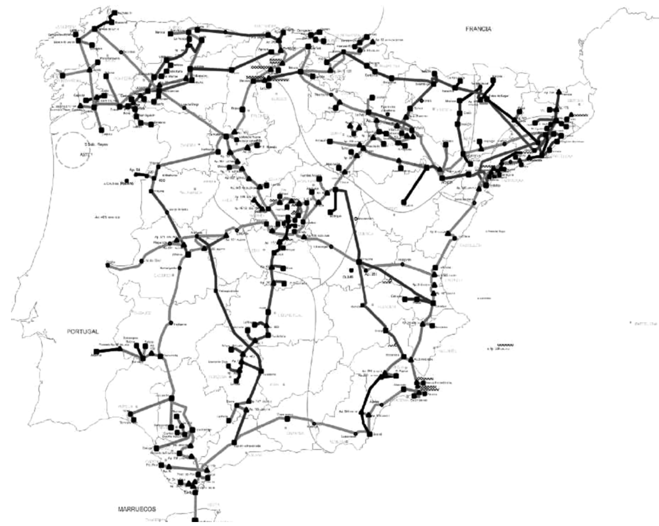

Fig. 5. REE Fiber Network.

TABLE I
OVERALL NETWORK UNAVAILABILITY IN A NETWORK WITH LINK AVAILABILITY $3,7410^{-5} \pm 2,4710^{-8}$ (MEAN $\pm$ STD DEV)

Optimization type	Unavailability
No. Hops	4,52 10^{-5}
Availability	3,53 10^{-5}
Disjoint Paths	1,18 10^{-5}
Connectivity	1,17 10^{-5}

the network elements (links and nodes), taking into account different effects, such as the link length or the geographical location. With the element availability provided by this model, a surrogate version of the target network was created and simulations were made on it. The model uses an inference machine which obtains path and service availability from links and nodes availability. Although node and link availability could sometimes be directly introduced, the developed algorithm based on statistical learning techniques has been used to predict availability [16].

The observed parameter has been the overall availability of the telecontrol service in the analysed network. Simulations were made on a model of REE transmission network considering only optical links, and not taking into account additional means such as power line carrier, VSAT, or radio links. The model included 465 nodes, and 1164 links. Results are shown in Table I for the overall unavailability. In addition, a surrogate network was created with an increase in its standard deviation, and unavailability from simulation results in this network are shown in Table II.

Hop optimization is very fast, and moreover, the algorithm converges quickly. Even by truncating the search to a maximum number of 10 paths, the algorithm usually reaches the optimum value without truncating, due to almost every node having two pairs of routes to its RFE in less than 10 links. By considering the two best availability paths, we obtain a slightly better result than with hop optimization, because when fault probabilities are similar, both criterion are also similar.

TABLE II
OVERALL NETWORK UNAVAILABILITY IN A SURROGATE NETWORK WITH LINK AVAILABILITY $5,7610^{-5} \pm 1,8210^{-4}$ (MEAN ± STD DEV)

Optimization type	Unavailability
No. Hops	5,41 10^{-5}
Availability	1,22 10^{-5}
Disjoint Paths	6,01 10^{-6}
Connectivity	5,39 10^{-6}

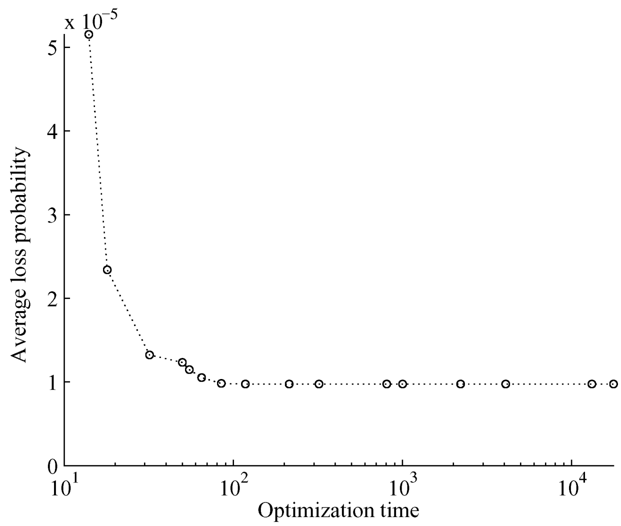

Fig. 6. Mean connectivity loss probability vs. computation elapsed time (in seconds).

Performance of the Connectivity and Disjoint Paths methods are similar in the proposed network. It has been observed an improvement of a 10% in the availability of a selected group of ten nodes, and a 1% in the whole network. As could be expected, the most reliable pair (connectivity optimization) is the best solution, improving both hop and availability optimization. Even when truncating the search to a maximum number of 10 paths, results are clearly better.

To guarantee a reasonable optimization time in our experiments, we implemented a premature stop, which happens as soon as a pre-established number, M, of complete paths has been explored. In our network, $M = 10$ provides quasi optimal pairs. Fig. 6 shows mean connectivity loss in all nodes, for the path pairs obtained with this algorithm with different complex levels. Horizontal axes represent computation elapsed time in seconds.

Simulations were also carried in a surrogate network with a mean failure probability of 5,76 10^{-5}, and a large variation in links failure probability. In this case, there was an improvement of 50% in the availability of a selected group of ten nodes, and 12% in the whole network.

VI. CONCLUSION

In this paper, the use of a first order approximation on the double route availability equations has allowed us to propose an availability-based cost, which has led to the use of classical algorithms for searching an optimal double route in a telecommunication network. More, a new algorithm, based on the use of a Bayesian Network framework, has allowed us to improve the performance in circumstances with sporadic high error rate elements being present. Simulations, and a real case study have shown the performance of the connectivity optimization by means of the two proposed algorithms. The results obtained were mainly coherent with intuition: availability improvement is huge when having two paths in comparison with just one, and even a precarious second path increases the availability by several magnitude orders. Another result coherent with experience was that if one link is shared between the two paths, this makes the availability decrease about a magnitude order, which clearly represents a situation to be avoided.

Special focus has been paid to the comparison between connectivity and disjoint path optimization. Connectivity algorithms have been specifically developed to find the best pair of routes in a transmission network similar to the one described, not considering the additive approximation of unavailability expressed in (3). Disjoint Path optimization based on Bhandari algorithm is a general purpose algorithm to be applied in graphs with an additive cost where its performance is unsurpassable in practice. In networks with low failure probability such as the one described (about 10^{-6}), where variability among links are almost negligible, the performance of the two algorithms is similar, and identical results are obtained in 97% of the cases. This fact indicates that differences are mainly due to the use of the additive approximation in (3), which is not so trustworthy with high variation in failure rates.

In summary, the proposed algorithms can be considered as useful tools to find the two best routes in a transmission network, as proposed when the availability of some links is expected to be low. One particular application could be as a cost-benefit analysis investment tool, which could quantify the increased availability for an investment in an additional link is going to produce in the network. Moreover, if additional information about the network behavior can be available, for example based on previous maintenance experience, then this information may be used to update and improve the model. Future research will be devoted to further exploit such flexibility of the Bayesian solution, with incorporation of additional information about other elements in the network apart from nodes and links.

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Juan Ramón Feijoo received the Telecommunication Engineering Degree from Universidad de Vigo, Vigo, Spain, in 1991. He has worked for France Telecom in telecommunication transmission network planning. Since 2003, he works in the Telecommunication Department in Red Eléctrica de España as the Head of the Network Design and Standarization Group. Since 2003, he has been also an Adjunct Professor in the Department of Signal Theory and Communications, Universidad Carlos III de Madrid, Spain. He has participated in several Research and Development Projects involving reliability and resiliency of large critical infrastructures. His main research interests include network reliability improvement, estimation and prediction methods based on statistical learning theory, neural networks, and Bayesian methods.

Jesús Cid-Sueiro (M'95–SM'08) received a degree in Telecommunications Engineering from Universidad de Vigo, Spain, in 1990, and the Ph.D. degree from Universidad Politécnica de Madrid, Spain, in 1994. He is a Professor in the Department of Signal Theory and Communications, Universidad Carlos III de Madrid, Spain. His main research interests include computational intelligence, neural networks, Bayesian methods and their applications to sensor networks, communications, and multimedia processing.

Juan Antonio García-López received the Telecomm Engineer degree from the Universidad de Barcelona, Barcelona, Spain, in 1977; and the PhD degree from Universidad Politécnica de Madrid, Spain, in 1987. Since 1985, he leads the Telecommunication Department in Red Eléctrica de España, and conducts investments in transmission, switching, fiber optics, and tcp/ip networks. His Department has intensively participated in different types of network deployment focusing in innovation and technological development. Special work lines have been targeted on telecommunication reliability improvement associated to critical services in the Spanish High Voltage electrical grid.

Patricia Conde received the Telecomm Engineer degree from Universidad de Vigo, Vigo, Spain, in 1999. She has worked in Comunitel Global on telecommunication network deployment, and she has been an Associate Professor in the Department of Signal Theory and Communications, Universidad de Vigo, Spain. Her main research interests include multimedia signal processing, and statistical learning methods.

José Luis Rojo-Álvarez (M'01) received the Telecommunication Engineering Degree from University of Vigo, Spain, in 1996 and the PhD in Telecommunication from Universidad Politécnica de Madrid, Spain, in 2000. He is an Associate Professor with the Department of Signal Theory and Communications, Universidad Rey Juan Carlos, Madrid, Spain. His main research interests include statistical learning theory, digital signal processing, and complex system modeling, with applications to digital communications, and to cardiac signal and image processing. He has published more than 50 papers, and more than 100 international conference communications on support vector machines and neural networks, robust analysis of time series and images, and cardiac signal and image analysis.