Important Formulas

- Newtons Second Law $F = ma$
- Formula of Weight $w = mg$
- Relation Between Force and Momentum $F = \frac{\Delta P}{T}$
- Centripetal Force $F_c = \frac{mv^2}{r}$
- Frictional Force $F_s = \mu mg$
- Relation Between Force and Momentum

$$F = \frac{\triangle p}{\triangle t}$$

- Impulse

$$Impulse = F \times \triangle t$$
$$Impulse = \frac{\triangle p}{\triangle t} \times \triangle t$$
$$Impulse = \triangle p$$
$$Impulse = Change\ in\ momentum$$

3.1. A $10\ kg$ block is placed on a smooth horizontal surface. A horizontal force of $5\ N$ is applied to the block. Find:

(a) the acceleration produced in the block.

(b) the velocity of block after $5\ seconds$.

Given Data

$$Mass\ of\ block = m = 10\ kg$$
$$Force = F = 5\ N$$
$$Initial\ velocity = v_i = 0\ ms^{-1}$$

To Find

$$Acceleration = a = ?$$
$$Final\ velocity = v_f = ?$$

Solution

According to second law of motion

$$F = ma$$
$$5 = (10)\ (a)$$
$$\frac{5}{10} = a$$
$$0.5 = a$$
$$\boldsymbol{a = 0.5\ ms^{-2}}$$

Now by using first equation of motion

$$v_f = v_i + at$$
$$v_f = 0 + (0.5)(5)$$
$$v_f = 0 + 2.5$$
$$\boldsymbol{v_f = 2.5\ ms^{-1}}$$

3.2. The mass of a person is $80\ kg$. What will be his weight on the Earth? What will be his weight on the Moon? The value of acceleration due to gravity of Moon is $1.6\ ms^{-2}$.

Given Data

$$Mass\ of\ body = m = 80\ kg$$
$$Value\ of\ g\ on\ the\ surface\ of\ Earth = g_E = 10\ ms^{-2}$$
$$Value\ of\ g\ on\ the\ surface\ of\ Moon = g_M = 1.6\ ms^{-2}$$

To Find

$$Weight\ on\ the\ surface\ of\ Earth = w_E = ?$$
$$Weight\ on\ the\ surface\ of\ Moon = w_M = ?$$

Solution

By using formula of weight $w = mg$

$$w_E = mg_E$$
$$w_E = (80)(10)$$
$$\boldsymbol{w_E = 800\ N}$$

Now again by using formula of weight $w = mg$

$$w_M = mg_M$$
$$w_M = (80)(1.6)$$
$$\boldsymbol{w_M = 128\ N}$$

3.3. What force is required to increase the velocity of $800\ kg$ car from $10\ ms^{-1}$ to $30\ ms^{-1}$ in $10\ seconds$?

Given Data

$$Mass\ of\ car = m = 800\ kg$$
$$Initial\ velocity = v_i = 10\ ms^{-1}$$
$$Final\ velocity = v_f = 30\ ms^{-1}$$
$$Time = t = 10\ s$$

To Find

$$Force = F = ?$$

Solution

According to second law of motion

$$F = ma$$
$$F = (m)\left(\frac{v_f - v_i}{t}\right)$$
$$F = (800)\left(\frac{30 - 10}{10}\right)$$
$$F = (800)\left(\frac{20}{10}\right)$$
$$F = (800)(2)$$
$$\boldsymbol{F = 1600\ N}$$

3.4. A $5\ g$ bullet is fired by a gun. The bullet moves with a velocity of $300\ ms^{-1}$. If the mass of the gun is $10\ kg$, find the recoil speed of the gun.

Given Data

$$Mass\ of\ bullet = m = 5\ g$$

$$m = \frac{5}{1000}\ kg$$
$$m = 0.005\ kg$$
$$Velocity\ of\ bullet = v = 300\ ms^{-1}$$
$$Mass\ of\ gun = M = 10\ kg$$

To Find

$$Recoil\ speed\ of\ the\ gun = V = ?$$

Solution

According to law of conservation of momentum

$$\begin{matrix} Total\ momentum \\ before\ firing \end{matrix} = \begin{matrix} Total\ momentum \\ after\ firing \end{matrix}$$
$$0 = MV + mv$$
$$0 = (10)V + (0.005)(300)$$
$$0 = 10V + 1.5$$
$$-1.5 = 10V$$
$$\frac{-1.5}{10} = V$$
$$-0.15 = V$$
$$\boldsymbol{V = -0.15\ ms^{-1}}$$

Negative sign indicates the gun recoils. $i.e$ move in backward direction opposite to the motion of bullet.

3.5. An astronaut weighs 70 kg. He throws a wrench of mass 300 g at a speed of $3.5\ ms^{-1}$. Determine:

(a) the speed of astronaut as he recoils away from the wrench.

(b) the distance covered by the astronaut in 30 minutes.

Given Data

$$Mass\ of\ astronaut = M = 70\ kg$$
$$Mass\ of\ wrench = m = 300\ g$$
$$m = \frac{300}{1000}\ kg$$
$$m = 0.3\ kg$$
$$Speed\ of\ wrench = v = 3.5\ ms^{-1}$$
$$Time = t = 30\ min.$$
$$t = 30 \times 60\ s$$
$$t = 1800\ s$$

To Find

$$Recoil\ speed\ of\ the\ astronaut = V = ?$$
$$Distance\ covered\ in\ 30\ minutes = S = ?$$

Solution

According to law of conservation of momentum

$$\begin{matrix} Total\ momentum \\ before\ throwing \end{matrix} = \begin{matrix} Total\ momentum \\ after\ throwing \end{matrix}$$
$$0 = MV + mv$$
$$0 = (70)V + (0.3)(3.5)$$
$$0 = 10V + 1.05$$
$$-1.05 = 70V$$
$$\frac{-1.05}{70} = V$$
$$-0.015 = V$$
$$V = -0.015\ ms^{-1}$$
$$\boldsymbol{V = -1.5 \times 10^{-2} ms^{-1}}$$

Negative sign indicates the astronaut recoils (moves in the opposite direction of the wrench).

Now by using formula of distance

$$S = Vt$$
$$S = (0.015)(1800)$$
$$\boldsymbol{S = 27\ m}$$

We don't take negative speed for distance because: Distance is always positive. The negative sign shows direction, not how far something moves.

3.6. A $6.5 \times 10^3\ kg$ bogie of a goods train is moving with a velocity of $0.8\ ms^{-1}$. Another bogie of mass $9.2 \times 10^3\ kg$ coming from behind with a velocity of $1.2\ ms^{-1}$ collides with the first one and couples to it. Find the common velocity of the two bogies after they become coupled.

Given Data

$$Mass\ of\ first\ bogie = m_1 = 6.5 \times 10^3\ kg$$
$$Velocity\ of\ first\ bogie = v_1 = 0.8\ ms^{-1}$$
$$Mass\ of\ second\ bogie = m_2 = 9.2 \times 10^3\ kg$$
$$Velocity\ of\ second\ bogie = v_2 = 1.2\ ms^{-1}$$

To Find

$$Common\ velocity\ after\ coupling = V = ?$$

Solution

According to law of conservation of momentum

$$\begin{matrix} Total\ momentum \\ before\ couple \end{matrix} = \begin{matrix} Total\ momentum \\ after\ couple \end{matrix}$$
$$m_1v_1 + m_2v_2 = m_1V + m_2V$$
$$m_1v_1 + m_2v_2 = V(m_1 + m_2)$$
$$(6.5 \times 10^3)(0.8) + (9.2 \times 10^3)(1.2) = V[(6.5 \times 10^3) + (9.2 \times 10^3)]$$
$$5200 + 11040 = V(15700)$$
$$\frac{16240}{15700} = V$$
$$1.03 = V$$
$$\boldsymbol{V = 1.03\ ms^{-1}}$$

3.7. A cyclist weighing 55 kg rides a bicycle of mass 5 kg. He starts from rest and applies a force of 90 N for 8 $seconds$. Then he continues at a constant speed for another 8 $seconds$. Calculate the total distance travelled by the cyclist.

Given Data

$$Mass\ of\ cyclist = m_1 = 55\ kg$$
$$Mass\ of\ bicycle = m_2 = 5\ kg$$
$$Total\ mass = m = 55\ kg + 5\ kg$$
$$m = 60\ kg$$

$$Force\ applied = F = 90\ N$$
$$Time\ of\ acceleration = t_1 = 8\ s$$
$$Time\ of\ constant\ speed = t_2 = 8\ s$$
$$Initial\ speed = v_i = 0\ ms^{-1}$$

To Find

$$Total\ distance\ travelled = S = ?$$

Solution

According to second law of motion

$$F = ma$$
$$90 = (60)\,(a)$$
$$\frac{90}{60} = a$$
$$1.5 = a$$
$$a = 1.5\ ms^{-2}$$

Now by using first equation of motion

$$v_f = v_i + at_1$$
$$v_f = 0 + (1.5)(8)$$
$$v_f = 0 + 12$$
$$v_f = 12\ ms^{-1}$$

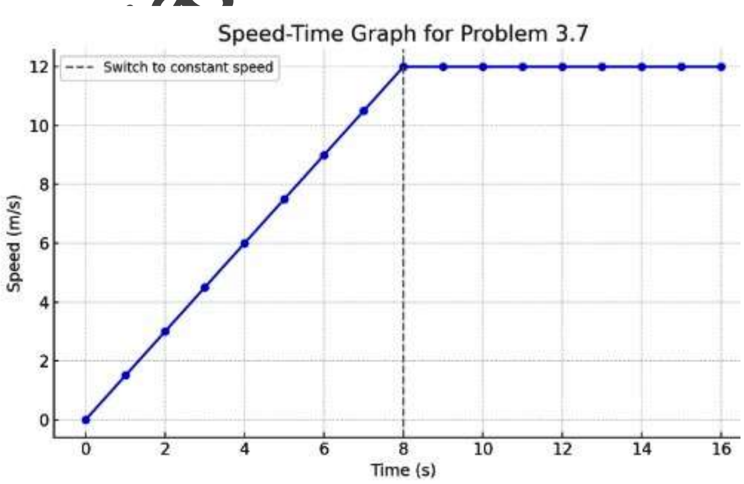
Distance covered **during acceleration** by using second equation of motion

$$S_1 = v_i t_1 + \frac{1}{2}a{t_1}^2$$
$$S_1 = (0)(8) + \frac{1}{2}(1.5)(8)^2$$
$$S_1 = 0 + \frac{1}{2}(1.5)(64)$$
$$S_1 = 0 + 48$$
$$S_1 = 48\ m$$

Distance covered at **constant speed** by formula $S = vt$

$$S_2 = v_f t_2$$
$$S_2 = (12)(8)$$
$$S_2 = 96\ m$$

$$Total\ distance\ travelled = S = S_1 + S_2$$
$$S = 48\ m + 96\ m$$
$$\boldsymbol{S = 144\ m}$$



3.8. A ball of mass $0.4\ kg$ is dropped on the floor from a height of $1.8\ m$. The ball rebounds straight upward to a height of $0.8\ m$. What is the magnitude and direction of the impulse applied to the ball by the floor?

Given Data

$$Mass\ of\ ball = m = 0.4\ kg$$
$$Drop\ height = h_1 = 1.8\ m$$
$$Rebound\ height = h_2 = 0.8\ m$$
$$Acceleration\ due\ to\ gravity = g = 10\ ms^{-2}$$

To Find

$$Impulse\ (magnitude\ and\ direction) = ?$$

Solution

Since the ball is dropped so, $v_i = 0\ ms^{-1}$

$$2gh_1 = v_f^2 - v_i^2$$
$$2(10)(1.8) = v_f^2 - (0)^2$$
$$36 = v_f^2$$
$$v_f^2 = 36$$
$$\sqrt{v_f^2} = \sqrt{36}$$
$$v_f = \pm 6\ ms^{-1}$$

But since the ball is moving downward, we select the negative root. $i.e$

$$v_f = v_{before} = -6\ ms^{-1}$$

At maximum rebound height, $v_f = 0\ ms^{-1}$

$$2gh_2 = v_f^2 - v_i^2$$
$$2(-10)(0.8) = (0)^2 - v_i^2\ \ (\because ball\ moving\ upward)$$
$$-16 = -v_i^2$$
$$v_i^2 = 16$$
$$\sqrt{v_i^2} = \sqrt{16}$$
$$v_i = \pm 4\ ms^{-1}$$

But since the ball is moving upward, we select the positive root. $i.e$

$$v_i = v_{after} = 4\ ms^{-1}$$

Now by using formula of impulse

$$Impulse = Change\ in\ momentum$$
$$= \triangle p$$
$$= p_f - p_i$$
$$= mv_f - mv_i$$
$$= m(v_f - v_i)$$
$$= m(v_{after} - v_{befor})$$
$$= (0.4)[4 - (-6)]$$
$$= (0.4)[10]$$
$$\boldsymbol{Impulse = 4\ Ns}$$

The **positive result** means the impulse is **upward** (floor pushes the ball up).

Note: In physics, we always choose a reference direction to be positive.

For vertical motion:

- We choose upward as positive (by convention).
- So, any velocity in the upward direction is positive.
- And any velocity in the downward direction is negative.

3.9. Two balls of masses $0.2\ kg$ and $0.4\ kg$ are moving towards each other with velocities $20\ ms^{-1}$ and $5\ ms^{-1}$ respectively. After collision, the velocity of $0.2\ kg$ ball becomes $6\ ms^{-1}$. What will be the velocity of $0.4\ kg$ ball?

Given Data

$$\begin{aligned} Mass\ of\ ball\ A &= m_1 = 0.2\ kg \\ Mass\ of\ ball\ B &= m_2 = 0.4\ kg \\ Initial\ velocity\ \ of\ ball\ A &= v_1 = 20\ ms^{-1} \\ Initial\ velocity\ \ of\ ball\ B &= v_2 = -5\ ms^{-1}(opposite\ direction) \\ Final\ velocity\ of\ ball\ A &= v_1' = 6\ ms^{-1} \end{aligned}$$

To Find

$$Final\ velocity\ of\ ball\ B = v_2' = ?$$

Solution

According to law of conservation of momentum

$$\begin{gathered} \begin{matrix} total\ momentum\ of\ the \\ system\ before\ collision \end{matrix} = \begin{matrix} total\ momentum\ of\ the \\ system\ after\ collision \end{matrix} \\ m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \\ (0.2)(20) + (0.4)(-5) = (0.2)(6) + (0.4)(v_2') \\ 4 - 2 = 1.2 + (0.4)(v_2') \\ 2 - 1.2 = 0.4v_2' \\ 0.8 = 0.4v_2' \\ \frac{0.8}{0.4} = v_2' \\ 2 = v_2' \\ \boldsymbol{v_2' = 2\ ms^{-1}} \end{gathered}$$