Important Formulas

- **Work Done** $W = FS \quad \Longrightarrow \quad W = FS\cos\theta$
- **Kinetic Energy** $E_k = \frac{1}{2}mv^2$
- **Potential Energy** $E_p = mgh$
- **Mass Energy Equation** $E = mc^2$
- **Power** $power = \frac{work}{time} \Rightarrow P = \frac{W}{t}$
- **Efficiency** $Efficiency = \frac{output}{input}$
- $\%\, Efficiency = \frac{output}{input} \times 100$
- **Weight** $w = mg$

5.1. A force of $20\ N$ acting at an angle of 60° to the horizontal is used to pull a box through a distance of $3\ m$ across a floor. How much work is done?

Given Data

$$Force = F = 20\ N$$
$$Angle = \theta = 60^\circ$$
$$Distance\ covered = S = 3\ m$$

To Find

$$Work\ done = W = ?$$

Solution

By using formula of work done

$$W = FS\cos\theta$$
$$W = (20)(3)\cos 60^\circ$$
$$W = (20)(3)(0.5)$$
$$\boldsymbol{W = 30\ J}$$

5.2. A body moves a distance of 5 $metres$ in a straight line under the action of a force of 8 $newtons$. If the work done is 20 $Joules$, find the angle which the force makes with the direction of motion of the body.

Given Data

$$Distance\ covered = S = 5\ m$$
$$Force = F = 8\ N$$
$$Work\ done = W = 20\ J$$

To Find

$$Angle = \theta = ?$$

Solution

By using formula of work done

$$W = FS\cos\theta$$
$$\cos\theta = \frac{W}{FS}$$
$$\cos\theta = \frac{20}{(8)(5)}$$
$$\cos\theta = \frac{20}{40}$$
$$\cos\theta = 0.5$$
$$\theta = \cos^{-1}(0.5)$$
$$\boldsymbol{\theta = 60^\circ}$$

5.3. An engine raises 100 kg of water through a height of 80 m in 25 s. What is the power of the engine?

Given Data

$$Mass\ of\ water = m = 100\ kg$$
$$Height\ raised = h = 80\ m$$
$$Time\ taken = t = 25\ s$$

To Find

$$Power = P = ?$$

Solution

By using formula of power

$$Power = \frac{Work}{time}$$
$$P = \frac{W}{t}$$
$$P = \frac{FS}{t}$$
$$P = \frac{mgh}{t} \qquad \because F = W = mg\ \&\ S = h$$
$$P = \frac{(100)(10)(80)}{25}$$
$$\boldsymbol{P = 3200\ W}$$

5.4. A body of mass 20 kg is at rest. A 40 N force acts on it for 5 $seconds$. What is the kinetic energy of the body at the end of this time?

Given Data

$$Mass\ of\ body = m = 20\ kg$$
$$Initial\ velocity = v_i = 0\ ms^{-1}$$
$$Force = F = 40\ N$$
$$Time\ = t = 5\ s$$

To Find

$$Kinetic\ energy = E_k = ?$$

Solution

By using second law of motion

$$F = ma$$
$$40 = (20)(a)$$
$$\frac{40}{20} = a$$
$$2 = a$$
$$a = 2\ ms^{-2}$$

For final velocity using first equation of motion

$$v_f = v_i + gt$$
$$v_f = 0 + (2)(5)$$
$$v_f = 10\ ms^{-1}$$
$$v = 10\ ms^{-1}$$

Now, by using formula of kinetic energy

$$E_k = \frac{1}{2}mv^2$$
$$E_k = \frac{1}{2}(20)(10)^2$$
$$E_k = \frac{1}{2}(20)(100)$$
$$\boldsymbol{E_k = 1000\ J}$$

5.5. A ball of mass 160 g is thrown vertically upward. The ball reaches a height of 20 m. Find the potential energy gained by the ball at this height.

Given Data

$$Mass\ of\ ball = m = 160\ g$$
$$m = \frac{160}{1000}\ kg$$
$$m = 0.16\ kg$$
$$Height\ reached = h = 20\ m$$
$$Gravitational\ acceleration\ = g = 10\ ms^{-2}$$

To Find

$$Potential\ energy = E_p = ?$$

Solution

By using formula of potential energy

$$E_p = mgh$$
$$E_p = (0.16)(10)(20)$$
$$\boldsymbol{E_p = 32\ J}$$

5.6. A $0.14\ kg$ ball is thrown vertically upward with an initial velocity of $35\ ms^{-1}$. Find the maximum height reached by the ball.

Given Data

$$Mass\ of\ ball = m = 0.14\ kg$$
$$Initial\ velocity = v = 35\ ms^{-1}$$
$$Gravitational\ acceleration\ = g = 10\ ms^{-2}$$

To Find

$$Maximum\ height\ reached = h = ?$$

Solution

At the *maximum height*, all the *kinetic energy* of the ball is converted into *potential energy*.

$$E_p = E_k$$
$$mgh = \frac{1}{2}mv^2$$
$$h = \frac{mv^2}{2mg}$$
$$h = \frac{v^2}{2g}$$
$$h = \frac{(35)^2}{2(10)}$$
$$h = \frac{1225}{20}$$
$$\boldsymbol{h = 61.25\ m}$$

5.7. A girl is swinging on a swing. At the lowest point of her swing, she is $1.2\ m$ from the ground, and at the highest point she is $2.0\ m$ from the ground. What is her maximum velocity and where?

Given Data

$$Height\ at\ lowest\ point = h_1 = 1.2\ m$$
$$Height\ at\ highest\ point = h_2 = 2.0\ m$$
$$Change\ in\ height = h = h_2 - h_1$$
$$h = 2.0 - 1.2$$
$$h = 0.8\ m$$
$$Gravitational\ acceleration\ = g = 10\ ms^{-2}$$

To Find

$$Maximum\ velocity\ = v = ?$$
$$Location\ of\ maximum\ velocity = ?$$

Solution

At the lowest point, all the *potential energy* is converted into *kinetic energy*. So,

$$E_p = E_k$$
$$mgh = \frac{1}{2}mv^2$$
$$\frac{2mgh}{m} = v^2$$
$$2gh = v^2$$
$$v^2 = 2gh$$
$$\sqrt{v^2} = \sqrt{2gh}$$
$$v = \sqrt{(2)(10)(0.8)}$$
$$v = \sqrt{16}$$
$$\boldsymbol{v = 4\ ms^{-1}}$$

The *maximum velocity* is $4\ ms^{-1}$, and it occurs at the *lowest point* of the swing.

5.8. A person pushes a lawn mower with a force of $50\ N$ making an angle of 45° with the horizontal. If the mower is moved through a distance of $20\ m$, how much work is done?

Given Data

$$Force = F = 50\ N$$
$$Angle = \theta = 45^\circ$$
$$Distance = S = 20\ m$$

To Find

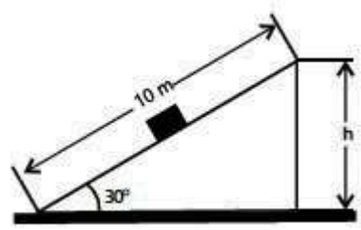
$$Work\ done = W = ?$$

Solution

By using formula of work done

$$W = FS\cos\theta$$
$$W = (50)(20)\cos 45^\circ$$
$$W = (50)(20)(0.707)$$
$$\boldsymbol{W = 707\ J}$$

5.9. Calculate the work done in



(i) Pushing a $5\ kg$ box up a frictionless inclined plane $10\ m$ long that makes an angle of 30° with the horizontal.

(ii) Lifting the box vertically up from the ground to the top of the inclined plane.

Given Data

$$Mass\ of\ box = m = 5\ kg$$
$$Length\ of\ inclined\ plane = S = 10\ m$$
$$Angle\ with\ horizontal = \theta = 30^\circ$$
$$Gravitational\ acceleration\ = g = 10\ ms^{-2}$$

To Find

$$Work\ done\ along\ the\ incline\ = W = ?$$
$$Work\ done\ in\ lifting\ the\ box\ vertically = W = ?$$

Solution (i)

The force needed to push the box up the slope is the component of weight along the incline:

$$F = mg\sin\theta$$

Using the formula for work

$$W = FS$$
$$W = (mg\sin\theta)(S)$$
$$W = (5)(10)(\sin 30^\circ)(10)$$
$$W = (5)(10)(0.5)(10)$$
$$\boldsymbol{W = 250\ J}$$

(ii) Given figure forms a right-angle triangle so for height use trigonometric ratio

$$\sin\theta = \frac{Perpendicular}{Hypotenuse}$$
$$\sin 30^\circ = \frac{h}{10}$$
$$(10)(\sin 30^\circ) = h$$
$$(10)(0.5) = h$$
$$5 = h$$
$$h = 5\ m$$

The work done against gravity is equal to the change in $potential\ energy$:

$$W = E_p$$
$$W = mgh$$
$$W = (5)(10)(5)$$
$$\boldsymbol{W = 250\ J}$$

5.10. A box of mass $10\ kg$ is pushed up along a ramp $15\ m$ long with a force of $80\ N$. If the box rises up a height of $5\ m$, what is the efficiency of the system?

Given Data

$$Mass\ of\ box = m = 10\ kg$$
$$Length\ of\ ramp = S = 15\ m$$
$$Force\ applied\ = F = 80\ N$$
$$Height\ raised = h = 5\ m$$
$$Gravitational\ acceleration\ = g = 10\ ms^{-2}$$

To Find

$$Efficiency\ of\ the\ system\ = ?$$

Solution

The total energy supplied is the work done $(input)$. By using formula of work done

$$W_{input} = FS$$
$$W_{input} = (80)(15)$$
$$W_{input} = 1200\ J$$

The useful energy is the work done $(output)$ to lift the box to height h.

$$W_{output} = mgh$$
$$W_{output} = (10)(10)(5)$$
$$W_{output} = 500\ J$$

By using formula of efficiency

$$\%\ Efficiency = \frac{W_{output}}{W_{input}} \times 100$$
$$\%\ Efficiency = \frac{500}{1200} \times 100$$
$$\%\ Efficiency = \frac{50000}{1200}$$
$$\%\ Efficiency = 41.7\ \%$$

5.11. A force of $600\ N$ acts on a box to push it $5\ m$ in $15\ s$. Calculate the power.

Given Data

$$Force = F = 600\ N$$
$$Distance = S = 5\ m$$
$$Time = t = 15\ s$$

To Find

$$Power = P = ?$$

Solution

By using formula of power

$$Power = \frac{Work}{time}$$
$$P = \frac{W}{t}$$
$$P = \frac{FS}{t}$$
$$P = \frac{(600)(5)}{15}$$
$$\boldsymbol{P = 200\ watt}$$

5.12. A 40 kg boy runs up-stair 10 m high in 8 s. What power he developed.

Given Data

$$Mass\ of\ boy = m = 40\ kg$$
$$Height = h = 10\ m$$
$$Time = t = 8\ s$$

To Find

$$Power = P = ?$$

Solution

By using formula of power

$$Power = \frac{Work}{time}$$
$$P = \frac{W}{t}$$
$$P = \frac{FS}{t}$$
$$P = \frac{mgh}{t} \qquad \because F = W = mg\ \&\ S = h$$
$$P = \frac{(40)(10)(10)}{5}$$
$$\boldsymbol{P = 500\ W}$$

5.13. A force F acts through a distance L on a body. The force is then increased to $2F$ that further acts through $2L$. Sketch a force-displacement graph and calculate the total work done.

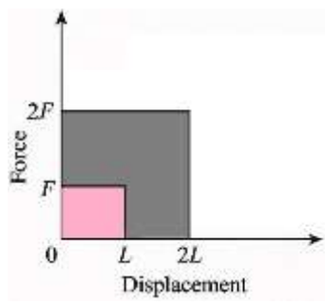
Given Data

$$Force\ for\ distance\ L = F$$
$$Force\ for\ distance\ 2L = 2F$$

To Find

$$Work\ done\ = W\ = ?$$

Solution



As, the area under a force-distance graph represents the work done by the force

$$Work\ done = Area\ under\ the\ graph$$
$$= (F \times L) + (2F \times 2L)$$
$$= FL + 4FL$$
$$= \boldsymbol{5FL}$$
$$\boldsymbol{Work\ done = 5\ unit}$$