Exercise 1.3

1. The sum of three consecutive integers is forty-two, find the three integers.

$$Let\ first\ integer = x$$
$$Second\ integer = x + 1$$
$$Third\ integer = x + 2$$

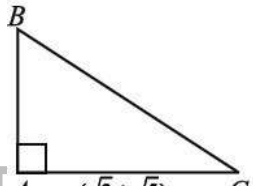
According to question

$$x + x + 1 + x + 2 = 42$$
$$3x + 3 = 42$$
$$3x = 42 - 3$$
$$3x = 39$$
$$x = \frac{39}{3}$$
$$x = 13$$

Hence

$$First\ integer = x = 13$$
$$Second\ integer = x + 1 = 13 + 1 = 14$$
$$Third\ integer = x + 2 = 13 + 2 = 15$$

2. The diagram shows right angled ΔABC in which the length of $\overline{AC}$ is $(\sqrt{3} + \sqrt{5})\ cm$. The area of ΔABC is $(1 + \sqrt{15})cm^2$. Find the length of $\overline{AB}$ in the form of $(a\sqrt{3} + b\sqrt{5})\ cm$, where a and b are integers.



$$\overline{AC} = (\sqrt{3} + \sqrt{5})\ cm$$
$$Area\ of\ \Delta ABC = (1 + \sqrt{15})\ cm^2$$
$$\overline{AB} = ?$$

$$Area\ of\ \Delta ABC = \frac{1}{2} \times base \times height$$
$$(1 + \sqrt{15}) = \frac{1}{2} \times (\sqrt{3} + \sqrt{5}) \times \overline{AB}$$
$$\frac{2(1 + \sqrt{15})}{\sqrt{3} + \sqrt{5}} = \overline{AB}$$
$$\overline{AB} = \frac{2(1 + \sqrt{15})}{\sqrt{3} + \sqrt{5}}$$
$$\overline{AB} = \frac{2 + 2\sqrt{15}}{\sqrt{3} + \sqrt{5}} \times \frac{\sqrt{3} - \sqrt{5}}{\sqrt{3} - \sqrt{5}}$$
$$\overline{AB} = \frac{2(\sqrt{3} - \sqrt{5}) + 2\sqrt{15}(\sqrt{3} - \sqrt{5})}{(\sqrt{3})^2 - (\sqrt{5})^2}$$
$$\overline{AB} = \frac{2\sqrt{3} - 2\sqrt{5} + 2\sqrt{45} - 2\sqrt{75}}{3 - 5}$$
$$\overline{AB} = \frac{2\sqrt{3} - 2\sqrt{5} + 2\sqrt{9 \cdot 5} - 2\sqrt{25 \cdot 3}}{-2}$$
$$\overline{AB} = \frac{2\sqrt{3} - 2\sqrt{5} + 2(3)\sqrt{5} - 2(5)\sqrt{3}}{-2}$$
$$\overline{AB} = \frac{2\sqrt{3} - 2\sqrt{5} + 6\sqrt{5} - 10\sqrt{3}}{-2}$$
$$\overline{AB} = \frac{-8\sqrt{3} + 4\sqrt{5}}{-2}$$
$$\overline{AB} = \frac{-8\sqrt{3}}{-2} + \frac{4\sqrt{5}}{-2}$$
$$\overline{AB} = 4\sqrt{3} - 2\sqrt{5}$$

3. A rectangle has sides of length $2 + \sqrt{18}\ m$ and $\left(5 - \frac{4}{\sqrt{2}}\right) m$. Express the area of rectangle in the form $a + b\sqrt{2}$, where a and b are integers.

$$Length\ of\ rectangle = l = 2 + \sqrt{18}\ m$$
$$l = 2 + \sqrt{9 \cdot 2}\ m$$
$$l = 2 + 3\sqrt{2}\ m$$
$$Breadth = b = 5 - \frac{4}{\sqrt{2}}\ m$$
$$b = 5 - \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}\ m$$
$$b = 5 - \frac{4\sqrt{2}}{(\sqrt{2})^2}\ m$$
$$b = 5 - \frac{4\sqrt{2}}{2}\ m$$
$$b = 5 - 2\sqrt{2}\ m$$
$$Area\ of\ \square = l \times b$$
$$= (2 + 3\sqrt{2}) \times (5 - 2\sqrt{2}\ )$$
$$= 2(5 - 2\sqrt{2}\ ) + 3\sqrt{2}(5 - 2\sqrt{2}\ )$$
$$= 10 - 4\sqrt{2} + 15\sqrt{2} - 6(\sqrt{2})^2$$
$$= 10 + 11\sqrt{2} - 6(2)$$
$$= 10 + 11\sqrt{2} - 12$$
$$= -2 + 11\sqrt{2}$$
$$= (11\sqrt{2} - 2)\ m^2$$

4. Find two numbers whose sum is 68 and difference is 22.

$$Let\ first\ numbers = x$$
$$Second\ number = y$$

According to question

$$x + y = 68 \quad \cdots (i)$$
$$x - y = 22$$
$$x = 22 + y \quad \cdots (ii)$$

Putting value of x in equation (i)

$$22 + y + y = 68$$
$$2y = 68 - 22$$
$$2y = 46$$

$$y = \frac{46}{2}$$
$$\boldsymbol{y = 23}$$

Putting value of y in equation (ii)

$$x = 22 + 23$$
$$\boldsymbol{x = 45}$$

5. The weather in Lahore was usually warm during the summer of 2024. The TV news reported temperature as high as 48℃. By using the formula, $\left(°\text{F} = \frac{9}{5}℃ + 32\right)$ find the temperature as Fahrenheit scale.

$$°\text{F} = \frac{9}{5}℃ + 32$$
$$= \frac{9}{5} \times 48 + 32$$
$$= 86.4 + 32$$
$$°\text{F} = \boldsymbol{118.4}°\text{F}$$

6. The sum of the ages of the father and son is 72 years. Six years ago, the father's age was 2 times the age of the son. What was son's age six years ago?

Let present

$$Age\ of\ son\ = x$$
$$Age\ of\ father\ = y$$

Six years ago

$$Age\ of\ son\ = x - 6$$
$$Age\ of\ father\ = y - 6$$

According to question,

$$x + y = 72 \quad \cdots (i)$$

And

$$y - 6 = 2(x - 6)$$
$$y = 2x - 12 + 6$$
$$y = 2x - 6$$

Putting value of y in equation (i)

$$x + (2x - 6) = 72$$
$$x + 2x - 6 = 72$$
$$3x = 72 + 6$$
$$3x = 78$$
$$x = \frac{78}{3}$$
$$x = 26$$
$$Six\ years\ ago\ age\ of\ son = x - 6$$
$$= 26 - 6$$
$$= \boldsymbol{20\ years}$$

7. Mirha bought a toy for $Rs.1500$ and sold for $Rs.1520$. What was her profit percentage?

$$CP = 1500\ Rs$$
$$SP = 1520\ Rs$$
$$Profit = SP - CP$$
$$Profit = 1520 - 1500$$
$$Profit = 20\ Rs$$
$$\%\ Profit = \frac{Profit}{CP} \times 100$$
$$= \frac{20}{1500} \times 100$$
$$= \boldsymbol{1.33}\ \%$$

8. The annual income of Tayyab is $Rs.9,60,000$ while the exempted amount is $Rs.1,30,000$. How much tax would he have to pay at the rate of 0.75%?

$$Annual\ Income = 960000\ Rs$$
$$Exempted\ Amount = 130000\ Rs$$
$$Taxable\ Income = 960000 - 130000$$
$$= 830000\ Rs$$
$$Tax\ rate\ = 0.75\ \%$$

$$Tax\ amount = 0.75\% \times 830000$$
$$= \frac{0.75}{100} \times 830000$$
$$= \boldsymbol{6225\ Rs}$$

9. Find the compound markup on $Rs.3,75,000$ for one year at the rate of 14% compounded annually.

$$Principal\ Amount\ = 3{,}75{,}000\ Rs$$
$$Time = 1\ year$$
$$Rate\ = 14\%$$
$$Compound\ Markup\ = ?$$

$$Compound\ Markup = \frac{Principal\ Amount \times time \times rate}{100}$$
$$= \frac{375000 \times 1 \times 14}{100}$$
$$= \boldsymbol{52500\ Rs}$$