

Exercise 1.1

1. Identify each of the following as a rational or irrational number: (i) 2.353535 (ii) $0.\overline{6}$ (iii) 2.236067... (iv) $\sqrt{7}$ (v) e (vi) π (vii) $5 + \sqrt{11}$ (viii) $\sqrt{3} + \sqrt{13}$ (ix) $\frac{15}{4}$ (x) $(2 - \sqrt{2})(2 + \sqrt{2})$

Sr	Number	Type	Reason
i	2.353535	Rational	Decimal repeats (35 repeats), so it's recurring.
ii	$0.\overline{6}$	Rational	Decimal repeats (6 repeats), so it's recurring.
iii	2.236067...	Irrational	Decimal doesn't repeat or end.
iv	$\sqrt{7}$	Irrational	Can't be written as $\frac{p}{q}$, decimal doesn't repeat or end.
v	e	Irrational	Decimal doesn't end or repeat. Known irrational number.
vi	π	Irrational	Decimal doesn't end or repeat. Known irrational number.
vii	$5 + \sqrt{11}$	Irrational	Rational + irrational = irrational.
viii	$\sqrt{3} + \sqrt{13}$	Irrational	Irrational + irrational = irrational (in most cases).
ix	$\frac{15}{4}$	Rational	In $\frac{p}{q}$ form where p & q are integers.
x	$(2 - \sqrt{2})(2 + \sqrt{2})$	Rational	This equals $4 - 2 = 2$, which is rational (special product formula).

2. Represent the following numbers on number line: (i) $\sqrt{2}$ (ii) $\sqrt{3}$ (iii) $4\frac{1}{3}$ (iv) $-2\frac{1}{7}$ (v) $\frac{5}{8}$ (vi) $2\frac{3}{4}$

(i) Locating $\sqrt{2} \approx 1.414$

- Draw a number line with $A = 0$ and $B = 1$
- At B , draw a perpendicular $BC = 1$ unit.
- Join A to C to form a right-angled $\triangle ABC$.

By Pythagoras theorem:

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 = (1)^2 + (1)^2$$

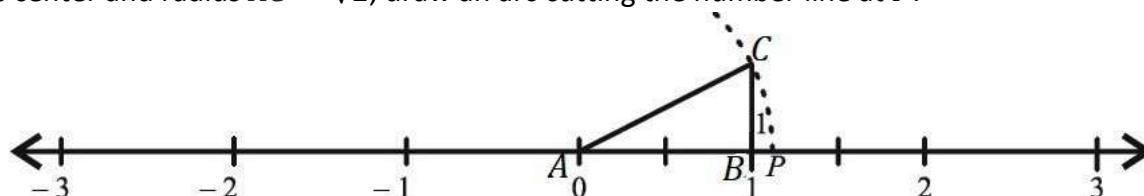
$$(AC)^2 = 1 + 1$$

$$(AC)^2 = 2$$

$$\sqrt{(AC)^2} = \sqrt{2}$$

$$AC = \sqrt{2}$$

With A as center and radius $AC = \sqrt{2}$, draw an arc cutting the number line at P .



(ii) Locating $\sqrt{3} \approx 1.732$

Step 1: Construct $\sqrt{2}$

- Draw a number line with $A = 0$ and $B = 1$
- At B , draw a perpendicular $BC = 1$ unit.
- Join A to C to form a right-angled $\triangle ABC$.

By Pythagoras theorem:

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 = (1)^2 + (1)^2$$

$$(AC)^2 = 1 + 1$$

$$\begin{aligned}(AC)^2 &= 2 \\ \sqrt{(AC)^2} &= \sqrt{2} \\ AC &= \sqrt{2}\end{aligned}$$

With A as center and radius $AC = \sqrt{2}$, draw an arc cutting the number line at D .

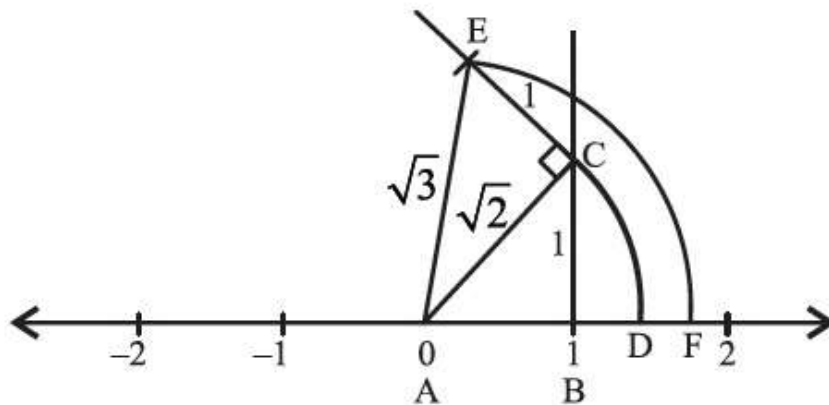
Step 2: Construct $\sqrt{3}$

- At C , draw a perpendicular $CE = 1$ unit (upwards).
- Join A to E to form a right-angled $\triangle ACE$

By Pythagoras theorem:

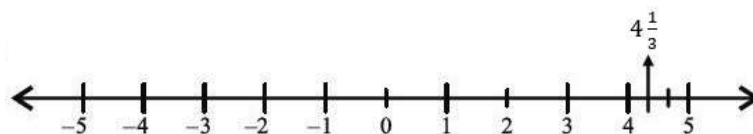
$$\begin{aligned}(\text{Hypotenuse})^2 &= (\text{Base})^2 + (\text{Perpendicular})^2 \\ (AE)^2 &= (AC)^2 + (CE)^2 \\ (AE)^2 &= (\sqrt{2})^2 + (1)^2 \\ (AE)^2 &= 2 + 1 \\ (AE)^2 &= 3 \\ \sqrt{(AE)^2} &= \sqrt{3} \\ AE &= \sqrt{3}\end{aligned}$$

With A as center and radius $AE = \sqrt{3}$, draw an arc cutting the number line at F .



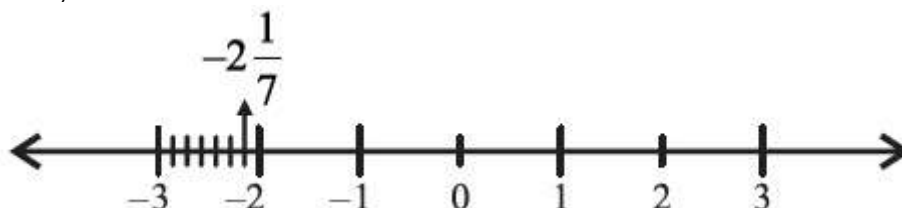
(iii) $4\frac{1}{3} = 4.33$

- Locate 4 and 5 on the number line
- Divide the space between them into 3 equal parts
- Mark the point $\frac{1}{3}$ of the way from 4 to 5



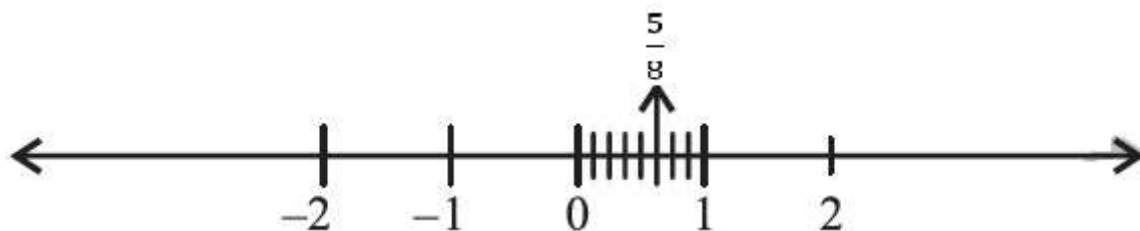
(iv) $-2\frac{1}{7} = -2.143$

- Locate -2 and -3 on the number line
- Divide the space between them into 7 equal parts
- Mark the point $\frac{1}{7}$ of the way from -2 to -3



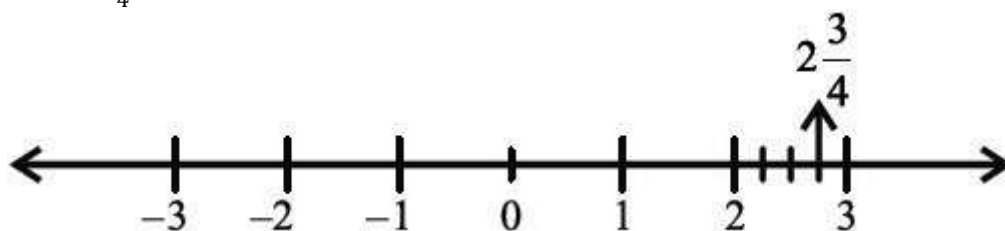
(v) $\frac{5}{8} = 0.625$

- Locate 0 and 1 on the number line
- Divide the space into 8 equal parts
- Mark the 5th division point.



(vi) $2\frac{3}{4} = 2.75$

- Locate 2 and 3 on the number line
- Divide the space into 4 equal parts
- Mark the point $\frac{3}{4}$ of the way from 2 to 3



3. Express the following as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$. (i) $0.\overline{4}$ (ii) $0.\overline{37}$ (iii) $0.\overline{21}$

(i) Let

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$$x = 0.\overline{4}$$

$$x = 0.444 \dots \dots (i)$$

$$10x = 10(0.444 \dots)$$

$$10x = 4.444 \dots \dots (ii)$$

Subtracting (i) from (ii)

$$10x - x = (4.444 \dots) - (0.444 \dots)$$

$$9x = 4$$

$$x = \frac{4}{9}$$

(ii) Let

$$x = 0.\overline{37}$$

$$x = 0.373737 \dots \dots (i)$$

$$100x = 100(0.373737 \dots)$$

$$100x = 37.373737 \dots \dots (ii)$$

Subtracting (i) from (ii)

$$100x - x = (37.373737 \dots) - (0.373737 \dots)$$

$$99x = 37$$

$$x = \frac{37}{99}$$

(iii) Let

$$x = 0.\overline{21}$$

$$x = 0.212121 \dots \dots (i)$$

$$100x = 100(0.212121 \dots)$$

$$100x = 21.212121 \dots \dots (ii)$$

Subtracting (i) from (ii)

$$100x - x = (21.212121 \dots) - (0.212121 \dots)$$

$$99x = 21$$

$$x = \frac{21}{99}$$

4. Name the property used in the following: (i) $(a + 4) + b = a + (4 + b)$ (ii) $\sqrt{2} + \sqrt{3} = \sqrt{3} + \sqrt{2}$ (iii) $x - x = 0$ (iv) $a(b + c) = ab + ac$ (v) $16 + 0 = 16$ (vi) $100 \times 1 = 100$ (vii) $4 \times (5 \times 8) = (4 \times 5) \times 8$ (viii) $ab = ba$

Sr	Expression	Property Name
i	$(a + 4) + b = a + (4 + b)$	Associative property over addition
ii	$\sqrt{2} + \sqrt{3} = \sqrt{3} + \sqrt{2}$	Commutative property over addition
iii	$x - x = 0$	Additive Inverse
iv	$a(b + c) = ab + ac$	Left distributive property of multiplication over addition
v	$16 + 0 = 16$	Additive Identity
vi	$100 \times 1 = 100$	Multiplicative Identity
vii	$4 \times (5 \times 8) = (4 \times 5) \times 8$	Associative Property under Multiplication
viii	$ab = ba$	Commutative Property under Multiplication

5. Name the property used in the following: (i) $-3 < -1 \Rightarrow 0 < 2$ (ii) If $a < b$ then $\frac{1}{a} > \frac{1}{b}$ (iii) If $a < b$ then $a + c < b + c$ (iv) If $ac < bc$ and $c > 0$ then $a < b$ (v) If $ac < bc$ and $c < 0$ then $a > b$ (vi) Either $a > b$ or $a = b$ or $a < b$

Sr	Expression	Property Name
i	$-3 < -1 \Rightarrow 0 < 2$	Additive Property of Inequality
ii	If $a < b$ then $\frac{1}{a} > \frac{1}{b}$	Reciprocal Property
iii	If $a < b$ then $a + c < b + c$	Additive Property (of order)
iv	If $ac < bc$ and $c > 0$ then $a < b$	Multiplicative Property (when $c > 0$)
v	If $ac < bc$ and $c < 0$ then $a > b$	Multiplicative Property (when $c < 0$)
vi	Either $a > b$ or $a = b$ or $a < b$	Trichotomy Property

6. Find two rational numbers between: (i) $\frac{1}{3}$ and $\frac{1}{4}$ (ii) 3 and 4 (iii) $\frac{3}{5}$ and $\frac{4}{5}$

(i) $\frac{1}{3}$ and $\frac{1}{4}$

$$\begin{aligned}
 1^{st} \text{ rational number} &= \left(\frac{1}{3} + \frac{1}{4} \right) \div 2 \\
 &= \left(\frac{4+3}{12} \right) \times \frac{1}{2} \\
 &= \frac{7}{12} \times \frac{1}{2} \\
 &= \frac{7}{24}
 \end{aligned}$$

$$\begin{aligned}
 2^{nd} \text{ rational number} &= \left(\frac{1}{3} + \frac{7}{24} \right) \div 2 \\
 &= \left(\frac{8+7}{24} \right) \times \frac{1}{2} \\
 &= \frac{15}{24} \times \frac{1}{2} \\
 &= \frac{15}{48}
 \end{aligned}$$

3	3,24
2	1,8
2	1,4
2	1,2
	1,1

(ii) 3 and 4

$$\begin{aligned}
 1^{\text{st}} \text{ rational number} &= (3 + 4) \div 2 \\
 &= (7) \times \frac{1}{2} \\
 &= \frac{7}{2}
 \end{aligned}$$

$$\begin{aligned}
 2^{\text{nd}} \text{ rational number} &= \left(3 + \frac{7}{2}\right) \div 2 \\
 &= \left(\frac{6 + 7}{2}\right) \times \frac{1}{2} \\
 &= \frac{13}{2} \times \frac{1}{2} \\
 &= \frac{13}{4}
 \end{aligned}$$

(iii) $\frac{3}{5}$ and $\frac{4}{5}$

$$\begin{aligned}
 1^{\text{st}} \text{ rational number} &= \left(\frac{3}{5} + \frac{4}{5}\right) \div 2 \\
 &= \left(\frac{3 + 4}{5}\right) \times \frac{1}{2} \\
 &= \frac{7}{5} \times \frac{1}{2} \\
 &= \frac{7}{10}
 \end{aligned}$$

$$\begin{aligned}
 2^{\text{nd}} \text{ rational number} &= \left(\frac{3}{5} + \frac{7}{10}\right) \div 2 \\
 &= \left(\frac{6 + 7}{10}\right) \times \frac{1}{2} \\
 &= \frac{13}{10} \times \frac{1}{2} \\
 &= \frac{13}{20}
 \end{aligned}$$

2	5, 10
5	5, 5
	1, 1

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