Exercise 3.1

1. Write the following sets in set builder notation:

(i) $\{1, 4, 9, 16, 25, 36, \ldots, 484\}$

$$\{1,4,9,16,25,36, \ldots,484\}$$
$$= \{x | x = n^2, n \in N \wedge\ 1 \leq n \leq 22\}$$

Note: This set consists of perfect squares from 1^2 to 22^2 (since $22^2 = 484$).

(ii) $\{2, 4, 8, 16, \ldots, 256\}$

$$\{2,4,8,16, \ldots,256\}$$
$$= \{x | x = 2^n, n \in N \wedge\ 1 \leq n \leq 8\}$$

Note: This set consists of powers of 2 from 2^1 to 2^8 (since $2^8 = 256$).

(iii) $\{0, \pm 1, \pm 2, \ldots, \pm 1000\}$

$$\{0, \pm 1, \pm 2, \ldots, \pm 1000\}$$
$$= \{x | x \in Z \wedge\ -1000 \leq x \leq 1000\}$$

Note: This set includes all integers from -1000 to 1000.

(iv) $\{6, 12, 18, \ldots, 120\}$

$$\{6,12,18, \ldots,120\}$$
$$= \{x | x = 6n, n \in N \wedge 1 \leq n \leq 20\}$$

Note: This set consists of multiples of 6 from 6×1 to 6×20 (since $6 \times 20 = 120$).

(v) $\{100, 102, 104, \ldots, 400\}$

$$\{100,102,104, \ldots,400\}$$
$$= \{x | x = 100 + 2n, n \in W \wedge\ 0 \leq n \leq 150\}$$

Note: This set includes even numbers from 100 to 400.

(vi) $\{1, 3, 9, 27, 81, \ldots\}$

$$\{1,3,9,27,81, \ldots\}$$
$$= \{x | x = 3^n, n \in W\}$$

Note: This set consists of powers of 3 starting from $3^0 = 1$.

(vii) $\{1, 2, 4, 5, 10, 20, 25, 50, 100\}$

$$\{1,2,4,5,10,20,25,50,100\}$$
$$= \{x | x\ is\ divisor\ of\ 100, n \in N \wedge\ 1 \leq n \leq 100\}$$

Note: This set includes all positive divisors of 100.

(viii) $\{5, 10, 15, \ldots, 100\}$

$$\{5,10,15, \ldots,100\}$$
$$= \{x | x = 5n, n \in N \wedge\ 1 \leq n \leq 20\}$$

Note: This set consists of multiples of 5 from 5×1 to 5×20 (since $5 \times 20 = 100$).

(ix) The set of all integers between -100 and 1000.

$$= \{x | x \in Z \wedge\ -100 \leq x \leq 1000\}$$

Note: This set includes all integers from -100 to 1000.

2. Write each of the following sets in tabular form:

(i) $\{x \mid x\ is\ a\ multiple\ of\ 3 \wedge x \leq 36\}$

$$\{x \mid x\ is\ a\ multiple\ of\ 3 \wedge x \leq 36\}$$
$$= \{0,3,6,9,12,15,18,21,24,27,30,33,36\}$$

(ii) $\{x \mid x \in R \wedge 2x + 1 = 0\}$

$$2x + 1 = 0$$
$$2x = -1$$
$$x = -\frac{1}{2}$$
$$\{x \mid x \in R \wedge 2x + 1 = 0\}$$
$$= \left\{-\frac{1}{2}\right\}$$

(iii) $\{x \mid x \in P \wedge x < 12\}$

$$\{x \mid x \in P \wedge x < 12\}$$
$$= \{2,3,5,7,11\}$$

(iv) $\{x \mid x\ is\ a\ divisor\ of\ 128\}$

$$\{x \mid x\ is\ a\ divisor\ of\ 128\}$$
$$= \{1,2,4,8,16,32,64,128\}$$

(v) $\{x | x = 2^n, n \in N \wedge\ n < 8\}$

$$\{x | x = 2^n, n \in N \wedge\ n < 8\}$$
$$= \{2,4,8,16,32,64,128\}$$

(vi) $\{x \mid x \in N \wedge x + 4 = 0\}$

$$\{x \mid x \in N \wedge x + 4 = 0\}$$
$$= \{\ \ \}$$

As $x + 4 = 0 \Rightarrow x = -4 \notin N$

(vii) $\{x \mid x \in N \wedge x = x\}$

$$\{x \mid x \in N \wedge x = x\}$$
$$= \{1,2,3, \ldots\}$$

(viii) $\{x \mid x \in Z \wedge 3x + 1 = 0\}$

$$\{x \mid x \in Z \wedge 3x + 1 = 0\}$$
$$= \{\ \ \}$$

As $3x + 1 = 0 \Rightarrow x = -\frac{1}{3} \notin Z$

3. Write two proper subsets of each of the following sets:

(i) $\{a, b, c\}$

The Proper subsets of $\{a, b, c\}$ are $\{a\}$, $\{b\}$

(ii) $\{0, 1\}$

The Proper subsets of $\{0, 1\}$ are $\{0\}$, $\{1\}$

(iii) N

The Proper subsets of N are Prime Numbers, Even Natural Numbers

(iv) Z

The Proper subsets of Z are Prime Numbers, Whole Numbers

(v) Q

The Proper subsets of Q are Integers, Natural Numbers

(vi) $\{x|x \in Q \wedge 0 < x \leq 2\}$

The Proper subsets of $\{x|x \in Q \wedge 0 < x \leq 2\}$ are $\left\{\frac{1}{2}\right\}$, $\left\{\frac{3}{4}\right\}$

4. Is there any set which has no proper subset? If so, name that set.

Yes, there exist a set which has no proper subset, that is called 'empty set' { }.

5. What is the difference between $\{a, b\}$ and $\{\{a, b\}\}$?

$\{a, b\}$ set has two elements: a and b. While $\{\{a, b\}\}$ set has only one element $\{a, b\}$.

6. What is the number of elements of the power set of each of the following sets?

(i) { }

$$No.\,of\ elements\ of\ the\ power\ set = 2^n$$
$$= 2^0$$
$$= 1$$

(ii) $\{0, 1\}$

$$No.\,of\ elements\ of\ the\ power\ set = 2^n$$
$$= 2^2$$
$$= 4$$

(iii) $\{1, 2, 3, 4, 5, 6, 7\}$

$$No.\,of\ elements\ of\ the\ power\ set = 2^n$$
$$= 2^7$$
$$= 128$$

(iv) $\{0, 1, 2, 3, 4, 5, 6, 7\}$

$$No.\,of\ elements\ of\ the\ power\ set = 2^n$$
$$= 2^8$$
$$= 256$$

(v) $\{a, \{b, c\}\}$

$$No.\,of\ elements\ of\ the\ power\ set = 2^n$$
$$= 2^2$$
$$= 4$$

(vi) $\{\{a, b\}, \{b, c\}, \{d, e\}\}$

$$No.\,of\ elements\ of\ the\ power\ set = 2^n$$
$$= 2^3$$
$$= 8$$

7. Write down the power set of each of the following sets:

(i) $\{9, 11\}$

Let

$$A = \{9, 11\}$$
$$P(A) = \{\emptyset, \{9\}, \{11\}, \{9, 11\}\}$$

(ii) $\{+, -, \times, \div\}$

Let

$$A = \{+, -, \times, \div\}$$
$$P(A) = \{\emptyset, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+, -\}, \{+, \times\}, \{+, \div\}, \{-, \times\}, \{-, \div\}, \{\times, \div\}, \{+, -, \times\}, \{+, -, \div\}, \{-, \times, \div\}, \{+, \times, \div\}, \{+, -, \times, \div\}\}$$

(iii) $\{\emptyset\}$

Let

$$A = \{\emptyset\}$$
$$P(A) = \{\emptyset, \{\emptyset\}\}$$

(iv) $\{a, \{b, c\}\}$

Let

$$A = \{a, \{b, c\}\}$$
$$P(A) = \{\emptyset, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$$