Review Exercise 2

2. Express the following numbers in scientific notation:

(i) 0.000567

$$0.000567 = 5.67 \times 10^{-4}$$

(ii) 734

$$734 = 7.34 \times 10^{2}$$

(iii) 0.33×10^{3}

$$\begin{aligned} 0.33 \times 10^{3} &= 3.3 \times 10^{-1} \times 10^{3} \\ &= 3.3 \times 10^{-1+3} \\ &= 3.3 \times 10^{2} \end{aligned}$$

3. Express the following numbers in ordinary notation:

(i) 2.6×10^{3}

$$2.6 \times 10^{3} = 2600$$

(ii) 8.974×10^{-4}

$$8.974 \times 10^{-4} = 0.0008974$$

(iii) 6×10^{-6}

$$6 \times 10^{-6} = 0.000006$$

4. Express each of the following in logarithmic form:

(i) $3^{7} = 2187$

$$3^{7} = 2187$$
$$\log_3 2187 = 7$$

(ii) $a^{b} = c$

$$a^{b} = c$$
$$\log_a c = b$$

(iii) $12^{2} = 144$

$$12^{2} = 144$$
$$\log_{12} 144 = 2$$

5. Express each of the following in exponential form:

(i) $\log_4 8 = x$

$$\log_4 8 = x$$
$$4^{x} = 8$$

(ii) $\log_9 729 = 3$

$$\log_9 729 = 3$$
$$9^{3} = 729$$

(iii) $\log_4 1024 = 5$

$$\log_4 1024 = 5$$
$$4^{5} = 1024$$

6. Find the value of x in the following:

(i) $\log_9 x = 0.5$

$$\log_9 x = 0.5$$
$$9^{0.5} = x$$
$$9^{\frac{5}{10}} = x$$

$$9^{\frac{1}{2}} = x$$
$$(3^2)^{\frac{1}{2}} = x$$
$$3 = x$$
$$\boldsymbol{x = 3}$$

(ii) $\left(\frac{1}{9}\right)^{3x} = 27$

$$\left(\frac{1}{9}\right)^{3x} = 27$$
$$\left(\frac{1}{3^2}\right)^{3x} = 3^3$$
$$(3^{-2})^{3x} = 3^3$$
$$3^{-6x} = 3^3$$
$$\Longrightarrow -6x = 3$$
$$x = \frac{3}{-6}$$
$$\boldsymbol{x = \frac{1}{-2}}$$

(iii) $\left(\frac{1}{32}\right)^{2x} = 64$

$$\left(\frac{1}{32}\right)^{2x} = 64$$
$$\left(\frac{1}{2^5}\right)^{2x} = 2^6$$
$$(2^{-5})^{2x} = 2^6$$
$$2^{-10x} = 2^6$$
$$\Longrightarrow -10x = 6$$
$$x = \frac{6}{-10}$$
$$\boldsymbol{x = -\frac{3}{5}}$$

7. Write the following as a single logarithm:
(i) $7\log x - 3\log y^2$

$$7\log x - 3\log y^2$$
$$= \log x^7 - \log(y^2)^3$$
$$= \log x^7 - \log y^6$$
$$= \log\frac{x^7}{y^6}$$

(ii) $3\log 4 - \log 32$

$$3\log 4 - \log 32$$
$$= \log 4^3 - \log 32$$
$$= \log\frac{4^3}{32}$$

(iii) $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$

$$\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$$
$$= \frac{1}{3}\log_5 2^3 + \frac{1}{3}\log_5 3^3 - \log_5 3$$

$$= \log_5 (2^3)^{\frac{1}{3}} + \log_5 (3^3)^{\frac{1}{3}} - \log_5 3$$
$$= \log_5 2 + \log_5 3 - \log_5 3$$
$$= \log_5 2$$

8. Expand the following using laws of logarithms:

(i) $\log(xyz^6)$

$$\log(xyz^6)$$
$$= \log x + \log y + \log z^6$$
$$= \log x + \log y + 6 \log z$$

(ii) $\log_3 \sqrt[6]{m^5 n^3}$

$$\log_3 \sqrt[6]{m^5 n^3}$$
$$= \log_3 (m^5 n^3)^{\frac{1}{6}}$$
$$= \frac{1}{6}[\log_3 m^5 n^3]$$
$$= \frac{1}{6}[\log_3 m^5 + \log_3 n^3]$$
$$= \frac{1}{6}[5\log_3 m + 3\log_3 n]$$

(iii) $\log \sqrt{8x^3}$

$$\log \sqrt{8x^3}$$
$$= \log(2^3 x^3)^{\frac{1}{2}}$$
$$= \frac{1}{2}[\log 2^3 x^3]$$
$$= \frac{1}{2}[\log 2^3 + \log x^3]$$
$$= \frac{1}{2}[3 \log 2 + 3 \log x]$$
$$= \frac{3}{2}[\log 2 + \log x]$$

9. Find the values of the following with the help of logarithm table:

(i) $\sqrt[3]{68.24}$

Let

$$x = \sqrt[3]{68.24}$$
$$\log x = \log \sqrt[3]{68.24}$$
$$\log x = \log(68.24)^{\frac{1}{3}}$$
$$\log x = \frac{1}{3}\log(68.24)$$
$$\log x = \frac{1}{3}(1.8340)$$
$$\log x = 0.6113$$
$$Anti\log(\log x) = Anti \log(0.6113)$$
$$\boldsymbol{x = 4.086}$$

(ii) 319.8×3.543

Let

$$x = 319.8 \times 3.543$$
$$\log x = \log(319.8 \times 3.543)$$
$$\log x = \log 319.8 + \log 3.543$$

$$\log x = 2.5049 + 0.5494$$
$$\log x = 3.0543$$
$$Anti\log(\log x) = Anti\log(3.0543)$$
$$\boldsymbol{x = 1133}$$

(iii) $\dfrac{36.12 \times 750.9}{113.2 \times 9.98}$

Let

$$x = \frac{36.12 \times 750.9}{113.2 \times 9.98}$$
$$\log x = \log\left[\frac{36.12 \times 750.9}{113.2 \times 9.98}\right]$$
$$\log x = \log 36.12 + \log 750.9 - \log 113.2 - \log 9.98$$
$$\log x = 1.5577 + 2.8756 - 2.0538 - 0.9991$$
$$\log x = 1.3804$$
$$Anti\log(\log x) = Anti\log(1.3804)$$
$$\boldsymbol{x = 24.01}$$

10. In the year 2016, the population of a city was 22 millions and was growing at a rate of 2.5% per year. The function $p(t) = 22(1.025)^t$ gives the population in millions, t years after 2016. Use the model to determine in which year the population will reach 35 millions. Round the answer to the nearest year.

$$Initial\ population\ in\ 2016 = 22\ millions$$
$$Population = p(t) = 35\ millions$$
$$Year\ when\ population\ will\ reach\ 35\ millions = ?$$

Since

$$p(t) = 22(1.025)^t$$
$$35 = 22(1.025)^t$$
$$\log 35 = \log 22(1.025)^t$$
$$\log 35 = \log 22 + \log(1.025)^t$$
$$\log 35 - \log 22 = t\log 1.025$$
$$1.5441 - 1.3424 = t(0.0107)$$
$$\frac{0.2017}{0.0107} = t$$
$$18.85 = t$$
$$t \approx 19\ years$$

Now

$$Year\ when\ population\ will\ be\ 35\ million = 2016 + 19$$
$$= \boldsymbol{2035}$$