Exercise 2.4

1. Without using a calculator, evaluate the following:

(i) $\log_2 18 - \log_2 9$

$$\begin{aligned} &\log_2 18 - \log_2 9 \\ &= \log_2 \frac{18}{9} \\ &= \log_2 2 \\ &= 1 \qquad \because \log_a a = 1 \end{aligned}$$

(ii) $\log_2 64 - \log_2 2$

$$\begin{aligned} &\log_2 64 - \log_2 2 \\ &= \log_2 64 \times 2 \\ &= \log_2 128 \\ &= \log_2 2^7 \\ &= 7\log_2 2 \\ &= 7(1) \qquad \because \log_a a = 1 \\ &= 7 \end{aligned}$$

(iii) $\frac{1}{3}\log_3 8 - \log_3 18$

$$\begin{aligned} &\frac{1}{3}\log_3 8 - \log_3 18 \\ &= \frac{1}{3}\log_3 2^3 - \log_3 2\cdot 3^2 \\ &= \frac{\cancel{3}}{\cancel{3}}\log_3 2 - [\log_3 2 + \log_3 3^2] \\ &= \cancel{\log_3 2} - \cancel{\log_3 2} - 2\log_3 3 \\ &= -2\log_3 3 \\ &= -2(1) \qquad \because \log_a a = 1 \\ &= -2 \end{aligned}$$

(iv) $2\log 2 + \log 25$

$$\begin{aligned} &2\log 2 + \log 25 \\ &= 2\log 2 + \log 5^2 \\ &= 2\log 2 + 2\log 5 \\ &= 2[\log 2 + \log 5] \\ &= 2\log 2 \times 5 \\ &= 2\log 10 \\ &= 2\log_{10} 10 \\ &= 2(1) \\ &= 2 \end{aligned}$$

(v) $\frac{1}{3}\log_3 64 + 2\log_5 25$

$$\begin{aligned} &\frac{1}{3}\log_4 64 + 2\log_5 25 \\ &= \frac{1}{3}\log_4 4^3 + 2\log_5 5^2 \\ &= \frac{3}{3}\log_4 4 + 2\times 2\log_5 5 \\ &= \log_4 4 + 4\log_5 5 \\ &= 1 + 4(1) \\ &= 1 + 4 \\ &= 5 \end{aligned}$$

(vi) $\log_3 12 + \log_3 0.25$

$$\begin{aligned} &\log_3 12 + \log_3 0.25 \\ &= \log_3(12 \times 0.25) \\ &= \log_3\left(12 \times \frac{25}{100}\right) \\ &= \log_3 3 \\ &= 1 \end{aligned}$$

2. Write the following as a single logarithm:

(i) $\frac{1}{2}\log 25 - 2\log 3$

$$\begin{aligned} &\frac{1}{2}\log 25 - 2\log 3 \\ &= \log(5^2)^{\frac{1}{2}} - \log 3^2 \\ &= \log 5 - \log 9 \\ &= \log\frac{5}{9} \end{aligned}$$

(ii) $\log 9 - \log\frac{1}{3}$

$$\begin{aligned} &\log 9 - \log\frac{1}{3} \\ &= \log\frac{9}{1/3} \\ &= \log\frac{9\times 3}{1} \\ &= \log 27 \end{aligned}$$

(iii) $\log_5 b^2 \cdot \log_a 5^3$

$$\begin{aligned} &\log_5 b^2 \cdot \log_a 5^3 \\ &= 2\log_5 b \cdot 3\log_a 5 \\ &= 6\log_5 b \cdot \log_a 5 \\ &= 6\frac{\log b}{\log 5}\cdot\frac{\log 5}{\log a} \qquad \because \log_b x = \frac{\log_a x}{\log_a b} \\ &= 6\frac{\log b}{\log a} \\ &= 6\log_a b \qquad \because \frac{\log_a x}{\log_a b} = \log_b x \end{aligned}$$

(iv) $2\log_3 x + \log_3 y$

$$\begin{aligned} &2\log_3 x + \log_3 y \\ &= \log_3 x^2 + \log_3 y \\ &= \log_3 x^2 y \end{aligned}$$

(v) $4\log_5 x - \log_5 y + \log_5 z$

$$\begin{aligned} &4\log_5 x - \log_5 y + \log_5 z \\ &= \log_5 x^4 - \log_5 y + \log_5 z \\ &= \log_5 \frac{x^4 z}{y} \end{aligned}$$

(vi) $2\ln a + 3\ln b - 4\ln c$

$$2\ln a + 3\ln b - 4\ln c$$

$$= \ln a^2 + \ln b^3 - \ln c^4$$
$$= \ln \frac{a^2 b^3}{c^4}$$

3. Expand the following using laws of logarithms:

(i) $\log\left(\frac{11}{5}\right)$

$$\log\left(\frac{11}{5}\right)$$
$$= \log 11 - \log 5$$

(ii) $\ln \frac{a^2 b}{c}$

$$\ln \frac{a^2 b}{c}$$
$$= \ln a^2 + \ln b - \ln c$$
$$= 2 \ln a + \ln b - \ln c$$