Review Exercise 1

2. if $a=\frac{3}{2}$, $b=\frac{5}{3}$ and $c=\frac{7}{5}$ then verify that (i) $a(b+c)=ab+ac$ (ii) $(a+b)c=ac+bc$

(i) $a(b+c)=ab+ac$

2	6,10
3	3, 5
5	1,5
	1,1

$$a(b+c)=ab+ac$$
$$\frac{3}{2}\left(\frac{5}{3}+\frac{7}{5}\right)=\left(\frac{3}{2}\right)\left(\frac{5}{3}\right)+\left(\frac{3}{2}\right)\left(\frac{7}{5}\right)$$
$$\frac{3}{2}\left(\frac{25+21}{15}\right)=\frac{15}{6}+\frac{21}{10}$$
$$\frac{3}{2}\left(\frac{46}{15}\right)=\frac{75+63}{30}$$
$$\frac{138}{30}=\frac{138}{30}\quad(Proved)$$

(ii) $(a+b)c=ac+bc$

2	10,15
3	5 ,15
5	5 , 5
	1 , 1

$$(a+b)c=ac+bc$$
$$\left(\frac{3}{2}+\frac{5}{3}\right)\frac{7}{5}=\left(\frac{3}{2}\right)\left(\frac{7}{5}\right)+\left(\frac{5}{3}\right)\left(\frac{7}{5}\right)$$
$$\left(\frac{9+10}{6}\right)\frac{7}{5}=\frac{21}{10}+\frac{35}{15}$$
$$\left(\frac{19}{6}\right)\frac{7}{5}=\frac{63+70}{30}$$
$$\frac{133}{30}=\frac{133}{30}\quad(Proved)$$

3. if $a=\frac{4}{3}$, $b=\frac{5}{2}$ and $c=\frac{7}{4}$ then verify the associative property of real numbers w.r.t addition and multiplication.

Associative property w.r.t addition

2	2,4
2	1,2
	1,1

2	4,6
2	2,3
3	1,3
	1,1

$$a+(b+c)=(a+b)+c$$
$$\frac{4}{3}+\left(\frac{5}{2}+\frac{7}{4}\right)=\left(\frac{4}{3}+\frac{5}{2}\right)+\frac{7}{4}$$
$$\frac{4}{3}+\left(\frac{10+7}{4}\right)=\left(\frac{8+15}{6}\right)+\frac{7}{4}$$
$$\frac{4}{3}+\frac{17}{4}=\frac{23}{6}+\frac{7}{4}$$
$$\frac{16+51}{12}=\frac{46+21}{12}$$
$$\frac{67}{12}=\frac{67}{12}\quad(Proved)$$

Associative property w.r.t multiplication

$$a(bc)=(ab)c$$
$$\frac{4}{3}\left(\frac{5}{2}\times\frac{7}{4}\right)=\left(\frac{4}{3}\times\frac{5}{2}\right)\frac{7}{4}$$
$$\frac{4}{3}\left(\frac{35}{8}\right)=\left(\frac{20}{6}\right)\frac{7}{4}$$
$$\frac{140}{24}=\frac{140}{24}\quad(Proved)$$

4. Is 0 a rational number? Explain.

Yes, zero (0) is a rational number. It satisfies the definition of rational numbers. For example, $\frac{0}{2}, \frac{0}{-9}$ both are rational numbers.

5. State trichotomy property of real numbers.

$\forall\ a, b \in R$, either $a=b$ or $a>b$ or $a<b$

6. Find two rational numbers between 4 and 5.

$$1^{st}\ rational\ number=(4+5)\div 2$$
$$=(9)\times\frac{1}{2}$$
$$=\frac{9}{2}$$

$$2^{nd}\ rational\ number=\left(4+\frac{9}{2}\right)\div 2$$
$$=\left(\frac{8+9}{2}\right)\times\frac{1}{2}$$
$$=\frac{17}{2}\times\frac{1}{2}$$
$$=\frac{17}{4}$$

7. Simplify the following:

(i) $\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}}$

$$\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}}$$
$$=\left(\frac{x^{15}y^{35}}{z^{20}}\right)^{\frac{1}{5}}$$
$$=\frac{(x^{15})^{\frac{1}{5}}(y^{35})^{\frac{1}{5}}}{(z^{20})^{\frac{1}{5}}}$$
$$=\boldsymbol{\frac{x^3y^7}{z^4}}$$

(ii) $\sqrt[3]{(27)^{2x}}$

$$\sqrt[3]{(27)^{2x}}$$
$$=[(27)^{2x}]^{1/3}$$
$$=(3^3)^{2x/3}$$
$$=\boldsymbol{3^{2x}}$$

(iii) $\frac{6(3)^{n+2}}{3^{n+1}-3^n}$

$$\frac{6(3)^{n+2}}{3^{n+1}-3^n}$$
$$=\frac{6\cdot 3^n\cdot 3^2}{3^n\cdot 3^1-3^n}$$
$$=\frac{6\cdot 3^n\cdot 9}{3^n(3-1)}$$

$$= \frac{54}{2}$$
$$= \mathbf{27}$$

8. The sum of three consecutive odd integers is 51. Find the three integers.

$$Let\ first\ integer = x$$
$$Second\ integer = x + 2$$
$$Third\ integer = x + 4$$

According to question

$$x + x + 2 + x + 4 = 51$$
$$3x + 6 = 51$$
$$3x = 51 - 6$$
$$3x = 45$$
$$x = \frac{45}{3}$$
$$x = 15$$

Hence

$$First\ integer = x = \mathbf{15}$$
$$Second\ integer = x + 2 = 15 + 2 = \mathbf{17}$$
$$Third\ integer = x + 4 = 15 + 4 = \mathbf{19}$$

9. Abdullah picked up 96 balls and placed them into two buckets. One bucket has twenty-eight more balls than the other bucket. How many balls were in each bucket?

$$Let\ balls\ in\ first\ bucket = x$$
$$Balls\ in\ scond\ bucket = x + 28$$
$$Total\ balls = 96$$

According to question

$$x + x + 28 = 96$$
$$2x + 28 = 96$$
$$2x = 96 - 28$$
$$2x = 68$$
$$x = \frac{68}{2}$$
$$x = 34$$

Hence

$$Balls\ in\ first\ bucket = x = \mathbf{34}$$
$$Balls\ in\ scond\ bucket = x + 28$$
$$= 34 + 28$$
$$= \mathbf{62}$$

10. Salma invested $Rs.\ 3,50,000$ in a bank, which paid simple profit at the rate of $7\frac{1}{4}\ \%$ per annum. After 2 years, the rate was increased to $8\ \%$ per annum. Find the amount she had at the end of 7 years.

For first 2 years:

$$Principal\ Amount\ = 350000\ Rs$$
$$Rate = 7\frac{1}{4}\ \%$$
$$= 7.25\ \%$$
$$Time = 2\ years$$
$$Profit = \frac{Principal\ Amount\ \times\ time\ \times\ rate}{100}$$
$$P_1 = \frac{350000 \times 2 \times 7.25}{100}$$
$$P_1 = 50750\ Rs.$$

For Next 5 years:

$$Principal\ Amount\ = 350000\ Rs$$
$$Rate = 8\ \%$$
$$Time = 2\ years$$
$$Profit = \frac{Principal\ Amount\ \times\ time\ \times\ rate}{100}$$
$$P_2 = \frac{350000 \times 2 \times 8}{100}$$
$$P_2 = 140000\ Rs.$$

At end of 7 years:

$$Total\ Amount\ = Principal\ Amount + P_1 + P_2$$
$$= 350000 + 50750 + 140000$$
$$= \mathbf{540750\ Rs.}$$