Important Formulas

- **Density** $Density = \frac{Mass}{Volume} \quad \Rightarrow \rho = \frac{m}{V}$
- **Pressure** $P = \frac{F}{A}$
- **Spring constant** $k = \frac{F}{x}$
- **Equation of Hydraulic Press** $\frac{F_1}{A_1} = \frac{F_2}{A_2}$
- **Volume** $V = L \times B \times H$
- **Pressure at a depth** $P = \rho g h$

6.1. A spring is stretched $20\,mm$ by a load of $40\,N$. Calculate the value of spring constant. If an object cause an extension of $16\,mm$, what will be its $weight$?

Given Data

$$Extension\ in\ spring = x_1 = 20\ mm$$
$$x_1 = 20 \times 10^{-3}\ m$$
$$x_1 = 0.02\ m$$
$$Force\ applied = F_1 = 40\ N$$
$$New\ extension = x_2 = 16\ mm$$
$$x_2 = 16 \times 10^{-3}\ m$$
$$x_1 = 0.016\ m$$

To Find

$$Spring\ constant = k = ?$$
$$Weight\ for\ extension\ of\ 16\ mm = F_2 = ?$$

Solution

By using formula of spring constant $k = \frac{F}{x}$

$$k = \frac{F_1}{x_1}$$
$$k = \frac{40}{0.02}$$
$$\boldsymbol{k = 2000\ Nm^{-1}}$$

As $k = \frac{F}{x} \Longrightarrow F = kx$, so

$$F_2 = kx_2$$
$$F_2 = (2000)(0.016)$$
$$\boldsymbol{F_2 = 32\ N}$$

6.2. The mass of 5 litres of milk is 4.5 kg. Find its density in SI units.

Given Data

$$Mass\ of\ milk = m = 4.5\ kg$$
$$Volume\ of\ milk = V = 5\ litres$$
$$V = 5 \times 10^{-3}\ m^3$$

To Find

$$Density = \rho = ?$$

Solution

By using formula of density

$$Density = \frac{Mass}{Volume}$$
$$\rho = \frac{m}{V}$$
$$\rho = \frac{4.5}{5 \times 10^{-3}}$$
$$\boldsymbol{\rho = 900\ kgm^{-3}}$$

Note: *A volume of* $1000\ litres$ *is the same as* $1\ cubic\ meter$ *of space.*

$$1000\ litres = 1\ m \times 1\ m \times 1\ m$$
$$1000\ litres = 1\ m^3$$
$$1\ litres = \frac{1}{1000}\ m^3$$
$$1\ litres = \frac{1}{10^3}\ m^3$$
$$1\ litres = 10^{-3} m^3$$

6.3. When a solid of mass $60\ g$ is lowered into a measuring cylinder, the level of water rises from $40\ cm^3$ to $44\ cm^3$. Calculate the density of the solid.

Given Data

$$Mass\ of\ solid = m = 60\ g$$
$$m = \frac{60}{1000}\ kg$$
$$m = 0.06\ kg$$
$$Initial\ volume\ of\ water = V_1 = 40\ cm^3$$
$$Final\ volume\ of\ water = V_2 = 44\ cm^3$$
$$Volume\ of\ solid = V = V_2 - V_1$$
$$V = 44 - 40$$
$$V = 4\ cm^3$$
$$V = 4\ (10^{-2})^3 m^3$$
$$V = 4 \times 10^{-6}\ m^3$$

To Find

$$Density = \rho = ?$$

Solution

By using formula of density

$$Density = \frac{Mass}{Volume}$$
$$\rho = \frac{m}{V}$$
$$\rho = \frac{0.06}{4 \times 10^{-6}}$$
$$\boldsymbol{\rho = 15000\ kgm^{-3}}$$
$$\boldsymbol{\rho = 15 \times 10^3\ kgm^{-3}}$$

6.4. A block of density $8 \times 10^3\ kgm^{-3}$ has a volume $60\ cm^3$. Find its mass.

Given Data

$$Density = \rho = 8 \times 10^3\ kgm^{-3}$$
$$Volume = V = 60\ cm^3$$
$$V = 60\ (10^{-2})^3 m^3$$
$$V = 60 \times 10^{-6}\ m^3$$

To Find

$$Mass\ of\ the\ block = m = ?$$

Solution

By using formula of density

$$\rho = \frac{m}{V}$$
$$m = \rho V$$
$$m = (8 \times 10^3)(60 \times 10^{-6})$$
$$\boldsymbol{m = 0.48\ kg}$$

6.5. A brick measures $5\ cm \times 10\ cm \times 20\ cm$. If its mass is $5\ kg$, calculate the maximum and minimum pressure which the brick can exert on a horizontal surface.

Given Data

$$Dimensions\ of\ the\ brick = V = 5\ cm \times 10\ cm \times 20\ cm$$
$$Mass\ of\ the\ brick = m = 5\ kg$$
$$Minimum\ area = A_{min} = 5\ cm \times 10\ cm$$
$$A_{min} = 50\ cm^2$$

$$A_{min} = 50\ (10^{-2})^2 m^2$$
$$A_{min} = 50\ \times 10^{-4}\ m^2$$
$$Maximum\ area = A_{max} = 10\ cm \times 20\ cm$$
$$A_{max} = 200\ cm^2$$
$$A_{max} = 200\ (10^{-2})^2 m^2$$
$$A_{max} = 200\ \times 10^{-4}\ m^2$$

To Find

$$Minimum\ pressure = P_{min} = ?$$
$$Maximum\ pressure = P_{max} = ?$$

Solution

As we know that force is equal to weight of brick, so

$$F = w$$
$$F = mg$$
$$F = (5)(10)$$
$$F = 50\ N$$

For minimum pressure, by using formula of pressure $P = \frac{F}{A}$

$$P_{min} = \frac{F}{A_{max}}$$
$$P_{min} = \frac{50}{200\ \times 10^{-4}}$$
$$P_{min} = 2500\ Nm^{-2}$$
$$\boldsymbol{P_{min} = 2.5 \times 10^3\ Nm^{-2}} \quad (\boldsymbol{Pa})$$

For maximum pressure, by using formula of pressure $P = \frac{F}{A}$

$$P_{max} = \frac{F}{A_{min}}$$
$$P_{max} = \frac{50}{50\ \times 10^{-4}}$$
$$P_{max} = 10000\ Nm^{-2}$$
$$\boldsymbol{P_{max} = 1.0 \times 10^4\ Nm^{-2}} \quad (\boldsymbol{Pa})$$

Note: Pressure is $minimum$ when the area is $\boldsymbol{maximum}$, and pressure is $maximum$ when the area is $\boldsymbol{minimum}$.

6.6. What will be the height of the column in barometer at sea level if mercury is replaced by water of density $\boldsymbol{1000\ kgm^{-3}}$, where density of mercury is $\boldsymbol{13.6 \times 10^3\ kgm^{-3}}$.

Given Data

$$Density\ of\ water = \rho_1 = 1000\ kgm^{-3}$$
$$Density\ of\ mercury = \rho_2 = 13.6 \times 10^3\ kgm^{-3}$$
$$height\ of\ mercury\ column = h_2 = 0.76\ m$$

To Find

$$Height\ of\ water\ column = h_1 = ?$$

Solution

Since the pressure at sea level remains the same, we equate the pressures for mercury and water columns. So

$$P_{water} = P_{mercury}$$
$$\rho_1 g h_1 = \rho_2 g h_2 \qquad \because P = \rho g h$$
$$h_1 = \frac{\rho_2 g h_2}{\rho_1 g}$$
$$h_1 = \frac{\rho_2 h_2}{\rho_1}$$
$$h_1 = \frac{(13.6 \times 10^3)(0.76)}{1000}$$
$$\boldsymbol{h_1 = 10.34\ m}$$

6.7. Suppose in the hydraulic brake system of a car, the force exerted normally on its piston of cross-sectional area of $\boldsymbol{5\ cm^2}$ is $\boldsymbol{500\ N}$. What will be the pressure transferred to the brake oil? What will be the force on the second piston of area of cross-section $\boldsymbol{20\ cm^2}$?

Given Data

$$Area\ of\ first\ piston = A_1 = 5\ cm^2$$
$$A_1 = 5\ (10^{-2})^2 m^2$$
$$A_1 = 5\ \times 10^{-4}\ m^2$$
$$Force\ on\ first\ piston = F_1 = 500\ N$$
$$Area\ of\ second\ piston = A_2 = 20\ cm^2$$
$$A_2 = 20\ (10^{-2})^2 m^2$$
$$A_2 = 20\ \times 10^{-4}\ m^2$$

To Find

$$Pressure\ transferred\ to\ brake\ oil = P_1 = ?$$
$$Force\ on\ second\ piston = F_2 = ?$$

Solution

By using formula of pressure $P = \frac{F}{A}$

$$P_1 = \frac{F_1}{A_1}$$
$$P_1 = \frac{500}{5\ \times 10^{-4}}$$
$$P_1 = 1000000$$
$$\boldsymbol{P_1 = 1.0 \times 10^6\ Nm^{-2}} \quad (\boldsymbol{Pa})$$

By using equation of hydraulic press

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$
$$F_2 = \frac{F_1 A_2}{A_1}$$
$$F_2 = \frac{(500)(20\ \times 10^{-4})}{5\ \times 10^{-4}}$$
$$\boldsymbol{F_2 = 2000\ N}$$

6.8. Find the water pressure on a deep-sea diver at a depth of $\boldsymbol{10\ m}$, where the density of sea water is $\boldsymbol{1030\ kgm^{-3}}$.

Given Data

$$Depth\ of\ water = h = 10\ m$$
$$Density\ of\ sea\ water = \rho = 1030\ kgm^{-3}$$
$$Gravitational\ acceleration = g = 10\ ms^{-2}$$

To Find

$$Pressure\ at\ depth\ h = P = ?$$

Solution

By using formula of pressure at a depth

$$P = \rho g h$$
$$P = (1030)(10)(10)$$
$$P = 103000\ Nm^{-2}$$
$$\boldsymbol{P = 1.03 \times 10^5\ Nm^{-2}} \quad (\boldsymbol{Pa})$$

6.9. The area of cross-section of the small and large pistons of a hydraulic press is respectively $\boldsymbol{10\ cm^2}$ and $\boldsymbol{100\ cm^2}$. What force should be exerted on the small piston in order to lift a car of weight $\boldsymbol{4000\ N}$?

Given Data

$$Area\ of\ small\ piston = A_1 = 10\ cm^2$$
$$A_1 = 10\ (10^{-2})^2 m^2$$
$$A_1 = 10 \times 10^{-4}\ m^2$$
$$Area\ of\ large\ piston = A_2 = 100\ cm^2$$
$$A_2 = 100\ (10^{-2})^2 m^2$$

$$A_2 = 100 \times 10^{-4}\, m^2$$
$$Weight\ to\ be\ lifted = F_2 = 4000\ N$$

To Find

$$Force\ on\ small\ piston = F_1 = ?$$

Solution

By using equation of hydraulic press

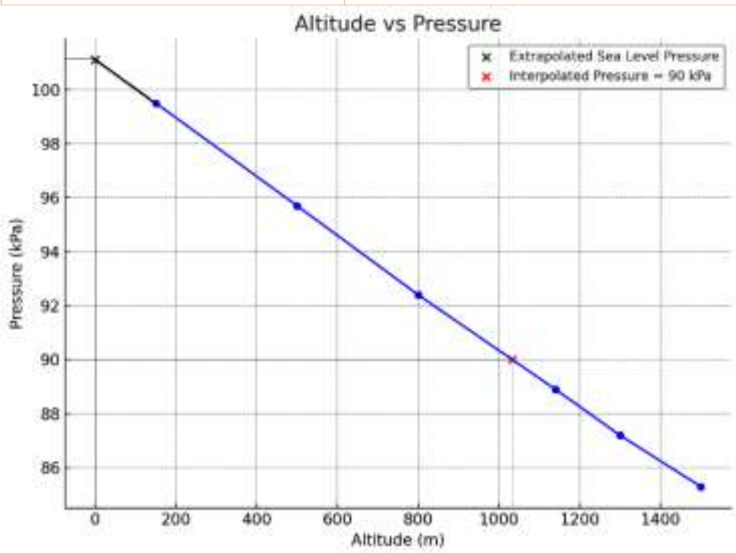
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$
$$F_1 = \frac{F_2 A_1}{A_2}$$
$$F_1 = \frac{(4000)(10 \times 10^{-4})}{100 \times 10^{-4}}$$
$$\boldsymbol{F_1 = 400\ N}$$

6.10. In a hot air balloon, the following data was recorded. Draw a graph between the altitude and pressure and find out:

(a) What would the air pressure have been at sea level?

(b) At what height the air pressure would have been 90 kPa?

Altitude (m)	Pressure (kPa)
150	99.5
500	95.7
800	92.4
1140	88.9
1300	87.2
1500	85.3



From the graph:

(a) At sea level ($0\ m$), the extrapolated air pressure is approximately $101.1\ kPa$.

(b) When the air pressure is $90\ kPa$, the interpolated altitude is approximately $1033\ m$.

6.11. If the pressure in a hydraulic press is increased by an additional $10\ Ncm^{-2}$, how much extra load will the output platform support if its cross-sectional area is $50\ cm^2$?

Given Data

$$Pressure\ increase = P = 10\ Ncm^{-2}$$
$$Cross-sectional\ area = A = 50\ cm^2$$

To Find

$$Extra\ load\ (Force)\ supported = F = ?$$

Solution

By using formula of pressure

$$P = \frac{F}{A}$$
$$F = PA$$
$$F = (10\ Ncm^{-2})(50\ cm^2)$$
$$\boldsymbol{F = 500\ N}$$

6.12. The force exerted normally on the hydraulic brake system of a car, with its piston of cross-sectional area $5\ cm^2$ is $500\ N$. What will be the:

(a) pressure transferred to the brake oil?

(b) force on the brake piston of area of cross section $20\ cm^2$? [$Same\ as\ 6.7$]

Given Data

$$Area\ of\ small\ piston = A_1 = 5\ cm^2$$
$$A_1 = 5\,(10^{-2})^2 m^2$$
$$A_1 = 5 \times 10^{-4}\ m^2$$
$$Force\ on\ small\ piston = F_1 = 500\ N$$
$$Area\ of\ large\ piston = A_2 = 20\ cm^2$$
$$A_2 = 20\,(10^{-2})^2 m^2$$
$$A_2 = 20 \times 10^{-4}\ m^2$$

To Find

$$Pressure\ transferred\ to\ brake\ oil = P_1 = ?$$
$$Force\ on\ the\ large\ piston = F_2 = ?$$

Solution

By using formula of pressure $P = \frac{F}{A}$

$$P_1 = \frac{F_1}{A_1}$$
$$P_1 = \frac{500}{5 \times 10^{-4}}$$
$$P_1 = 1000000$$
$$\boldsymbol{P_1 = 1.0 \times 10^6\ Nm^{-2}} \qquad \boldsymbol{(Pa)}$$

By using equation of hydraulic press

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$
$$F_2 = \frac{F_1 A_2}{A_1}$$
$$F_2 = \frac{(500)(20 \times 10^{-4})}{5 \times 10^{-4}}$$
$$\boldsymbol{F_2 = 2000\ N}$$