

Exercise 1.2

Important Formulas:

- (i) $(x + y)^2 = x^2 + 2xy + y^2$
- (ii) $(x - y)^2 = x^2 - 2xy + y^2$
- (iii) $x^2 - y^2 = (x + y)(x - y)$
- (iv) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
- (v) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
- (vi) $a^m \cdot a^n = a^{m+n}$
- (vii) $(a^m)^n = a^{mn}$
- (viii) $(ab)^n = a^n b^n$
- (ix) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- (x) $\frac{a^m}{a^n} = a^{m-n}$
- (xi) $a^0 = 1$
- (xii) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
- (xiii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- (xiv) $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- (xv) $(\sqrt[n]{a})^n = (a^{\frac{1}{n}})^n = a$

1. Rationalize the denominator of the following:

(i) $\frac{13}{4+\sqrt{13}}$

$$\begin{aligned}\frac{13}{4+\sqrt{13}} &= \frac{13}{4+\sqrt{13}} \times \frac{4-\sqrt{13}}{4-\sqrt{13}} \\ &= \frac{13(4-\sqrt{13})}{(4)^2 - (\sqrt{13})^2} \\ &= \frac{13(4-\sqrt{13})}{16-13} \\ &= \frac{13(4-\sqrt{13})}{3} \\ &= 4 - \sqrt{13}\end{aligned}$$

(ii) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

$$\begin{aligned}\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} &= \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{\sqrt{2 \cdot 3} + \sqrt{5 \cdot 3}}{(\sqrt{3})^2} \\ &= \frac{\sqrt{6} + \sqrt{15}}{3}\end{aligned}$$

(iii) $\frac{\sqrt{2}-1}{\sqrt{5}}$

$$\frac{\sqrt{2}-1}{\sqrt{5}} = \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$\begin{aligned}&= \frac{\sqrt{2 \cdot 5} - \sqrt{5}}{(\sqrt{5})^2} \\ &= \frac{\sqrt{10} - \sqrt{5}}{5}\end{aligned}$$

(iv) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

$$\begin{aligned}\frac{6-4\sqrt{2}}{6+4\sqrt{2}} &= \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}} \\ &= \frac{6(6-4\sqrt{2}) - 4\sqrt{2}(6-4\sqrt{2})}{(6)^2 - (4\sqrt{2})^2} \\ &= \frac{36 - 24\sqrt{2} - 24\sqrt{2} + 16(\sqrt{2})^2}{36 - (4)^2(\sqrt{2})^2} \\ &= \frac{36 - 48\sqrt{2} + 16(2)}{36 - (16)(2)} \\ &= \frac{36 - 48\sqrt{2} + 32}{36 - 32} \\ &= \frac{68 - 48\sqrt{2}}{4} \\ &= \frac{68}{4} - \frac{48\sqrt{2}}{4} \\ &= 17 - 12\sqrt{2}\end{aligned}$$

(v) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$\begin{aligned}\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} &= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} \\ &= \frac{\sqrt{3}(\sqrt{3}-\sqrt{2}) - \sqrt{2}(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2 - (\sqrt{2})^2} \\ &= \frac{(\sqrt{3})^2 - \sqrt{6} - \sqrt{6} + (\sqrt{2})^2}{3-2} \\ &= \frac{3 - 2\sqrt{6} + 2}{1} \\ &= 5 - 2\sqrt{6}\end{aligned}$$

(vi) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

$$\begin{aligned}\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} &= \frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} \\ &= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{(\sqrt{7})^2 - (\sqrt{5})^2} \\ &= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{7-5} \\ &= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{2}\end{aligned}$$

$$= 2\sqrt{3}(\sqrt{7} - \sqrt{5})$$

2. Simplify the following:

$$(i) \left(\frac{81}{16}\right)^{-\frac{3}{4}}$$

$$\begin{aligned} & \left(\frac{81}{16}\right)^{-\frac{3}{4}} \\ &= \left(\frac{16}{81}\right)^{\frac{3}{4}} \\ &= \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}} \\ &= \frac{(2^4)^{\frac{3}{4}}}{(3^4)^{\frac{3}{4}}} \\ &= \frac{2^3}{3^3} \\ &= \frac{8}{27} \end{aligned}$$

$$(ii) \left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^3 \times \frac{16}{27}$$

$$\begin{aligned} & \left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^3 \times \frac{16}{27} \\ &= \left(\frac{4}{3}\right)^2 \div \left(\frac{4}{3^2}\right)^3 \times \frac{4^2}{3^3} \\ &= \frac{4^2}{3^2} \div \frac{4^3}{3^6} \times \frac{4^2}{3^3} \\ &= \frac{4^2}{3^2} \div \frac{4^3}{3^6} \times \frac{4^2}{3^3} \\ &= \frac{4^2}{3^2} \times \frac{3^6}{4^3} \times \frac{4^2}{3^3} \\ &= 4^{2+2-3} \times 3^{6-2-3} \\ &= 4^1 \times 3^1 \\ &= 4 \times 3 \\ &= 12 \end{aligned}$$

$$(iii) (0.027)^{-\frac{1}{3}}$$

$$\begin{aligned} & (0.027)^{-\frac{1}{3}} \\ &= \left(\frac{27}{1000}\right)^{-\frac{1}{3}} \\ &= \left(\frac{1000}{27}\right)^{\frac{1}{3}} \\ &= \left(\frac{10^3}{3^3}\right)^{\frac{1}{3}} \\ &= \frac{(10^3)^{\frac{1}{3}}}{(3^3)^{\frac{1}{3}}} \\ &= \frac{10}{3} \end{aligned}$$

$$(iv) \sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} \times z^7}}$$

$$\begin{aligned} & \sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} \times z^7}} \\ &= \left(\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} \times z^7}\right)^{\frac{1}{7}} \\ &= (x^{14} \cdot y^{21-14} \cdot z^{35-7})^{\frac{1}{7}} \\ &= (x^{14} \cdot y^7 \cdot z^{28})^{\frac{1}{7}} \\ &= (x^{14})^{\frac{1}{7}} \cdot (y^7)^{\frac{1}{7}} \cdot (z^{28})^{\frac{1}{7}} \\ &= x^2 y z^4 \end{aligned}$$

$$(v) \frac{5 \cdot (25)^{n+1} - 25 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (25)^{n+1}}$$

$$\begin{aligned} & \frac{5 \cdot (25)^{n+1} - 25 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (25)^{n+1}} \\ &= \frac{5 \cdot (5^2)^{n+1} - 5^2 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (5^2)^{n+1}} \\ &= \frac{5 \cdot 5^{2n+2} - 5^2 \cdot (5)^{2n}}{5 \cdot 5^{2n+3} - 5^{2n+2}} \\ &= \frac{5^{1+2n+2} - 5^{2+2n}}{5^{1+2n+3} - 5^{2n+2}} \\ &= \frac{5^{2n+3} - 5^{2n+2}}{5^{2n+3} - 5^{2n+2}} \\ &= \frac{5^{2n+2}(5 - 1)}{5^{2n+2}(5 - 1)} \\ &= \frac{5 - 1}{5 - 1} \\ &= \frac{4}{4} \\ &= 1 \end{aligned}$$

$$(vi) \frac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}}$$

$$\begin{aligned} & \frac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}} \\ &= \frac{(2^4)^{x+1} + 20(2^2)^{2x}}{2^{x-3} \times (2^3)^{x+2}} \\ &= \frac{2^{4x+4} + 20 \cdot 2^{4x}}{2^{x-3} \times 2^{3x+6}} \\ &= \frac{2^{4x+4} + 20 \cdot 2^{4x}}{2^{x-3+3x+6}} \\ &= \frac{2^{4x+4} + 20 \cdot 2^{4x}}{2^{4x+3}} \\ &= \frac{2^{4x}(2^4 + 20)}{2^{4x} \cdot 2^3} \\ &= \frac{16 + 20}{2^3} \end{aligned}$$

$$= \frac{36}{8}$$

$$(vii) (64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$$

$$\begin{aligned} & (64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}} \\ &= (4^3)^{-\frac{2}{3}} \div (3^2)^{-\frac{3}{2}} \\ &= 4^{-2} \div 3^{-3} \\ &= \frac{4^{-2}}{3^{-3}} \\ &= \frac{3^3}{4^2} \\ &= \frac{27}{16} \end{aligned}$$

$$(viii) \frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$$

$$\begin{aligned} & \frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}} \\ &= \frac{3^n \times (3^2)^{n+1}}{3^{n-1} \times (3^2)^{n-1}} \\ &= \frac{3^n \times 3^{2n+2}}{3^{n-1} \times 3^{2n-2}} \\ &= \frac{3^{n+2n+2-n+1-2n+2}}{3^5} \\ &= 243 \end{aligned}$$

$$(ix) \frac{5^{n+3} - 6 \cdot 5^{n+1}}{9 \times 5^n - 4 \times 5^n}$$

$$\begin{aligned} & \frac{5^{n+3} - 6 \cdot 5^{n+1}}{9 \times 5^n - 4 \times 5^n} \\ &= \frac{5^n(5^3 - 6 \cdot 5^1)}{5^n(9 - 4)} \\ &= \frac{125 - 30}{9 - 4} \\ &= \frac{95}{5} \\ &= 19 \end{aligned}$$

3. If $x = 3 + \sqrt{8}$ then find the values of

$$(i) x + \frac{1}{x} = ?$$

$$\begin{aligned} x &= 3 + \sqrt{8} \\ \frac{x}{1} &= \frac{3 + \sqrt{8}}{1} \\ \frac{1}{x} &= \frac{1}{3 + \sqrt{8}} \\ \frac{1}{x} &= \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}} \\ \frac{1}{x} &= \frac{3 - \sqrt{8}}{(3)^2 - (\sqrt{8})^2} \end{aligned}$$

$$\frac{1}{x} = \frac{3 - \sqrt{8}}{9 - 8}$$

$$\frac{1}{x} = \frac{3 - \sqrt{8}}{1}$$

$$\frac{1}{x} = 3 - \sqrt{8}$$

$$\begin{aligned} x + \frac{1}{x} &= (3 + \sqrt{8}) + (3 - \sqrt{8}) \\ &= 3 + \sqrt{8} + 3 - \sqrt{8} \end{aligned}$$

$$x + \frac{1}{x} = 6$$

$$(ii) x - \frac{1}{x} = ?$$

$$\begin{aligned} x - \frac{1}{x} &= (3 + \sqrt{8}) - (3 - \sqrt{8}) \\ &= 3 + \sqrt{8} - 3 + \sqrt{8} \end{aligned}$$

$$x - \frac{1}{x} = 2\sqrt{8}$$

$$(iii) x^2 + \frac{1}{x^2} = ?$$

As

$$x + \frac{1}{x} = 6$$

Taking square on both sides

$$\left(x + \frac{1}{x}\right)^2 = (6)^2$$

$$(x)^2 + \left(\frac{1}{x}\right)^2 + 2(x)\left(\frac{1}{x}\right) = 36$$

$$x^2 + \frac{1}{x^2} + 2 = 36$$

$$x^2 + \frac{1}{x^2} = 36 - 2$$

$$x^2 + \frac{1}{x^2} = 34$$

$$(iv) x^2 - \frac{1}{x^2} = ?$$

$$\begin{aligned} x^2 - \frac{1}{x^2} &= (x)^2 - \left(\frac{1}{x}\right)^2 \\ &= \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) \\ &= (6)(2\sqrt{8}) \\ &= 12\sqrt{8} \end{aligned}$$

$$(v) x^4 + \frac{1}{x^4} = ?$$

As

$$x^2 + \frac{1}{x^2} = 34$$

Taking square on both sides

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$$\begin{aligned}\left(x^2 + \frac{1}{x^2}\right)^2 &= (34)^2 \\ (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x^2)\left(\frac{1}{x^2}\right) &= 1156 \\ x^4 + \frac{1}{x^4} + 2 &= 1156 \\ x^4 + \frac{1}{x^4} &= 1156 - 2 \\ x^4 + \frac{1}{x^4} &= \mathbf{1154}\end{aligned}$$

(ii) $\left(x - \frac{1}{x}\right)^2 = ?$

As

$$x - \frac{1}{x} = 2\sqrt{8}$$

Taking square on both sides

$$\begin{aligned}\left(x - \frac{1}{x}\right)^2 &= (2\sqrt{8})^2 \\ \left(x - \frac{1}{x}\right)^2 &= (2)^2 (\sqrt{8})^2 \\ \left(x - \frac{1}{x}\right)^2 &= (4)(8) \\ \left(x - \frac{1}{x}\right)^2 &= \mathbf{32}\end{aligned}$$

4. Find the rational numbers p and q such that

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}$$

$$\begin{aligned}\frac{8-3\sqrt{2}}{4+3\sqrt{2}} &= p + q\sqrt{2} \\ \frac{8-3\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} &= p + q\sqrt{2} \\ \frac{8(4-3\sqrt{2}) - 3\sqrt{2}(4-3\sqrt{2})}{(4)^2 - (3\sqrt{2})^2} &= p + q\sqrt{2} \\ \frac{32 - 24\sqrt{2} - 12\sqrt{2} + 9(\sqrt{2})^2}{16 - 9(\sqrt{2})^2} &= p + q\sqrt{2} \\ \frac{32 - 36\sqrt{2} + 9(2)}{16 - 9(2)} &= p + q\sqrt{2} \\ \frac{32 - 36\sqrt{2} + 18}{16 - 18} &= p + q\sqrt{2} \\ \frac{50 - 36\sqrt{2}}{-2} &= p + q\sqrt{2} \\ \frac{50}{-2} - \frac{36\sqrt{2}}{-2} &= p + q\sqrt{2} \\ -25 + 18\sqrt{2} &= p + q\sqrt{2}\end{aligned}$$

By comparing we get,

$$p = -25 \text{ and } q = 18$$

5. Simplify the following:

(i) $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

$$\begin{aligned}&\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}} \\ &= \frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}}} \\ &= \frac{5^3 \times 3^3}{2^5 \times 2^4} \\ &= \frac{125 \times 27}{32 \times 16} \\ &= \mathbf{\frac{3375}{512}}\end{aligned}$$

(ii) $\frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

$$\begin{aligned}&\frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})} \\ &= \frac{54 \times [(3^3)^{2x}]^{\frac{1}{3}}}{(3^2)^{x+1} + 216(3^{2x-1})} \\ &= \frac{54 \times (3^3)^{\frac{2x}{3}}}{3^{2x+2} + 216(3^{2x-1})} \\ &= \frac{54 \times 3^{2x}}{3^{2x}[3^2 + 216(3^{-1})]} \\ &= \frac{54}{3^2 + \frac{216}{3}} \\ &= \frac{54}{9 + 72} \\ &= \frac{54}{81} \\ &= \mathbf{\frac{2}{3}}\end{aligned}$$

(iii) $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$

$$\begin{aligned}&\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}} \\ &= \sqrt{\frac{(6^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{3}{2}}}}\end{aligned}$$

$$\begin{aligned}
&= \sqrt{\frac{(6)^2 \times (5)^1}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}} \\
&= \sqrt{\frac{6^2 \times 5}{(25)^{\frac{3}{2}}}} \\
&= \sqrt{\frac{6^2 \times 5}{(5^2)^{\frac{3}{2}}}} \\
&= \sqrt{\frac{6^2 \times 5}{5^3}} \\
&= \sqrt{\frac{6^2}{5^{3-1}}} \\
&= \sqrt{\frac{6^2}{5^2}} \\
&= \frac{\sqrt{6^2}}{\sqrt{5^2}} \\
&= \frac{6}{5}
\end{aligned}$$

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$$\begin{aligned}
&\text{(iv)} \left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right) \\
&\quad \left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right) \\
&= \left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \left[\left(a^{\frac{1}{3}}\right)^2 - \left(a^{\frac{1}{3}}\right)\left(b^{\frac{2}{3}}\right) + \left(b^{\frac{2}{3}}\right)^2\right] \\
&= \left(a^{\frac{1}{3}}\right)^3 + \left(b^{\frac{2}{3}}\right)^3 \\
&= a + b^2
\end{aligned}$$