Unit 3 Sets and Functions

1. Write the following sets in set builder notation:

(i) $\{1,4,9,16,25,36,\dots,484\}$

$$\{1,4,9,16,25,36,\dots,484\}$$
$$=\{x|x=n^2, n\in N\wedge\ 1\le n\le 22\}$$

Note: This set consists of perfect squares from 1^2 to 22^2 (since $22^2=484$).

(ii) $\{2,4,8,16,\dots,256\}$

$$\{2,4,8,16,\dots,256\}$$
$$=\{x|x=2^n, n\in N\wedge\ 1\le n\le 8\}$$

Note: This set consists of powers of 2 from 2^1 to 2^8 (since $2^8=256$).

(iii) $\{0,\pm1,\pm2,\dots,\pm1000\}$

$$\{0,\pm1,\pm2,\dots,\pm1000\}$$
$$=\{x|x\in Z\wedge\ -1000\le x\le 1000\}$$

Note: This set includes all integers from -1000 to 1000.

(iv) $\{6,12,18,\dots,120\}$

$$\{6,12,18,\dots,120\}$$
$$=\{x|x=6n, n\in N\wedge\ 1\le n\le 20\}$$

Note: This set consists of multiples of 6 from 6×1 to 6×20 (since $6\times20=120$).

(v) $\{100,102,104,\dots,400\}$

$$\{100,102,104,\dots,400\}$$
$$=\{x|x=100+2n, n\in W\wedge\ 0\le n\le 150\}$$

Note: This set includes even numbers from 100 to 400.

(vi) $\{1,3,9,27,81,\dots\}$

$$\{1,3,9,27,81,\dots\}$$
$$=\{x|x=3^n, n\in W\}$$

Note: This set consists of powers of 3 starting from $3^0=1$.

(vii) $\{1,2,4,5,10,20,25,50,100\}$

$$\{1,2,4,5,10,20,25,50,100\}$$
$$=\{x|x\ is\ divisor\ of\ 100, n\in N\wedge\ 1\le n\le 100\}$$

Note: This set includes all positive divisors of 100.

(viii) $\{5,10,15,\dots,100\}$

$$\{5,10,15,\dots,100\}$$
$$=\{x|x=5n, n\in N\wedge\ 1\le n\le 20\}$$

Note: This set consists of multiples of 5 from 5×1 to 5×20 (since $5\times20=100$).

(ix) The set of all integers between -100 and 1000.

$$=\{x|x\in Z\wedge\ -100\le x\le 1000\}$$

Note: This set includes all integers from -100 to 1000.

2. Write each of the following sets in tabular form:

(i) $\{x\mid x\ is\ a\ multiple\ of\ 3\wedge x\le 36\}$

$$\{x\mid x\ is\ a\ multiple\ of\ 3\wedge x\le 36\}$$
$$=\{0,3,6,9,12,15,18,21,24,27,30,33,36\}$$

(ii) $\{x\mid x\in R\wedge 2x+1=0\}$

$$2x+1=0$$
$$2x=-1$$
$$x=-\frac{1}{2}$$
$$\{x\mid x\in R\wedge 2x+1=0\}$$
$$=\left\{-\frac{1}{2}\right\}$$

(iii) $\{x\mid x\in P\wedge x<12\}$

$$\{x\mid x\in P\wedge x<12\}$$
$$=\{2,3,5,7,11\}$$

(iv) $\{x\mid x\ is\ a\ divisor\ of\ 128\}$

$$\{x\mid x\ is\ a\ divisor\ of\ 128\}$$
$$=\{1,2,4,8,16,32,64,128\}$$

(v) $\{x|x=2^n, n\in N\wedge\ n<8\}$

$$\{x|x=2^n, n\in N\wedge\ n<8\}$$
$$=\{2,4,8,16,32,64,128\}$$

(vi) $\{x\mid x\in N\wedge x+4=0\}$

$$\{x\mid x\in N\wedge x+4=0\}$$
$$=\{\ \ \}$$

As $x+4=0\Rightarrow x=-4\notin N$

(vii) $\{x\mid x\in N\wedge x=x\}$

$$\{x\mid x\in N\wedge x=x\}$$
$$=\{1,2,3,\dots\}$$

(viii) $\{x\mid x\in Z\wedge 3x+1=0\}$

$$\{x\mid x\in Z\wedge 3x+1=0\}$$
$$=\{\ \ \}$$

As $3x+1=0\Rightarrow x=-\frac{1}{3}\notin Z$

3. Write two proper subsets of each of the following sets:

(i) $\{a,b,c\}$

The Proper subsets of $\{a,b,c\}$ are $\{a\}$, $\{b\}$

(ii) $\{0,1\}$

The Proper subsets of $\{0,1\}$ are $\{0\}$, $\{1\}$

(iii) N

The Proper subsets of N are Prime Numbers, Even Natural Numbers

(iv) Z

The Proper subsets of Z are Prime Numbers, Whole Numbers

(v) Q

The Proper subsets of Q are Integers, Natural Numbers

(vi) $\{x|x \in Q \wedge 0 < x \leq 2\}$

The Proper subsets of $\{x|x \in Q \wedge 0 < x \leq 2\}$ are $\left\{\frac{1}{2}\right\}$, $\left\{\frac{3}{4}\right\}$

4. Is there any set which has no proper subset? If so, name that set.

Yes, there exist a set which has no proper subset, that is called 'empty set' $\{\quad\}$.

5. What is the difference between $\{a,b\}$ and $\{\{a,b\}\}$?

$\{a,b\}$ set has two elements: a and b. While $\{\{a,b\}\}$ set has only one element $\{a,b\}$.

6. What is the number of elements of the power set of each of the following sets?

(i) $\{\quad\}$

$$\begin{aligned} No.of\ elements\ of\ the\ power\ set &= 2^n \\ &= 2^0 \\ &= 1 \end{aligned}$$

(ii) $\{0,1\}$

$$\begin{aligned} No.of\ elements\ of\ the\ power\ set &= 2^n \\ &= 2^2 \\ &= 4 \end{aligned}$$

(iii) $\{1,2,3,4,5,6,7\}$

$$\begin{aligned} No.of\ elements\ of\ the\ power\ set &= 2^n \\ &= 2^7 \\ &= 128 \end{aligned}$$

(iv) $\{0,1,2,3,4,5,6,7\}$

$$\begin{aligned} No.of\ elements\ of\ the\ power\ set &= 2^n \\ &= 2^8 \\ &= 256 \end{aligned}$$

(v) $\{a,\{b,c\}\}$

$$\begin{aligned} No.of\ elements\ of\ the\ power\ set &= 2^n \\ &= 2^2 \\ &= 4 \end{aligned}$$

(vi) $\{\{a,b\},\{b,c\},\{d,e\}\}$

$$\begin{aligned} No.of\ elements\ of\ the\ power\ set &= 2^n \\ &= 2^3 \\ &= 8 \end{aligned}$$

7. Write down the power set of each of the following sets:

(i) $\{9,11\}$

Let

$$A = \{9,11\}$$
$$P(A) = \{\emptyset,\{9\},\{11\},\{9,11\}\}$$

(ii) $\{+,-,\times,\div\}$

Let

$$A = \{+,-,\times,\div\}$$
$$\begin{aligned} P(A) = \{&\emptyset,\{+\},\{-\},\{\times\},\{\div\}, \\ &\{+,-\},\{+,\times\},\{+,\div\}, \\ &\{-,\times\},\{-,\div\},\{\times,\div\}, \\ &\{+,-,\times\},\{+,-,\div\},\{-,\times,\div\}, \\ &\{+,\times,\div\},\{+,-,\times,\div\}\} \end{aligned}$$

(iii) $\{\emptyset\}$

Let

$$A = \{\emptyset\}$$
$$P(A) = \{\emptyset,\{\emptyset\}\}$$

(iv) $\{a,\{b,c\}\}$

Let

$$A = \{a,\{b,c\}\}$$
$$P(A) = \{\emptyset,\{a\},\{\{b,c\}\},\{a,\{b,c\}\}\}$$