Review Exercise 3

2. Write each of the following sets in tabular forms:

(i) $\{x \mid x = 2n, n \in N\}$

$$\{x \mid x = 2n, n \in N\}$$
$$= \{2,4,6,8, \dots\}$$

(ii) $\{x \mid x = 2m + 1, m \in N\}$

$$\{x \mid x = 2m + 1, m \in N\}$$
$$= \{3,5,7,9,11, \dots\}$$

(iii) $\{x \mid x = 11n, n \in W \wedge n < 11\}$

$$\{x \mid x = 11n, n \in W \wedge n < 11\}$$
$$= \{0,11,22,33,44,55,66,77,88,99,110\}$$

(iv) $\{x \mid x \in E \wedge 4 < x < 6\}$

$$\{x \mid x \in E \wedge 4 < x < 6\}$$
$$= \{\quad\}$$

(v) $\{x \mid x \in Q \wedge 5 < x < 7\}$

$$\{x \mid x \in Q \wedge 5 < x < 7\}$$
$$= \{\quad\} \qquad (book\ answer)$$

Note: This set includes all rational numbers between 5 and 7. Since there are infinitely many, we cannot list them all.

(vi) $\{x \mid x \in Q \wedge x^2 = 2\}$

$$\{x \mid x \in Q \wedge x^2 = 2\}$$
$$= \{\quad\}$$

Note: There are no rational numbers whose square is 2.

(vii) $\{x \mid x \in Q \wedge x = -x\}$

$$\{x \mid x \in Q \wedge x = -x\}$$
$$= \{0\}$$

(viii) $\{x \mid x \in R \wedge x \notin Q'\}$

$$\{x \mid x \in R \wedge x \notin Q'\}$$
$$= Q$$

3.Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5\}$, $C = \{1, 3, 5, 7, 9\}$ **List the members of each of the following sets:**

(i) A'

$$A' = U - A$$
$$= \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}$$
$$= \{1,3,5,7,9\}$$

(ii) B'

$$B' = U - A$$
$$B' = \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5\}$$
$$= \{6,7,8,9,10\}$$

(iii) $A \cup B$

$$A \cup B = \{2,4,6,8,10\} \cup \{1,2,3,4,5\}$$
$$= \{1,2,3,4,5,6,8,10\}$$

(iv) $A - B$

$$A \cup B = \{2,4,6,8,10\} - \{1,2,3,4,5\}$$
$$= \{6,8,10\}$$

(v) $A \cap C$

$$A \cup C = \{2,4,6,8,10\} \cap \{1,3,5,7,9\}$$
$$= \{\quad\}$$

(vi) $A' \cup C'$

$$A' = U - A$$
$$= \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}$$
$$= \{1,3,5,7,9\}$$

$$C' = U - C$$
$$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,3,5,7,9\}$$
$$= \{2,4,6,8,10\}$$

$$A' \cup C' = \{1,3,5,7,9\} \cup \{2,4,6,8,10\}$$
$$= \{1,2,3,4,5,6,7,8,9,10\}$$

(vii) $A' \cup C$

$$A' = U - A$$
$$= \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}$$
$$= \{1,3,5,7,9\}$$

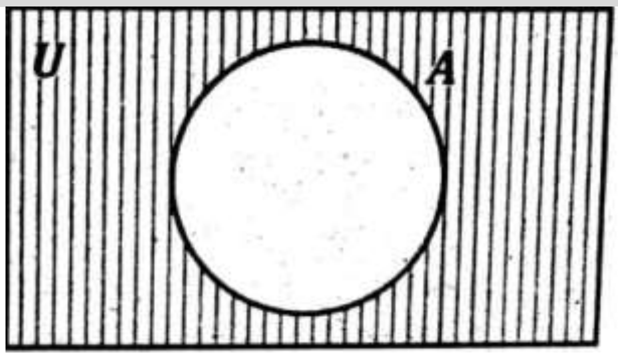
$$A' \cup C = \{1,3,5,7,9\} \cup \{1,3,5,7,9\}$$
$$= \{1,3,5,7,9\}$$

(viii) U'

$$U' = U - A$$
$$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5,6,7,8,9,10\}$$
$$= \{\quad\}$$

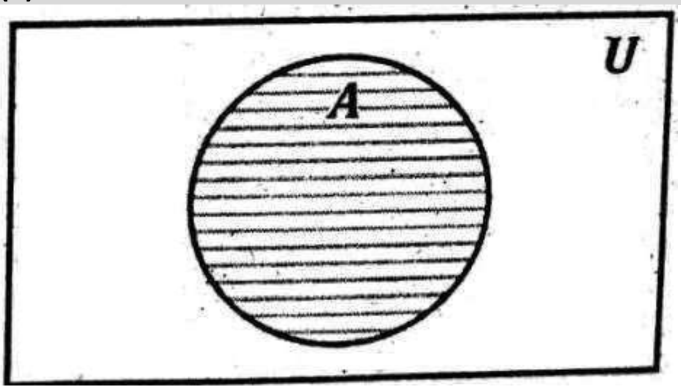
4. Using the Venn diagrams, if necessary, find the single sets equal to the following:

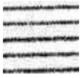
(i) A'



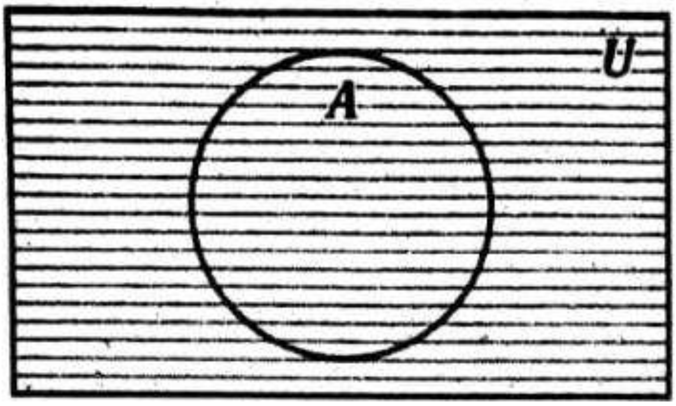
$A' =$ 

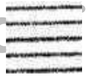
(ii) $A \cap U$



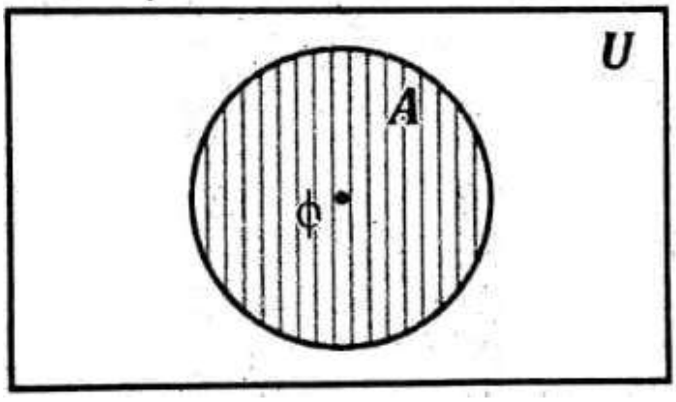
$A \cap U =$ 

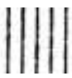
(iii) $A \cup U$



$A \cup U =$ 

(iv) $A \cup \emptyset$



$A \cup \emptyset =$ 

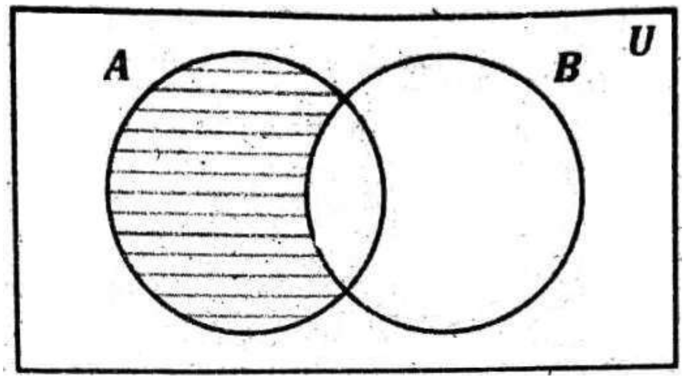
(v) $\emptyset \cup \emptyset$

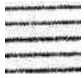
The intersection of the empty set $\emptyset$ with itself is the set of elements common to both sets. Since the empty set contains no elements, its intersection with itself also results in the empty set.

5. Use Venn diagrams to verify the following:

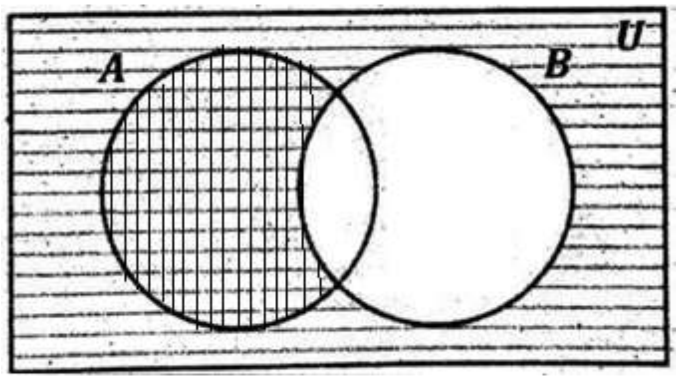
(i) $A - B = A \cap B'$

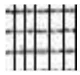
$$L.H.S = A - B$$



$A - B =$  .... (i)

$$R.H.S = A \cap B'$$



$A \cap B' =$  .... (ii)

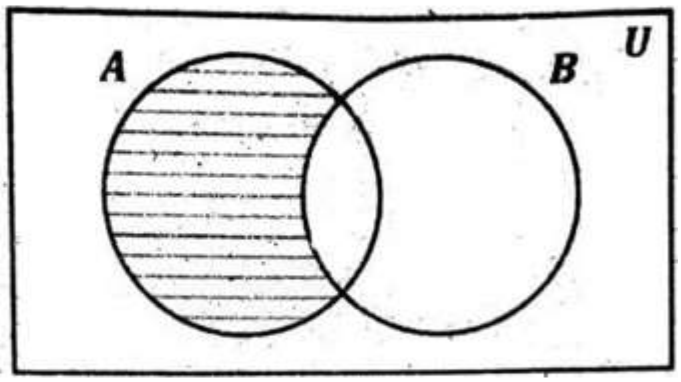
From (i) and (ii)

$$L.H.S = R.H.S$$

(ii) $(A - B)' \cap B = B$

$$L.H.S = (A - B)' \cap B$$

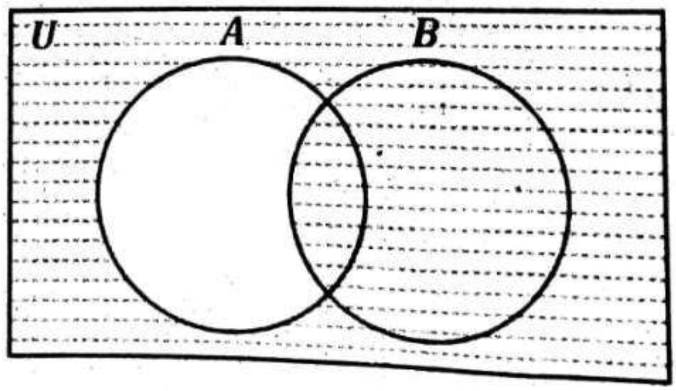
Venn diagram of $A - B$

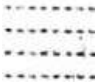


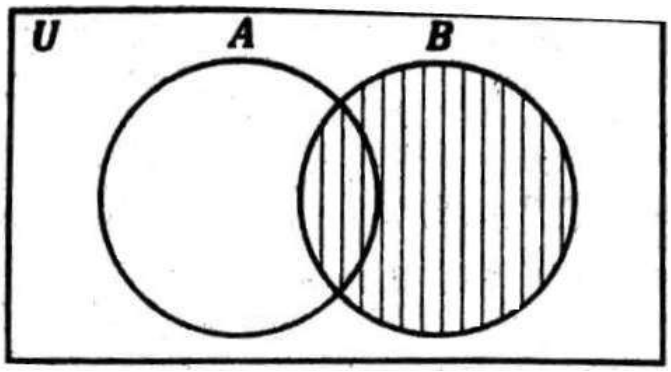
$A - B =$ 

Now for

$$(A - B)' = U - (A - B)$$

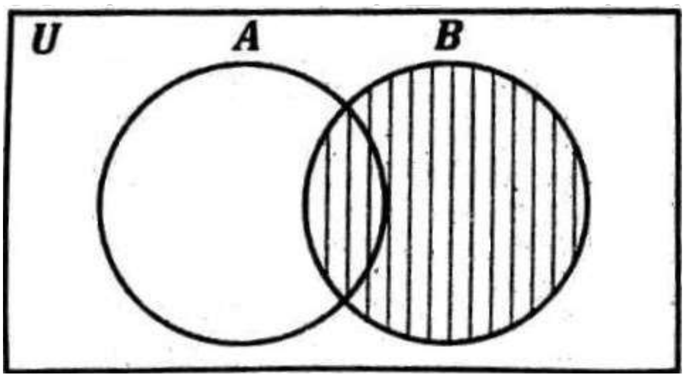


$(A-B)' =$ 



$(A-B)' \cap B =$  .... (i)

$R.H.S = B$



$B =$  .... (ii)

From (i) and (ii)

$$L.H.S = R.H.S$$

6. Verify the properties for the sets A, B, and C given below:
(i) Associativity of union.
(ii) Associativity of intersection.
(iii) Distributivity of union over intersection.
(iv) Distributivity of intersection over union.
(a) $A = \{1,2,3,4\}$, $B = \{3,4,5,6,7,8\}$, $C = \{5,6,7,9,10\}$

Associativity of union:

$$(\boldsymbol{A} \cup \boldsymbol{B}) \cup \boldsymbol{C} = \boldsymbol{A} \cup (\boldsymbol{B} \cup \boldsymbol{C})$$

$$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6,7,8\}$$
$$= \{1,2,3,4,5,6,7,8\}$$

$$B \cup C = \{3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$$
$$B \cup C = \{3,4,5,6,7,8,9,10\}$$

$$L.H.S = (A \cup B) \cup C$$
$$= \{1,2,3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$$
$$= \{1,2,3,4,5,6,7,8,9,10\}$$

$$R.H.S = A \cup (B \cup C)$$
$$= \{1,2,3,4\} \cup \{3,4,5,6,7,8,9,10\}$$
$$= \{1,2,3,4,5,6,7,8,9,10\}$$

Hence

$$L.H.S = R.H.S$$

Associativity of intersection:

$$(\boldsymbol{A} \cap \boldsymbol{B}) \cap \boldsymbol{C} = \boldsymbol{A} \cap (\boldsymbol{B} \cap \boldsymbol{C})$$

$$A \cap B = \{1,2,3,4\} \cap \{3,4,5,6,7,8\}$$
$$= \{3,4\}$$

$$B \cap C = \{3,4,5,6,7,8\} \cap \{5,6,7,9,10\}$$
$$B \cap C = \{5,6,7\}$$

$$L.H.S = (A \cap B) \cap C$$
$$= \{3,4\} \cap \{5,6,7,9,10\}$$
$$= \{\ \}$$

$$R.H.S = A \cap (B \cap C)$$
$$= \{1,2,3,4\} \cap \{5,6,7\}$$
$$= \{\ \}$$

Hence

$$L.H.S = R.H.S$$

Distributivity of union over intersection:

$$\boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C}) = (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$$

$$B \cap C = \{3,4,5,6,7,8\} \cap \{5,6,7,9,10\}$$
$$= \{5,6,7\}$$

$$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6,7,8\}$$
$$= \{1,2,3,4,5,6,7,8\}$$

$$A \cup C = \{1,2,3,4\} \cup \{5,6,7,9,10\}$$
$$= \{1,2,3,4,5,6,7,9,10\}$$

$$L.H.S = A \cup (B \cap C)$$
$$= \{1,2,3,4\} \cup \{5,6,7\}$$
$$= \{1,2,3,4,5,6,7\}$$

$$R.H.S = (A \cup B) \cap (A \cup C)$$
$$= \{1,2,3,4,5,6,7,8\} \cap \{1,2,3,4,5,6,7,9,10\}$$
$$= \{1,2,3,4,5,6,7\}$$

Hence

$$L.H.S = R.H.S$$

Distributivity of intersection over union:

$$\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C})$$

$$B \cup C = \{3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$$

$$= \{3,4,5,6,7,8,9,10\}$$

$$A \cap B = \{1,2,3,4\} \cap \{3,4,5,6,7,8\}$$
$$= \{3,4\}$$

$$A \cap C = \{1,2,3,4\} \cap \{5,6,7,9,10\}$$
$$= \{\ \ \}$$

$$L.H.S = A \cap (B \cup C)$$
$$= \{1,2,3,4\} \cap \{3,4,5,6,7,8,9,10\}$$
$$= \{3,4\}$$

$$R.H.S = (A \cap B) \cup (A \cap C)$$
$$= \{3,4\} \cup \{\ \ \}$$
$$= \{3,4\}$$

Hence

$$L.H.S = R.H.S$$

(b) $\boldsymbol{A = \emptyset = \{\ \ \}, B = \{0\}, C = \{0,1,2\}}$

Associativity of union:

$$(\boldsymbol{A} \cup \boldsymbol{B}) \cup \boldsymbol{C} = \boldsymbol{A} \cup (\boldsymbol{B} \cup \boldsymbol{C})$$
$$A \cup B = \{\ \ \} \cup \{0\}$$
$$= \{0\}$$

$$B \cup C = \{0\} \cup \{0,1,2\}$$
$$= \{0,1,2\}$$

$$L.H.S = (A \cup B) \cup C$$
$$= \{0\} \cup \{0,1,2\}$$
$$= \{0,1,2\}$$

$$R.H.S = A \cup (B \cup C)$$
$$= \{\ \ \} \cup \{0,1,2\}$$
$$= \{0,1,2\}$$

Hence

$$L.H.S = R.H.S$$

Associativity of intersection:

$$(\boldsymbol{A} \cap \boldsymbol{B}) \cap \boldsymbol{C} = \boldsymbol{A} \cap (\boldsymbol{B} \cap \boldsymbol{C})$$
$$A \cap B = \{\ \ \} \cap \{0\}$$
$$= \{\ \ \}$$

$$B \cap C = \{0\} \cap \{0,1,2\}$$
$$B \cap C = \{0\}$$

$$L.H.S = (A \cap B) \cap C$$
$$= \{\ \ \} \cap \{0,1,2\}$$
$$= \{\ \ \}$$

$$R.H.S = A \cap (B \cap C)$$
$$= \{\ \ \} \cap \{0\}$$
$$= \{\ \ \}$$

Hence

$$L.H.S = R.H.S$$

Distributivity of union over intersection:

$$\boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C}) = (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$$
$$B \cap C = \{0\} \cap \{0,1,2\}$$
$$= \{0\}$$

$$A \cup B = \{\ \ \} \cup \{0\}$$
$$= \{0\}$$

$$A \cup C = \{\ \ \} \cup \{0,1,2\}$$
$$= \{0,1,2\}$$

$$L.H.S = A \cup (B \cap C)$$
$$= \{\ \ \} \cup \{0\}$$
$$= \{0\}$$

$$R.H.S = (A \cup B) \cap (A \cup C)$$
$$= \{0\} \cap \{0,1,2\}$$
$$= \{0,1,2\}$$

Hence

$$L.H.S = R.H.S$$

Distributivity of intersection over union:

$$\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C})$$
$$B \cup C = \{0\} \cup \{0,1,2\}$$
$$= \{0,1,2\}$$

$$A \cap B = \{\ \ \} \cap \{0\}$$
$$= \{\ \ \}$$

$$A \cap C = \{\ \ \} \cap \{0,1,2\}$$
$$= \{\ \ \}$$

$$L.H.S = A \cap (B \cup C)$$
$$= \{\ \ \} \cap \{0,1,2\}$$
$$= \{\ \ \}$$

$$R.H.S = (A \cap B) \cup (A \cap C)$$
$$= \{\ \ \} \cup \{\ \ \}$$
$$= \{\ \ \}$$

Hence

$$L.H.S = R.H.S$$

(c) $\boldsymbol{A = N, B = Z, C = Q}$

Note: $N \subset Z \sqsubset Q$

Associativity of union:

$$(\boldsymbol{A} \cup \boldsymbol{B}) \cup \boldsymbol{C} = \boldsymbol{A} \cup (\boldsymbol{B} \cup \boldsymbol{C})$$
$$A \cup B = N \cup Z$$
$$= Z$$

$$B \cup C = Z \cup Q$$
$$B \cup C = Q$$

$$L.H.S = (A \cup B) \cup C$$
$$= Z \cup Q$$
$$= Q$$

$$R.H.S = A \cup (B \cup C)$$

$$= N \cup Q$$
$$= Q$$

Hence

$$L.H.S = R.H.S$$

Associativity of intersection:

$$(\boldsymbol{A} \cap \boldsymbol{B}) \cap \boldsymbol{C} = \boldsymbol{A} \cap (\boldsymbol{B} \cap \boldsymbol{C})$$

$$A \cap B = N \cap Z$$
$$= N$$

$$B \cap C = Z \cap Q$$
$$= Z$$

$$L.H.S = (A \cap B) \cap C$$
$$= N \cap Q$$
$$= N$$

$$R.H.S = A \cap (B \cap C)$$
$$= N \cap Z$$
$$= N$$

Hence

$$L.H.S = R.H.S$$

Distributivity of union over intersection:

$$\boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C}) = (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$$

$$B \cap C = Z \cap Q$$
$$= Z$$

$$A \cup B = N \cup Z$$
$$= Z$$

$$A \cup C = N \cup Q$$
$$= Q$$

$$L.H.S = A \cup (B \cap C)$$
$$= N \cup Z$$
$$= Z$$

$$R.H.S = (A \cup B) \cap (A \cup C)$$
$$= Z \cap Q$$
$$= Z$$

Hence

$$L.H.S = R.H.S$$

Distributivity of intersection over union:

$$\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C})$$

$$B \cup C = Q \cup Z$$
$$= Q$$

$$A \cap B = N \cap Q$$
$$= N$$

$$A \cap C = N \cap Q$$
$$= N$$

$$L.H.S = A \cap (B \cup C)$$
$$= N \cap Q$$
$$= N$$

$$R.H.S = (A \cap B) \cup (A \cap C)$$
$$= N \cup N$$
$$= N$$

Hence

$$L.H.S = R.H.S$$

7. Verify De Morgan's Laws for the following sets: $\boldsymbol{U} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \dots, \mathbf{20}\}$, $\boldsymbol{A} = \{\mathbf{2}, \mathbf{4}, \mathbf{6}, \dots, \mathbf{20}\}$ and $\boldsymbol{B} = \{\mathbf{1}, \mathbf{3}, \mathbf{5}, \dots, \mathbf{19}\}$

De Morgan's Laws: (i) $(\boldsymbol{A} \cup \boldsymbol{B})' = \boldsymbol{A}' \cap \boldsymbol{B}'$ (ii) $(\boldsymbol{A} \cap \boldsymbol{B})' = \boldsymbol{A}' \cup \boldsymbol{B}'$

(i) $(\boldsymbol{A} \cup \boldsymbol{B})' = \boldsymbol{A}' \cap \boldsymbol{B}'$

$$A \cup B = \{2,4,6, \dots ,20\} \cup \{1,3,5, \dots ,19\}$$
$$= \{1,2,3, \dots ,20\}$$

$$A' = U - A$$
$$A' = \{1,2,3, \dots ,20\} - \{2,4,6, \dots ,20\}$$
$$A' = \{1,3,5, \dots ,19\}$$

$$B' = U - B$$
$$B' = \{1,2,3, \dots ,20\} - \{1,3,5, \dots ,19\}$$
$$B' = \{2,4,6, \dots ,20\}$$

$$L.H.S = (A \cup B)'$$
$$= U - (A \cup B)$$
$$= \{1,2,3, \dots ,20\} - \{1,2,3, \dots ,20\}$$
$$L.H.S = \{\quad\}$$

$$R.H.S = A' \cap B'$$
$$= \{1,3,5, \dots ,19\} \cap \{2,4,6, \dots ,20\}$$

$$R.H.S = \{\ \ \}$$

Hence Proved

$$L.H.S = R.H.S$$

(i) $(A \cap B)' = A' \cup B'$

$$A \cap B = \{2,4,6,\dots,20\} \cap \{1,3,5,\dots,19\}$$
$$= \{\ \ \}$$

$$A' = U - A$$
$$A' = \{1,2,3,\dots,20\} - \{2,4,6,\dots,20\}$$
$$A' = \{1,3,5,\dots,19\}$$

$$B' = U - B$$
$$B' = \{1,2,3,\dots,20\} - \{1,3,5,\dots,19\}$$
$$B' = \{2,4,6,\dots,20\}$$

$$L.H.S = (A \cap B)'$$
$$= U - (A \cap B)$$
$$= \{1,2,3,\dots,20\} - \{\ \ \}$$
$$L.H.S = \{1,2,3,\dots,20\}$$

$$R.H.S = A' \cup B'$$
$$= \{1,3,5,\dots,19\} \cup \{2,4,6,\dots,20\}$$
$$R.H.S = \{1,2,3,\dots,20\}$$

Hence Proved

$$L.H.S = R.H.S$$

8. Consider the set $P = \{x \mid x = 5m, m \in N\}$ and $Q = \{x \mid x = 2m, m \in N\}$. Find $P \cap Q$.

$$P = \{x \mid x = 5m, m \in N\}$$
$$P = \{5,10,15,20,\dots\}$$
$$Q = \{x \mid x = 2m, m \in N\}$$
$$Q = \{2,4,6,8,\dots\}$$

$$P \cap Q = \{5,10,15,20,\dots\} \cap \{2,4,6,8,\dots\}$$
$$= \{10,20,30,\dots\}$$

9. From suitable properties of union and intersection, deduce the following results:
(i) $A \cap (A \cup B) = A \cup (A \cap B)$

$$L.H.S = A \cap (A \cup B)$$
$$= (A \cap A) \cup (A \cap B)$$
$$= A \cup (A \cap B) \qquad \because A \cap A = A$$
$$= R.H.S$$

(ii) $A \cup (A \cap B) = A \cap (A \cup B)$

$$L.H.S = A \cup (A \cap B)$$
$$= (A \cup A) \cap (A \cup B)$$
$$= A \cap (A \cup B) \qquad \because A \cup A = A$$
$$= R.H.S$$

10. If $g(x) = 7x - 2$ and $s(x) = 8x^2 - 3$, then find:
(i) $g(0)$

Since

$$g(x) = 7x - 2$$

Put $x = 0$

$$\begin{aligned} g(0) &= 7(0) - 2 \\ &= 0 - 2 \\ &= -2 \end{aligned}$$

(ii) $g(-1)$

Since

$$g(x) = 7x - 2$$

Put $x = -1$

$$\begin{aligned} g(-1) &= 7(-1) - 2 \\ &= -7 - 2 \\ &= -9 \end{aligned}$$

(iii) $g\left(-\frac{5}{3}\right)$

Since

$$g(x) = 7x - 2$$

Put $x = -\frac{5}{3}$

$$\begin{aligned} g\left(-\frac{5}{3}\right) &= 7\left(-\frac{5}{3}\right) - 2 \\ &= \frac{-35 - 6}{3} \\ &= \frac{-41}{3} \end{aligned}$$

(iv) $s(1)$

Since

$$s(x) = 8x^2 - 3$$

Put $x = 1$

$$\begin{aligned} s(1) &= 8(1)^2 - 3 \\ &= 8 - 3 \\ &= 5 \end{aligned}$$

(v) $s(-4)$

Since

$$s(x) = 8x^2 - 3$$

Put $x = -9$

$$\begin{aligned} s(-9) &= 8(-9)^2 - 3 \\ &= 8(81) + 1 \\ &= 648 - 3 \\ &= 645 \end{aligned}$$

(v) $s\left(\frac{7}{2}\right)$

Since

$$s(x) = 8x^2 - 3$$

Put $x = \frac{7}{2}$

$$\begin{aligned} s\left(\frac{7}{2}\right) &= 8\left(\frac{7}{2}\right)^2 - 3 \\ &= 8\left(\frac{49}{4}\right) - 3 \\ &= \frac{392}{4} - 3 \end{aligned}$$

$$= \frac{392 - 12}{4}$$
$$= \frac{380}{4}$$
$$= 95$$

11. Given that $f(x) = ax + b$, where a and b are constant numbers. If $f(-2) = 3$ and $f(4) = 10$, then find the values of a and b.

Since
$$f(x) = ax + b$$
Put $x = -2$
$$f(-2) = a(-2) + b$$
$$3 = -2a + b \quad \dots \quad (i) \qquad \because f(-2) = 3$$
Now
$$f(x) = ax + b$$
Put $x = 4$
$$f(4) = a(4) + b + 1$$
$$10 = 4a + b \quad \dots \quad (ii) \qquad \because f(4) = 10$$
Subtract equation (i) from equation (ii)
$$10 = 4a + b$$
$$\pm 3 = \mp 2a \pm b$$

$$7 = 6a$$
$$\mathbf{\frac{7}{6} = a}$$
Putting $a = \frac{7}{6}$ in equation (i)

$$3 = -2\left(\frac{7}{6}\right) + b$$
$$3 = \frac{-14}{6} + b$$
$$3 + \frac{14}{6} = b$$
$$\frac{18 + 14}{6} = b$$
$$\frac{32}{6} = b$$
$$\mathbf{\frac{16}{3} = b}$$

12. Consider the function defined by $k(x) = 7x - 5$. If $k(x) = 100$, find the value of x.

Since
$$k(x) = 7x - 5$$
Put $k(x) = 100$
$$100 = 7x - 5$$
$$100 + 5 = 7x$$
$$105 = 7x$$
$$\frac{105}{7} = x$$
$$15 = x$$
$$\mathbf{x = 15}$$

13. Consider the function $g(x) = mx^2 + n$, where m and n are constant numbers. If $g(4) = 20$ and $g(0) = 5$, find the values of m and n.

Since

$$g(x) = mx^2 + n$$

Put $x = 0$

$$\begin{aligned} g(0) &= mx^2 + n \\ 5 &= m(0)^2 + n \qquad \because g(0) = 5 \\ 5 &= 0 + n \\ \boldsymbol{n} &= \mathbf{5} \end{aligned}$$

Now

$$g(x) = mx^2 + n$$

Put $x = 4$

$$\begin{aligned} g(4) &= m(4)^2 + n \\ 20 &= 16m + 5 \qquad \because g(4) = 20 \,\&\, n = 5 \\ 20 - 5 &= 16m \\ 15 &= 16m \\ \frac{15}{16} &= m \\ \boldsymbol{m} &= \frac{\mathbf{15}}{\mathbf{16}} \end{aligned}$$

14. A shopping mall has 100 products from various categories labeled 1 to 100, representing the universal set U. The products are categorized as follows:

- **Set A: Electronics, consisting of 30 products labeled from 1 to 30.**
- **Set B: Clothing, comprising 25 products labeled from 31 to 55.**
- **Set C: Beauty Products, comprising 25 products labeled from 76 to 100.**

Tabular Form of Sets:

$$\begin{aligned} A &= \{1,2,3, \dots, 30\} \\ B &= \{31,32,33, \dots, 55\} \\ C &= \{76,77,78, \dots, 100\} \end{aligned}$$

Union of All Three Sets:

$$\begin{aligned} A \cup B \cup C &= \{1,2,3, \dots, 30\} \cup \{31,32,33, \dots, 55\} \cup \{76,77,78, \dots, 100\} \\ &= \{1,2, \dots, 55,76,77,78, \dots, 100\} \end{aligned}$$

15. Out of the 180 students who appeared in the annual examination, 120 passed the math test, 90 passed the science test, and 60 passed both the math and science tests.

(a) How many passed either the math or science test?

Let

$$\begin{aligned} M &= Set\ of\ students\ who\ passed\ math \\ S &= Set\ of\ students\ who\ passed\ science \end{aligned}$$

According to question: $n(U) = 180,\ n(M) = 120,\ n(S) = 90,\ n(M \cap S) = 60$

Using Inclusion-Exclusion Principle,

$$\begin{aligned} n(M \cup S) &= n(M) + n(S) - n(M \cap S) \\ n(M \cup S) &= 120 + 90 - 60 \\ \boldsymbol{n(M \cup S)} &= \mathbf{150}\ \boldsymbol{students} \end{aligned}$$

(b) How many did not pass either of the two tests.

$$\begin{aligned} n(M \cup S)' &= n(U) - n(M \cup S) \\ &= 180 - 150 \\ &= \mathbf{30}\ \boldsymbol{students} \end{aligned}$$

(c) How many passed the science test but not the math test?

$$\begin{aligned} Students\ passed\ science\ only\ but\ not\ math &= n(S) - n(M \cap S) \\ &= 90 - 60 \\ &= \mathbf{30}\ \boldsymbol{students} \end{aligned}$$

(d) How many failed the science test?

$$\begin{aligned} Students\ who\ failed\ the\ science\ test &= n(U) - n(S) \\ &= 180 - 90 \\ &= \mathbf{90}\ \boldsymbol{students} \end{aligned}$$

16. In a Software house of city with 300 software developers, a survey was conducted to determine which programming languages are liked more. The survey revealed the following statistics:

- **150 developers like Python.**
- **130 developers like Java.**
- **120 developers like PHP.**
- **70 developers like both Python and Java.**
- **60 developers like both Python and PHP.**
- **50 developers like both Java and PHP.**
- **40 developers like all three languages: Python, Java and PHP.**

(a) How many developers use at least one of these languages?

Let

$$\begin{aligned} P &= Set\ of\ developers\ who\ like\ Python \\ J &= Set\ of\ developers\ who\ like\ Java \\ H &= Set\ of\ developers\ who\ like\ PHP \end{aligned}$$

According to question: $n(U) = 300$, $n(P) = 150$, $n(J) = 130$, $n(H) = 120$, $n(P \cap J) = 70$, $n(P \cap H) = 60$, $n(J \cap H) = 50$, $n(P \cap J \cap H) = 40$

Using Inclusion-Exclusion Principle,

$$n(P \cup J \cup H) = n(P) + n(J) + n(H) - n(P \cap J) - n(P \cap H) - n(J \cap H) + n(P \cap J \cap H)$$
$$n(P \cup J \cup H) = 150 + 130 + 120 - 70 - 60 - 50 + 40$$
$$\boldsymbol{n(P \cup J \cup H) = 260}$$

(b) How many developers use only one of these languages?

$$\begin{aligned} Developers\ use\ only\ Python &= n(P) - n(P \cap J) - n(P \cap H) + n(N \cap B \cap K) \\ &= 150 - 70 - 60 + 40 \\ &= 60 \end{aligned}$$

$$\begin{aligned} Developers\ use\ only\ Java &= n(J) - n(P \cap J) - n(J \cap H) + n(N \cap B \cap K) \\ &= 130 - 70 - 50 + 40 \\ &= 50 \end{aligned}$$

$$\begin{aligned} Developers\ use\ only\ PHP &= n(H) - n(P \cap H) - n(J \cap H) + n(N \cap B \cap K) \\ &= 120 - 60 - 50 + 40 \\ &= 50 \end{aligned}$$

$$\boldsymbol{Total\ = 60 + 50 + 50 = 160\ \ developers}$$

(c) How many developers do not use any of these languages?

$$\begin{aligned} n(P \cup J \cup H)' &= n(U) - n(P \cup J \cup H) \\ &= 300 - 260 \\ \boldsymbol{n(N \cup B \cup K)'} &= \boldsymbol{40\ \ developers} \end{aligned}$$

(d) How many developers use only PHP?

$$Developers\ use\ only\ PHP = n(H) - n(P \cap H) - n(J \cap H) + n(N \cap B \cap K)$$

$$= 120 - 60 - 50 + 40$$

$$\boldsymbol{Developers\ use\ only\ PHP = 50}$$