Review Exercise 3

2. Write each of the following sets in tabular forms:

(i) $\{x \mid x = 2n, n \in N\}$

$$\{x \mid x = 2n, n \in N\}$$
$$= \{2,4,6,8, \dots\}$$

(ii) $\{x \mid x = 2m + 1, m \in N\}$

$$\{x \mid x = 2m + 1, m \in N\}$$
$$= \{3,5,7,9,11, \dots\}$$

(iii) $\{x \mid x = 11n, n \in W \wedge n < 11\}$

$$\{x \mid x = 11n, n \in W \wedge n < 11\}$$
$$= \{0,11,22,33,44,55,66,77,88,99,110\}$$

(iv) $\{x \mid x \in E \wedge 4 < x < 6\}$

$$\{x \mid x \in E \wedge 4 < x < 6\}$$
$$= \{\ \ \}$$

(v) $\{x \mid x \in Q \wedge 5 < x < 7\}$

$$\{x \mid x \in Q \wedge 5 < x < 7\}$$
$$= \{\ \ \} \qquad (book\ answer)$$

Note: This set includes all rational numbers between 5 and 7. Since there are infinitely many, we cannot list them all.

(vi) $\{x \mid x \in Q \wedge x^2 = 2\}$

$$\{x \mid x \in Q \wedge x^2 = 2\}$$
$$= \{\ \ \}$$

Note: There are no rational numbers whose square is 2.

(vii) $\{x \mid x \in Q \wedge x = -x\}$

$$\{x \mid x \in Q \wedge x = -x\}$$
$$= \{0\}$$

(viii) $\{x \mid x \in R \wedge x \notin Q'\}$

$$\{x \mid x \in R \wedge x \notin Q'\}$$
$$= Q$$

3.Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5\}$, $C = \{1, 3, 5, 7, 9\}$ **List the members of each of the following sets:**

(i) A'

$$A' = U - A$$
$$= \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}$$
$$= \{1,3,5,7,9\}$$

(ii) B'

$$B' = U - A$$
$$B' = \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5\}$$
$$= \{6,7,8,9,10\}$$

(iii) $A \cup B$

$$A \cup B = \{2,4,6,8,10\} \cup \{1,2,3,4,5\}$$
$$= \{1,2,3,4,5,6,8,10\}$$

(iv) $A - B$

$$A \cup B = \{2,4,6,8,10\} - \{1,2,3,4,5\}$$
$$= \{6,8,10\}$$

(v) $A \cap C$

$$A \cup C = \{2,4,6,8,10\} \cap \{1,3,5,7,9\}$$
$$= \{\ \ \}$$

(vi) $A' \cup C'$

$$A' = U - A$$
$$= \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}$$
$$= \{1,3,5,7,9\}$$

$$C' = U - C$$
$$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,3,5,7,9\}$$
$$= \{2,4,6,8,10\}$$

$$A' \cup C' = \{1,3,5,7,9\} \cup \{2,4,6,8,10\}$$
$$= \{1,2,3,4,5,6,7,8,9,10\}$$

(vii) $A' \cup C$

$$A' = U - A$$
$$= \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}$$
$$= \{1,3,5,7,9\}$$

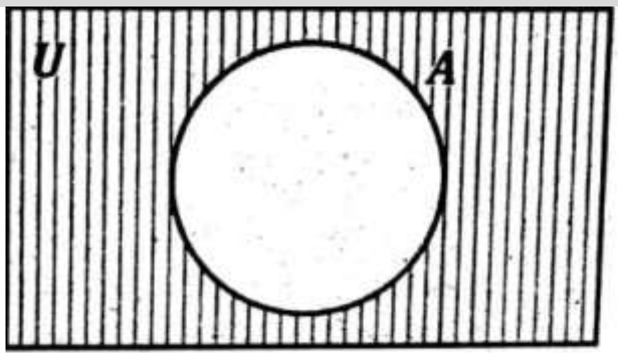
$$A' \cup C = \{1,3,5,7,9\} \cup \{1,3,5,7,9\}$$
$$= \{1,3,5,7,9\}$$

(viii) U'

$$U' = U - A$$
$$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5,6,7,8,9,10\}$$
$$= \{\ \ \}$$

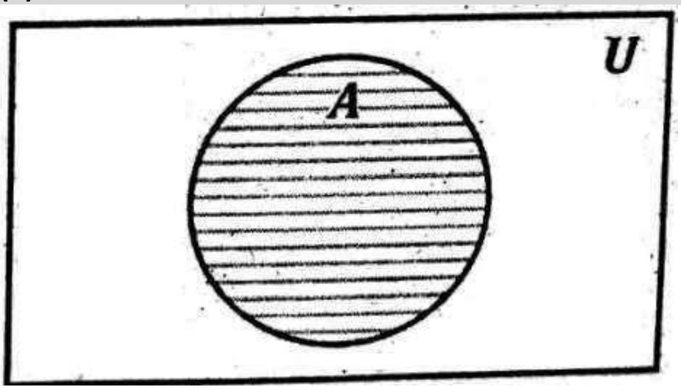
4. Using the Venn diagrams, if necessary, find the single sets equal to the following:

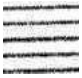
(i) A'



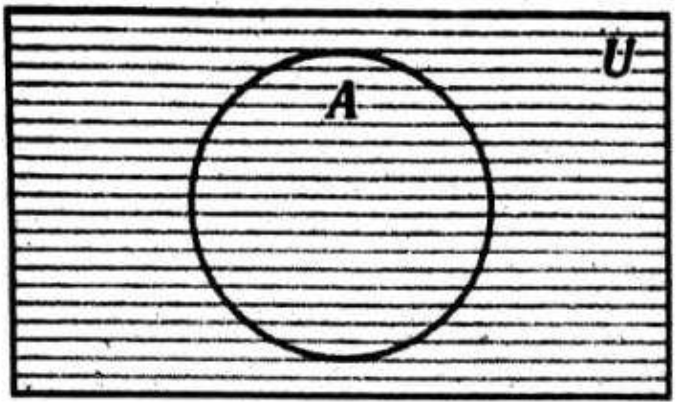
$A' =$ 

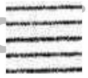
(ii) $A \cap U$



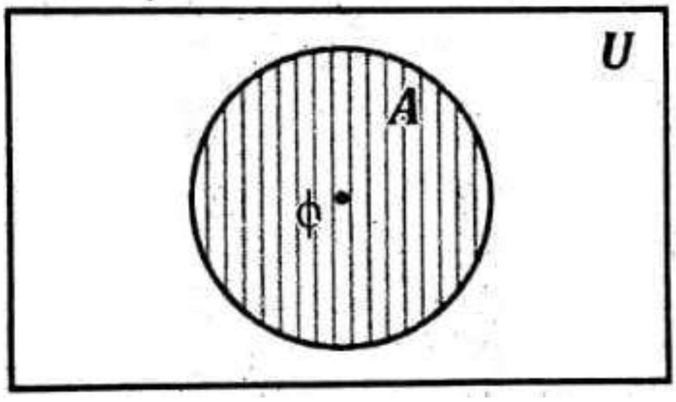
$A \cap U =$ 

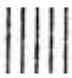
(iii) $A \cup U$



$A \cup U =$ 

(iv) $A \cup \emptyset$



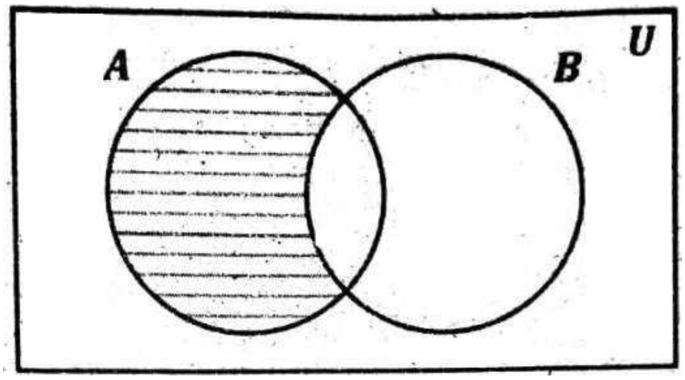
$A \cup \emptyset =$ 

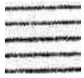
(v) $\emptyset \cup \emptyset$

The intersection of the empty set $\emptyset$ with itself is the set of elements common to both sets. Since the empty set contains no elements, its intersection with itself also results in the empty set.

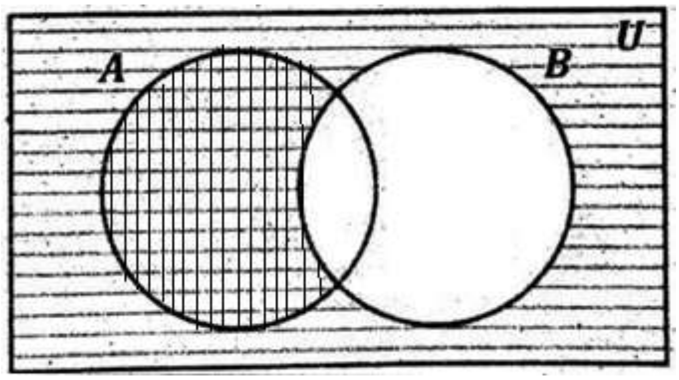
5. Use Venn diagrams to verify the following:
(i) $A - B = A \cap B'$

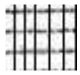
$$L.H.S = A - B$$



$A - B =$  .... (i)

$$R.H.S = A \cap B'$$



$A \cap B' =$  .... (ii)

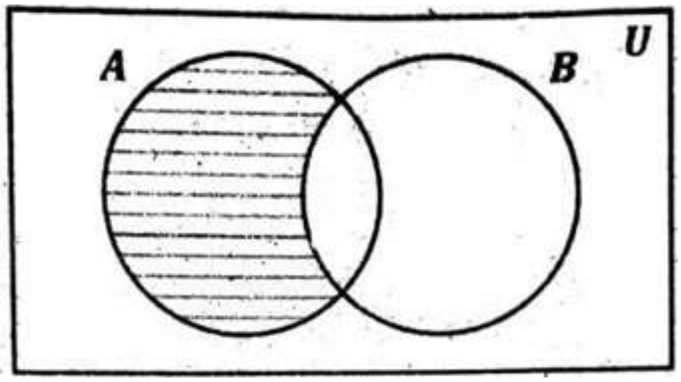
From (i) and (ii)

$$L.H.S = R.H.S$$

(ii) $(A - B)' \cap B = B$

$$L.H.S = (A - B)' \cap B$$

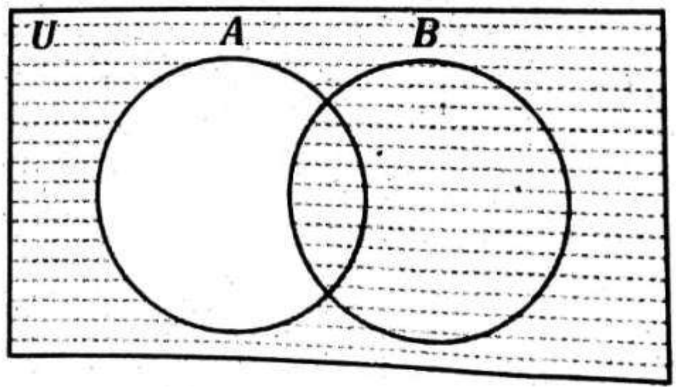
Venn diagram of $A - B$

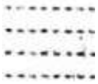


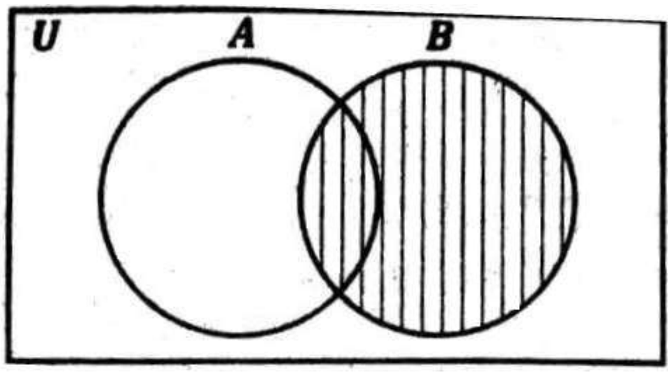
$A - B =$ 

Now for

$$(A - B)' = U - (A - B)$$

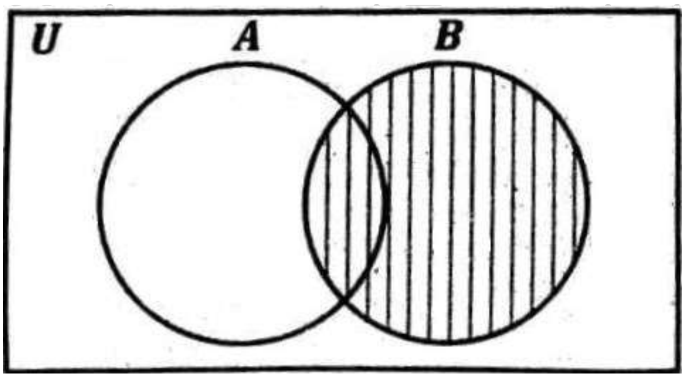


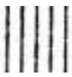
$(A-B)' =$ 



$(A-B)' \cap B =$  .... (i)

$R.H.S = B$



$B =$  .... (ii)

From (i) and (ii)

$$L.H.S = R.H.S$$

6. Verify the properties for the sets A, B, and C given below:

(i) Associativity of union.

(ii) Associativity of intersection.

(iii) Distributivity of union over intersection.

(iv) Distributivity of intersection over union.

(a) $A = \{1,2,3,4\}$, $B = \{3,4,5,6,7,8\}$, $C = \{5,6,7,9,10\}$

Associativity of union:

$$(\boldsymbol{A} \cup \boldsymbol{B}) \cup \boldsymbol{C} = \boldsymbol{A} \cup (\boldsymbol{B} \cup \boldsymbol{C})$$

$$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6,7,8\}$$
$$= \{1,2,3,4,5,6,7,8\}$$

$$B \cup C = \{3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$$
$$B \cup C = \{3,4,5,6,7,8,9,10\}$$

$$L.H.S = (A \cup B) \cup C$$
$$= \{1,2,3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$$
$$= \{1,2,3,4,5,6,7,8,9,10\}$$

$$R.H.S = A \cup (B \cup C)$$
$$= \{1,2,3,4\} \cup \{3,4,5,6,7,8,9,10\}$$
$$= \{1,2,3,4,5,6,7,8,9,10\}$$

Hence

$$L.H.S = R.H.S$$

Associativity of intersection:

$$(\boldsymbol{A} \cap \boldsymbol{B}) \cap \boldsymbol{C} = \boldsymbol{A} \cap (\boldsymbol{B} \cap \boldsymbol{C})$$

$$A \cap B = \{1,2,3,4\} \cap \{3,4,5,6,7,8\}$$
$$= \{3,4\}$$

$$B \cap C = \{3,4,5,6,7,8\} \cap \{5,6,7,9,10\}$$
$$B \cap C = \{5,6,7\}$$

$$L.H.S = (A \cap B) \cap C$$
$$= \{3,4\} \cap \{5,6,7,9,10\}$$
$$= \{\ \}$$

$$R.H.S = A \cap (B \cap C)$$
$$= \{1,2,3,4\} \cap \{5,6,7\}$$
$$= \{\ \}$$

Hence

$$L.H.S = R.H.S$$

Distributivity of union over intersection:

$$\boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C}) = (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$$

$$B \cap C = \{3,4,5,6,7,8\} \cap \{5,6,7,9,10\}$$
$$= \{5,6,7\}$$

$$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6,7,8\}$$
$$= \{1,2,3,4,5,6,7,8\}$$

$$A \cup C = \{1,2,3,4\} \cup \{5,6,7,9,10\}$$
$$= \{1,2,3,4,5,6,7,9,10\}$$

$$L.H.S = A \cup (B \cap C)$$
$$= \{1,2,3,4\} \cup \{5,6,7\}$$
$$= \{1,2,3,4,5,6,7\}$$

$$R.H.S = (A \cup B) \cap (A \cup C)$$
$$= \{1,2,3,4,5,6,7,8\} \cap \{1,2,3,4,5,6,7,9,10\}$$
$$= \{1,2,3,4,5,6,7\}$$

Hence

$$L.H.S = R.H.S$$

Distributivity of intersection over union:

$$\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C})$$

$$B \cup C = \{3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$$

$$= \{3,4,5,6,7,8,9,10\}$$

$$A \cap B = \{1,2,3,4\} \cap \{3,4,5,6,7,8\}$$
$$= \{3,4\}$$

$$A \cap C = \{1,2,3,4\} \cap \{5,6,7,9,10\}$$
$$= \{\quad\}$$

$$L.H.S = A \cap (B \cup C)$$
$$= \{1,2,3,4\} \cap \{3,4,5,6,7,8,9,10\}$$
$$= \{3,4\}$$

$$R.H.S = (A \cap B) \cup (A \cap C)$$
$$= \{3,4\} \cup \{\quad\}$$
$$= \{3,4\}$$

Hence

$$L.H.S = R.H.S$$

(b) $A = \emptyset = \{\quad\}, B = \{0\}, C = \{0, 1, 2\}$

Associativity of union:

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$A \cup B = \{\quad\} \cup \{0\}$$
$$= \{0\}$$

$$B \cup C = \{0\} \cup \{0,1,2\}$$
$$= \{0,1,2\}$$

$$L.H.S = (A \cup B) \cup C$$
$$= \{0\} \cup \{0,1,2\}$$
$$= \{0,1,2\}$$

$$R.H.S = A \cup (B \cup C)$$
$$= \{\quad\} \cup \{0,1,2\}$$
$$= \{0,1,2\}$$

Hence

$$L.H.S = R.H.S$$

Associativity of intersection:

$$(A \cap B) \cap C = A \cap (B \cap C)$$
$$A \cap B = \{\quad\} \cap \{0\}$$
$$= \{\quad\}$$

$$B \cap C = \{0\} \cap \{0,1,2\}$$
$$B \cap C = \{0\}$$

$$L.H.S = (A \cap B) \cap C$$
$$= \{\quad\} \cap \{0,1,2\}$$
$$= \{\quad\}$$

$$R.H.S = A \cap (B \cap C)$$
$$= \{\quad\} \cap \{0\}$$
$$= \{\quad\}$$

Hence

$$L.H.S = R.H.S$$

Distributivity of union over intersection:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$B \cap C = \{0\} \cap \{0,1,2\}$$
$$= \{0\}$$

$$A \cup B = \{\quad\} \cup \{0\}$$
$$= \{0\}$$

$$A \cup C = \{\quad\} \cup \{0,1,2\}$$
$$= \{0,1,2\}$$

$$L.H.S = A \cup (B \cap C)$$
$$= \{\quad\} \cup \{0\}$$
$$= \{0\}$$

$$R.H.S = (A \cup B) \cap (A \cup C)$$
$$= \{0\} \cap \{0,1,2\}$$
$$= \{0,1,2\}$$

Hence

$$L.H.S = R.H.S$$

Distributivity of intersection over union:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$B \cup C = \{0\} \cup \{0,1,2\}$$
$$= \{0,1,2\}$$

$$A \cap B = \{\quad\} \cap \{0\}$$
$$= \{\quad\}$$

$$A \cap C = \{\quad\} \cap \{0,1,2\}$$
$$= \{\quad\}$$

$$L.H.S = A \cap (B \cup C)$$
$$= \{\quad\} \cap \{0,1,2\}$$
$$= \{\quad\}$$

$$R.H.S = (A \cap B) \cup (A \cap C)$$
$$= \{\quad\} \cup \{\quad\}$$
$$= \{\quad\}$$

Hence

$$L.H.S = R.H.S$$

(c) $A = N, B = Z, C = Q$

Note: $N \subset Z \sqsubset Q$

Associativity of union:

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$A \cup B = N \cup Z$$
$$= Z$$

$$B \cup C = Z \cup Q$$
$$B \cup C = Q$$

$$L.H.S = (A \cup B) \cup C$$
$$= Z \cup Q$$
$$= Q$$

$$R.H.S = A \cup (B \cup C)$$

$$= N \cup Q$$
$$= Q$$

Hence

$$L.H.S = R.H.S$$

Associativity of intersection:

$$(\boldsymbol{A \cap B}) \cap \boldsymbol{C} = \boldsymbol{A} \cap (\boldsymbol{B \cap C})$$

$$A \cap B = N \cap Z$$
$$= N$$

$$B \cap C = Z \cap Q$$
$$= Z$$

$$L.H.S = (A \cap B) \cap C$$
$$= N \cap Q$$
$$= N$$

$$R.H.S = A \cap (B \cap C)$$
$$= N \cap Z$$
$$= N$$

Hence

$$L.H.S = R.H.S$$

Distributivity of union over intersection:

$$\boldsymbol{A} \cup (\boldsymbol{B \cap C}) = (\boldsymbol{A \cup B}) \cap (\boldsymbol{A \cup C})$$

$$B \cap C = Z \cap Q$$
$$= Z$$

$$A \cup B = N \cup Z$$
$$= Z$$

$$A \cup C = N \cup Q$$
$$= Q$$

$$L.H.S = A \cup (B \cap C)$$
$$= N \cup Z$$
$$= Z$$

$$R.H.S = (A \cup B) \cap (A \cup C)$$
$$= Z \cap Q$$
$$= Z$$

Hence

$$L.H.S = R.H.S$$

Distributivity of intersection over union:

$$\boldsymbol{A} \cap (\boldsymbol{B \cup C}) = (\boldsymbol{A \cap B}) \cup (\boldsymbol{A \cap C})$$

$$B \cup C = Q \cup Z$$
$$= Q$$

$$A \cap B = N \cap Q$$
$$= N$$

$$A \cap C = N \cap Q$$
$$= N$$

$$L.H.S = A \cap (B \cup C)$$
$$= N \cap Q$$
$$= N$$

$$R.H.S = (A \cap B) \cup (A \cap C)$$
$$= N \cup N$$
$$= N$$

Hence

$$L.H.S = R.H.S$$

7. Verify De Morgan's Laws for the following sets: $\boldsymbol{U = \{1, 2, 3, \ldots, 20\}}$, $\boldsymbol{A = \{2, 4, 6, \ldots, 20\}}$ and $\boldsymbol{B = \{1, 3, 5, \ldots, 19\}}$

De Morgan's Laws: (i) $(\boldsymbol{A \cup B})' = \boldsymbol{A' \cap B'}$ (ii) $(\boldsymbol{A \cap B})' = \boldsymbol{A' \cup B'}$

(i) $(\boldsymbol{A \cup B})' = \boldsymbol{A' \cap B'}$

$$A \cup B = \{2,4,6, \ldots ,20\} \cup \{1,3,5, \ldots ,19\}$$
$$= \{1,2,3, \ldots ,20\}$$

$$A' = U - A$$
$$A' = \{1,2,3, \ldots ,20\} - \{2,4,6, \ldots ,20\}$$
$$A' = \{1,3,5, \ldots ,19\}$$

$$B' = U - B$$
$$B' = \{1,2,3, \ldots ,20\} - \{1,3,5, \ldots ,19\}$$
$$B' = \{2,4,6, \ldots ,20\}$$

$$L.H.S = (A \cup B)'$$
$$= U - (A \cup B)$$
$$= \{1,2,3, \ldots ,20\} - \{1,2,3, \ldots ,20\}$$
$$L.H.S = \{\quad\}$$

$$R.H.S = A' \cap B'$$
$$= \{1,3,5, \ldots ,19\} \cap \{2,4,6, \ldots ,20\}$$

$$R.H.S = \{\ \ \}$$

Hence Proved

$$L.H.S = R.H.S$$

(i) $(\boldsymbol{A} \cap \boldsymbol{B})' = \boldsymbol{A}' \cup \boldsymbol{B}'$

$$\begin{aligned} A \cap B &= \{2,4,6, \dots ,20\} \cap \{1,3,5, \dots ,19\} \\ &= \{\ \ \} \end{aligned}$$

$$\begin{aligned} A' &= U - A \\ A' &= \{1,2,3, \dots ,20\} - \{2,4,6, \dots ,20\} \\ A' &= \{1,3,5, \dots ,19\} \end{aligned}$$

$$\begin{aligned} B' &= U - B \\ B' &= \{1,2,3, \dots ,20\} - \{1,3,5, \dots ,19\} \\ B' &= \{2,4,6, \dots ,20\} \end{aligned}$$

$$\begin{aligned} L.H.S &= (A \cap B)' \\ &= U - (A \cap B) \\ &= \{1,2,3, \dots ,20\} - \{\ \ \} \\ L.H.S &= \{1,2,3, \dots ,20\} \end{aligned}$$

$$\begin{aligned} R.H.S &= A' \cup B' \\ &= \{1,3,5, \dots ,19\} \cup \{2,4,6, \dots ,20\} \\ R.H.S &= \{1,2,3, \dots ,20\} \end{aligned}$$

Hence Proved

$$L.H.S = R.H.S$$

8. Consider the set $\boldsymbol{P} = \{\boldsymbol{x} \mid \boldsymbol{x} = 5\boldsymbol{m}, \boldsymbol{m} \in \boldsymbol{N}\}$ and $\boldsymbol{Q} = \{\boldsymbol{x} \mid \boldsymbol{x} = 2\boldsymbol{m}, \boldsymbol{m} \in \boldsymbol{N}\}$. Find $\boldsymbol{P} \cap \boldsymbol{Q}$.

$$\begin{aligned} P &= \{x \mid x = 5m, m \in N\} \\ P &= \{5,10,15,20, \dots\} \\ Q &= \{x \mid x = 2m, m \in N\} \\ Q &= \{2,4,6,8, \dots\} \end{aligned}$$

$$\begin{aligned} P \cap Q &= \{5,10,15,20, \dots\} \cap \{2,4,6,8, \dots\} \\ &= \{10,20,30, \dots\} \end{aligned}$$

9. From suitable properties of union and intersection, deduce the following results:

(i) $\boldsymbol{A} \cap (\boldsymbol{A} \cup \boldsymbol{B}) = \boldsymbol{A} \cup (\boldsymbol{A} \cap \boldsymbol{B})$

$$\begin{aligned} L.H.S &= A \cap (A \cup B) \\ &= (A \cap A) \cup (A \cap B) \\ &= A \cup (A \cap B) && \because A \cap A = A \\ &= R.H.S \end{aligned}$$

(ii) $\boldsymbol{A} \cup (\boldsymbol{A} \cap \boldsymbol{B}) = \boldsymbol{A} \cap (\boldsymbol{A} \cup \boldsymbol{B})$

$$\begin{aligned} L.H.S &= A \cup (A \cap B) \\ &= (A \cup A) \cap (A \cup B) \\ &= A \cap (A \cup B) && \because A \cup A = A \\ &= R.H.S \end{aligned}$$

10. If $\boldsymbol{g}(\boldsymbol{x}) = 7\boldsymbol{x} - 2$ and $\boldsymbol{s}(\boldsymbol{x}) = 8\boldsymbol{x}^2 - 3$, then find:

(i) $\boldsymbol{g}(\mathbf{0})$

Since

$$g(x) = 7x - 2$$

Put $x = 0$

$$
\begin{aligned}
g(0) &= 7(0) - 2 \\
&= 0 - 2 \\
&= -2
\end{aligned}
$$

(ii) $g(-1)$

Since

$$g(x) = 7x - 2$$

Put $x = -1$

$$
\begin{aligned}
g(-1) &= 7(-1) - 2 \\
&= -7 - 2 \\
&= -9
\end{aligned}
$$

(iii) $g\left(-\frac{5}{3}\right)$

Since

$$g(x) = 7x - 2$$

Put $x = -\frac{5}{3}$

$$
\begin{aligned}
g\left(-\frac{5}{3}\right) &= 7\left(-\frac{5}{3}\right) - 2 \\
&= \frac{-35 - 6}{3} \\
&= \frac{-41}{3}
\end{aligned}
$$

(iv) $s(1)$

Since

$$s(x) = 8x^2 - 3$$

Put $x = 1$

$$
\begin{aligned}
s(1) &= 8(1)^2 - 3 \\
&= 8 - 3 \\
&= 5
\end{aligned}
$$

(v) $s(-4)$

Since

$$s(x) = 8x^2 - 3$$

Put $x = -9$

$$
\begin{aligned}
s(-9) &= 8(-9)^2 - 3 \\
&= 8(81) + 1 \\
&= 648 - 3 \\
&= 645
\end{aligned}
$$

(v) $s\left(\frac{7}{2}\right)$

Since

$$s(x) = 8x^2 - 3$$

Put $x = \frac{7}{2}$

$$
\begin{aligned}
s\left(\frac{7}{2}\right) &= 8\left(\frac{7}{2}\right)^2 - 3 \\
&= 8\left(\frac{49}{4}\right) - 3 \\
&= \frac{392}{4} - 3
\end{aligned}
$$

$$= \frac{392 - 12}{4}$$
$$= \frac{380}{4}$$
$$= 95$$

11. Given that $f(x) = ax + b$, where a and b are constant numbers. If $f(-2) = 3$ and $f(4) = 10$, then find the values of a and b.

Since
$$f(x) = ax + b$$
Put $x = -2$
$$f(-2) = a(-2) + b$$
$$3 = -2a + b \quad \dots \quad (i) \qquad \because f(-2) = 3$$
Now
$$f(x) = ax + b$$
Put $x = 4$
$$f(4) = a(4) + b + 1$$
$$10 = 4a + b \quad \dots \quad (ii) \qquad \because f(4) = 10$$
Subtract equation (i) from equation (ii)
$$10 = 4a + b$$
$$\pm 3 = \mp 2a \pm b$$
$$7 = 6a$$
$$\boldsymbol{\frac{7}{6} = a}$$
Putting $a = \frac{7}{6}$ in equation (i)

$$3 = -2\left(\frac{7}{6}\right) + b$$
$$3 = \frac{-14}{6} + b$$
$$3 + \frac{14}{6} = b$$
$$\frac{18 + 14}{6} = b$$
$$\frac{32}{6} = b$$
$$\boldsymbol{\frac{16}{3} = b}$$

12. Consider the function defined by $k(x) = 7x - 5$. If $k(x) = 100$, find the value of x.

Since
$$k(x) = 7x - 5$$
Put $k(x) = 100$
$$100 = 7x - 5$$
$$100 + 5 = 7x$$
$$105 = 7x$$
$$\frac{105}{7} = x$$
$$15 = x$$
$$\boldsymbol{x = 15}$$