1. Define set.
A collection of well-defined distinct object is called set. It is denoted by capital letters A, B, C etc. For example, $A = \{1,2,3,4\}$

2. How can a set be described using different methods? Explain each with an example.
A set can be described using three different methods:

(i) Descriptive Form: A set is described in words without listing its elements. For example, the set of all vowels in the English alphabet.

(ii) Tabular Form (Roster Form): A set is described by listing its elements within curly brackets { }. For example, if A is the set of vowels, we write:

$$A = \{a, e, i, o, u\}$$

(iii) Set-Builder Form: A set is described by stating a property that all its elements share. For example, the set of vowels can be written as:

$$A = \{x | x \text{ is a vowel of the English alphabet}\}$$

This means "A is the set of all x such that x is a vowel of the English alphabet."

Note: In algebra, we usually deal with sets of numbers. Such sets, along with their names, are given below:

- $\boldsymbol{N} = \text{The set of natural numbers} = \{1,2,3,4, \cdots\}$
- $\boldsymbol{W} = \text{The set of whole numbers} = \{0,1,2,3,4, \cdots\}$
- $\boldsymbol{Z} = \text{The set of integers} = \{0, \pm1, \pm2, \pm3, \cdots\}$
- $\boldsymbol{O} = \text{The set of odd integers} = \{\pm1, \pm3, \pm5, \cdots\}$
- $\boldsymbol{E} = \text{The set of even integers} = \{0, \pm2, \pm4, \pm6, \cdots\}$
- $\boldsymbol{P} = \text{The set of prime numbers} = \{2, 3, 5, 7, 11, 13, 17, \ldots\}$
- $\boldsymbol{Q} = \text{The set of all rational numbers} = \left\{x | x = \frac{p}{q}\ ,\ p, q \in Z\ \wedge q \neq 0\right\}$
- $\boldsymbol{Q'} = \text{The set of all irrational numbers} = \left\{x | x \neq \frac{p}{q}\ ,\ p, q \in z\ \wedge q \neq 0\right\}$
- $\boldsymbol{R} = \text{The set of all real numbers} = \boldsymbol{Q} \cup \boldsymbol{Q'}$

3. Define singleton set.
A set with only one element is called a singleton set. For example, $\{3\}$, $\{a\}$, and $\{Saturday\}$ are singleton sets.

4. Define empty set.
The set with no elements (zero number of elements) is called an empty set, null set, or void set. The empty set is denoted by the symbol ∅ or { }.

5. Define equal sets.
Two sets A and B are equal if they have exactly the same elements or if every element of set A is an element of set B. If two sets A and B are equal, we write $A = B$.

Thus, the sets $\{1, 2, 3\}$ and $\{2, 1, 3\}$ are equal.

6. Define equivalent sets.
Two sets A and B are equivalent if they have the same number of elements. For example, if $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4, 5\}$, then A and B are equivalent sets. The symbol $\sim$ is used to represent equivalent sets. Thus, we can write $A \sim B$.

7. Define subset.
If every element of a set A is an element of set B, then A is a subset of B. Symbolically, this is written as $A \subseteq B$ $(A\ is\ a\ subset\ of\ B)$.

In such a case, we say B is a superset of A. Symbolically, this is written as: $B \supseteq A$ $(B\ is\ a\ superset\ of\ A)$

8. Define proper subset.
If A is a subset of B and B contains at least one element that is not an element of A, then A is said to be a proper subset of B. In such a case, we write:

$$A \subset B\ (A\ is\ a\ proper\ subset\ of\ B)$$

9. Define improper subset.
If A is a subset of B and $A = B$, then we say that A is an improper subset of B. From this definition, it also follows that every set is a subset of itself and is called an improper subset.

For example, let $A = \{a, b, c\}$, $B = \{c, a, b\}$ and $C = \{a, b, c, d\}$, then clearly: $A \subset C, B \subset C$ but $A = B$.

Notice that each of sets A and B is an improper subset of the other because $A = B$

10. Define universal set.
The set that contains all objects or elements under consideration is called the universal set or the universe of discourse. It is denoted by U.

11. Define power set.
The power set of a set S denoted by $P(S)$ is the set containing all the possible subsets of S. If **S** is a finite set with $n(S) = m$, representing the number of elements in set S, then:

$$n\{P(S)\} = 2^m$$

This represents the number of elements in the power set.

Note:

- The subset of a set can also be stated as follows: $\boldsymbol{A} \subseteq \boldsymbol{B}$ if $\forall\, \boldsymbol{x} \in \boldsymbol{A} \rightarrow \boldsymbol{x} \in \boldsymbol{B}$
- When we do not want to distinguish between proper and improper subsets, we may use the symbol $\subseteq$ for the relationship. It is easy to see that: $\boldsymbol{N} \subseteq \boldsymbol{W} \subseteq \boldsymbol{Z} \subseteq \boldsymbol{Q} \subseteq \boldsymbol{R}$

12. Define union of sets.
The union of two sets A and B, denoted by $A \cup B$, is the set of all elements that belong to A or B. Symbolically,

$$A \cup B = \{x | x \in A\ \vee\ x \in B\}$$

For example, if $A = \{1,2\}$ and $B = \{1,3\}$, then

$$A \cup B = \{1,2\} \cup \{1,3\}$$
$$A \cup B = \{1,2,3\}$$

13. Define intersection of two sets.
The intersection of two sets A and B, written as $A \cap B$, is the set of all elements that belong to both A and B. Symbolically,

$$A \cap B = \{x | x \in A\ \wedge\ x \in B\}$$

For example, if $A = \{1,2\}$ and $B = \{1,3\}$, then

$$A \cap B = \{1,2\} \cap \{1,3\}$$
$$A \cap B = \{1\}$$

14. Define difference of sets.

The set difference of A and B denoted by $A - B$, consists of all elements that belong to A but do not belong to B. Symbolically,

$$A - B = \{x | x \in A \ \wedge \ x \notin B\}$$
$$and \qquad B - A = \{x | x \in B \ \wedge \ x \notin A\}$$

For example, if $A = \{1,2\}$ and $B = \{1,3\}$, then

$$A - B = \{1,2\} - \{1,3\}$$
$$A - B = \{2\}$$

15. Define complement of a set.

The complement of a set A, denoted by A' or A^c, relative to the universal set U is the set of all elements of U that do not belong to A. Symbolically,

$$A' = U - A$$
$$A' = \{x | x \in U \ \wedge \ x \notin A\}$$

For example, if $U = \{1,2,3,4,5\}$ and $A = \{1,3\}$, then

$$A' = U - A$$
$$A' = \{1,2,3,4,5\} \cup \{1,3\}$$
$$A' = \{2,4,5\}$$

16. What are disjoint sets?

If the intersection of two sets is the empty set, the sets are said to be disjoint. For example, if:

$$S_1 = \textit{The set of odd natural numbers}$$
$$S_2 = \textit{The set of even natural numbers}$$

Then, S_1 and S_2 are disjoint sets because they have no common elements. Similarly, the set of **arts students** and the set of **science students** in a school are **disjoint sets**.

17. What are overlapping sets?

If the intersection of two sets is non-empty but neither is a subset of the other, the sets are called overlapping sets. For example: If

$$L = \{2, 3, 4, 5, 6, 7, 8, 9\}$$
$$M = \{5, 6, 7, 8, 9, 10\}$$

then L and M are overlapping sets because they have common elements.

18. Define Venn diagram (closed figures).

British mathematician john Venn (1834-1923) introduced rectangle for a universal set U and its subsets A and B as closed figures inside this rectangle.

19. What is the cardinality of a set? How is it represented?

The cardinality of a set is defined as the total number of elements of a set. It represents the size of the set. For a non-empty set, the cardinality of a set is denoted by $n(A)$. For example, if

$$A = \{1,3,5,7,9,11\}$$

then $n(A) = 6$ because the set has 6 elements.

20. What is the Inclusion-Exclusion Principle? State it for two and three sets.

To find the cardinality of a set, we use a rule called the Inclusion-Exclusion Principle, which helps calculate the number of elements in the union of two or more sets by avoiding overcounting.

Inclusion-Exclusion Principle for Two Sets: If A and B are finite sets, then:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Note: $A \cup B$ and $A \cap B$ are also finite sets.

Inclusion-Exclusion Principle for Three Sets: If A, B, and C are finite sets, then:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Note: $A \cup B \cup C$, $A \cap B$, $A \cap C$, $B \cap C$, and $A \cap B \cap C$ are also finite sets.

21. What is a relation in mathematics? Provide examples.
In everyday use, a relation refers to an abstract connection between two people or objects, such as: $(Teacher, Pupil)$, $(Mother, Son)$, $(Husband, Wife)$, $(Brother, Sister)$, $(Friend, Friend)$ etc.

In mathematics, a relation is any set of ordered pairs. The relationship between the components of an ordered pair may or may not be mentioned.

Examples of Mathematical Relations:

1. Greater than $(>)$: $(5,4)$
2. Square: $(25,5)$
3. Square Root: $(2,4)$
4. Equal (=): $(2 \times 2, 4)$

22. Define Cartesian product.
Let A and B be two non-empty sets. The Cartesian product is the set of all ordered pairs (x, y) such that $x \in A$ and $y \in B$ and is denoted by $A \times B$. Symbolically,

$$A \times B = \{x | x \in A \ \wedge \ y \in B\}$$

23. Define binary relation.
Any subset of the Cartesian product $A \times B$ is called a binary relation or simply a relation, from A to B. It is usually represented by the letter r.

24. What are the domain and range of a relation?
Domain: The domain of a relation is the set of first elements of the ordered pairs. It is written as $Dom\, r$.

Range: The range of a relation is the set of second elements of the ordered pairs. It is written as $Ran\, r$.

Example: if $A = \{1,2,3\}$ and $B = \{2,3\}$, then relation $r: A \to B$ such that $r = \{(x, y) | x < y\}$

$$A \times B = \{1,2,3\} \times \{2,3\}$$
$$A \times B = \{(1,2), (1,3), (2,2), (2,3), (3,2), (3,3)\}$$

Since $r = \{(x, y) | x < y\}$, so

$$r = \{(1,2), (1,3), (2,3)\}$$
$$Dom\, r = \{1,2\}$$
$$Ran\, r = \{2,3\}$$

25. What is a relation on a set?
If A is a non-empty set, any subset of $A \times A$ is called a relation on A.

26. Define function?
A very important particular type of relation is a function defined as below:
Let A and B be two non-empty sets such that:

(i) f is a relation from A to B, that is, f is a subset of $A \times B$.
(ii) $Domain\, f = A$.
(iii) First element of no two pairs of f are equal, then f is said to be a function from A to B.

The function f is also written as: $f: A \to B$

Example: If $A = \{0, 1, 2, 3, 4\}$ and $B = \{3, 5, 7, 9, 11\}$ define a function $f: A \to B$ where:

$$\boldsymbol{f = \{(x, y) \mid y = 2x + 3, x \in A, y \in B\}}$$

Find the value of function f, its domain, co-domain, and range.

Given: $y = 2x + 3$

Now, calculating for each x in A:

x (Domain A)	$y = 2x + 3$ (Range B)
0	$2(0) + 3 = 3$
1	$2(1) + 3 = 5$
2	$2(2) + 3 = 7$
3	$2(3) + 3 = 9$
4	$2(4) + 3 = 11$

Thus, the function is:

$$f = \{(0,3), (1,5), (2,7), (3,9), (4,11)\}$$

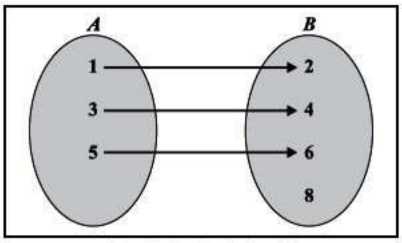
- **Domain**: $A = \{0,1,2,3,4\}$
- **Co-domain**: $B = \{3,5,7,9,11\}$
- **Range**: $\{3,5,7,9,11\} \subseteq B$

27. What is meant by domain and co-domain of a function.

If $f: A \to B$ is a function, then A is called the domain of f and B is called co-domain of f.

28. What is an Into Function?

If a function $f: A \to B$ is such $that\ Range\ f \subset B$ i.e., $Range\ f \neq B$, then f is said to be a function from A into B.

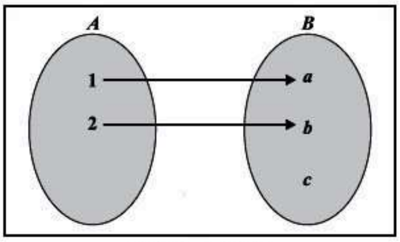


$f = \{(1, 2), (3, 4), (5, 6)\}$

In figure, f is clearly a function. But $Range\ f \neq B$. Therefore, f is an into function from A into B.

29. What is a One-One Function (Injective Function)?

If a function $f: A \to B$ is such that second elements of no two of its ordered pairs are the same, then it is called an injective function.



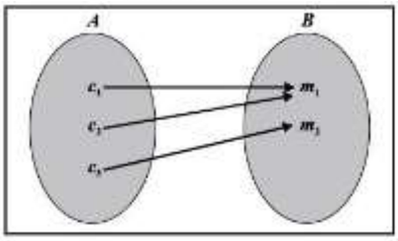
$f = \{(1, a), (2, b)\}$

The function shown in figure is such a function.

30. What is an Onto Function (or Surjective Function)?

If a function $f: A \to B$ is such that $Range\ f = B$,

$i.e.$, every element of B is the image of some element of A, then f is called an onto function or a surjective function.



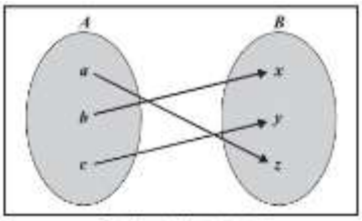
$f = \{(c_1, m_1), (c_2, m_1), (c_3, m_2)\}$

31. What is a One-One and Onto Function (or Bijective Function)?

A function $f: A \to B$ is said to be a Bijective function if it is both $one - one$ and $onto$. Such a function is also called a $(1 - 1)$ correspondence between the sets A and B.

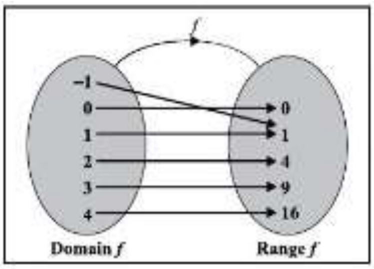
Example:

(a, z), (b, x), and (c, y) are the pairs of corresponding elements. In this case, $f = \{(a, z), (b, x), (c, y)\}$ is a bijective function or $(1 - 1)$ correspondence between the sets A and B.



$f = \{(a, z), (b, x), (c, y)\}$

32. What is the Notation of a Function?



We know that set-builder notation is more suitable for infinite sets. The same applies to a function comprising an infinite number of ordered pairs.

Example: Consider the function:

$$f = \{(-1,1), (0,0), (1,1), (2,4), (3,9), (4,16), \dots\}$$

$$Domain\ f \ = \{-1, 0, 1, 2, 3, 4, \dots\}$$

$$Range\ f \ = \{0, 1, 4, 9, 16, \dots\}$$

This function may be written as:

$$f = \{(x, y) | y = x^2, x \in N\}$$

The mapping diagram for this function is shown in figure.

33. Define a linear function.

A function of the form $\{(x, y) | y = mx + c\}$ is called a linear function because its geometric representation is a straight line.

The equation $y = mx + c$ represents a straight line, where m is the slope and $\boldsymbol{c}$ is the $y\text{-}intercept$.

34. What is the general form of a quadratic function?

A function of the form $\{(x, y) \mid y = ax^2 + bx + c\}$ is called a quadratic function. It represents a parabolic curve in geometric representation.