Exercise 3.1

1. Write the following sets in set builder notation:

(i) $\{1, 4, 9, 16, 25, 36, \dots, 484\}$

$$\{1,4,9,16,25,36,\dots,484\}$$
$$= \{x|x = n^2, n \in N \wedge\ 1 \le n \le 22\}$$

Note: This set consists of perfect squares from 1^2 to 22^2 (since $22^2 = 484$).

(ii) $\{2, 4, 8, 16, \dots, 256\}$

$$\{2,4,8,16,\dots,256\}$$
$$= \{x|x = 2^n, n \in N \wedge\ 1 \le n \le 8\}$$

Note: This set consists of powers of 2 from 2^1 to 2^8 (since $2^8 = 256$).

(iii) $\{0, \pm 1, \pm 2, \dots, \pm 1000\}$

$$\{0, \pm 1, \pm 2, \dots, \pm 1000\}$$
$$= \{x|x \in Z \wedge\ -1000 \le x \le 1000\}$$

Note: This set includes all integers from -1000 to 1000.

(iv) $\{6, 12, 18, \dots, 120\}$

$$\{6,12,18,\dots,120\}$$
$$= \{x|x = 6n, n \in N \wedge 1 \le n \le 20\}$$

Note: This set consists of multiples of 6 from 6×1 to 6×20 (since $6 \times 20 = 120$).

(v) $\{100, 102, 104, \dots, 400\}$

$$\{100,102,104,\dots,400\}$$
$$= \{x|x = 100 + 2n, n \in W \wedge\ 0 \le n \le 150\}$$

Note: This set includes even numbers from 100 to 400.

(vi) $\{1, 3, 9, 27, 81, \dots\}$

$$\{1,3,9,27,81,\dots\}$$
$$= \{x|x = 3^n, n \in W\}$$

Note: This set consists of powers of 3 starting from $3^0 = 1$.

(vii) $\{1, 2, 4, 5, 10, 20, 25, 50, 100\}$

$$\{1,2,4,5,10,20,25,50,100\}$$
$$= \{x|x\ is\ divisor\ of\ 100, n \in N \wedge\ 1 \le x \le 100\}$$

Note: This set includes all positive divisors of 100.

(viii) $\{5, 10, 15, \dots, 100\}$

$$\{5,10,15,\dots,100\}$$
$$= \{x|x = 5n, n \in N \wedge\ 1 \le n \le 20\}$$

Note: This set consists of multiples of 5 from 5×1 to 5×20 (since $5 \times 20 = 100$).

(ix) The set of all integers between -100 and 1000.

$$= \{x|x \in Z \wedge\ -100 \le x \le 1000\}$$

Note: This set includes all integers from -100 to 1000.

2. Write each of the following sets in tabular form:

(i) $\{x \mid x\ is\ a\ multiple\ of\ 3 \wedge x \le 36\}$

$$\{x \mid x\ is\ a\ multiple\ of\ 3 \wedge x \le 36\}$$
$$= \{3,6,9,12,15,18,21,24,27,30,33,36\}$$

(ii) $\{x \mid x \in R \wedge 2x + 1 = 0\}$

$$2x + 1 = 0$$
$$2x = -1$$
$$x = -\frac{1}{2}$$
$$\{x \mid x \in R \wedge 2x + 1 = 0\}$$
$$= \left\{-\frac{1}{2}\right\}$$

(iii) $\{x \mid x \in P \wedge x < 12\}$

$$\{x \mid x \in P \wedge x < 12\}$$
$$= \{2,3,5,7,11\}$$

(iv) $\{x \mid x\ is\ a\ divisor\ of\ 128\}$

$$\{x \mid x\ is\ a\ divisor\ of\ 128\}$$
$$= \{1,2,4,8,16,32,64,128\}$$

(v) $\{x|x = 2^n, n \in N \wedge\ n < 8\}$

$$\{x|x = 2^n, n \in N \wedge\ n < 8\}$$
$$= \{2,4,8,16,32,64,128\}$$

(vi) $\{x \mid x \in N \wedge x + 4 = 0\}$

$$\{x \mid x \in N \wedge x + 4 = 0\}$$
$$= \{\ \ \}$$

As $x + 4 = 0 \Rightarrow x = -4 \notin N$

(vii) $\{x \mid x \in N \wedge x = x\}$

$$\{x \mid x \in N \wedge x = x\}$$
$$= \{1,2,3,\dots\}$$

(viii) $\{x \mid x \in Z \wedge 3x + 1 = 0\}$

$$\{x \mid x \in Z \wedge 3x + 1 = 0\}$$
$$= \{\ \ \}$$

As $3x + 1 = 0 \Rightarrow x = -\frac{1}{3} \notin Z$

3. Write two proper subsets of each of the following sets:

(i) $\{a, b, c\}$

The Proper subsets of $\{a, b, c\}$ are $\{a\}$, $\{b\}$

(ii) $\{0,1\}$

The Proper subsets of $\{0,1\}$ are $\{0\}$, $\{1\}$

(iii) N

The Proper subsets of N are Prime Numbers, Even Natural Numbers

(iv) Z

The Proper subsets of Z are Prime Numbers, Whole Numbers

(v) Q

The Proper subsets of Q are Integers, Natural Numbers

(vi) $\{x|x \in Q \wedge 0 < x \leq 2\}$

The Proper subsets of $\{x|x \in Q \wedge 0 < x \leq 2\}$ are $\left\{\frac{1}{2}\right\}$, $\left\{\frac{3}{4}\right\}$

4. Is there any set which has no proper subset? If so, name that set.

Yes, there exist a set which has no proper subset, that is called 'empty set' $\{\quad\}$.

5. What is the difference between $\{a,b\}$ and $\{\{a,b\}\}$?

$\{a,b\}$ set has two elements: a and b. While $\{\{a,b\}\}$ set has only one element $\{a,b\}$.

6. What is the number of elements of the power set of each of the following sets?

(i) $\{\quad\}$

$$\begin{aligned} No.of\ elements\ of\ the\ power\ set &= 2^n \\ &= 2^0 \\ &= 1 \end{aligned}$$

(ii) $\{0,1\}$

$$\begin{aligned} No.of\ elements\ of\ the\ power\ set &= 2^n \\ &= 2^2 \\ &= 4 \end{aligned}$$

(iii) $\{1,2,3,4,5,6,7\}$

$$\begin{aligned} No.of\ elements\ of\ the\ power\ set &= 2^n \\ &= 2^7 \\ &= 128 \end{aligned}$$

(iv) $\{0,1,2,3,4,5,6,7\}$

$$\begin{aligned} No.of\ elements\ of\ the\ power\ set &= 2^n \\ &= 2^8 \\ &= 256 \end{aligned}$$

(v) $\{a,\{b,c\}\}$

$$\begin{aligned} No.of\ elements\ of\ the\ power\ set &= 2^n \\ &= 2^2 \\ &= 4 \end{aligned}$$

(vi) $\{\{a,b\},\{b,c\},\{d,e\}\}$

$$\begin{aligned} No.of\ elements\ of\ the\ power\ set &= 2^n \\ &= 2^3 \\ &= 8 \end{aligned}$$

7. Write down the power set of each of the following sets:

(i) $\{9,11\}$

Let

$$A = \{9,11\}$$
$$P(A) = \{\emptyset,\{9\},\{11\},\{9,11\}\}$$

(ii) $\{+,-,\times,\div\}$

Let

$$A = \{+,-,\times,\div\}$$
$$\begin{aligned} P(A) = \{&\emptyset,\{+\},\{-\},\{\times\},\{\div\}, \\ &\{+,-\},\{+,\times\},\{+,\div\}, \\ &\{-,\times\},\{-,\div\},\{\times,\div\}, \\ &\{+,-,\times\},\{+,-,\div\},\{-,\times,\div\}, \\ &\{+,\times,\div\},\{+,-,\times,\div\}\} \end{aligned}$$

(iii) $\{\emptyset\}$

Let

$$A = \{\emptyset\}$$
$$P(A) = \{\emptyset,\{\emptyset\}\}$$

(iv) $\{a,\{b,c\}\}$

Let

$$A = \{a,\{b,c\}\}$$
$$P(A) = \{\emptyset,\{a\},\{\{b,c\}\},\{a,\{b,c\}\}\}$$