Exercise 1.2

Important Formulas:

(i) $(x+y)^2 = x^2 + 2xy + y^2$

(ii) $(x-y)^2 = x^2 - 2xy + y^2$

(iii) $x^2 - y^2 = (x+y)(x-y)$

(iv) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

(v) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

(vi) $a^m . a^n = a^{m+n}$

(vii) $(a^m)^n = a^{mn}$

(viii) $(ab)^n = a^n a^n$

(ix) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(x) $\frac{a^m}{a^n} = a^{m-n}$

(xi) $a^0 = 1$

(xii) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

(xiii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(xiv) $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

(xv) $\left(\sqrt[n]{a}\right)^n = \left(a^{\frac{1}{n}}\right)^n = a$

1. Rationalize the denominator of the following:

(i) $\frac{13}{4+\sqrt{13}}$

$$\frac{13}{4+\sqrt{3}} = \frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}}$$
$$= \frac{13\left(4-\sqrt{3}\right)}{(4)^2 - \left(\sqrt{3}\right)^2}$$
$$= \frac{13\left(4-\sqrt{3}\right)}{16-3}$$
$$= \frac{13\left(4-\sqrt{3}\right)}{13}$$
$$= \mathbf{4-\sqrt{3}}$$

(ii) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{\sqrt{2\cdot 3}+\sqrt{5\cdot 3}}{\left(\sqrt{3}\right)^2}$$
$$= \frac{\mathbf{\sqrt{6}+\sqrt{15}}}{\mathbf{3}}$$

(iii) $\frac{\sqrt{2}-1}{\sqrt{5}}$

$$\frac{\sqrt{2}-1}{\sqrt{5}} = \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$
$$= \frac{\sqrt{2\cdot 5}-\sqrt{5}}{\left(\sqrt{5}\right)^2}$$
$$= \frac{\mathbf{\sqrt{10}-\sqrt{5}}}{\mathbf{5}}$$

(iv) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

$$\frac{6-4\sqrt{2}}{6+4\sqrt{2}} = \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}}$$
$$= \frac{6\left(6-4\sqrt{2}\right) - 4\sqrt{2}\left(6-4\sqrt{2}\right)}{(6)^2 - \left(4\sqrt{2}\right)^2}$$
$$= \frac{36 - 24\sqrt{2} - 24\sqrt{2} + 16\left(\sqrt{2}\right)^2}{36 - (4)^2\left(\sqrt{2}\right)^2}$$
$$= \frac{36 - 48\sqrt{2} + 16(2)}{36 - (16)(2)}$$
$$= \frac{36 - 48\sqrt{2} + 32}{36-32}$$
$$= \frac{68 - 48\sqrt{2}}{4}$$
$$= \frac{68}{4} - \frac{48\sqrt{2}}{4}$$
$$= \mathbf{17 - 12\sqrt{2}}$$

(v) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$
$$= \frac{\sqrt{3}\left(\sqrt{3}-\sqrt{2}\right) - \sqrt{2}\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$
$$= \frac{\left(\sqrt{3}\right)^2 - \sqrt{6} - \sqrt{6} + \left(\sqrt{2}\right)^2}{3-2}$$
$$= \frac{3 - 2\sqrt{6} + 2}{1}$$
$$= \frac{5 - 2\sqrt{6}}{1}$$
$$= \mathbf{5 - 2\sqrt{6}}$$

(vi) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

$$\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} = \frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$
$$= \frac{4\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{\left(\sqrt{7}\right)^2 - \left(\sqrt{5}\right)^2}$$
$$= \frac{4\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{7-5}$$
$$= \frac{4\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{2}$$

$$= 2\sqrt{3}(\sqrt{7}-\sqrt{5})$$

2. Simplify the following:

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}}$$
$$= \left(\frac{16}{81}\right)^{\frac{3}{4}}$$
$$= \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}}$$
$$= \frac{(2^4)^{\frac{3}{4}}}{(3^4)^{\frac{3}{4}}}$$
$$= \frac{2^3}{3^3}$$
$$= \frac{8}{27}$$

(ii) $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^3 \times \frac{16}{27}$

$$\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^3 \times \frac{16}{27}$$
$$= \left(\frac{4}{3}\right)^2 \div \left(\frac{4}{3^2}\right)^3 \times \frac{4^2}{3^3}$$
$$= \frac{4^2}{3^2} \div \frac{4^3}{(3^2)^3} \times \frac{4^2}{3^3}$$
$$= \frac{4^2}{3^2} \div \frac{4^3}{3^6} \times \frac{4^2}{3^3}$$
$$= \frac{4^2}{3^2} \times \frac{3^6}{4^3} \times \frac{4^2}{3^3}$$
$$= 4^{2+2-3} \times 3^{6-2-3}$$
$$= 4^1 \times 3^1$$
$$= 4 \times 3$$
$$= \mathbf{12}$$

(iii) $(0.027)^{-\frac{1}{3}}$

$$(0.027)^{-\frac{1}{3}}$$
$$= \left(\frac{27}{1000}\right)^{-\frac{1}{3}}$$
$$= \left(\frac{1000}{27}\right)^{\frac{1}{3}}$$
$$= \left(\frac{10^3}{3^3}\right)^{\frac{1}{3}}$$
$$= \frac{(10^3)^{\frac{1}{3}}}{(3^3)^{\frac{1}{3}}}$$
$$= \frac{\mathbf{10}}{\mathbf{3}}$$

(iv) $\sqrt[7]{\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}\times z^7}}$

$$\sqrt[7]{\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}\times z^7}}$$
$$= \left(\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}\times z^7}\right)^{\frac{1}{7}}$$
$$= \left(x^{14}\cdot y^{21-14}\cdot z^{35-7}\right)^{\frac{1}{7}}$$
$$= (x^{14}\cdot y^7\cdot z^{28})^{\frac{1}{7}}$$
$$= (x^{14})^{\frac{1}{7}}\cdot (y^7)^{\frac{1}{7}}\cdot (z^{28})^{\frac{1}{7}}$$
$$= \boldsymbol{x^2yz^4}$$

(v) $\frac{5\cdot(25)^{n+1}-25\cdot(5)^{2n}}{5\cdot(5)^{2n+3}-(25)^{n+1}}$

$$\frac{5\cdot(25)^{n+1}-25\cdot(5)^{2n}}{5\cdot(5)^{2n+3}-(25)^{n+1}}$$
$$= \frac{5\cdot(5^2)^{n+1}-5^2\cdot(5)^{2n}}{5\cdot(5)^{2n+3}-(5^2)^{n+1}}$$
$$= \frac{5\cdot 5^{2n+2}-5^2\cdot(5)^{2n}}{5\cdot(5)^{2n+3}-5^{2n+2}}$$
$$= \frac{5^{1+2n+2}-5^{2+2n}}{5^{1+2n+3}-5^{2n+2}}$$
$$= \frac{5^{2n+3}-5^{2n+2}}{5^{2n+4}-5^{2n+2}}$$
$$= \frac{5^{2n+2}(5-1)}{5^{2n+2}(5^2-1)}$$
$$= \frac{5-1}{25-1}$$
$$= \frac{4}{24}$$
$$= \frac{\mathbf{1}}{\mathbf{6}}$$

(vi) $\frac{(16)^{x+1}+20(4^{2x})}{2^{x-3}\times 8^{x+2}}$

$$\frac{(16)^{x+1}+20(4^{2x})}{2^{x-3}\times 8^{x+2}}$$
$$= \frac{(2^4)^{x+1}+20(2^2)^{2x}}{2^{x-3}\times(2^3)^{x+2}}$$
$$= \frac{2^{4x+4}+20\cdot 2^{4x}}{2^{x-3}\times 2^{3x+6}}$$
$$= \frac{2^{4x+4}+20\cdot 2^{4x}}{2^{x-3+3x+6}}$$
$$= \frac{2^{4x+4}+20\cdot 2^{4x}}{2^{4x+3}}$$
$$= \frac{2^{4x}(2^4+20)}{2^{4x}\cdot 2^3}$$
$$= \frac{16+20}{2^3}$$

$$= \mathbf{\frac{36}{8}}$$

(vii) $(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$

$$(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$$
$$= (4^3)^{-\frac{2}{3}} \div (3^2)^{-\frac{3}{2}}$$
$$= 4^{-2} \div 3^{-3}$$
$$= \frac{4^{-2}}{3^{-3}}$$
$$= \frac{3^3}{4^2}$$
$$= \mathbf{\frac{27}{16}}$$

(viii) $\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

$$\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$$
$$= \frac{3^n \times (3^2)^{n+1}}{3^{n-1} \times (3^2)^{n-1}}$$
$$= \frac{3^n \times 3^{2n+2}}{3^{n-1} \times 3^{2n-2}}$$
$$= 3^{n+2n+2-n+1-2n+2}$$
$$= 3^5$$
$$= \mathbf{243}$$

(ix) $\frac{5^{n+3} - 6 \cdot 5^{n+1}}{9 \times 5^n - 4 \times 5^n}$

$$\frac{5^{n+3} - 6 \cdot 5^{n+1}}{9 \times 5^n - 4 \times 5^n}$$
$$= \frac{5^n(5^3 - 6 \cdot 5^1)}{5^n(9-4)}$$
$$= \frac{125 - 30}{9 - 4}$$
$$= \frac{95}{5}$$
$$= 19$$

3. If $x = 3 + \sqrt{8}$ then find the values of

(i) $x + \frac{1}{x} = ?$

$$x = 3 + \sqrt{8}$$
$$\frac{x}{1} = \frac{3 + \sqrt{8}}{1}$$
$$\frac{1}{x} = \frac{1}{3 + \sqrt{8}}$$
$$\frac{1}{x} = \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$$
$$\frac{1}{x} = \frac{3 - \sqrt{8}}{(3)^2 - (\sqrt{8})^2}$$
$$\frac{1}{x} = \frac{3 - \sqrt{8}}{9 - 8}$$
$$\frac{1}{x} = \frac{3 - \sqrt{8}}{1}$$
$$\mathbf{\frac{1}{x} = 3 - \sqrt{8}}$$

$$x + \frac{1}{x} = (3 + \sqrt{8}) + (3 - \sqrt{8})$$
$$= 3 + \sqrt{8} + 3 - \sqrt{8}$$
$$\mathbf{x + \frac{1}{x} = 6}$$

(ii) $x - \frac{1}{x} = ?$

$$x - \frac{1}{x} = (3 + \sqrt{8}) - (3 - \sqrt{8})$$
$$= 3 + \sqrt{8} - 3 + \sqrt{8}$$
$$\mathbf{x - \frac{1}{x} = 2\sqrt{8}}$$

(iii) $x^2 + \frac{1}{x^2} = ?$

As

$$x + \frac{1}{x} = 6$$

Taking square on both sides

$$\left(x + \frac{1}{x}\right)^2 = (6)^2$$
$$(x)^2 + \left(\frac{1}{x}\right)^2 + 2(x)\left(\frac{1}{x}\right) = 36$$
$$x^2 + \frac{1}{x^2} + 2 = 36$$
$$x^2 + \frac{1}{x^2} = 36 - 2$$
$$\mathbf{x^2 + \frac{1}{x^2} = 34}$$

(iv) $x^2 - \frac{1}{x^2} = ?$

$$x^2 - \frac{1}{x^2} = (x)^2 - \left(\frac{1}{x}\right)^2$$
$$= \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$
$$= (6)(2\sqrt{8})$$
$$= \mathbf{12\sqrt{8}}$$

(v) $x^4 + \frac{1}{x^4} = ?$

As

$$x^2 + \frac{1}{x^2} = 34$$

Taking square on both sides

$$\left(x^2+\frac{1}{x^2}\right)^2=(34)^2$$
$$(x^2)^2+\left(\frac{1}{x^2}\right)^2+2(x^2)\left(\frac{1}{x^2}\right)=1156$$
$$x^4+\frac{1}{x^4}+2=1156$$
$$x^4+\frac{1}{x^4}=1156-2$$
$$\boldsymbol{x^4+\frac{1}{x^4}=1154}$$

(ii) $\left(\boldsymbol{x-\frac{1}{x}}\right)^{\mathbf{2}}=?$

As

$$x-\frac{1}{x}=2\sqrt{8}$$

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2=\left(2\sqrt{8}\right)^2$$
$$\left(x-\frac{1}{x}\right)^2=(2)^2\left(\sqrt{8}\right)^2$$
$$\left(x-\frac{1}{x}\right)^2=(4)(8)$$
$$\boldsymbol{\left(x-\frac{1}{x}\right)^2=32}$$

4. Find the rational numbers $\boldsymbol{p}$ and $\boldsymbol{q}$ such that $\boldsymbol{\frac{8-3\sqrt{2}}{4+3\sqrt{2}}=p+q\sqrt{2}}$

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}}=p+q\sqrt{2}$$
$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}}\times\frac{4-3\sqrt{2}}{4-3\sqrt{2}}=p+q\sqrt{2}$$
$$\frac{8\left(4-3\sqrt{2}\right)-3\sqrt{2}\left(4-3\sqrt{2}\right)}{(4)^2-\left(3\sqrt{2}\right)^2}=p+q\sqrt{2}$$
$$\frac{32-24\sqrt{2}-12\sqrt{2}+9\left(\sqrt{2}\right)^2}{16-9\left(\sqrt{2}\right)^2}=p+q\sqrt{2}$$
$$\frac{32-36\sqrt{2}+9(2)}{16-9(2)}=p+q\sqrt{2}$$
$$\frac{32-36\sqrt{2}+18}{16-18}=p+q\sqrt{2}$$
$$\frac{50-36\sqrt{2}}{-2}=p+q\sqrt{2}$$
$$\frac{50}{-2}-\frac{36\sqrt{2}}{-2}=p+q\sqrt{2}$$
$$-25+18\sqrt{2}=p+q\sqrt{2}$$

By comparing we get,

$$\boldsymbol{p=-25\ and\ q=18}$$

5. Simplify the following:

(i) $\boldsymbol{\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}}$

$$\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$$
$$=\frac{(5^2)^{\frac{3}{2}}\times(3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}}\times(2^3)^{\frac{4}{3}}}$$
$$=\frac{5^3\times3^3}{2^5\times2^4}$$
$$=\frac{125\times27}{32\times16}$$
$$=\boldsymbol{\frac{3375}{512}}$$

(ii) $\boldsymbol{\frac{54\times\sqrt[3]{(27)^{2x}}}{9^{x+1}+216(3^{2x-1})}}$

$$\frac{54\times\sqrt[3]{(27)^{2x}}}{9^{x+1}+216(3^{2x-1})}$$
$$=\frac{54\times[(3^3)^{2x}]^{\frac{1}{3}}}{(3^2)^{x+1}+216(3^{2x-1})}$$
$$=\frac{54\times(3^3)^{\frac{2x}{3}}}{3^{2x+2}+216(3^{2x-1})}$$
$$=\frac{54\times3^{2x}}{3^{2x}[3^2+216(3^{-1})]}$$
$$=\frac{54}{3^2+\frac{216}{3}}$$
$$=\frac{54}{9+72}$$
$$=\frac{54}{81}$$
$$=\boldsymbol{\frac{2}{3}}$$

(iii) $\boldsymbol{\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}}$

$$\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$$
$$=\sqrt{\frac{\left(6^3\right)^{\frac{2}{3}}\times\left(5^2\right)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{3}{2}}}}$$

$$= \sqrt{\frac{(6)^2 \times (5)^1}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{(25)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{\left(5^2\right)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{5^3}}$$

$$= \sqrt{\frac{6^2}{5^{3-1}}}$$

$$= \sqrt{\frac{6^2}{5^2}}$$

$$= \frac{\sqrt{6^2}}{\sqrt{5^2}}$$

$$= \boldsymbol{\frac{6}{5}}$$

(iv) $\left(\boldsymbol{a^{\frac{1}{3}} + b^{\frac{2}{3}}}\right) \times \left(\boldsymbol{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}}\right)$

$$\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$$

$$= \left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right)\left[\left(a^{\frac{1}{3}}\right)^2 - \left(a^{\frac{1}{3}}\right)\left(b^{\frac{2}{3}}\right) + \left(b^{\frac{2}{3}}\right)^2\right]$$

$$= \left(a^{\frac{1}{3}}\right)^3 + \left(b^{\frac{2}{3}}\right)^3$$

$$= \boldsymbol{a + b^2}$$