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Ranking of Pareto-optimal solutions and selecting the best solution in multi- and many-objective optimization problems using R-method

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ABSTRACT

This paper presents a new multi-attribute decision-making (MADM) method, named as R-method, for ranking of Pareto-optimal solutions and selecting the best solution in multi- and many-objective optimization problems. The compromise among the optimization objectives is different for each Pareto-optimal solution and, hence, the solution that has the best compromise among the objectives can be considered as the best solution. The proposed R-method is used to identify such best compromise solution. The method ranks the objectives based on their importance for the given optimization problem and ranks the alternative solutions (i.e. Pareto-optimal solutions) based on their data corresponding to the objectives. The ranks assigned to the objectives and the ranks assigned to the alternative solutions with respect to each of the objectives are converted to appropriate weights and the final composite scores of the alternative solutions are computed using these weights. The final ranking of alternative solutions is done based on the composite scores. The steps of the proposed method are described along with a pseudocode. Three examples are considered to demonstrate and validate the proposed method. The first example contains 4-objectives and 50 alternative solutions, the second example contains 6-objectives and 30 alternative solutions, and the third example contains 3-objectives and 25 alternative solutions. The results of the proposed method are compared with those of the other widely used MADM methods for the three examples considered. Also, the proposed method is compared with four well-known ranking methods to demonstrate its rationality in assigning weights to the ranks of the objectives and the alternative solutions. The proposed method is comparatively easier, more logical, and can be used for choosing the best compromise solution in multi- and many-objective optimization problems.

1. Introduction

Multi-criteria decision-making (MCDM), also called multi-criteria optimization, is about decision-making in the presence of multiple, generally conflicting criteria. The MCDM problems are subdivided into multi-attribute decision-making (MADM) and multi-objective decision-making (MODM).The MADM methods are generally discrete and aim to select a right alternative from amongst the predetermined number of alternatives. The MODM methods, also called multi-objective optimization (MOO) methods, have continuous or integer decision variable values with a large number of alternative choices, the best of which should satisfy the decision-maker's constraints and preferences [1]. Since last few years, the multi-objective decision-making problems are called many-objective decision-making problems if the number of objectives is 4 or more.

The constituents of the multi- or many-objective optimization problem are: the number of objectives expressed in terms of the design or decision variables, the number of inequality or equality constraints expressed in terms of the design or decision variables, and the ranges of the variables (i.e. the bounds). The general form of multi- or many-objective optimization problems is as given below.

$$Min/Maxf_i(X), l = 1, 2, \ldots, s \tag{1}$$

Subject to the constraints:

$$p_i(X) \le 0,\ \ i = 1, 2, \ldots., m \tag{2}$$

$$q_i(X) = 0,\ \ i = 1, 2, \ldots.., r \tag{3}$$

$$x_k^{(L)} \le x_k \le x_k^{(u)},\ \ k = 1, 2, \ldots.., d \tag{4}$$

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Where, X is an d-dimensional vector called the design or decision variables vector, $f_l(X)$are the objective functions, and $p_i(X)$ and $q_j(X)$ are the inequality and equality constraints, respectively. There need not be any relation between the number of objectives s, the number of variables d and the number of constraints m and r.

In a single-objective optimization problem, the alternative solutions obtained by using an optimization algorithm are compared with each other and the alternative solution giving the best value (i.e. minimum value in the case of minimization or maximum value in the case of maximization) can be chosen by the decision-maker. However, in the case of multi- or many-objective optimization problems, it is not easy to choose a particular alternative solution as the better solution, as this solution may not be better with respect to all the objectives (i.e. an alternative solution may be better in the case of a few objectives but may not be better in the case of other objectives). A set of alternative solutions that are not dominated by any member solution of that set is called non-dominated solution set. The non-dominated set of the complete feasible solution space is called the Pareto-optimal solutions set. The boundary formed by all the solutions mapped from the Pareto-optimal solutions set is called Pareto-optimal front [2].

The well-known multi- or many-objective optimization algorithms include non-dominated sorting genetic algorithm-II (NSGA-II), NSGA-III, multi-objective genetic algorithm (MOGA), niched Pareto genetic algorithm (NPGA), strength Pareto evolutionary algorithm (SPEA), multi-objective versions of various advanced optimization algorithms such as simulated annealing (SA), ant colony optimization (ACO), particle swarm optimization (PSO), artificial bee colony (ABC), differential evolution (DE), grey wolf optimizer (GWO), teaching-learning-based optimization (TLBO), firefly optimizer, whale optimizer, ant lion optimizer, Jaya optimizer, etc. These algorithms have been applied successfully to solve the optimization problems in many fields of engineering and sciences, where optimal decisions need to be taken in the presence of trade-offs between two or more objectives that may be in conflict.

It may be noted that once a set of Pareto-optimal solutions is obtained, all the solutions can be considered as equivalent, as these solutions are non-dominated in nature. However, the compromise among the objectives is different for each solution. Hence, the solution that has the best compromise among the objectives can be considered as the best solution. Hence, the researchers have started using different MADM methods since last few yearsto identify the solution with the best compromise.The common questions faced by the researchers and decision-makers are: how to assign the weights to the objectives, how to make use of the performance data of various alternative solutions corresponding to the objectives, and how to arrive at a final decision (i.e. choosing a best compromise alternative solution).

The MADM methods being used by the researchers to select the best Pareto solution from among the available non-dominated solutions include Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [3-7], Grey Relational Analysis (GRA) [8,9], VIšekriterijumsko KOmpromisno Rangiranje (VIKOR) [10], ELimination Et Choix TRaduisant la REalité (ELECTRE) [11,12], Analytic Hierarchy Process (AHP) [13], Data Envelopment Analysis (DEA) [6,14], Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE) [15,16], Cosine similarity measure [17,18], etc. These methods have been proved successful in different decision-making situations. However, these methods have their own merits and limitations. For example, the TOPSIS method used by many of the researchers involves lengthy calculations which become more complex with the increase in the number of alternatives and the attributes. Different methods of normalizing the data in TOPSIS method lead to different rankings of the alternatives. The GRA method involves more computation and the use of different values of distinguishing coefficient lead to different rankings of thealternative solutions. The VIKOR method involves more computation. Further-more, the weight of the strategy of 'the majority of attributes' used in VIKOR method can change from 0 to 1 and different ranking lists can result for the same weights of the attributes. The ELECTRE method uses the concept of outranking relationship and the computational procedure is quite complex. The method involves lengthy calculations of net concordance and net discordance values for various alternatives. The AHP method involves comparisons of attributes and alternatives on a scale of (1/9) to 9, leading to many comparison matrices. With the increase in the number of alternatives and attributes, the number and size of the comparison matrices increases fast. Moreover, the weights of the attributes found using arithmetic mean, geometric mean, etc. will be different and lead to different rankings of the alternatives. The DEA method requires more computation and if the number of attributes is more and the number of alternatives less, then DEA cannot differentiate between good and poor alternatives. Again, DEA works only if the decision-maker is familiar with the linear programming concepts. Similarly, the methods like PROMETHEE have their own merits and limitations.

The weights of the objectives used in taking a decision about the best compromise alternative solution can be derived by subjective approaches or objective approaches. In the case of objective approaches, the weights of the objectives can be obtained using methods like entropy method, standard deviation method, etc. The weights thus obtained are called the objective weights and decision-maker has no role to play regarding his preferences. In the case of subjective approaches, the decision-maker may assign the weights either arbitrarily, based on his intuition or experience or preference, or the weights may be decided by using methods like AHP [13] or ranking methods such as equal weights, rank exponent, rank sum, rank reciprocal and centroid weights. A detailed review of techniques of weights elicitation is available in [19]. The ranking methods, except centroid method, take a more heuristic approach used to determine the weights of the attributes. However, the weights assigned by the centroid method are much steeper; the most important attribute gets assigned a relatively very high weight and the least important attribute gets assigned a relatively very low weight.

There is a need to develop a simple MADM methodto derive the weights of the objectives logically and to evaluate the measures of performance of the alternative solutions with respect to the objectives so that the best compromise alternative solution from among the available Pareto-optimal solutions in the multi- or many-objective optimization problems can be chosen with relative ease. Recently, Rao and Lakshmi [20] proposed R-method as a simple and effective MADM method that can provide effective solutions to the complex decision-making problems. In the currentpaper, the R-method is extended to the MODM problems involving a large number of Pareto-optimal alternative solutions and objectives. The objectives in MODM problems correspond to attributes in the MADM problems. The proposed R-method becomes useful in situations of limited time availability and decision-maker's limited attention and capability to process the information. The details of the proposed method are given in the next section.

2. Proposed R-method for ranking of Pareto-optimal solutions and selecting the best solution in multi- or many-objective optimization

After the non-dominated Pareto-optimal solutions are obtained in the multi- or many-objective optimization problems using any optimization algorithm (e.g. NSGA-II or NSGA-III or SPEA or MOGA or multi-objective versions of any other advanced optimization algorithms such as SA, ACO, PSO, ABC, DE, TLBO, grey wolf optimizer, firefly optimizer, whale optimizer, ant lion optimizer, Jaya optimizer, etc.), the decision table has the objectives and the measures of performance of the non-dominated alternative solutions (i.e. Pareto-optimal solutions) corresponding to the objectives. Table 1 shows the general form of the decision table. The decision-maker's task is to find the best alternative solution from among the available alternative solutions.

The decision table shows the non-dominated alternative solutions, A_j

Table 1
General form of decision table.

Non-dominated alternative solutions	Objectives O_1 (w_1)	O_2 (w_2)	O_3 (w_3)	–	–	O_M (w_M)
A_1	m_{11}	m_{12}	m_{13}	–	–	m_{1M}
A_2	m_{21}	m_{22}	m_{23}	–	–	m_{2M}
A_3	m_{31}	m_{32}	m_{33}	–	–	m_{3M}
–				–	–	
–				–	–	
A_N	m_{N1}	m_{N2}	m_{N3}	–	–	m_{NM}

(for $j = 1, 2,\ldots, N$); objectives, O_i (for $i = 1, 2,\ldots, M$); weights of objectives, w_i(for $i = 1, 2,\ldots, M$); and the measures of performance of alternative solutions, m_{ji} (for $j = 1, 2,\ldots, N$; $i = 1, 2,\ldots, M$). Given the decision table information and a decision making method, the task of the decision maker is to find the best alternative solution by ranking the entire set of alternatives. The steps of the proposed R-method for ranking of Pareto-optimal alternative solutions to find the best compromise solution in multi- or many-objective optimization are given below.

Step 1: Obtain the non-dominated alternative solutions to the given multi- or many-objective optimization problem using any of the advanced optimization algorithms and prepare a decision table containing the performance data of the alternative solutions corresponding to the objectives.

Step 2: Rank the objectives based on their importance, as perceived by the decision-maker in terms of 1, 2, 3, etc. If two or more objectives are considered equally important, then an average rank is assigned to those objectives.

Step 3: Rank the alternative solutions (i.e. Pareto-optimal solutions) in terms of 1, 2, 3, etc. based on their performance data related to the objectives. If two or more alternative solutions have the same value corresponding to an objective, then an average rank is assigned to those alternative solutions.

Table 2
Pareto-optimal solutions provided by MO-Jaya algorithm in a single simulation run for electro-discharge machining process [21].

Solution No.	Process Parameters V_g (V)	I_p (A)	T_{on} (µs)	N (rpm)	Objectives MRR (0.1 mg/s)	TWR(0.1 mg/s)	Θ (degree)	DF
1	25	10	1913.724	200	1.2453(50)	0.0965(01)	3.3476(48)	1.1574(29)
2	25.0495	10	1844.116	200	1.2865(49)	0.0986(02)	3.0562(47)	1.1558(28)
3	25	10	1757.623	200	1.3199(48)	0.0996(03)	2.7192(46)	1.1536(27)
4	26.2683	10	2000	200	1.4191(47)	0.1162(04)	3.7259(49)	1.1603(30)
5	25	10	300	200	1.4245(46)	0.2215(05)	0.0811(01)	1.079(02)
6	31.7003	10	2000	200	2.5179(45)	0.2405(06)	3.8046(50)	1.1629(31)
7	28.5	10	932.73	212.1907	3.0999(44)	0.2672(07)	0.6472(11)	1.1259(18)
8	33.8835	10	980.8407	214.6995	5.0426(43)	0.4827(08)	0.7417(14)	1.13(19)
9	39.4565	10	1366.835	200	5.5058(42)	0.5499(09)	1.7016(26)	1.1488(26)
10	39.5125	10	893.006	200	6.1636(41)	0.6041(10)	0.6878(13)	1.1325(21)
11	43.1006	10	785.4233	214.3395	9.0452(40)	1.0027(11)	0.5488(08)	1.1238(17)
12	60.5423	10	300	200	9.4074(39)	1.871(18)	0.159(02)	1.0949(08)
13	50.252	10	951.2899	200	10.0145(38)	1.1314(12)	0.943(18)	1.1347(22)
14	50.4624	10	1094.945	209.8391	11.047(37)	1.3154(13)	1.1924(23)	1.1359(24)
15	95	10	300	370.8176	11.201(36)	1.547(15)	0.5758(09)	1.0749(01)
16	53.9205	10	1193.568	203.7774	11.3644(35)	1.3979(14)	1.5711(24)	1.1404(25)
17	61.7591	10	417.817	200	11.776(34)	1.8677(17)	0.269(03)	1.1054(11)
18	52.3786	10	997.8612	216.286	12.7649(33)	1.5735(16)	0.9942(19)	1.1302(20)
19	59.0602	10	1199.503	212.3769	14.2355(32)	1.9259(19)	1.6368(25)	1.1357(23)
20	62.0647	10	782.3541	212.8126	16.3417(31)	2.1735(20)	0.7642(15)	1.1214(16)
21	57.6466	10	899.7264	241.1095	17.1198(30)	2.3377(21)	0.8244(17)	1.1187(15)
22	78.1695	10	300	303.9107	18.6777(29)	3.184(23)	0.3371(06)	1.0817(03)
23	63.9669	10	721.4555	233.2439	19.5525(28)	2.7339(22)	0.6496(12)	1.1129(14)
24	81.4454	10.4816	300	263.1196	20.3185(27)	4.6091(30)	0.3181(04)	1.0889(07)
25	82.047	10.242	300	276.3105	20.4793(26)	4.1102(27)	0.3279(05)	1.0849(05)
26	81.5354	10	407.3847	289.2469	22.0527(25)	3.4706(24)	0.4306(07)	1.0863(06)
27	93.3095	10	460.6347	290.8624	23.1194(24)	3.5922(25)	0.5781(10)	1.0837(04)
28	77.2987	10	847.0946	243.5197	23.7081(23)	3.6683(26)	1.0182(20)	1.1097(12)
29	84.271	11.0556	628.0503	247.9736	24.8563(22)	5.7193(31)	0.7768(16)	1.1108(13)
30	95	10	680.5518	230.7705	25.9749(21)	4.2886(28)	1.0235(21)	1.101(10)
31	95	10	726.217	247.0427	26.8204(20)	4.4199(29)	1.0638(22)	1.0983(09)
32	63.1759	35.7338	815.4502	250.6803	26.8784(19)	141.1848(32)	1.9473(28)	1.2461(34)
33	46.8665	45	704.2118	262.2001	26.8928(18)	168.1049(36)	2.1411(31)	1.2377(32)
34	66.0972	36.3491	644.0377	251.924	27.0320(17)	153.657(33)	1.8928(27)	1.2522(36)
35	63.4694	37.0986	865.4543	259.8581	27.2314(16)	155.9124(34)	2.0329(29)	1.2489(35)
36	65.7791	37.3432	876.1797	259.7034	27.5357(15)	164.2039(35)	2.1005(30)	1.2531(37)
37	48.7786	45	750.8355	259.6498	27.6576(14)	176.157(37)	2.1703(32)	1.2431(33)
38	53.8153	45	571.8286	249.4247	27.9858(13)	200.6127(39)	2.1753(33)	1.2581(40)
39	55.3277	45	591.4365	277.3743	28.3591(12)	208.3648(41)	2.2865(34)	1.2568(39)
40	52.1831	45	875.1416	246.9805	28.454(11)	185.9188(38)	2.3039(35)	1.254(38)
41	55.6714	45	867.8184	251.2169	29.5533(10)	205.7847(40)	2.3407(37)	1.2608(41)
42	56.978	45	895.9178	245.2088	29.6352(09)	208.5368(42)	2.4141(40)	1.265(42)
43	59.1992	45	664.6343	264.8876	29.8885(08)	224.7545(44)	2.3068(36)	1.2662(44)
44	58.1049	45	835.4176	253.878	30.121(07)	218.1782(43)	2.3559(39)	1.2653(43)
45	60.3879	45	738.0855	255.0349	30.420(06)	228.327(45)	2.3521(38)	1.27(45)
46	62.1815	45	846.2731	248.4064	30.6445(05)	233.4057(46)	2.473(41)	1.2745(46)
47	64.936	45	937.1024	251.8109	30.7501(04)	246.4366(48)	2.577(43)	1.278(47)
48	64.7866	45	770.7681	249.6904	30.8257(03)	243.3128(47)	2.4831(42)	1.2792(48)
49	69.817	45	810.0259	259.0902	30.8293(02)	260.9632(50)	2.5826(44)	1.2849(50)
50	68.0958	45	836.1816	252.7234	31.0207(01)	256.4056(49)	2.5864(45)	1.2836(49)
Ranks assigned to the objectives→					1.5	1.5	3.5	3.5

The numbers written in parentheses indicate the ranks of the alternative solutions corresponding to the respective objectives.

Table 3
Weights assigned to the alternative solutions and the objectives, composite scores, and composite ranks of the alternative solutions for electro-discharge machining process.

Solution No.	*MRR*	*TWR*	*Θ*	*DF*	Composite Score	Composite Rank
1	0.014539	0.065415	0.014671	0.016512	0.030688	2
2	0.014604	0.04361	0.01474	0.016657	0.024	5
3	0.014671	0.035681	0.014811	0.01681	0.021609	10
4	0.01474	0.031399	0.014604	0.016374	0.020182	16
5	0.014811	0.028649	0.065415	0.04361	0.034217	**1**
6	0.014884	0.0267	0.014539	0.016243	0.018735	23
7	0.01496	0.025229	0.021661	0.018716	0.02013	17
8	0.015038	0.024069	0.020118	0.018439	0.019449	20
9	0.015139	0.023123	0.016971	0.016971	0.018308	28
10	0.015202	0.022334	0.02057	0.017945	0.018954	22
11	0.015289	0.021661	0.024069	0.019018	0.019644	19
12	0.015379	0.018716	0.04361	0.024069	0.023444	7
13	0.015472	0.02108	0.018716	0.017724	0.018255	30
14	0.015569	0.02057	0.017517	0.017324	0.017822	37
15	0.01567	0.019714	0.023123	0.065415	0.027815	4
16	0.015775	0.020118	0.017324	0.017142	0.017675	38
17	0.015884	0.019018	0.035681	0.021661	0.021725	9
18	0.015999	0.019349	0.018439	0.018182	0.017916	35
19	0.016118	0.018439	0.017142	0.017517	0.017298	42
20	0.016243	0.018182	0.019714	0.019349	0.018096	33
21	0.016374	0.017945	0.019018	0.019714	0.018	34
22	0.016512	0.017517	0.0267	0.035681	0.022414	8
23	0.016657	0.017724	0.02108	0.020118	0.018489	26
24	0.01681	0.016374	0.031399	0.025229	0.021057	13
25	0.016971	0.01681	0.028649	0.028649	0.021369	11
26	0.017142	0.017324	0.025229	0.0267	0.020559	15
27	0.017324	0.017142	0.022334	0.031399	0.020902	14
28	0.017517	0.016971	0.018182	0.02108	0.018153	31
29	0.017724	0.016243	0.019349	0.02057	0.018117	32
30	0.017945	0.016657	0.017945	0.022334	0.018382	27
31	0.018182	0.016512	0.017724	0.023123	0.018519	25
32	0.018439	0.016118	0.016657	0.015884	0.016895	49
33	0.018716	0.01567	0.016243	0.016118	0.016807	50
34	0.019018	0.015999	0.01681	0.01567	0.017025	48
35	0.019349	0.015884	0.016512	0.015775	0.017055	47
36	0.019714	0.015775	0.016374	0.015569	0.017069	46
37	0.020118	0.015569	0.016118	0.015999	0.017164	44
38	0.02057	0.015379	0.015999	0.015289	0.017087	45
39	0.02108	0.015202	0.015844	0.015379	0.017177	43
40	0.021661	0.015472	0.015775	0.015472	0.017445	41
41	0.022334	0.015289	0.015569	0.015202	0.017506	40
42	0.023123	0.015139	0.015289	0.015139	0.017639	39
43	0.024069	0.01496	0.01567	0.01496	0.017915	36
44	0.025229	0.015038	0.015379	0.015038	0.018257	29
45	0.0267	0.014884	0.015472	0.014884	0.018654	24
46	0.028649	0.014811	0.015202	0.014811	0.019169	21
47	0.031399	0.014671	0.015038	0.01474	0.019932	18
48	0.035681	0.01474	0.015119	0.014671	0.021281	12
49	0.04361	0.014539	0.01496	0.014539	0.023618	6
50	0.065415	0.014604	0.014884	0.014604	0.030385	**3**
Weights assigned to the objectives→	0.309545	0.309545	0.190454	0.190454		

Step 4: Convert the ranks assigned to the objectives and the alternative solutions into the corresponding weights. Table A1,given in the Appendix A, can be used for this purpose. Table A1 shows the weights assigned to 35 ranks (corresponding to 35 alternative solutions or objectives). However, the weights can be assigned to any number of ranks by using Eq. (5) proposed below.

$$w_j = \frac{\left(\frac{1}{\sum_{k=1}^{j}\left(\frac{1}{r_k}\right)}\right)}{\sum_{j=1}^{n}\left(\frac{1}{\sum_{k=1}^{j}\left(\frac{1}{r_k}\right)}\right)} \tag{5}$$

w_j = weight of objective/alternative j (j = 1, 2, 3, ….., n)
r_k = rank of objective/alternative k (k = 1, 2, 3, ….., j)
n = number of objectives/alternatives

Step 5: Compute the composite scores of the alternative solutions by summing up the products of the weights of the objectives with the corresponding weights of the alternative solutions.

Step 6: Determine the composite ranks of the alternative solutions based on the composite scores. The alternative solution having the highest composite score is considered as the best compromise Pareto-optimal solution. The decision-maker can go ahead with the best compromise Pareto-optimal solution for implementation.

The pseudocode of the method is given at the end of this section. The code in MATLAB 2015a is given in the supplementary material.

3. Demonstration of applications of the proposed R-method for ranking of Pareto-optimal solutions and selecting the best solution

The proposed R-method is demonstrated by means of three examples and the performance is compared with four widely used MADM methods in this section.

3.1. Example 1: Optimization of electro-discharge machining process

Rao *et al.*[21] conducted experiments on electro-discharge machining process and developed mathematical models for the objective functions of maximizing the metal removal rate (*MRR)*, minimizing the tool wear rate (*TWR*), minimizing the taper angle (Θ) and minimizing the delamination factor (*DF*). The process parameters considered for optimization were the gap voltage (V_g), pulse current (I_p), pulse-on time (T_{on}) and tool rotation speed (*N*). The process parameter bounds were: 25 V $\leq V_g \leq$ 95 V; 10 A $\leq I_p \leq$ 45 A; 300 µs $\leq T_{on} \leq$ 2000 µs; and 200 rpm $\leq N \leq$ 400 rpm. The experiments were conducted with these four process parameters considering each parameter at 5 levels. A total of 30 experiments were conducted and the values of the 4 objectives, i.e. *MRR, TWR,* Θ and *DF*, were measured. The design of experiments and the details are available in Rao et al. [21]. The mathematical models for the4 objectives were developed in terms of the process parameters of V_g, I_p, T_{on} and *N* using regression analysis. Subsequently, a multi-objective Jaya (MO-Jaya) algorithm was used to simultaneously optimize the 4 objective functions using a non-dominated sorting approach. The 50 non-dominated alternative solutions along with the optimum values of process parameters are given in Table 2. Each non-dominated alternative solution is a set of values of *MRR, TWR,* Θ and *DF* corresponding to particular combination of process parameters of V_g, I_p, T_{on}, and *N*. Now, to find the best compromise solution from among the 50 alternative solutions, the steps of the proposed R-method are carried out as explained below.

Step 1: Table 2 shows the 50 non-dominated alternative solutions to the given many-objective optimization problem obtained using MO-Jaya algorithm. The performance data of the 50 alternative solutions corresponding to the 4 objectives is also shown in Table 2. These solutions are non-dominated in nature, and all the solutions can be considered as equivalent. However, the compromise among the objectives is different for each solution. The solution that has the best compromise among the objectives can be considered as the best solution. Hence, to identify the solution with the best compromise, the proposed R-methodis used.

Step 2: The 4 objectives are ranked based on their importance for the considered electro-discharge machining process. Let the decision-maker (i.e. process planner) considers *MRR* and *TWR* equally important in the electro-discharge machining process. Then an average rank of 1.5 (i.e. average of rank 1 and rank 2) is assigned to both *MRR* and *TWR*. Similarly, let the decision maker considers Θ and *DF* equally important. In such a situation, the average rank of 3.5 (i.e. average of rank 3 and rank 4) is assigned to both Θ and *DF*.

Step 3: The alternative solutions based on their performance data related to the objectives are also ranked and shown in parentheses next to their corresponding performance measures in Table 2. The objective *MRR* is beneficial objective and higher values are desirable. However, lower values are desirable in the case of the other three objectives of *TWR,* Θ and *DF*. Hence, from *MRR* point of view, alternative solution no. 50 is assigned rank 1. From *TWR*viewpoint, alternative solution no. 1 is assigned rank 1. Similarly, in the case of Θ and *DF*, the alternative solution no. 5 is assigned rank 1. Other alternative solutions are ranked based on their corresponding values with respect to the objectives.

Step 4: The ranks assigned to the 4 objectives and 50 alternative solutions are converted into the corresponding weights. Following Eq. (5) given in Section 2, the weights are developed and are shown in Table 3. It may be noted that in the case of objectives *MRR* and *TWR*, as the average rank is 1.5, an average weight of 0.309545 (i.e. average of 0.3714543 and 0.2476362) is assigned to both *MRR* and *TWR*. Similarly, in the case of objectives Θ and *DF*, as the average rank is 3.5, an average weight of 0.190454 (i.e. average of 0.202611436 and 0.178298064) is assigned to both Θ and *DF*.

The weights of the ranks of the 50 alternative solutions are calculated as given below.

1/reciprocal of rank 1: $1/\ (1/1) = 1.000000$
1/reciprocals of ranks up to 2: $1/(1/1 + 1/2) = 0.666666$
1/reciprocals of ranks up to 3: $1/(1/1 + 1/2 + 1/3) = 0.545454$
1/reciprocals of ranks up to 4: $1/(1/1 + 1/2 + 1/3 + 1/4) = 0.48$
1/reciprocals of ranks up to 5: $1/(1/1 + 1/2 + 1/3 + 1/4 + 1/5) = 0.437956$
1/reciprocals of ranks up to 6: $1/(1/1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6) = 0.408163$
1/reciprocals of ranks up to 50: $1/(1/1 + 1/2 + \ldots + \ldots + 1/50) = 0.2222614$
Total $= 1.000000 + 0.666666 + 0.545454 + \ldots.. + \ldots.. + 0.2222614 = 15.287014$

Therefore, the weight assigned to rank 1 = 1.000000/15.287014 = 0.065415; weight assigned to rank 2 = 0.666666/15.287014 = 0.04361; ….., and weight assigned to rank 50 = 0.2222614/15.287014 = 0.014539.

Similarly, the weights of the ranks of the 4 objectives are calculated as given below.

1/reciprocal of rank 1: $1/\ (1/1) = 1.000000$
1/reciprocals of ranks up to 2: $1/(1/1 + 1/2) = 0.666666$
1/reciprocals of ranks up to 3: $1/(1/1 + 1/2 + 1/3) = 0.545454$
1/reciprocals of ranks up to 4: $1/(1/1 + 1/2 + 1/3 + 1/4) = 0.48$
Total $= 1.000000 + 0.666666 + 0.545454 + 0.48 = 2.69212$

Therefore, weight assigned to rank 1 = 1.000000/2.69212 = 0.37145; Weight assigned to rank 2 = 0.666666/2.69212 = 0.24763; Weight assigned to rank 3 = 0.545454/2.69212 = 0.20261; and Weight assigned to rank 4 = 0.48/2.69212 = 0.17829. These values are same as those given in Table A1.

Step 5: The composite scores of the alternative solutions are computed by summing up the products of the weights of the objectives with the corresponding weights of the alternative solutions. The composite scores are shown in Table 3.

Step 6: The composite ranks of the alternative solutions are determined based on the composite scores. The alternative solution no. 5 which is having the highest composite score is considered as the best choice. The second choice is solution no. 1 and the last choice is solution no. 33.

To compare the performance of the proposed R-method, four different MADM methods are considered and applied to the same example with the same weights of the objectives and the data given in Table 2. The MADM methods are: (i). simple additive weighing (SAW) method, (ii). Weighted product method (WPM), (iii). TOPSIS, and (iv). PROMETHEE method. The readers may refer to [1] for details about the working steps of these four MADM methods. The rankings of the alternative solutions based on the composite scores given by these four

Table 4
The Pareto-optimal solutions obtained by the SAP-Rao algorithm for the compression ignition biodiesel engine with an exhaust gas recirculation system case study[23].

Solution No.	Design Variables				Objectives					
	ER (%)	*EL (%)*	*ES (rpm)*	*BP (%)*	*P (kW)*	*BSFC (g/kWh)*	*CO (%)*	*NO_x (ppm)*	*HC (ppm)*	*S_m (1/m)*
1	0.000	41.608	1800.000	15.000	1.874(22)	319.726(09)	0.072(02)	288.515(23)	16.017(01)	2.706(04)
2	2.573	62.466	1800.000	9.000	2.598(15)	218.952(01)	0.418(14)	368.509(29)	51.897(12)	6.838(15)
3	30.000	25.000	2164.146	15.000	1.087(28)	564.355(29)	0.092(03)	86.943(01)	46.614(10)	3.750(08)
4	0.000	30.081	1800.000	10.765	1.320(25)	420.639(23)	0.125(04)	210.699(10)	22.879(02)	2.304(01)
5	0.039	71.261	1800.000	7.199	2.887(11)	235.347(04)	0.745(19)	386.066(30)	80.418(16)	9.799(19)
6	30.000	25.000	1895.402	15.000	1.207(26)	535.929(27)	0.00044(1)	114.501(03)	32.575(06)	3.033(06)
7	0.000	25.000	2156.082	15.000	0.896(29)	576.845(30)	0.195(10)	110.599(02)	45.015(09)	2.894(05)
8	0.000	25.000	2006.552	5.339	0.866(30)	540.802(28)	0.144(07)	128.730(04)	41.406(08)	2.600(03)
9	30.000	75.000	2400.000	0.000	3.523(01)	424.208(24)	2.160(30)	256.076(19)	249.948(30)	17.390(30)
10	21.019	74.220	2400.000	8.614	3.298(07)	380.535(15)	1.746(25)	250.382(16)	197.299(25)	14.726(25)
11	30.000	74.574	2400.000	15.000	3.324(06)	390.991(17)	1.582(24)	202.916(08)	178.579(24)	13.891(24)
12	16.915	75.000	2400.000	5.040	3.336(05)	395.523(18)	1.903(26)	268.393(21)	214.100(26)	15.729(26)
13	30.000	75.000	2400.000	1.607	3.505(02)	412.975(22)	2.087(290	251.746(17)	240.594(29)	16.922(29)
14	29.825	75.000	2400.000	4.381	3.472(03)	398.013(20)	1.969(28)	243.709(14)	225.472(28)	16.174(28)
15	19.350	65.053	2229.668	10.244	2.735(14)	332.716(11)	0.936(20)	252.536(18)	125.951(20)	9.861(20)
16	30.000	67.251	2349.973	12.677	3.008(10)	349.942(13)	1.183(22)	225.179(12)	145.346(22)	11.154(21)
17	29.077	51.512	2136.747	7.645	2.215(19)	332.489(10)	0.414(13)	236.483(13)	82.577(17)	6.201(14)
18	10.248	68.575	1800.000	9.327	2.865(12)	227.390(03)	0.570(16)	351.145(27)	73.983(15)	8.769(17)
19	30.000	75.000	2255.077	15.000	3.216(08)	389.408(16)	1.290(23)	194.374(07)	158.665(23)	12.925(23)
20	0.000	57.276	1800.000	9.685	2.402(17)	225.008(02)	0.290(12)	363.642(28)	36.694(07)	5.371(13)
21	2.573	48.956	1800.000	10.333	2.096(20)	254.435(06)	0.141(06)	325.837(25)	25.783(04)	3.819(09)
22	0.000	26.266	1800.000	8.369	1.110(27)	466.624(26)	0.165(08)	178.753(06)	27.945(05)	2.380(02)
23	23.501	67.410	1899.793	7.779	2.812(13)	270.468(07)	0.640(17)	292.914(24)	97.236(18)	9.191(18)
24	30.000	75.000	1893.304	4.722	3.113(09)	311.412(08)	1.039(21)	280.125(22)	140.763(21)	12.457(22)
25	0.000	52.570	1800.000	12.297	2.262(18)	243.823(05)	0.170(09)	345.434(26)	24.795(03)	4.182(10)
26	29.825	53.749	2305.836	6.440	2.445(16)	339.513(12)	0.740(18)	245.849(15)	110.963(19)	7.828(16)
27	18.792	32.541	2150.875	9.740	1.324(24)	457.749(25)	0.130(05)	157.341(05)	51.722(11)	3.591(07)
28	26.329	46.882	2168.578	13.360	2.017(21)	358.695(14)	0.255(11)	207.096(09)	62.543(13)	5.007(12)
29	14.377	40.087	2400.000	13.640	1.756(23)	402.843(21)	0.428(15)	210.913(11)	67.840(14)	4.899(11)
30	20.360	74.794	2400.000	4.152	3.371(4)	396.828(19)	1.937(27)	263.050(20)	220.051(27)	15.967(27)
Ranks assigned to the objectives→					1.5	1.5	4.5	4.5	4.5	4.5

The numbers written in parentheses indicate the ranks of the alternative solutions corresponding to the respective objectives.

MADM methods and the proposed R-method are given below (entire ranking is not shown for space reasons and it is not necessary also to show the rankings of all alternative solutions).

SAW: 5-1-2-3————————————9
WPM: 5-12-7-17———————————49
TOPSIS: 29-31-30-27——————————49
PROMETHEE: 5-26-15-22——————————49
R: 5-1-50-15——————————33

In decision-making situations, the decision-maker is always interested in knowing the first best choice. In the present example, it may be noted that in this example the proposed R-method, SAW, WPM, and PROMETHEE suggested alternative solution no. 5 as the first best choice. The alternative solution no. 1 is suggested as the second choice by the proposed R-method and SAW. The TOPSIS method has suggested alternative solution no. 29 as the first choice. However, a close look at the values of *MRR, TWR, Θ*, and *DF* of alternative solutions 5 and 29 reveal that the alternative solution no. 5 is much better than solution no. 29 in case of three objectives (i.e. *TWR, Θ*, and *DF*) out of four objectives and solution no. 29 is better than solution no. 5 only in the case of *MRR*. Furthermore, the combined weight of three objectives *TWR, Θ*, and *DF*for which solution no. 5 is much better than solution no. 29 (i.e. $0.309545 + 0.190454 + 0.190454 = 0.690453$) is higher. Thus, proposing solution no. 5 as the first choice is more rational and logical. Hence, the decision-maker (i.e. process planner) can choose the process parameters corresponding to solution no. 5 (i.e. 25 V, 10 A, 300 μs, and 200 rpm) for the electro-discharge machining process considered.

3.2. Example 2: Optimization of design variables of acompression ignition biodiesel engine with an exhaust gas recirculation system

To further demonstrate the working of proposed R-method, a many-objective case study of a compression ignition biodiesel engine with an exhaust gas recirculation system presented by Jaliliantabar *et al.*[22] and Rao and Keesari [23] is considered. The exhaust gas recirculation rate (*ER*), engine load percentage (*EL*), engine speed (*ES*) in rpm, and biodiesel percentage (*BP*) were the design variables with the ranges $0 \le ER \le 30$, $25 \le EL \le 75$, $1800 \le ES \le 2400$, and $0 \le BP \le 15$. The objective functions of this case study were maximization of power output (*P*), and minimization of brake specific-fuel consumption (*BSFC),* carbon monoxide *(CO),* nitrogen oxides (*NOx*), unburned hydrocarbons (*HC*), and smoke capacity (*Sm*). Jaliliantabar *et al.*[22] and Rao and Keesari [23] presented the regression models for the 6 objectives in terms of the design variables.

Jaliliantabar *et al.* [22] presented the optimum design parameters using the NSGA-II algorithm and reported only a single optimal solution. However, Rao and Keesari [23] proposed self-adaptive population Rao (SAP-Rao) algorithm and presented the results along with the results of Jaya, AMTPG-Jaya, and Rao algorithms(Rao-1, Rao-2, Rao-3 and Rao-4) to identify the optimum design parameters through many-objective optimization (six-objectives in this example). Rao and Keesari [21] reported that the self-adaptive population Rao algorithm (SAP-Rao) has produced a better set of 30 non-dominated alternative solutions

Table 5
Weights assigned to the alternative solutions and the objectives, composite scores, and composite ranks of the alternative solutions for thecompression ignition biodiesel engine with an exhaust gas recirculation system case study.

Solution No.	*P*	*T*	*BSFC*	*CO*	NO_x	*HC*	Composite Score	Composite Rank
1	0.025537	0.033317	0.062836	0.02524	0.094253	0.045242	0.043954887	1
2	0.028405	0.094253	0.028987	0.023791	0.030373	0.028405	0.043640477	2
3	0.024	0.023791	0.051411	0.094253	0.03218	0.034679	0.039359401	6
4	0.0247	0.02524	0.045242	0.03218	0.062836	0.094253	0.042773184	3
5	0.031211	0.045242	0.026567	0.023593	0.02788	0.026567	0.031839307	15
6	0.024453	0.024221	0.094253	0.051411	0.038471	0.038471	0.040900721	5
7	0.023791	0.023593	0.03218	0.062836	0.033317	0.041279	0.033589079	12
8	0.023593	0.024	0.036351	0.045242	0.034679	0.051411	0.033383216	14
9	0.094253	0.024961	0.023593	0.026567	0.023593	0.023593	0.040950241	4
10	0.036351	0.028405	0.0247	0.02788	0.0247	0.0247	0.028737029	27
11	0.038471	0.027403	0.024961	0.034679	0.024961	0.024961	0.03000297	18
12	0.041279	0.026967	0.024453	0.025856	0.024453	0.024453	0.029193359	24
13	0.062836	0.025537	0.023791	0.027403	0.023791	0.023791	0.033875638	11
14	0.051411	0.026198	0.024	0.028987	0.024	0.024	0.031632868	16
15	0.028987	0.031211	0.026198	0.026967	0.026198	0.026198	0.028137069	29
16	0.03218	0.029638	0.025537	0.030373	0.025537	0.025856	0.028748971	26
17	0.026567	0.03218	0.029638	0.029638	0.027403	0.028987	0.029131593	25
18	0.030373	0.051411	0.02788	0.024221	0.028405	0.027403	0.03353152	13
19	0.034679	0.02788	0.02524	0.036351	0.02524	0.02524	0.029554003	22
20	0.027403	0.062836	0.030373	0.024	0.036351	0.029638	0.03716961	7
21	0.026198	0.038471	0.038471	0.0247	0.045242	0.033317	0.033972999	10
22	0.024221	0.024453	0.034679	0.038471	0.041279	0.062836	0.034904918	8
23	0.029638	0.036351	0.027403	0.024961	0.026967	0.026967	0.02959843	21
24	0.033317	0.034679	0.025856	0.025537	0.025856	0.025537	0.029606701	20
25	0.026967	0.041279	0.033317	0.024453	0.051411	0.03218	0.034766662	9
26	0.02788	0.030373	0.026967	0.028405	0.026567	0.02788	0.02824205	28
27	0.024961	0.0247	0.041279	0.041279	0.031211	0.036351	0.031547777	17
28	0.025856	0.028987	0.031211	0.033317	0.029638	0.030373	0.029385466	23
29	0.02524	0.025856	0.028405	0.031211	0.028987	0.031211	0.027878145	30
30	0.045242	0.026567	0.024221	0.026198	0.024221	0.024221	0.029985708	19
Weights of the objectives→	0.235521	0.235521	0.132238	0.132238	0.132238	0.132238		

compared to the other algorithms such as Jaya, AMTPG-Jaya, Rao-1, Rao-2, Rao-3, Rao-4 and NSGA-II. The coverage and spacing values corresponding to SAP-Rao algorithm were also reported better compared to the other algorithms. Now, to find the best compromise solution from among the 30 Pareto-optimal solutions, the steps of the proposed R-method are carried out as explained below.

Step 1: Table 4 shows 30 Pareto-optimal non-dominated alternative solutions obtained by using the SAP-Rao algorithm to the given many-objective optimization problem of the compression ignition biodiesel engine with an exhaust gas recirculation system. The performance data of the 30 alternative solutions corresponding to the 6 objectives is also shown in Table 4. These solutions are non-dominated in nature, and all the solutions can be considered as equivalent. Hence, to identify the solution with the best compromise, the proposed R-method is used.

Step 2: The 6 objectives are ranked based on their importance for the considered design optimization problem of the compression ignition biodiesel engine with an exhaust gas recirculation system. Let the decision-maker (i.e. designer) assigns equal importance to both *P* and *BSFC*. Hence the average rank assigned to *P* and *BSFC* is 1.5 each (i.e. average of rank 1 and rank 2). The objectives *CO, NOx, HC* and *Sm* are considered equally important and hence the average rank of 4.5 (i.e. average of 3, 4, 5, and 6) is assigned to *CO, NOx, HC* and *Sm*.

Step 3: The alternative solutions based on their performance data related to the objectives are also ranked and shown in parentheses next to their corresponding performance measures in Table 4. Higher values *P* and lower values of *BSFC, CO, NOx, HC* and *Sm* are desirable. Hence, from *P* point of view, alternative solution 9 which has the highest value is assigned rank 1. From *BSFC* point of view, alternative solution no. 2 is assigned rank 1. Similarly, the other alternative solutions are ranked based on their corresponding values with respect to the objectives.

Step 4: The ranks assigned to the 6 objectives and 30 alternative solutions are converted into the corresponding weights using Table A1 and are given in Table 5. It may be noted that in the case of objectives *CO, NOx, HC*, and *Sm* the average rank is assigned as 4.5, and hence an average weight of 0.132238 (i.e. average of 0.154159, 0.135660, 0.123777, and 0.115357) assigned to each of *CO, NOx, HC* and *Sm*.

Step 5: The composite scores of the alternative solutions are computed by summing up the products of the weights of the objectives with the corresponding weights of the alternative solutions. The composite scores are shown in Table 5.

Step 6: The composite ranks of the alternative solutions are determined based on the composite scores. The alternative solution no. 1 which is having the highest composite score is considered as the best choice. The second choice is solution no. 2, the third choice is solution no. 4 and the last choice is solution no. 29. Hence, the decision-maker (i.e. designer) can choose the design variables corresponding to solution no. 1 (i.e.0% of *ER*, 41.608% of *EL*, 1800rpm of *ES*, and 15% of *BP*) for the performance optimization of the the compression ignition biodiesel engine with an exhaust gas recirculation system.

To compare the performance of the proposed R-method, four different MADM methods are also applied to the same example problem with the same weights of the objectives and the data given in Table 4. The rankings of the alternative solutions based on the composite scores given by these methods and the proposed R-method are given below.

SAW: 6-1-25-21——————————29

Table 6
The Pareto-optimal solutions obtained by the SAP-Rao algorithm for the solar-assisted Brayton engine case study [26].

Solution No.	Design Variables						Objectives		
	ε_H	ε_L	ε_R	T_H (K)	T_L (K)	T_I (K)	P (kW)	η_m	F
1	0.7	0.7	0.8	1000	400	612.224	71.60(1.5)	0.22769(25)	0.29950(25)
2	0.7	0.7	0.8	1000	400	566.306	67.87(24)	0.23776(01)	0.31443(2.5)
3	0.7	0.7	0.8	1000	400	563.262	67.32(25)	0.23771(03)	0.31450(01)
4	0.7	0.7	0.8	1000	400	608.822	71.59(03)	0.22901(23)	0.30133(23)
5	0.7	0.7	0.8	1000	400	596.782	71.21(10)	0.23304(16)	0.30700(16)
6	0.7	0.7	0.8	1000	400	605.909	71.54(04)	0.23008(22)	0.30282(22)
7	0.7	0.7	0.8	1000	400	610.018	71.60(1.5)	0.22856(24)	0.30070(24)
8	0.7	0.7	0.8	1000	400	603.221	71.47(06)	0.23101(20)	0.30413(20)
9	0.7	0.7	0.8	1000	400	601.597	71.42(07)	0.23155(19)	0.30489(19)
10	0.7	0.7	0.8	1000	400	586.947	70.52(16)	0.23550(10)	0.31059(10)
11	0.7	0.7	0.8	1000	400	589.335	70.72(15)	0.23498(11)	0.30981(11)
12	0.7	0.7	0.8	1000	400	599.993	71.36(08)	0.23207(18)	0.30562(18)
13	0.7	0.7	0.8	1000	400	594.434	71.08(11)	0.23370(15)	0.30795(15)
14	0.7	0.7	0.8	1000	400	590.984	70.85(14)	0.23459(12)	0.30924(12)
15	0.7	0.7	0.8	1000	400	570.475	68.54(22)	0.23766(04)	0.31411(04)
16	0.7	0.7	0.8	1000	400	605.053	71.52(05)	0.23038(21)	0.30325(21)
17	0.7	0.7	0.8	1000	400	567.980	68.15(23)	0.23775(02)	0.31433(2.5)
18	0.7	0.7	0.8	1000	400	592.524	70.96(13)	0.23420(13)	0.30868(13)
19	0.7	0.7	0.8	1000	400	598.676	71.30(09)	0.23247(17)	0.30620(17)
20	0.7	0.7	0.8	1000	400	582.310	70.08(19)	0.23637(07)	0.31192(07)
21	0.7	0.7	0.8	1000	400	583.643	70.21(18)	0.23614(08)	0.31156(08)
22	0.7	0.7	0.8	1000	400	585.205	70.37(17)	0.23585(09)	0.31112(09)
23	0.7	0.7	0.8	1000	400	580.282	69.85(20)	0.23669(06)	0.31242(06)
24	0.7	0.7	0.8	1000	400	593.177	71.00(12)	0.23403(14)	0.30843(14)
25	0.7	0.7	0.8	1000	400	571.973	68.77(21)	0.23758(05)	0.31393(05)
Ranks assigned to the objectives→							1.5	1.5	3

The numbers written in parentheses indicate the ranks of the alternative solutions corresponding to the respective objectives.

WPM: 6-1-21-25————————————9
TOPSIS: 25-21-20-1————————————9
PROMETHEE: 1-25-21-2————————————9
R: 1-2-4-9————————————29

As mentioned already in example-1, the decision-maker is always interested in knowing the first best choice in decision-making situations. In the present example, it may be noted that the proposed R-method and PROMETHEE method have suggested alternative solution no. 1 as the first best choice. The SAW and WPM have suggested alternative solution no. 6 as the first best choice. The TOPSIS method has suggested alternative solution no. 21 as the first choice. A close look at the values of *P, BSFC, CO, NOx, HC* and *Sm* for the alternative solutions 1 and 6 reveal that the alternative solution no. 1 is much better than 6 in case of four objectives (i.e. *P,BSFC, HC,* and *Sm*) out of six objectives and solution no. 6 is better than solution no. 1 only in the case of two objectives (i.e. *CO* and *NOx*). Furthermore, the combined weight of four objectives *P,BSFC, HC,* and *Sm* for which solution no. 1 is much better than solution no. 6(i. e. 0.235521 + 0.235521 + 0.132238 + 0.132238 = 0.735518) is much higher than the combined weight of the remaining two objectives *CO*

Table 7
Weights assigned to the alternative solutions and the objectives, composite scores, and composite ranks of the alternative solutions for the solar-assisted Brayton engine case study.

Solution No.	P	η_m	F	Composite Score	Composite Rank
1	0.08926	0.02807	0.02807	0.051120604	5
2	0.028367	0.107113	0.0649	0.067039064	**1**
3	0.02807	0.058425	0.107113	0.058994384	**2**
4	0.058425	0.028684	0.028684	0.039887445	8
5	0.03657	0.031683	0.031683	0.033523665	24
6	0.051414	0.029022	0.029022	0.037457	10
7	0.08926	0.028367	0.028367	0.051305718	4
8	0.04372	0.029772	0.029772	0.035026053	15
9	0.041311	0.030192	0.030192	0.034380337	17
10	0.031683	0.03657	0.03657	0.034728653	16
11	0.03228	0.035469	0.035469	0.034267317	18
12	0.039411	0.030647	0.030647	0.03394818	19
13	0.035469	0.03228	0.03228	0.033481005	25
14	0.032942	0.034517	0.034517	0.033923337	20
15	0.029022	0.051414	0.051414	0.042978196	6
16	0.046911	0.029383	0.029383	0.035985679	13
17	0.028684	0.071409	0.0649	0.053708427	3
18	0.033682	0.033682	0.033682	0.033681663	21
19	0.037863	0.031142	0.031142	0.033673556	22
20	0.030192	0.041311	0.041311	0.037121948	11
21	0.030647	0.039411	0.039411	0.036109119	12
22	0.031142	0.037863	0.037863	0.035330753	14
23	0.029772	0.04372	0.04372	0.038465212	9
24	0.034517	0.032942	0.032942	0.033534989	23
25	0.029383	0.046911	0.046911	0.040307558	7
Weights of the objectives→	0.37671	0.37671	0.24657		

and *NOx* for which solution no. 6 is better than solution no. 1 (i.e. $0.235521 + 0.132238 = 0.367759$). Similar logic can be extended while comparing solution no. 1 with solution no. 25 suggested by the TOPSIS method. Thus, proposing solution no. 1 as the first choice by the R-method is rational and logical.

3.3. Example 3: Optimization of design variables of a solar-assisted Brayton engine system

The proposed R-method is demonstrated further by considering a case study presented by Li *et al.* [24].The three objective functions considered were:power output (*P*), thermal efficiency (η_m), and non-dimensional thermo-economic performance function (*F*) of a solar-assisted Brayton heat engine system. These three objective functions are maximization functions. The decision variables are: the temperature of the hot reservoir (T_H), the temperature of the cold reservoir (T_L), the temperature of the working fluid at state 1 of Brayton cycle (T_1), hot side heat exchange effectiveness (ε_H), cold side heat exchange effectiveness (ε_L), and regenerator effectiveness (ε_R) with the lower and upper bounds 700K $\leq T_H \leq$ 1000 K, 400 K $\leq T_L \leq$ 500 K, $T_L \leq T_1 \leq T_H$, $0.5 \leq \varepsilon_H \leq 0.7$, $0.5 \leq \varepsilon_L \leq 0.7$,and $0.5 \leq \varepsilon_R \leq 0.8$.

Li *et al.* [24] used NSGA-II algorithm to obtain the alternative solutions and then used Shannon-entropy, LINMAP, and TOPSISmethods for ranking the alternative solutions. However, Rao and Keesari [25] and Keesari [26] considered the regression models of the three objective functions expressed in terms of the six design variables (presented by Li *et al.* [24]) and attempted the same optimization problem with the same design variables and objective functions using different algorithms such as Jaya algorithm, Rao algorithms and their variants to find the optimal parameters and reported that the self-adaptive population Rao algorithm (SAP-Rao) has produced a better set of 25 non-dominated alternative solutions compared to all other algorithms including NSGA-II.The solutions of the SAP-Rao algorithm have better compromise among the power output, thermal efficiency, and non-dimensional thermo-economic function. It was reported that by using the SAP-Rao algorithm solution, the power output was increased by 2.72%, the thermal efficiency was increased by 1.4%, and the thermo-economic function of the system was increased by 1.4% when compared to that of the NSGA-II algorithm solutions using Shannon-entropy, LINMAP, and TOPSIS methods. Furthermore, the spacing value achieved by the SAP-Rao algorithms was found much better than that achieved by other algorithms. Also, the SAP-Rao algorithm's performance in terms of hypervolume and coverage metrics was better than the other algorithms as well as the NSGA-II algorithm.

Table 6 shows the Pareto-optimal solutions obtained by the SAP-Rao algorithm. Following the steps of the proposed R-method, the ranks are assigned to the alternative solutions and the objectives (shown in Table 6). Table 7 shows the weights assigned to the alternative solutions and the objectives, composite scores, and composite ranks of the alternative solutions for the solar-assisted Brayton engine system case study. The alternative solution no. 2 which is having the highest composite score is considered as the best choice. The second choice is solution no. 3 and the last choice is solution no. 13. Hence, the decision-maker (i.e. designer) can choose the design variables corresponding to solution no. 2 (i.e. $\varepsilon_H = 0.7$, $\varepsilon_L = 0.7$,$\varepsilon_R = 0.8$, $T_H = 1000$ K, $T_L = 400$ K, and $T_1 = 566.306$ K for implementation.

To compare the performance of the proposed R-method, four different MADM methods are also applied to the same problem with the same weights of the objectives and the data given in Table 6. The rankings given by these methods and the proposed R-method are given below.

SAW: 22-21-20-10————————1
WPM: 22-21-20-10————————1
TOPSIS: 11-10-14-22————————3
PROMETHEE:2-17-15-3————————1

Table 8
The execution time in seconds required by the proposed R-method for different number of objectives and alternative solutions.

No. of alternative solutions	No. of objectives 2	3	4	5	10	20	30	40	50	60	70	80	90	100
2	0.000373	0.000496	0.000481	0.000812	0.001343	0.003889	0.007762	0.014631	0.018497	0.023034	0.031546	0.040346	0.047906	0.061182
3	0.000417	0.000529	0.000511	0.000853	0.001784	0.005118	0.009195	0.012154	0.017501	0.022402	0.031848	0.038535	0.048347	0.058522
4	0.000503	0.000522	0.000843	0.001014	0.002095	0.005209	0.008271	0.011007	0.019063	0.023522	0.032592	0.043977	0.051128	0.062186
5	0.000826	0.000698	0.000983	0.001412	0.002042	0.00638	0.009104	0.013605	0.017828	0.025597	0.033699	0.043371	0.061283	0.061306
10	0.001121	0.001382	0.001967	0.001558	0.0034	0.006054	0.01043	0.014742	0.020218	0.027996	0.03574	0.045859	0.055751	0.068702
20	0.003601	0.005132	0.004309	0.003626	0.005644	0.007562	0.011606	0.017864	0.0248	0.031564	0.041048	0.051005	0.062598	0.074736
30	0.009739	0.011105	0.010175	0.009334	0.010152	0.009009	0.01455	0.020276	0.027636	0.035272	0.045802	0.057852	0.069389	0.082052
40	0.014705	0.015049	0.015888	0.01628	0.014063	0.014971	0.01792	0.024688	0.034501	0.039663	0.053739	0.065069	0.078991	0.090343
50	0.023723	0.025253	0.02313	0.025153	0.026613	0.023909	0.02589	0.027358	0.03656	0.048379	0.057266	0.073929	0.089314	0.099848
60	0.018908	0.017975	0.017884	0.018809	0.018496	0.01868	0.022792	0.016731	0.023594	0.032875	0.043753	0.050089	0.05981	0.0749
70	0.044534	0.043734	0.043455	0.044369	0.043402	0.04289	0.044383	0.042355	0.048651	0.057484	0.073559	0.085173	0.100861	0.120572
80	0.071274	0.075209	0.07543	0.068869	0.07336	0.072268	0.069789	0.07398	0.068022	0.071203	0.081425	0.095075	0.117153	0.131395
90	0.091211	0.098326	0.092372	0.088583	0.092522	0.090444	0.092065	0.08826	0.09078	0.088501	0.091634	0.106587	0.122274	0.153315
100	0.115437	0.114766	0.117109	0.113617	0.111449	0.116885	0.112887	0.114158	0.112082	0.124036	0.12516	0.123542	0.131536	0.149838

Table 9
The weights obtained by five different ranking methods.

No. of objectives	Reciprocal Weight (RW) method	Equal Weight (EW) method	Rank Sum (RS) method	Rank Order Centroid (ROC) method	Proposed R-method
2	w_1=0.6666 w_2=0.3333	w_1=0.5 w_2=0.5	w_1=0.6666 w_2=0.3333	w_1=0.75 w_2=0.25	w_1=0.6 w_2=0.4
3	w_1=0.5454 w_2=0.2727 w_3=0.1818	w_1=0.3333 w_2=0.3333 w_3=0.3333	w_1=0.5 w_2=0.3333 w_3=0.1666	w_1=0.6111 w_2=0.2727 w_3=0.1111	w_1=0.4521 w_2=0.3014 w_3=0.2465
4	w_1=0.48 w_2=0.24 w_3=0.16 w_4=0.12	w_1=0.25 w_2=0.25 w_3=0.25 w_4=0.25	w_1=0.4 w_2=0.3 w_3=0.2 w_4=0.1	w_1=0.5208 w_2=0.2708 w_3=0.1458 w_4=0.0624	w_1=0.3714 w_2=0.2476 w_3=0.2026 w_4=0.1782
5	w_1=0.4379 w_2=0.2189 w_3=0.1459 w_4=0.1094 w_5=0.0875	w_1=0.2 w_2=0.2 w_3=0.2 w_4=0.2 w_5=0.2	w_1=0.3333 w_2=0.2666 w_3=0.2 w_4=0.1333 w_5=0.0666	w_1=0.4566 w_2=0.2566 w_3=0.1566 w_4=0.09 w_5=0.04	w_1=0.3195 w_2=0.2130 w_3=0.1743 w_4=0.1533 w_5=0.1399
6	w_1=0.4081 w_2=0.2040 w_3=0.1360 w_4=0.1020 w_5=0.0816 w_6=0.0680	w_1=0.1666 w_2=0.1666 w_3=0.1666 w_4=0.1666 w_5=0.1666 w_6=0.1666	w_1=0.2857 w_2=0.2380 w_3=0.1904 w_4=0.1428 w_5=0.0952 w_6=0.0476	w_1=0.4083 w_2=0.2416 w_3=0.1583 w_4=0.1027 w_5=0.0611 w_6=0.0277	w_1=0.2826 w_2=0.1884 w_3=0.1542 w_4=0.1357 w_5=0.1238 w_6=0.1153
7	w_1=0.3856 w_2=0.1928 w_3=0.1285 w_4=0.0964 w_5=0.0771 w_6=0.0642 w_7=0.0551	w_1=0.1428 w_2=0.1428 w_3=0.1428 w_4=0.1428 w_5=0.1428 w_6=0.1428 w_7=0.1428	w_1=0.25 w_2=0.2142 w_3=0.1785 w_4=0.1428 w_5=0.1071 w_6=0.0714 w_7=0.0357	w_1=0.3704 w_2=0.2275 w_3=0.1561 w_4=0.1085 w_5=0.0727 w_6=0.0442 w_7=0.0204	w_1=0.2548 w_2=0.1699 w_3=0.1390 w_4=0.1223 w_5=0.1116 w_6=0.1040 w_7=0.0983
8	w_1=0.3679 w_2=0.1839 w_3=0.1226 w_4=0.0919 w_5=0.0735 w_6=0.0613 w_7=0.0525 w_8=0.0459	w_1=0.125 w_2=0.125 w_3=0.125 w_4=0.125 w_5=0.125 w_6=0.125 w_7=0.125 w_8=0.125	w_1=0.2222 w_2=0.1944 w_3=0.1666 w_4=0.1388 w_5=0.1111 w_6=0. 0833 w_7=0.0555 w_8=0.0277	w_1=0.3397 w_2=0.2147 w_3=0.1522 w_4=0.1105 w_5=0.0791 w_6=0.0543 w_7=0.0334 w_8=0.0156	w_1=0.2329 w_2=0.1553 w_3=0.1271 w_4=0.1118 w_5=0.1020 w_6=0.0951 w_7=0.0898 w_8=0.0857
9	w_1=0.3534 w_2=0.1767 w_3=0.1178 w_4=0.0883 w_5=0.0706 w_6=0.0589 w_7=0.0504 w_8=0.0441 w_9=0.0392	w_1=0.1111 w_2=0.1111 w_3=0.1111 w_4=0.1111 w_5=0.1111 w_6=0.1111 w_7=0.1111 w_8=0.1111 w_9=0.1111	w_1=0.2 w_2=0.1777 w_3=0.1555 w_4=0.1333 w_5=0.1111 w_6=0.0888 w_7=0.0666 w_8=0.0444 w_9=0.0222	w_1=0.3143 w_2=0.2032 w_3=0.1476 w_4=0.1106 w_5=0.0828 w_6=0.0606 w_7=0.0421 w_8=0.0262 w_9=0.0123	w_1=0.2152 w_2=0.1435 w_3=0.1174 w_4=0.1033 w_5=0.0942 w_6=0.0878 w_7=0.0830 w_8=0.0792 w_9=0.0761
10	w_1=0.3414 w_2=0.1707 w_3=0.1138 w_4=0.0853 w_5=0.0682 w_6=0.0569 w_7=0.0487 w_8=0.0426 w_9=0.0379 w_{10}=0.0341	w_1=0.1 w_2=0.1 w_3=0.1 w_4=0.1 w_5=0.1 w_6=0.1 w_7=0.1 w_8=0.1 w_9=0.1 w_{10}=0.1	w_1=0.1818 w_2=0.1636 w_3=0.1454 w_4=0.1272 w_5=0.1090 w_6=0.0909 w_7=0.0727 w_8=0.0545 w_9=0.0363 w_{10}=0.0181	w_1=0.2928 w_2=0.1928 w_3=0.1428 w_4=0.1095 w_5=0.0845 w_6=0.0645 w_7=0.0478 w_8=0.0336 w_9=0.0211 w_{10}=0.001	w_1=0.2005 w_2=0.1337 w_3=0.1094 w_4=0.0962 w_5=0.0878 w_6=0.0818 w_7=0.0773 w_8=0.0737 w_9=0.0708 w_{10}=0.0685

R: 2-3-17-7——————————————-13

In the present example, it may be noted that the proposed R-method and PROMETHEE method have suggested alternative solution no. 2 as the first best choice. The SAW and WPM have suggested alternative solution no. 22 as the first best choice. The TOPSIS method has suggested alternative solution no. 11 as the first choice. A close look at the values of *P*,η_m, and *F* for the alternative solutions 2 and 22 reveal that the alternative solution no. 2 is much better than solution no. 22 in the case of two objectives (i.e. η_m and *F*) out of three objectives and solution no. 22 is better than solution no. 2 only in the case of objective *P*. Furthermore, the combined weight of two objectives η_m and *F* for which solution no. 2 is much better than solution no. 22(i.e. 0.37671 + 0.24657 = 0.62328) is much higher. Similar logic can be extended while comparing solution no. 2 with solution no. 11 suggested by the TOPSIS method. Thus, proposing solution no. 2 as the first choice by the R-method is more rational and logical. Similarly, the comparison of solution no. 3 which is suggested by R-method as the second best choice is better than solution no. 21 suggested by SAW and WPM methods and solution no. 10 suggested by the TOPSIS method. Thus, it can be said that the proposed R-method providesa more logical way of dealing with the decision-making problems.

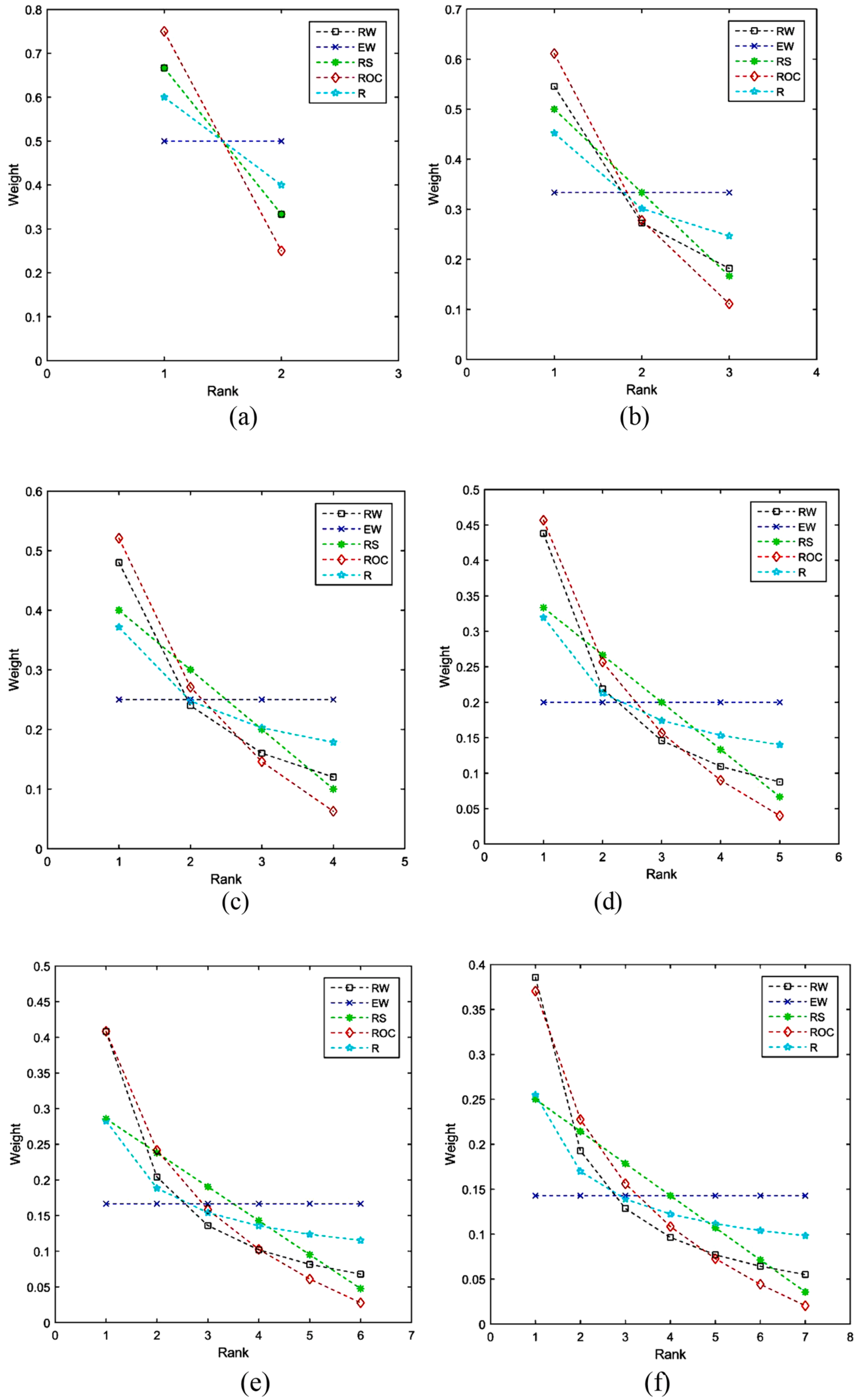


Fig. 1. Graphical comparisons of weights obtained by the RW, EW, RS, ROC and the proposed R-method in the case of (a). 2, (b). 3, (c). 4, (d). 5, (e).6, (f). 7, (g). 8, (h). 9, and (i). 10 ranks.

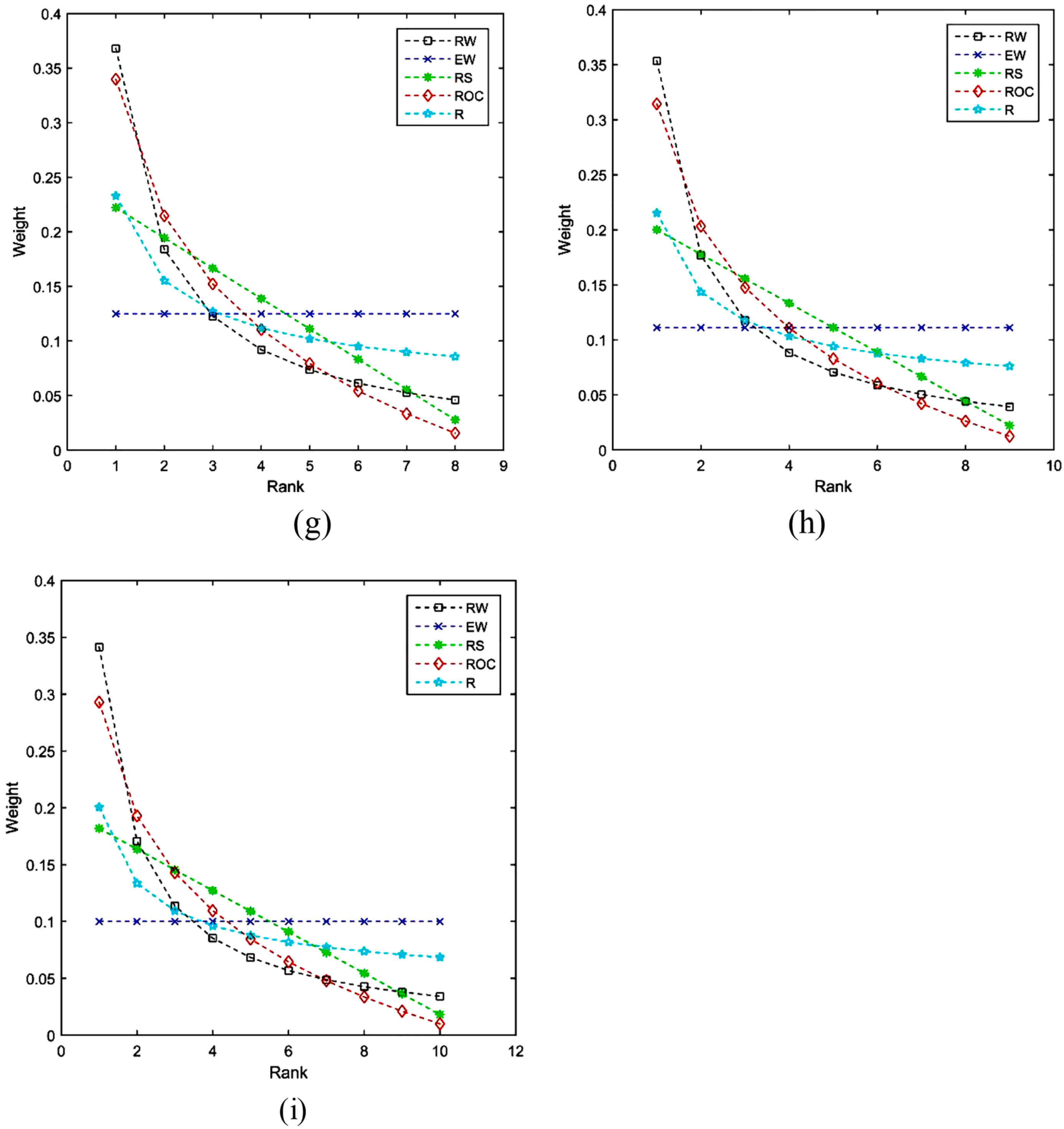


Fig. 1. (*continued*).

4. Discussion on the proposed R-method and comparisons with other ranking methods

The differences in the rankings of alternative solutions are due to the influence of several factors: the performance data of the alternative solutions, the weights of the objectives, and the calculation procedure of each method. It may be mentioned here that SAW, WPM, TOPSIS, and PROMETHEE methods have been proved successful in different decision-making situations. However, these methods have their own merits and limitations. For example, the TOPSIS method used by many of the researchers involves lengthy calculations which become more complex with the increase in the number of alternatives and the objectives. Different methods of normalizing the data in TOPSIS method lead to different rankings of the alternatives. The PROMETHEE method uses the concept of outranking relationship and the computational procedure is quite complex. Different versions of PROMETHEE method may give different rankings to the alternatives.The WPM provides a conservative aggregative method. However, it depends upon the exponential relative

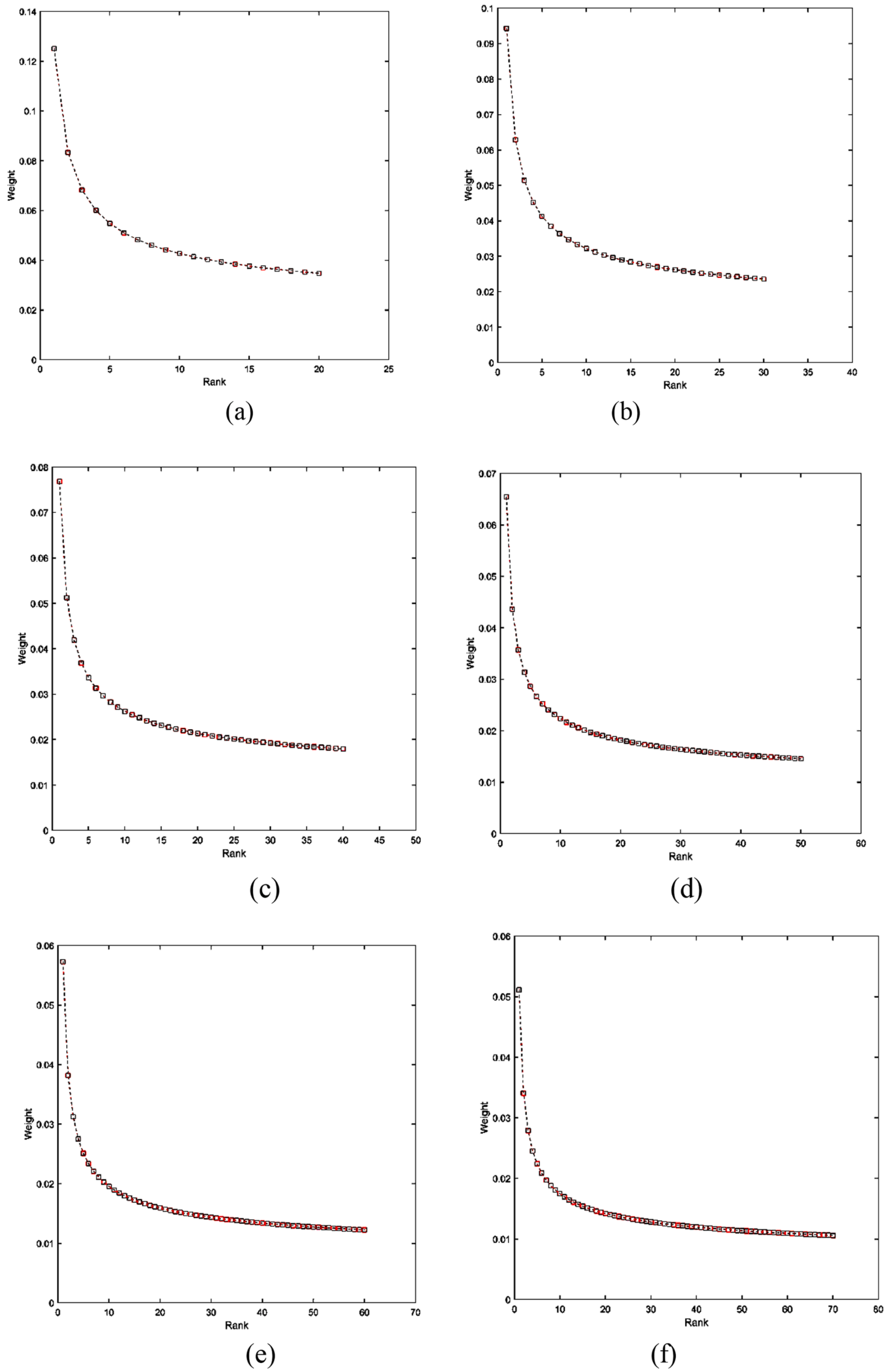
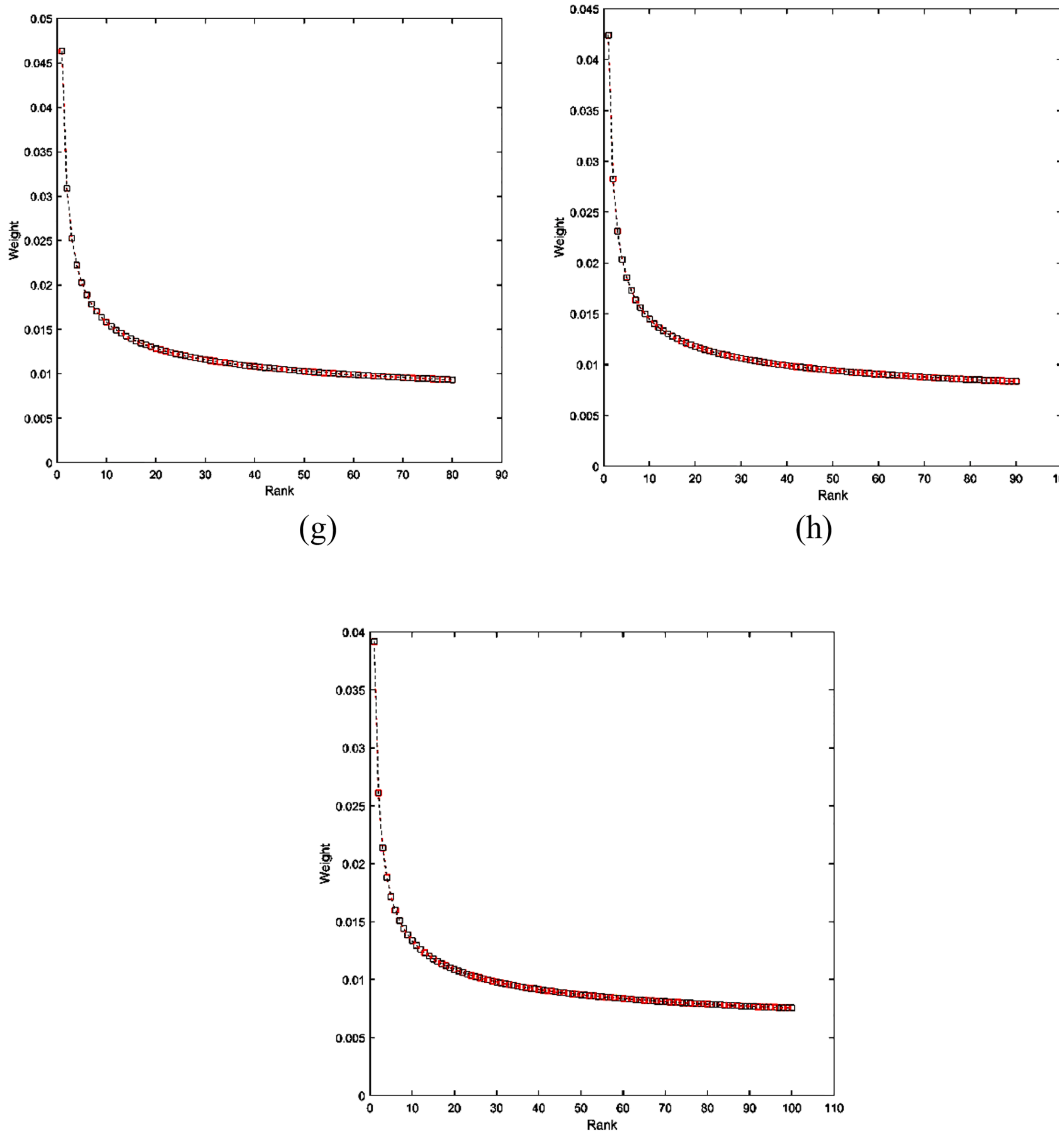


Fig. 2. Graphical representation of weights obtained by the proposed R-method in the case of (a). 20, (b). 30, (c). 40, (d). 50, (e).60, (f). 70, (g). 80, (h). 90, and (i). 100 ranks.



(g) (h) (i)

Fig. 2. (*continued*).

importance, and not proportional between the objectives. In terms of execution time, the SAW and the proposed R-method require shorter execution time than WPM, TOPSIS and PROMETHEE methods in all the three examples considered.

The SAW method and the proposed R-method are comparatively simpler. This is because of the reason that the SAW and the R-method have the simpler calculation processes than the other MADM methods. However, if a comparison is made between the SAW method and the R-method, it can be noted that the SAW method involves normalization of the objective values to make them dimension-less in the calculations and it takes time. Moreover, the weights of the objectives are to be assigned in SAW method by the decision maker using subjective or objective approaches. However, such exercise is not needed in the R-method. The R-method ranks the objectives based on their importance as perceived by the decision maker and the alternative solutions are also ranked based on their performance data corresponding to the objectives. The ranks assigned to the objectives and the alternative solutions are converted to weights. In fact, the decision maker can make use of the readily available table such as that given in Appendix A for this purpose. The composite scores are then calculated using the procedure as explained in Section 2. The code in MATLAB R2015a is given in the supplementary materialand the users can make use of it.

The proposed R-method is comparatively simple and less complex. The execution time required by the method is very less. The execution time in seconds required by the proposed R-method for different number of objectives and alternative solutions (up to 100 objectives and 100 alternative solutions) is given in Table 8. The details of the processor used for execution are: HP Pavilion 15 Notebook PC Processor = intel(R) core(TM) i5-5200u CPU@ 2.20 GHz, Windows 10, 8 Gb RAM, Hard drive = 1 TB, and 64 kb operating system.

From Table 8 it can be understood that the proposed R-method takes a very small fraction of a second to execute the code. The user has to simply input the ranks of the objectives (i.e. 1, 2, 3,…..) and ranks of the alternative solutions (i.e. 1, 2, 3,…..) and then has to execute the code.

It is to be noted here that the proposed R-method is not an algorithm like genetic algorithm (GA) or differential evolution algorithm (DE) or particle swarm optimization (PSO), or any such optimization algorithm. It is like a multi-attribute decision-making (MADM) method in which the number of attributes and the number of alternatives are finite and known. The objectives of the multi-objective optimization problems can be considered as the attributes and the alternative solutions given by the application of an optimization algorithm can be considered as the alternatives in multi-attribute decision-making (MADM). Once a set of alternative solutions are generated by any optimization algorithm (i.e. any meta-heuristic), then the task of the proposed R-method is to find the best alternative solution out of the available alternative solutions. The R-method acts like a decision-making method and it is not an optimization algorithm itself like GA, NSGA, DE, PSO, etc. The concept of "computational complexity" or "time complexity" used to compare different optimization algorithms (i.e. meta-heuristics) is not appropriate for a decision-making method like the proposed R-method as it does not involve any iterations and variables. Hence, the "computational complexity" of the proposed R-method has to be understood only from the time taken by the method to execute the given task (as shown in Table 8). It can be observed from Table 8 that the proposed R-method takes only 0.149838 seconds to give the ranking of solutions for a large problem having 100 objectives and 100 alternative solutions.

To compare the weights obtained by the proposed R-method, four other ranking methods known as "Reciprocal Weight (RW)", "Equal Weights (EW)", "Rank Sum (RS)", "Rank Order Centroid (ROC)" are considered. Eqs.(6–9) are used for computing the weight w_j of objective j in the case of RW, EW, RS, and ROC methods, respectively [19].

$$w_j(RW) = \frac{\left(\frac{1}{r_k}\right)}{\sum_{k=1}^{n}\left(\frac{1}{r_k}\right)} \tag{6}$$

$$w_j(EW) = \frac{1}{n} \tag{7}$$

$$w_j(RS) = \frac{n - r_j + 1}{\sum_{k=1}^{n} n - r_k + 1} \tag{8}$$

$$w_j(ROC) = \left(\frac{1}{n}\right)\sum_{k=1}^{n}\left(\frac{1}{r_k}\right) \tag{9}$$

Table 9 shows weights obtained by the proposed R-method using Eq. (5) and the other four methods using Eqs. (6–9).

Algorithm 1
Pseudocode for R-method

```
Begin
Identify the objectives (M) and alternatives (N)
Prepare the decision table (P) based on the performances data of alternative solutions
For each objective i= 1 to M
For each alternative solution j = 1 to N
P (i,j) = Assign rank from 1,2,3, … N, based on the performance values of alternative
   solutions for the objective i. If the performance measure for more than one
   alternative solution is identical, then calculate the average of the ranks and assign
   the average value to those alternative solutions
End For
P_O (i) = Assign rank to the objective i from 1,2,3, … M, based on their importance, as
   perceived by the decision maker
End For
Sorted_P_O = Sort the ranks of the objectives in ascending order.
For each objective i= 1 to M
Calculate Sum = Σ_{k=1}^{i} 1/Sorted_P_O(k)
Calculate w_O (i) = 1/Sum
End For
For each objective i= 1 to M
Sorted_P = Sort the ranks of the alternatives for the objective i in ascending order.
For each alternative solution j = 1 to N
Calculate Sum = Σ_{k=1}^{j} 1/Sorted_P(i,k) (calculate based on the rank value)
Calculate w (i, j) = 1/Sum (calculate based on the rank value)
End For
End For
CalculateSum_M = Σ_{k=1}^{M} w_O(k)
For each objective i= 1 to M
CalculateSum_N (i) = Σ_{k=1}^{N} w(i,k)
End For
For each objective i= 1 to M
For each alternative solution j = 1 to N
Calculate weight W (i, j) = w(i , j)/Sum_N (j)
End For
Calculate weight W_O(i) = w_O(i)/Sum_N (j)
End For
For each alternative j = 1 to N
Calculate composite scores for each alternative by summing up the products of the
   weights of the objectives with the corresponding weights of the alternative solutions
End For
Sort the composite scores in descending order.
The alternative with highest composite score is considered as the best choice.
End
```

The decision about weight determination will strongly influence the final results of the decision-making. A review of literature on ranking methods [19] reveals that the weights obtained by the ROC method, as compared to the RW, EW, and RS methods, are uniformly distributed on the simplex of rank order weights $w_{r1} \geq w_{r2} \geq \ldots \geq w_{rn}$, where $w_{r1} + w_{r2} + \ldots + w_{rn} = 1$ and r_i is a rank position of w_{ri}. It was reported [19] that the ROC approach to rank order has clear statistical basis and interpretation whereas the RW, EW, and RS methods follow somewhat a more heuristic approach. A choice between the ranking methods depends partly on the belief of the decision-maker about the steepness of the weights. Even though the ROC method is considered as better than the RW, EW, and RS methods, it can be observed from Table 9 that the weights obtained by ROC method are "steeper" assigning a bigger weight to the most important objective and smaller weight to the least important objective. On the other hand, the weights obtained by the RS method are much "flatter" than those given by the ROC method. The RS weights are reduced linearly lower from the most important to the least important. The weights obtained by the RW method descend aggressively after the most important, but in the least important end the ROC weights are the lowest ones.

It can be observed from Table 9 that the weights obtained by the proposed R-method are more uniform and stable compared to the weights obtained by all other methods including the ROC method. For example, in the case of 2 objectives case, the ROC method has assigned weights of 0.75 and 0.25 to the objectives which is a very steep step. On the other hand, the proposed R-method has assigned the weights of 0.6 and 0.4 which is more logical. Similarly, a close look at the values of the weights obtained by the ROC and the proposed R-method in the case of 3, 4, 5, 6, 7, 8, 9, and 10 objectives reveals that the weights obtained are more uniform and stable. The way how the reciprocals are taken in the proposed R-method provides comparatively more uniform and stable weights. With more number of alternative solutions and the objectives, the error will be much less and the uniformity and stability will increase.

Fig. 1(a) – 1(i) show the graphical comparisons of weights obtained by the RW, EW, RS, ROC and the proposed R-method n the case of 2, 3, 4, 5, 6, 7, 8, 9, and 10 ranks. Fig. 2(a) – 2(i) show the graphical representations of weights obtained by the proposed R-method in the case of 20, 30, 40, 50, 60, 70, 80, 90, and 100 ranks. It can be observed that the variation in the weights obtained by the proposed R-method is very smooth with the increase in the number of ranks (i.e. no. of objectives/ alternatives).

It is very clear from the examples presented in this paper that the proposed R-method is very simple and the decision-maker can easily apply this methodology to choose the best compromise solution from among the Pareto-optimal solutions. In general, the weights of the objectives do not depend on the considered problems but depend on the number of the objectives and position of an objective in the ranking order. However, the proposed method has a provision for the opinions of the decision-maker to assign weights of importance to the objectives (instead of assigning ranks). For example, in all the above three examples, if the decision-maker wishes to assign the weights to the objectives based on his intuition or understanding about the objectives, he/she can do so (i.e. without assigning ranks like 1, 2, 3, etc.). After that, the alternative solutions can be ranked with ranks 1, 2, 3, 4, etc. and can be converted to weights using Eq. (5). The weights of the objectives (assigned as per the opinion of the decision-maker) can then be multiplied with the corresponding weights of the alternative solutions and the composite scores of the alternative solutions can be computed.

It is to be noted that the ranking methods RW, EW, RS, and ROC are used by the researchers only for obtaining the weights of the attributes (i.e. objectives) in MADM problems and are not used for ranking the alternatives and the decision-maker may apply any of the established decision-making methods like SAW, TOPSIS, PROMETHEE, etc. to deal with the quantitative or qualitative data of the alternatives. The advantage of the proposed R-method is that it determines not only the weights of the objectives but also the weights of the alternative solutions based the performance data. The proposed methodology can simultaneously consider any number of objectives and alternative solutions and helps to obtain the composite scores for ranking and selection purpose.

Algorithm 1

5. Conclusions

There are a variety of situations where it is reasonable to use ranked weights, and there have been various ranking methods such as reciprocal weights, equal weights, rank sum, and centroid weights. Using ranks to elicit weights is more reliable than just directly assigning weights to the objectives because usually decision makers are more confident about the ranks of some objectives than their weights, and they (experts or non-experts) can agree on ranks more easily. Rank ordering the importance of objectives may be easier than describing other imprecise weights such as bounded weights.

A simple and new multi-attribute decision-making (MADM) method, named as R-method, is proposed in this paper for ranking of Pareto-optimal solutions in multi- and many-objective optimization problems. Once the Pareto-optimal solutions are obtained by using any of the advanced optimization algorithms, the proposed R-method requires only the ranking of objectives and ranking of Pareto-optimal solutions with respect to each of the objectives. The rank ordering the importance of objectives (and the alternatives corresponding to each objective) is easier and convenient. The proposed method is demonstrated by means of three examples. The first example is related to process parameter optimization of an electro-discharge machining process containing 4 objectives and 50 Pareto-optimal solutions. The second example is related to design optimization of a compression ignition biodiesel engine with exhaust gas recirculation system containing 6 objectives and 30 Pareto-optimal solutions. The third example is related to design optimization of a solar-assisted Brayton engine containing 3 objectives and 25 Pareto-optimal solutions.

An interesting feature of the proposed R-method is that even if the decision-maker is interested to assign particular weights of importance to the objectives (instead of the weights obtained by using the proposed methodology), he/she can do it and the remaining procedure, same as that suggested by the proposed methodology, can be followed to get the composite scores and ranks of the alternatives. The weights suggested by the proposed method are better and logical than the other ranking methods such as reciprocal weights, equal weights, rank sum, and centroid weights. The other ranking methods, except centroid method, take a more heuristic approach used to determine the weights of the attributes. However, the weights assigned by the centroid method are much steeper; the most important objective gets assigned a relatively very high weight and the least important objective gets assigned a relatively very low weight. The proposed R-method overcomes this drawback of rank centroid method and the weights suggested are not steeper and are distributed logically. The weights obtained in this manner for the alternative solutions and the objectives are very stable. With more number of alternative solutions and the objectives, the error for ranked alternative solutions and the objectives will be much less.

A point to be noted is that the existing ranking methods available in literature are used to determine the weights of the objectives only. But the proposed R-method determines not only the weights of the objectives but also the weights of the alternative solutions based the performance data. The proposed methodology can simultaneously consider any number of objectives and alternative solutions and helps to obtain the composite scores which evaluates and ranks the alternative solutions for a given multi- or many-objective optimization problem to find the best alternative solution.

The proposed R-method offers a general decision-making methodology and is more suitable in the situations of limited time availability and decision-maker's limited attention and capability to process the information. The results presented in this work are very useful for process planners and designers in the industrial environment. The idea of the proposed R-method is simple, straightforward, and effective. The method will be analyzed further and the results will be reported soon. The proposed method can act as a multi-attribute decision-making method to suggest the best alternative solution out of a large number of alternative solutions given by an optimization algorithm (i.e. metaheuristic) in the multi- and many-objective optimization problems of different fields of engineering and sciences.

Appendix A

Table A1

Table A1
Weights assigned to different ranks of the alternative solutions and the objectives.

	No. of alternative solutions or objectives to be ranked													
	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Rank*↓	Weights assigned													
1	0.6	0.452054795	0.3714543	0.319480916	0.282626336	0.254847479	0.232999618	0.215269575	0.200531189	0.188044339	0.177300512	0.167937568	0.159689863	0.152357647
2	0.4	0.301369863	0.2476362	0.212987277	0.188417557	0.169898319	0.155333078	0.14351305	0.133687459	0.125362893	0.118200342	0.111958378	0.106459908	0.101571764
3		0.246575342	0.202611436	0.174262318	0.15415982	0.139007716	0.1270907	0.117419768	0.109380649	0.10256964	0.09670937	0.09160231	0.087103561	0.083104171
4			0.178298064	0.15335084	0.135660641	0.12232679	0.111839816	0.103329396	0.096254971	0.090261283	0.085104246	0.080610032	0.076651134	0.07313167
5				0.139918649	0.123777957	0.111612034	0.102043628	0.094278646	0.087823878	0.082355185	0.077649859	0.0735493	0.069937166	0.066725977
6					0.115357688	0.104019379	0.095101885	0.087865133	0.081849465	0.076752791	0.072367556	0.068545946	0.065179536	0.062186795
7						0.098288284	0.089862111	0.083024078	0.077339852	0.072523988	0.068380363	0.06476931	0.061588377	0.058760525
8							0.085729163	0.079205625	0.073782829	0.069188456	0.065235405	0.061790432	0.058755797	0.056058004
9								0.07609473	0.07088492	0.066470997	0.062673207	0.059363539	0.056448093	0.053856259
10									0.068464787	0.064201563	0.060533436	0.057336766	0.054520858	0.052017514
11										0.062268866	0.058711163	0.055610725	0.052879586	0.050451601
12											0.057134539	0.054117359	0.051459562	0.049096778
13												0.052808335	0.050214826	0.047909194
14													0.049111734	0.046856751
15														0.04591535

	No. of alternative solutions or objectives to be ranked																			
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Rank*↓	Weights assigned																			
1	0.14579	0.13986	0.13448	0.12957	0.12506	0.12091	0.11708	0.11352	0.11021	0.10711	0.10422	0.10149	0.09894	0.09653	0.09425	0.09209	0.09005	0.08811	0.08627	0.08451
2	0.09719	0.09324	0.08965	0.08638	0.08337	0.08061	0.07805	0.07568	0.07347	0.07141	0.06948	0.06767	0.06596	0.06435	0.06284	0.06139	0.06004	0.05874	0.05751	0.05634
3	0.07952	0.07629	0.07335	0.07067	0.06822	0.06595	0.06386	0.06192	0.06011	0.05842	0.05685	0.05536	0.05397	0.05265	0.05141	0.05023	0.04912	0.04806	0.04705	0.04609
4	0.06998	0.06713	0.06455	0.06219	0.06003	0.05804	0.05619	0.05449	0.05289	0.05141	0.05002	0.04872	0.04749	0.04633	0.04524	0.04421	0.04323	0.04229	0.04141	0.04056
5	0.06385	0.06125	0.05889	0.05674	0.05477	0.05295	0.05128	0.04972	0.04827	0.04691	0.04564	0.04445	0.04333	0.04227	0.04127	0.04033	0.03944	0.03859	0.03778	0.03701
6	0.05950	0.05708	0.05488	0.05288	0.05104	0.04935	0.04778	0.04633	0.04498	0.04372	0.04253	0.04142	0.04038	0.0394	0.03847	0.03759	0.03675	0.03596	0.03521	0.03449
7	0.05622	0.05394	0.05186	0.04997	0.04823	0.04663	0.04515	0.04378	0.04250	0.04131	0.04019	0.03914	0.03815	0.03723	0.03635	0.03552	0.03473	0.03398	0.03327	0.03259
8	0.05364	0.05145	0.04948	0.04767	0.04601	0.04448	0.04307	0.04176	0.04054	0.03941	0.03834	0.03734	0.03640	0.03551	0.03467	0.03388	0.03313	0.03242	0.03174	0.03109
9	0.05153	0.04943	0.04753	0.0458	0.04420	0.04274	0.04138	0.04012	0.03895	0.03786	0.03683	0.03587	0.03497	0.03412	0.03331	0.03255	0.03183	0.03114	0.03049	0.02987
10	0.04977	0.04775	0.04591	0.04423	0.04269	0.04128	0.03997	0.03875	0.03762	0.03657	0.03558	0.03465	0.03378	0.03295	0.03218	0.03144	0.03074	0.03008	0.02945	0.02885
11	0.04827	0.04631	0.04453	0.04290	0.04141	0.04004	0.03876	0.03759	0.03649	0.03546	0.03451	0.03361	0.03276	0.03196	0.03121	0.03049	0.02982	0.02917	0.02856	0.02798
12	0.04698	0.04506	0.04333	0.04175	0.04030	0.03896	0.03772	0.03658	0.03551	0.03451	0.03358	0.03270	0.03188	0.03110	0.03037	0.02967	0.02902	0.02839	0.0278	0.02723
13	0.04584	0.04397	0.04228	0.04074	0.03932	0.03802	0.03681	0.03569	0.03465	0.03368	0.03277	0.03191	0.03111	0.03035	0.02963	0.02896	0.02831	0.02770	0.02712	0.02657
14	0.04483	0.04301	0.04135	0.03984	0.03846	0.03718	0.03600	0.03491	0.03389	0.03294	0.03205	0.03121	0.03042	0.02968	0.02898	0.02832	0.02769	0.02709	0.02653	0.02599
15	0.04393	0.04214	0.04052	0.03904	0.03769	0.03644	0.03528	0.03421	0.03321	0.03228	0.03140	0.03058	0.02981	0.02909	0.02840	0.02775	0.02713	0.02655	0.02599	0.02546
16	0.04312	0.04137	0.03977	0.03832	0.03699	0.03576	0.03463	0.03357	0.03259	0.03168	0.03082	0.03002	0.02926	0.02855	0.02788	0.02724	0.02663	0.02606	0.02551	0.02499
17		0.04066	0.03909	0.03767	0.03636	0.03515	0.03403	0.03300	0.03204	0.03114	0.0303	0.02951	0.02876	0.02806	0.02740	0.02677	0.02618	0.02561	0.02508	0.02457
18			0.03847	0.03707	0.03578	0.03459	0.03349	0.03248	0.03153	0.03064	0.02981	0.02904	0.02830	0.02761	0.02696	0.02635	0.02576	0.02521	0.02468	0.02417
19				0.03652	0.03525	0.03408	0.03300	0.03199	0.03106	0.03019	0.02937	0.02860	0.02788	0.02720	0.02656	0.02596	0.02538	0.02483	0.02431	0.02382
20					0.03476	0.03361	0.03254	0.03155	0.03063	0.02977	0.02896	0.02821	0.02750	0.02683	0.02619	0.02559	0.02503	0.02449	0.02397	0.02349
21						0.03317	0.03211	0.03114	0.03023	0.02938	0.02858	0.02784	0.02714	0.02648	0.02585	0.02526	0.02470	0.02417	0.02366	0.02318
22							0.03172	0.03076	0.02986	0.02902	0.02823	0.0275	0.02681	0.02615	0.02554	0.02495	0.0244	0.02387	0.02337	0.02289
23								0.03039	0.02951	0.02868	0.02791	0.02718	0.02649	0.02585	0.02524	0.02466	0.02412	0.02359	0.02310	0.02263
24									0.02918	0.02836	0.0276	0.02688	0.02620	0.02556	0.02496	0.02439	0.02385	0.02333	0.02284	0.02238
25										0.02807	0.02731	0.02659	0.02592	0.02529	0.0247	0.02413	0.02359	0.02309	0.02261	0.02214
26											0.02703	0.02633	0.02567	0.02504	0.02445	0.02389	0.02336	0.02286	0.02238	0.02192
27												0.02608	0.02542	0.02481	0.02422	0.02366	0.02314	0.02264	0.02217	0.02171
28													0.02519	0.02458	0.024	0.02345	0.02293	0.02243	0.02196	0.02152
29														0.02436	0.02379	0.02324	0.02273	0.02224	0.02177	0.02133
30															0.02359	0.02305	0.02254	0.02205	0.02159	0.02115
31																0.02286	0.02236	0.02187	0.02142	0.02098
32																	0.02219	0.02171	0.02125	0.02082
33																		0.02155	0.02109	0.02066
34																			0.02095	0.02052
35																				0.02038

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