QBIO 401 HW 6- Hosseini

November 1, 2023

1 QBIO 401 Homework 6

1.1 Hirad Hosseini, Fall 2023

[1]: !pip install numpy

!pip install pandas

dateutil>=2.8.2->pandas) (1.16.0)

The format is the same as we discussed in class: rows are SNPs, columns 0:9 (Python notation, not including 9) are details about the SNPs, columns 9:90 (Python notation, not including 90) are individuals from Europe, and columns 90:179 (Python notation, not including 179) are individuals from Africa. Note: the INFO column is not informative for this assignment, since the file I have given you does not have all the columns and rows from the original data file.

1. It is possible that a SNP is polymorphic in the world-wide sample, but is not polymorphic in a given population sample. Count the number of SNPs in "abbgen1k.csv" for which the ALT allele does NOT have zero frequency in the sample of Europeans individuals. Also count this number for the sample of African individuals. Due to the out-of-Africa hypothesis, we expect that this number is greater for the African sample than the European sample. Is this what we observe?

```
!pip install matplotlib
Requirement already satisfied: numpy in
/home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (1.26.0)
Requirement already satisfied: pandas in
/home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (2.1.1)
Requirement already satisfied: numpy>=1.22.4 in
/home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from pandas)
Requirement already satisfied: python-dateutil>=2.8.2 in
/home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from pandas)
Requirement already satisfied: pytz>=2020.1 in
/home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from pandas)
(2023.3.post1)
Requirement already satisfied: tzdata>=2022.1 in
/home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from pandas)
Requirement already satisfied: six>=1.5 in
/home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from python-
```

```
Requirement already satisfied: matplotlib in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (3.8.1)
    Requirement already satisfied: contourpy>=1.0.1 in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from matplotlib)
    (1.1.1)
    Requirement already satisfied: cycler>=0.10 in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from matplotlib)
    (0.12.1)
    Requirement already satisfied: fonttools>=4.22.0 in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from matplotlib)
    (4.43.1)
    Requirement already satisfied: kiwisolver>=1.3.1 in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from matplotlib)
    (1.4.5)
    Requirement already satisfied: numpy<2,>=1.21 in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from matplotlib)
    (1.26.0)
    Requirement already satisfied: packaging>=20.0 in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from matplotlib)
    (23.1)
    Requirement already satisfied: pillow>=8 in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from matplotlib)
    Requirement already satisfied: pyparsing>=2.3.1 in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from matplotlib)
    Requirement already satisfied: python-dateutil>=2.7 in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from matplotlib)
    Requirement already satisfied: importlib-resources>=3.2.0 in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from matplotlib)
    Requirement already satisfied: zipp>=3.1.0 in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from importlib-
    resources>=3.2.0->matplotlib) (3.11.0)
    Requirement already satisfied: six>=1.5 in
    /home/hiradh/miniconda3/envs/rna/lib/python3.9/site-packages (from python-
    dateutil>=2.7->matplotlib) (1.16.0)
[2]: import numpy as np
     import pandas as pd
     import matplotlib.pyplot as plt
```

[3]: data = pd.read_csv("abbgen1k.csv")

data

```
[3]:
              CHROM
                            POS
                                             ID REF ALT
                                                           QUAL FILTER \
     0
                  22
                      16050115
                                  rs587755077
                                                   G
                                                        Α
                                                             100
                                                                    PASS
     1
                  22
                      16050213
                                  rs587654921
                                                   С
                                                        Т
                                                             100
                                                                    PASS
     2
                  22
                      16050783
                                  rs587743568
                                                        G
                                                             100
                                                                    PASS
                                                   Α
     3
                      16050840
                                                        G
                                                             100
                  22
                                  rs587616822
                                                   C
                                                                    PASS
     4
                  22
                      16050958
                                  rs587636807
                                                        Т
                                                             100
                                                                    PASS
                                                   Α
                                     15907
                  22
                      18097177
                                  rs545154757
                                                   C
                                                        Τ
                                                             100
                                                                    PASS
     15908
                                                        С
                                                             100
                                                                    PASS
                  22
                      18097179
                                   rs28539556
                                                   Τ
     15909
                  22
                      18097181
                                  rs575863671
                                                   G
                                                        Α
                                                             100
                                                                    PASS
                  22
                                                   G
                                                             100
                                                                    PASS
     15910
                      18097196
                                     rs5747273
                                                        Α
     15911
                  22
                      18097202 rs144996543
                                                   C
                                                        G
                                                             100
                                                                    PASS
                                                                    INFO FORMAT NAO6984 ...
     0
              AC=32; AF=0.00638978; AN=5008; NS=2504; DP=11468; E...
                                                                                     010
                                                                            GT
                                                                                     010
     1
              AC=38; AF=0.00758786; AN=5008; NS=2504; DP=15092; E...
                                                                            GT
              AC=39; AF=0.00778754; AN=5008; NS=2504; DP=24717; E...
     2
                                                                            GT
                                                                                     0|0
     3
              AC=26; AF=0.00519169; AN=5008; NS=2504; DP=23832; E...
                                                                            GT
                                                                                     0|0
     4
              AC=1; AF=0.000199681; AN=5008; NS=2504; DP=22326; E...
                                                                            GT
                                                                                     0|0
             AC=16; AF=0.00319489; AN=5008; NS=2504; DP=15476; E...
     15907
                                                                            GT
                                                                                    0 0
     15908
              AC=270; AF=0.0539137; AN=5008; NS=2504; DP=15582; E...
                                                                                     0|0
                                                                            GT
     15909
              AC=19; AF=0.00379393; AN=5008; NS=2504; DP=15627; E...
                                                                            GT
                                                                                     0 0
     15910
             AC=1412; AF=0.281949; AN=5008; NS=2504; DP=15124; E...
                                                                            GT
                                                                                     0|0
     15911
             AC=42; AF=0.00838658; AN=5008; NS=2504; DP=14659; E...
                                                                            GT
                                                                                     0|0
             NA19143 NA19144 NA19146 NA19147 NA19113 NA19114 NA19256 NA19257 NA19117
     0
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                  010
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                                                                                              0 | 0
     2
                  010
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             NA19118
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     4
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                  0 | 0
```

```
15910
                 0 | 0
                 010
     15911
     [15912 rows x 179 columns]
[4]:
     data.shape
[4]: (15912, 179)
     european_subset = data.iloc[:, 0:90]
     european_subset
[5]:
             CHROM
                           POS
                                          ID REF ALT
                                                        QUAL FILTER \
                                rs587755077
     0
                 22
                     16050115
                                                         100
                                                G
                                                    Α
                                                                PASS
     1
                 22
                     16050213
                                rs587654921
                                                С
                                                     Т
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     2
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                     16050783
                                rs587743568
                                                Α
                                                         100
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                 22
                     16050840
                                rs587616822
                                                C
                                                                PASS
     4
                 22
                     16050958
                                rs587636807
                                                     Т
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                                  15907
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                     18097177
                                rs545154757
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                     18097179
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                                                Τ
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     15909
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                     18097181
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     15910
                     18097196
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                     18097202 rs144996543
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                                                                INFO FORMAT NAO6984 ...
     0
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             AC=32; AF=0.00638978; AN=5008; NS=2504; DP=11468; E...
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     1
             AC=38; AF=0.00758786; AN=5008; NS=2504; DP=15092; E...
                                                                        GT
                                                                                0|0 ...
     2
             AC=39; AF=0.00778754; AN=5008; NS=2504; DP=24717; E...
                                                                        GT
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     3
             AC=26; AF=0.00519169; AN=5008; NS=2504; DP=23832; E...
                                                                        GT
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             AC=1; AF=0.000199681; AN=5008; NS=2504; DP=22326; E...
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             AC=16; AF=0.00319489; AN=5008; NS=2504; DP=15476; E...
                                                                        GT
     15907
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             AC=270; AF=0.0539137; AN=5008; NS=2504; DP=15582; E...
     15908
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     15909
             AC=19; AF=0.00379393; AN=5008; NS=2504; DP=15627; E...
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     15910
             AC=1412; AF=0.281949; AN=5008; NS=2504; DP=15124; E...
                                                                        GT
                                                                                0|0
            AC=42; AF=0.00838658; AN=5008; NS=2504; DP=14659; E...
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     15911
            NA12762 NA12763 NA12775 NA12776 NA12777 NA12778 NA12812 NA12814 NA12815
     0
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010

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            0|1
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                                0|0
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15910
                                                     1 | 1
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15911
            010
                      010
                                 010
                                           010
                                                     010
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                                                                                   010
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```

```
NA12827
             0|0
0
             010
1
2
             0 | 0
3
             010
4
             010
            0 | 0
15907
15908
             010
            0 | 0
15909
             1 | 1
15910
15911
            0 | 0
```

[15912 rows x 90 columns]

```
[6]: def countPolymorphicSites(file_name: str) -> dict:
         data = pd.read_csv(file_name)
         europe_subset = data.iloc[:, 9:90]
         africa_subset = data.iloc[:, 90:179]
         africa polymorphisms = sum(africa subset.apply(checkIfPolymorphic, axis = 11
      →1))
         europe_polymorphisms = sum(europe_subset.apply(checkIfPolymorphic, axis =_
      →1))
         return {"African subset non-polymorphic SNPs": africa polymorphisms,
                "European subset non-polymorphic SNPs": europe_polymorphisms}
     def checkIfPolymorphic(row): #checks if all columns in given SNP row are 0/0, u
      ⇒which would mean that there are no ALT genes
         #and thus the SNP is nonpolymorphic
         numNonpolymorphs = sum(row[0:] == "0|0")
         if numNonpolymorphs == row.count():
             return False
         else:
             return True
```

```
[7]: print(countPolymorphicSites("abbgen1k.csv"))
```

{'African subset non-polymorphic SNPs': 12705, 'European subset non-polymorphic SNPs': 7959}

In concordance with what we would expect under the out-of-Africa hypothesis, we see that the number of SNPs which are polymorphic is greater in the African subset of

the population compared to the European subset.

The allele frequency spectrum (see slide 64 in the Population Genetics lecture) shows the number of SNPs for which there are k haplotypes that have the ALT allele. For example, when k=0 this is the number of SNPs such that there are no haplotypes with the ALT allele (all haplotypes have the REF allele), when k=1 this is the number of SNPs such that exactly one haplotype has the ALT allele (and all other haplotypes have the REF allele), when k=2 this is the number of SNPs such that exactly two haplotypes have the ALT allele (and all other haplotypes have the REF allele), ... For the small example above, there is one SNP (SNP D) with k=0 ALT alleles, there is one SNP (SNP A) with k=1 ALT alleles, and there are two SNPs (SNPs B and C) with k=3 ALT alleles. The allele frequency spectrum is then a barplot with the k numbers on the x-axis and the number of SNPs with k=1 (the k=0 case are those sites that are not polymorphic, the number of SNPs with k=0 ALT alleles is much higher than for all the other k numbers).

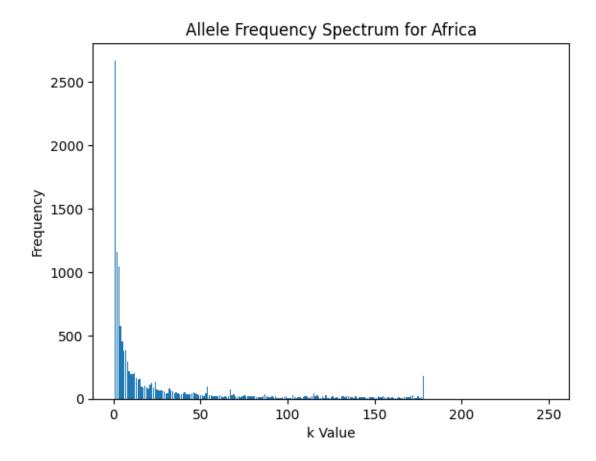
2. Generalize what you did in #1 to compute the allele frequency spectrum. Make two separate plots: the allele frequency spectrum for the European individuals and the allele frequency spectrum for the African individuals. Comment on any similarities or differences between the plots.

```
[8]: def extractAlleleFreqSpect(file_name: str): #subset dataframe into europe &
      ⇔africa, then apply getALleleFreq function to return k-counts
         data = pd.read_csv(file_name)
         europe subset = data.iloc[:, 9:90]
         african_subset = data.iloc[:, 90:179]
         africanKCounts = [0]*250
         europeanKCounts = [0]*250
         african subset.apply(lambda row: getAlleleFreq(row, africanKCounts), axis = 11
      →1)
         europe_subset.apply(lambda row: getAlleleFreq(row, europeanKCounts), axis =__
      →1)
         return africanKCounts[1:], europeanKCounts[1:]
     def getAlleleFreq(row, kCountsList): #sums each SNP row by the number of ALT_
      →haplotypes
         alleleFreq = 2 * sum(row[0:] == "1|1") + sum(row[0:] == "1|0") + sum(row[0:]
      \Rightarrow == "0|1")
         kCountsList[alleleFreq] += 1
     def plotAlleleFreqSpect(alleleFreqList, subpopName): #utilize pyplot to_
      →construct allele frequency spectra
         plt.bar(range(1,250), alleleFreqList)
         plt.xlabel("k Value")
         plt.ylabel ("Frequency")
         plt.title(f"Allele Frequency Spectrum for {subpopName}")
```

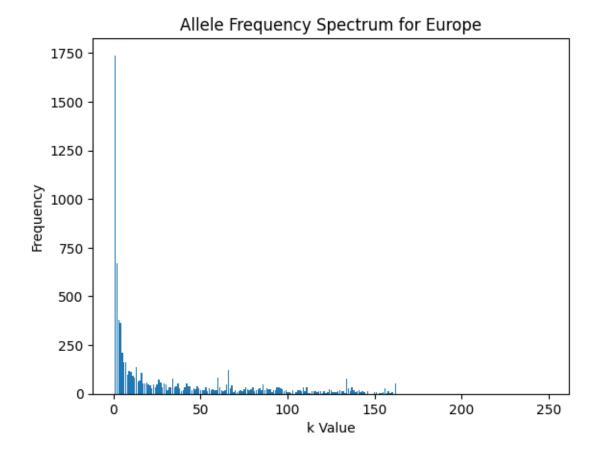
```
[9]: africa, europe = extractAlleleFreqSpect("abbgen1k.csv")
print(africa, europe)
```

[2672, 1156, 1046, 577, 456, 377, 383, 297, 219, 198, 198, 200, 163, 151, 161, 93, 87, 102, 88, 84, 110, 126, 92, 133, 77, 69, 67, 65, 61, 40, 45, 83, 66, 62, 42, 54, 45, 35, 37, 44, 52, 39, 33, 36, 44, 49, 43, 34, 26, 27, 26, 24, 45, 98, 25, 28, 23, 21, 19, 23, 26, 18, 14, 17, 12, 18, 76, 31, 34, 22, 15, 17, 14, 19, 29, 20, 18, 17, 23, 21, 20, 10, 15, 10, 16, 18, 33, 18, 11, 16, 19, 10, 21, 7, 9, 9, 13, 16, 23, 6, 9, 6, 31, 11, 6, 14, 10, 8, 14, 17, 18, 9, 11, 19, 40, 17, 25, 12, 6, 24, 4, 28, 6, 6, 12, 18, 9, 15, 7, 9, 21, 18, 12, 20, 20, 15, 10, 4, 19, 6, 14, 11, 12, 12, 4, 9, 10, 15, 11, 9, 6, 19, 13, 14, 17, 10, 6, 10, 2, 16, 2, 9, 5, 13, 9, 6, 13, 14, 11, 11, 24, 28, 8, 3, 20, 8, 15, 183, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] [1738, 668, 379, 364, 210, 163, 163, 95, 119, 111, 91, 85, 137, 61, 66, 107, 54, 52, 57, 46, 43, 27, 48, 32, 46, 74, 56, 34, 51, 48, 20, 35, 32, 77, 31, 39, 52, 29, 15, 16, 35, 53, 36, 39, 20, 30, 24, 39, 26, 19, 18, 16, 32, 13, 26, 18, 25, 17, 20, 81, 34, 16, 12, 16, 47, 120, 30, 44, 10, 16, 10, 12, 19, 11, 21, 31, 24, 18, 21, 35, 15, 18, 21, 30, 19, 49, 18, 28, 23, 21, 10, 17, 16, 33, 34, 28, 25, 14, 20, 10, 9, 6, 16, 0, 9, 16, 17, 13, 34, 15, 31, 5, 5, 13, 14, 11, 10, 11, 12, 5, 11, 5, 7, 23, 20, 10, 10, 8, 13, 20, 13, 14, 5, 80, 28, 12, 35, 18, 9, 15, 16, 7, 15, 7, 1, 12, 1, 1, 1, 10, 9,

[10]: plotAlleleFreqSpect(africa, "Africa")







Both plots demonstrate a geometric distribution which has a high frequency for lower k counts and quickly declines to relativley low frequencies for higher k counts. For the African allele frequency spectrum, we observe a higher initial frequency at k=1 than the European spectrum. We also observe higher possible k values for the African subset as the plot shows some non-zero frequencies near k=175 while Europe has k frequencies near k=160. Overall, there appears to be higher frequency counts for Africa compared to Europe which may be explained in part by the higher number of polymorphic sites observed for Africa (as observed in Question 1).

Next I want you to compute the bootstrap 95% confidence interval for the average pairwise diversity statistic. Below are instructions to compute the average pairwise diversity statistic: a. Randomly pick two haplotypes. For our small example, say we pick the left haplotype of NA1 (1,0,1,0) and the right haplotype of NA2 (0,0,1,0). This pair of haplotypes disagree at the first SNP but agree at the other three SNPs. So = 0.25 (the fraction of SNPs for which the pair disagrees). b. Repeat step (a) 50 times (randomly selecting two haplotypes each time). Take the average of these 50 values. This is our estimate of the average pairwise diversity. To get the bootstrap 95% confidence interval repeat steps (a) and (b) 100 times, then sort the 100 average pairwise diversity estimates, the estimates at the 2nd and the 98th positions make the bootstrap 95% confidence interval. Note: You can't use the "myboot" function from lecture because that function used a vector as input, while for this problem you need two-dimensional matrices.

3. I want you to compute three separate bootstrap 95% confidence intervals for the average pairwise diversity statistic: one within the European samples, one within the African samples, and one between the European and African samples. To get the three confidence intervals, you have to repeat the entire procedure above three times: to get the confidence intervals within the European samples all the randomly chosen haplotypes are from Europeans, to get the confidence interval within the African samples all the randomly haplotypes are from Africans, and to get the confidence interval between the European and African samples all the randomly chosen pairs of haplotypes contain one from a European and one from an African. Due to the out-of-Africa hypothesis, we expect that the average pairwise diversity within Europe will be less than the other two. Is this what we observe?

Note: You cannot eliminate all loops for this problem, but do try to eliminate unnecessary loops (which can really slow your code down)

```
[12]: def calculateAvgPairwiseDiversity(popDataframe, reps: int, mixedSamples: bool)
       →→> float: #obtains average pi value across 50 reps
          sumPiValues = 0
          for replication in range(reps):
              if mixedSamples == False: #checks if we want Afro-European sampling
                  haplotype_pairs = popDataframe.sample(n = 2, axis = 1, replace =___
       →True)
              else:
                  haplotype_pairs = pd.concat([popDataframe.iloc[:, 9:90].sample(n = _
       41, axis = 1, replace = True), popDataframe.iloc[:, 90:179].sample(n = 1,
       →axis = 1, replace = True)], axis = 1)
             hapOneCol = np.random.randint(2) #selects one of either maternal or_
       ⇒paternal haplotype
             hapOne = haplotype_pairs.iloc[:,0].apply(lambda row: row.
       ⇔split("|")[hapOneCol].strip()).to_numpy()
             hapTwoCol = np.random.randint(2)
             hapTwo = haplotype_pairs.iloc[:,1].apply(lambda row: row.
       ⇔split("|")[hapTwoCol].strip()).to_numpy()
             numPairwiseMatches = sum(hapOne != hapTwo) # Numpy's != operator;
       returns boolean vector where True corresponds to rows
              # where all columns match. We sum this value to obtain all the SNPs,
       →that match for our two randomly sampled haplotypes.
             repPiValue = float(numPairwiseMatches)/len(hapOne)
              sumPiValues += repPiValue
          averagePiValue = float(sumPiValues)/reps #obtain average Pi value across
       ⇒all reps
          return averagePiValue
      def bootstrapPairwiseDiversity(popDataframe, numTrials: int, mixedSamples:
       ⇒bool) -> list:
          bootstrapValues = []
          for trial in range(numTrials): #conducts n trials of k reps
```

```
bootstrapValues.append(calculateAvgPairwiseDiversity(popDataframe, 50, usintedSamples))
bootstrapValues.sort() #sorts average trial Pi values from smallest touslargest
return [bootstrapValues[2], bootstrapValues[98]] #selects valuesuscorresponding to 95% confidence interval
```

```
[13]: data = pd.read_csv("abbgen1k.csv")
    african_subset = data.iloc[:, 9:90]
    print("African subset CI: " + str(bootstrapPairwiseDiversity(african_subset, u))
```

African subset CI: [0.08894419306184016, 0.09707642031171446]

```
[14]: european_subset = data.iloc[:, 90:179]
print("European subset CI: " + str(bootstrapPairwiseDiversity(european_subset, u + 100, False)))
```

European subset CI: [0.10880718954248364, 0.11700854700854696]

```
[15]: afroeuropean_subset = data
print("Afro-European subset CI: " +

→str(bootstrapPairwiseDiversity(afroeuropean_subset, 100, True)))
```

Afro-European subset CI: [0.11547762694821519, 0.1211236802413273]

In concordance with the out-of-Africa hypothesis, we do observe a lower average pairwise diversity within Europe compared to the Africa and Europe-Africa population compositions based on our bootstrapping results. The European and Europe-Africa population compositions have similar overlapping confidence intervals whereas the African sample clearly has a lower confidence interval than both.

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    Your version must be at least (2.14.2) but less than (4.0.0).
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Refer to https://pandoc.org/installing.html.
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