CS 520

Assignment 3

Probabilistic Search and Destroy

Adhish Shrivastava, Hiral Nagda, Yash Nisar

A Stationary Target:

1. **Given observations up to time t ($Observations_t$), and a failure searching Cell j ($Observations_{t+1}$ = $Observations_t$ ∧ Failure in $Cell_j$), how can Bayes' theorem be used to efficiently update the belief state, i.e., compute:**

P(Target in $Cell_i$ | $Observations_t$ ∧ Failure in $Cell_j$)

Ans 1:

Initially, at time t = 0, all the cells are equally likely to hold the target. The probability that any cell selected at random will contain the target is equal to 1/(n*n) where n is the dimension of the grid. In our case, we are considering a (50*50) grid. Therefore, the *P(Target in $Cell_i$)* = (1/2500).

Thereafter, the cell which is most likely to be a target is selected and searched. There can be 2 cases in this situation:

Case 1: If the cell which is most likely to be a target is the target and we don't have a false negative report, we end the search indicating that the target was found.

Case 2: Otherwise, we have a failure, i.e. the target is not found.

If we are not able to find the target in the most probable cell (Case 2), we use this information to statistically infer the location of the target in other cells. This is done by updating the belief value of the current (most probable) cell. Since we've noted a failure, the updation results in decrease of the probability of finding a target in that cell, which is done as follows:

*P(Target in $Cell_i$) = P(Target in $Cell_i$) * P(False negative rate of terrain)*

*P(Target in $Cell_i$) = P(Target in $Cell_i$) * P(Target not found in $Cell_i$ | Target is in $Cell_i$)*

This in turn increases the probability of finding the target in other cells (relatively) because the probability of finding the target in the current cell has decreased. The sum of new probabilities is not equal to 1 so we normalize the belief matrix by individually dividing each probability by the sum of all the probabilities. This in turn results in the sum of all individual probabilities of the belief matrix to 1.

P_{norm}(Target in $Cell_i$) = P(Target in $Cell_i$)/Sum(P(Target in $Cell_i$))

Additionally, we can also redistribute the change in probability on the explored cell to all the other cells i.e. decrease in *P(Cell explored)* and increase in *P(Cells other than the current explored cell)*. The distribution will be proportionate to their current probabilities.

2. **Given the observations up to time t, the belief state captures the current probability the target is in a given cell. What is the probability that the target will be found in Cell i if it is searched:**

P(Target found in $Cell_i$ | $Observations_t$)?

Ans 2:

The probability that the target will be found on the cell currently being explored depends on two factors: is the target there on this cell and the search on the cell returns a positive result.

*P(Target found in $Cell_i$ | Target in $Cell_i$) * P(Target in $Cell_i$ | Observations)*

Depending on the type of the terrain, the chance of getting a positive search result if the target is present in that cell will be 0.9, 0.7, 0.3, and 0.1 for flat land, hilly terrain, forested land and maze of caves.

Everytime a cell is explored, based on the result the probabilities are updated in the knowledgebase. This probability is $P(Target\ in\ Cell_i \mid Observations)$ and at t=0 its value is 1/(dim*dim) (here, 1/2500) for all the cells.

$$P(Target\ found\ in\ Cell_i \mid Observations) = P(Target\ found\ in\ Cell_i \mid Target\ in\ Cell_i) * P(Target\ in\ Cell_i \mid Observations_{t-1})$$

$$P(Target\ in\ Cell_i \mid Observations_0) = 1/2500$$

3. **Consider comparing the following two decision rules:**
    - **Rule 1: At any time, search the cell with the highest probability of containing the target.**
    - **Rule 2: At any time, search the cell with the highest probability of finding the target.**

    For either rule, in the case of ties between cells, consider breaking ties arbitrarily. How can these rules be interpreted / implemented in terms of the known probabilities and belief states? For a fixed map, consider repeatedly using each rule to locate the target (replacing the target at a new, uniformly chosen location each time it is discovered). On average, which performs better (i.e., requires less searches), Rule 1 or Rule 2? Why do you think that is? Does that hold across multiple maps?

Ans 3:

For Rule 1, as we need to consider cell with highest probability of containing the target, we pick the cell which has maximum belief value.

For Rule 2, we need to consider cell with highest probability of finding the target.

Using below calculations we get probability of target found in $Cell_i$ equal to one minus the false negative rate of the terrain of that cell. We scale the belief matrix by this probability of finding target in $Cell_i$. That is flat land will be multiplied by 0.9, Hilly by 0.7, Forested by 0.3 and Maze of Caves by 0.1

$$P(\ Target\ found\ in\ Cell_i\ ) = P\ (Target\ found\ in\ Cell_i \mid Target\ is\ in\ Cell_i) + P(Target\ found\ in\ Cell_i \mid Target\ not\ in\ Cell_i)$$

As $P(Target\ found\ in\ Cell_i \mid Target\ not\ in\ Cell_i) = 0$

$$P(Target\ found\ in\ Cell_i) = P(Target\ found\ in\ Cell_i \mid Target\ in\ Cell_i)$$

$$P(Target\ found\ in\ Cell_i \mid Target\ in\ Cell_i) = 1\text{-}\ P(Target\ not\ found\ in\ Cell_i \mid Target\ in\ Cell_i)$$

The avg. number of searches + moves on a 50*50 grid for 100 iterations using rule 1 = 10273.2
The avg. number of searches + moves on a 50*50 grid for 100 iterations using rule 2 = 19548.1

4. **Consider modifying the problem in the following way: at any time, you may only search the cell at your current location, or move to a neighboring cell (up/down, left/right). Search or motion each constitute a single 'action'. In this case, the 'best' cell to search by the previous rules may be out of reach, and require travel. One possibility is to simply move to the cell indicated by the previous rules and search it, but this may incur a large cost in terms of required travel. How can you use the belief state and your current location to determine whether to search or move (and where to move), and minimize the total number of actions required? Derive a decision rule based on the**

current belief state and current location, and compare its performance to the rule of simply always traveling to the next cell indicated by Rule 1 or Rule 2. Discuss.

Ans 4:

To choose a cell to search we select the cell which has maximum value of the term given below.

*P(Target in Cell$_i$) * P(Target found in Cell$_i$ | Target is in Cell$_i$) / Cost required to travel to the cell.*

We consider two aspects to choose a cell to search which are: how far away is the cell and how likely is success at a cell. When a cell is far way the value will be less due to the travel cost. For a greater value, chances of success are greater as target value can be found quickly.

The cost is calculated on the basis of number of steps needed to reach to that cell. We calculate the cost using Manhattan Distance between current cell and Cell$_i$.

We start our search from (0, 0) and move or search on basis of rules given above.

The performance for this rule will be worse than Rule 1 and Rule 2 as for each move we are restricted to move to a cell with less travel cost.

5. **An old joke goes something like the following:**

A policeman sees a drunk man searching for something under a streetlight and asks what the drunk has lost. He says he lost his keys and they both look under the streetlight together. After a few minutes the policeman asks if he is sure he lost them here, and the drunk replies, no, and that he lost them in the park. The policeman asks why he is searching here, and the drunk replies, "the light is better here".

In light of the results of this project, discuss.

Ans 5:

In the above situation, the behavior of the drunk man is similar to Rule 2 mentioned in question 3. The rule states that "At any time, search where it is more likely to find the target, given that the target can be there". The man is aware that he has lost the keys in the park, still he chooses to search under the streetlight as he thinks it is more likely that he will find the keys under the streetlight, if they are there. Although, finding the keys under the streetlight may seem hopeful, but if the chances of the keys being there is minimal, the probability that the man will find a key there is also very less.

From the results we obtained in question 3, for any situation the best step to solve is to have the true probability of success, i.e. in our case the true probability of keys being in the park or under the streetlight and the success of finding the keys in the park or under the streetlight. If we know the chance of finding the keys under the streetlight is equal to or greater than the chance of finding the keys in the park, the man is more likely to search under the light first, as it is easier. The decision to search under the streetlight can also be dependent on the position he is in, if he is nearer to the streetlight than he might search it before going to the park which is some distance away. If he follows this method, he will be searching more efficiently and not wasting time and steps.

A Moving Target:

Ans:

In this scenario, we know that the target is moving. But the extra information available to us that is case is the transition boundary where the target is changing cells. All the other cell probabilities are made zero. After we have transformed the knowledgebase, we normalize it so that the sum is equal to 1.

As we know the transition boundary, the cells not containing these boundaries cannot have the target or in other words the probability that the target is present in such cells would be 0. The boundary of the target moving is set to be non-zero and are normalized. Then we go for the cell which has the boundary same as that of the target transition and has max probability of the target being present there (the max can be calculated by using rule 1 or rule 2).

The search is run 100 times and the average number of searches taken using rule 1 is 39 and rule 2 took 33 searches.