CS 370 Spring 2017: Assignment 1

Due Thursday May 25, 5:00 PM, in the Assignment Boxes, 4th Floor MC.

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Lectures: MWF 1:30, 3:30 | MC2035 | Office Hours: Tues 11:00-12:00

1. **(10 marks)**

   Consider a system of floating point numbers F of our standard form

   $$0 \text{ or } \quad \pm 0.d_1 d_2..d_t \times 2^p \text{ for } -20 \leq p \leq 20 \text{ and } d_1 \neq 0.$$

   Suppose you have a number system with base 2 and that the distance between 7 and the next largest floating point number is 2^{-10}.

   (a) What is the distance between 70 and the next largest floating point number?

   (b) What is the smallest positive normalized floating point number?

   (c) What is the largest value of n such that $n!$ can be represented exactly? Show your work.

2. **(15 marks)**

   (a) Consider the recurrence sequence $\{p_n\}$ defined by

   $$81p_n = -81p_{n-1} + 10p_{n-2}, \quad n \geq 2. \tag{1}$$

   Show that if

   $$p_0 = 1, \quad p_1 = \frac{1}{9}$$

   then

   $$p_n = \left(\frac{1}{9}\right)^n$$

   is an exact solution of (1) is for all $n \geq 0$.

   (b) Now, write a Matlab function to evaluate (1) for $n = 2, \ldots, 50$. What happens?

   (c) Carry out a stability analysis of this recursion, similar to what we did in class and explain what you see. Note that for any two term recursion

   $$q_n + aq_{n-1} + bq_{n-2} = 0$$

   with $a,\ b$ constant, the solution is given by

   $$q_n = c_1(r_1)^n + c_2(r_2)^n$$

   where c_1, c_2 are constants, and r_1, r_2 are roots of the equation $x^2 + ax + b = 0$.

3. **(10 marks)** Suppose we have a natural spline:

   $$S(x) = \begin{cases} 28 + a_1 x + 9x^2 + x^3 & x \in [-3, -1] \\ a_2 + 19x + a_3 x^2 - x^3 & x \in [-1, 0] \\ 26 + 19x + 3x^2 + a_4 x^3 & x \in [0, 3] \end{cases}$$

(a) Determine the values of a_1, a_2, $\cdots$, a_4.

(b) Using the Lagrange basis, construct a cubic interpolation polynomial which interpolates the values of $S(x)$ at $-3, -1, 0, 3$.

4. **(15 marks)**

(a) Write down the linear system of equations needed to determine the derivative values s_1, s_2, s_3, s_4, s_5 for the cubic spline $S(x)$ going through the points $(1, 2)$, $(2, 2)$, $(4, 1)$, $(6, 4)$ and $(8, 3)$ and having natural boundary conditions.

(b) Use the results of part (a) to determine the coefficients of $S_1(x)$, $S_2(x)$ $S_3(x)$ and $S_4(x)$ where

$$\begin{aligned}
S_1(x) &= a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3 \quad 1 \le x \le 2, \\
S_2(x) &= a_2 + b_2(x-2) + c_2(x-2)^2 + d_2(x-2)^3 \quad 2 \le x \le 4, \\
S_3(x) &= a_3 + b_3(x-4) + c_3(x-4)^2 + d_3(x-4)^3 \quad 4 \le x \le 6 \\
S_4(x) &= a_4 + b_4(x-6) + c_4(x-6)^2 + d_4(x-6)^3 \quad 6 \le x \le 8,
\end{aligned} \tag{2}$$

represent the piecewise polynomials of the cubic spline. Note that you can use Matlab or Maple to solve the linear system if you wish.

(c) Graph this spline. That is, write a short Matlab script to produce a plot of the resulting smooth curve over the range [1,8]. Submit the plot with your written part, and include your code in your electronic submission.

5. **(20 marks)** For this question, you will use MATLAB to create parametric curves representing the first letters in your first and last name, separated by an ampersand enclosed in the outline of a hand drawn heart. Each letter should be in upper or lower case. Create parametric curves representing the outline of an image of a heart with your first and last initial separated by an ampersand inside the heart, written in cursive (handwriting).

For example,

Follow the steps below:

(a) Start by downloading the script file **init.m** from the course website, which can be modified and used to initialize data arrays for your drawing. Use Matlab's **help** command to learn any commands that are unfamiliar to you. Given an image, you can load it and trace the image of the object directly using mouse clicks. To add your initials, write them on a piece of paper, and then hold it up to the computer screen to trace it similarly. Create your data using the mouse to select a few dozen points outlining the object and your initials. Hitting enter once begins a new sequence of points; hitting enter twice terminates the input. Points to note: You can modify the script as needed. The script calls **ginput** for each segment (i.e., each pen stroke), and uses *cell arrays* to store the data. You may also find the **save** and **load** commands useful.

(b) Using the resulting data arrays produced above, generate parametric curve representations based on smooth parametric curve interpolation as described in the course notes (see sections 3.1 and 3.2). At least one region of the sequence should be curved enough to be interesting (if not, you may change your initials). Show the output using piecewise linear and cubic splines. Specifically, you will do:

Prepare three Matlab.m scripts, one for each of the following tasks (along with any functions you choose to define). *Use the same axis scaling v from* **init.m** *for subsequent plots* (i.e., **axis(v)**).

(**part1.m**) Create a plot of the interpolation data points corresponding to the crude initial shape, plotted with the '$*$' symbol. The plot should have a title, and display both the axes and gridlines.

(**part2.m**) Create a plot of your initials and the object created by joining the original data points with straight lines. (The curves are not expected to look very smooth.) Include a title, and both axis and gridlines.

(**part3.m**) Create *two* smooth plots of your initials and the object using spline interpolation, refining the parameter partition by a factor of 5. For the first plot, use natural end conditions and for the second plot use not-a-knot end conditions (see functions **csape** and **fnval**). Include titles, but no gridlines or axes.