Convex Hull Algorithms

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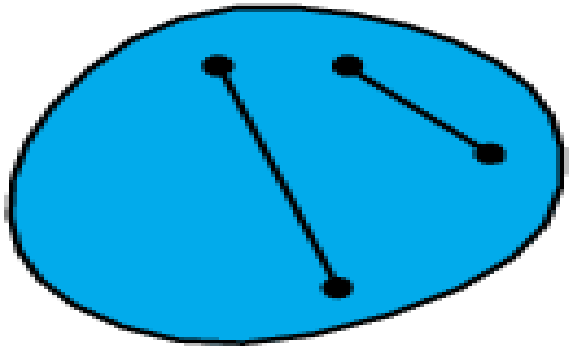
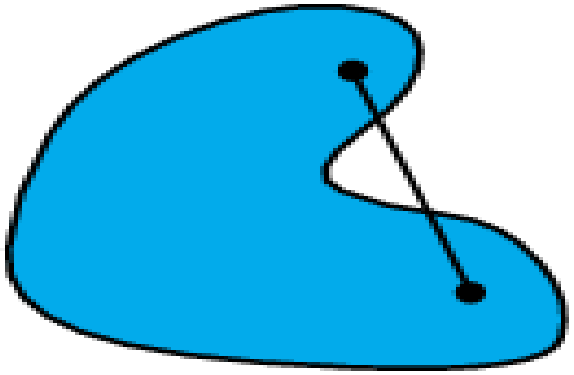
Formal Definitions

(Convex Set)

- A set S is convex if x in S and y in S implies that the segment xy is a subset of S
- Example in 2D:



convex

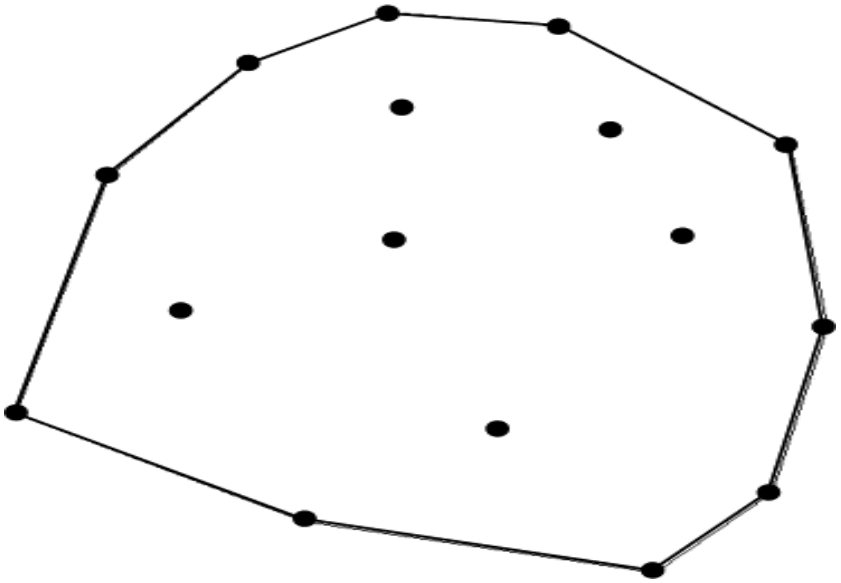


concave

Formal Definitions

(Convex Hull of a Set of Points)

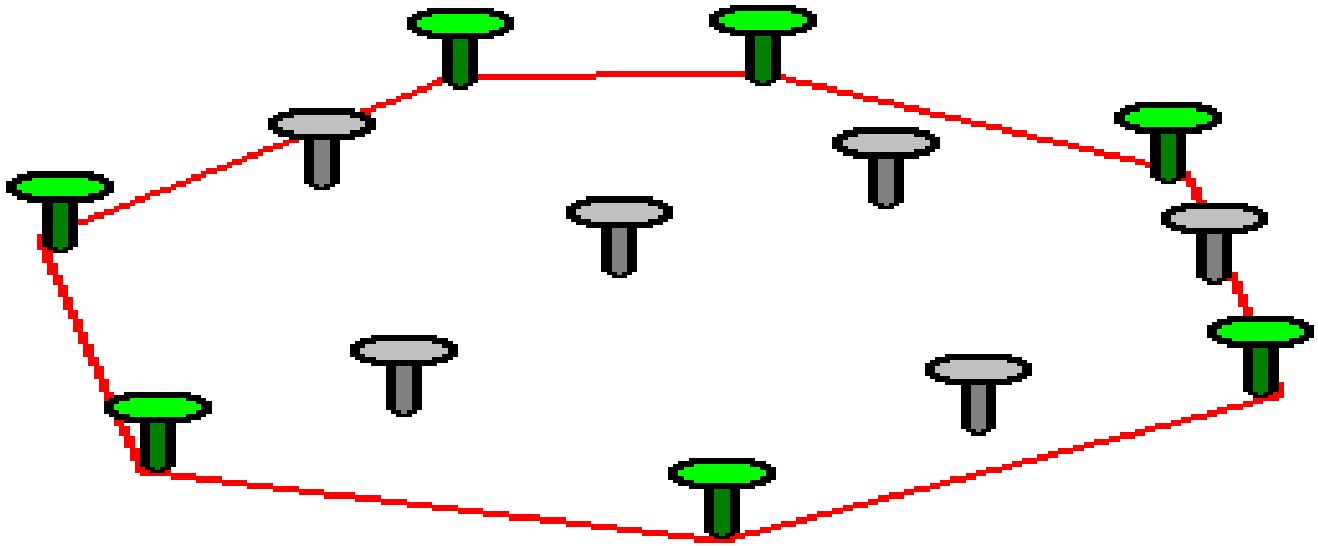
- The convex hull of a set S of points is the smallest convex set containing all the points in S
- Example in 2D:



Intuitive Appreciations

(Convex Hull of a Set of Points in 2D)

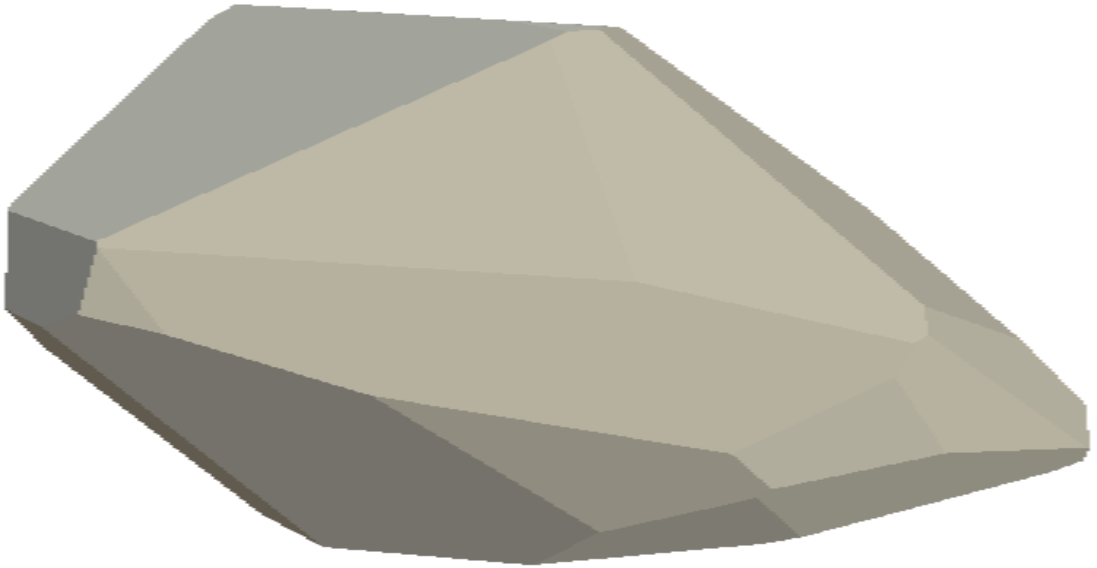
- The convex hull of a set of points in two dimensions is the shape taken by a rubber band stretched around nails pounded into the plane at each point
- Example:



Intuitive Appreciations

(Convex Hull of a Set of Points in 3D)

- The boundary of the convex hull of points in three dimensions is the shape taken by plastic wrap stretched tightly around the points
- Example:

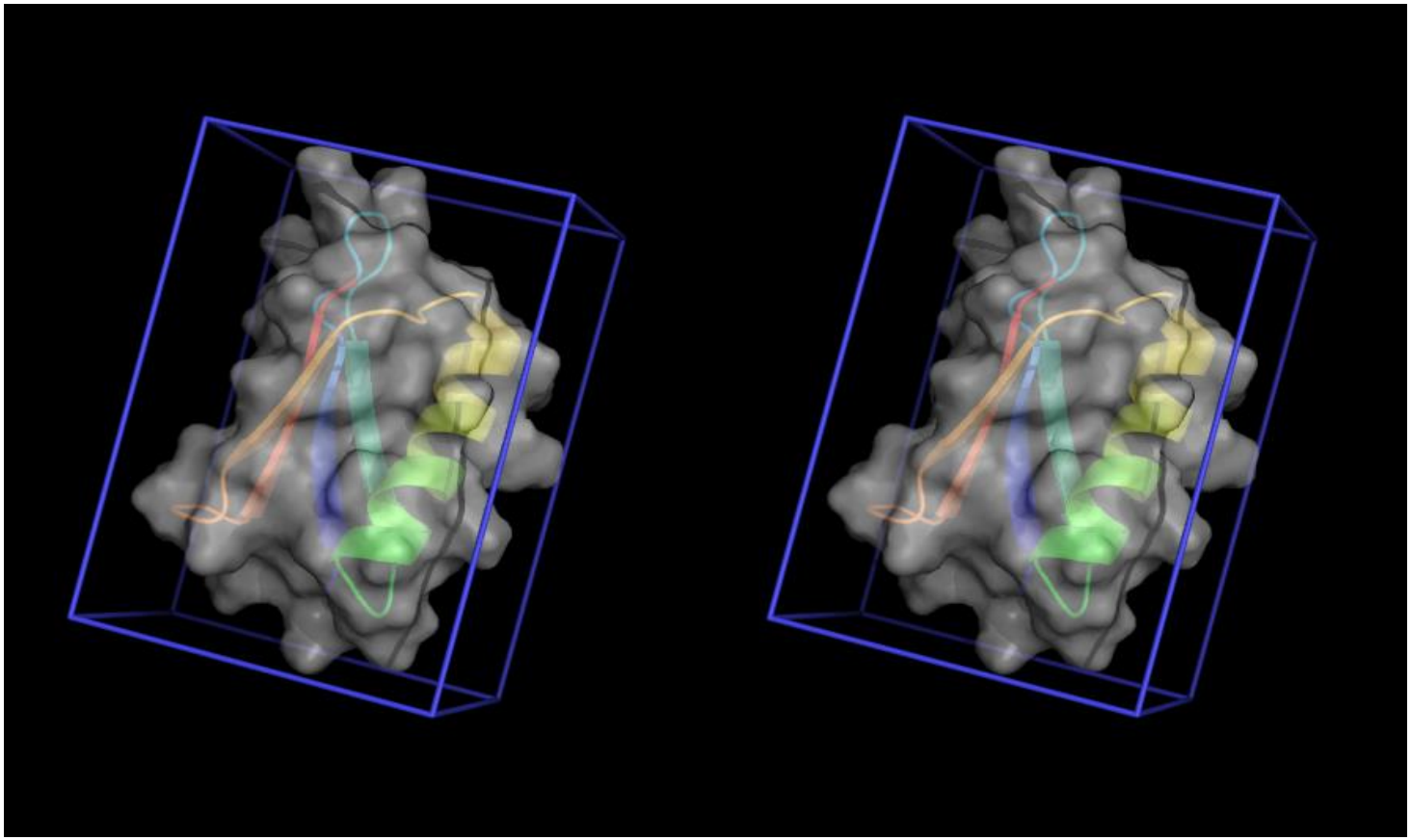


Applications

- Collision Avoidance

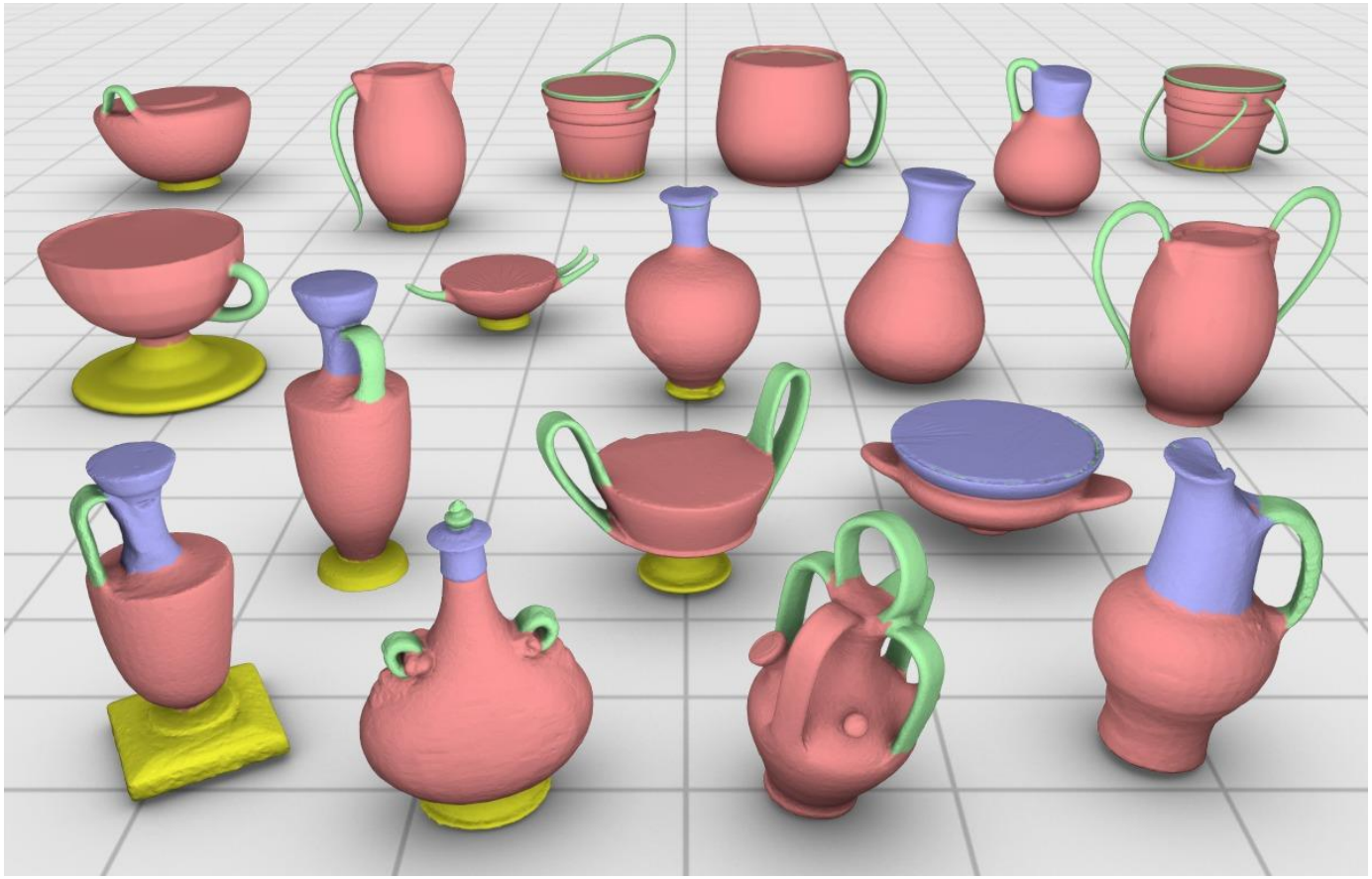
Applications

- Minimum Bounding Box



Applications

- Shape Analysis



Existing Convex Hull Algorithms

- Gift Wrapping (1970)
- Graham Scan (1972)
- Quick Hull (1977)
- Divide and Conquer (1977)
- Monotone Chain (1979)
- Incremental (1984)
- Marriage before Conquest (1986)
- Chan (1996)

Interior Points Algorithm

Based on the following Lemma

A point is non-extreme if and only if it is inside some (closed) triangle whose vertices are points of the set and is not itself a corner of that triangle

Interior Points Algorithm

Algorithm: INTERIOR POINTS

```
for each  i do
    for each  j !=  i do
        for each  k != i != j do
            for each  l !=  k != i != j do
                if p(l) in Triangle{ p(i), p(j), p(k) }
                    then  p(l) is non-extreme
```

Any Questions?

Thank You!