Homework #2

Due: Friday, February 8, 2019

1. Horizon in a matter-filled universe. [15 points]
Consider a flat universe filled with only non-relativistic matter and no cosmological constant. Take the Hubble constant today to be H_0. We showed in class that for this model universe, $a(t) \propto t^{2/3}$.

(a) [6 points] Solve for the expansion history of the Universe $a(t)$, and show that the present age of the Universe is $t_0 = 2/(3H_0)$. What is this in years if $H_0 = 70$ km/s/Mpc?

(b) [6 points] Evaluate the conformal age of the universe today, η_0. Consider the most distant visible objects in the universe (i.e. those for which the light they emitted just after the Big Bang is reaching our telescopes today). What is the present-day distance to those objects, in light-years?

(c) [3 points] Explain, in words or equations that would make sense to a first-year undergraduate, why the answer to (b) in light-years is greater than the answer to (a) in years.

2. Distance measures. [20 points]
Write a computer program or script in the language of your choice to compute $D_A(z)$ and $D_L(z)$ for a given choice of Ω_m and Ω_K. Attach your code.

Make a plot of $D_A(z)$ and $D_L(z)$ over the range $0 < z < 1.5$ for the cases of (i) an $\Omega_\mathrm{m} = 1$, $\Omega_K = 0$ cosmology (Einstein-de Sitter); (ii) an $\Omega_\mathrm{m} = 0.3$, $\Omega_K = 0$ cosmology (Λ+dark matter); and (iii) an $\Omega_\mathrm{m} = 0.3$, $\Omega_K = 0.7$ (open, low-density, no Λ) cosmology.

You might want to numerically check some of your values against Ned Wright's online calculator.