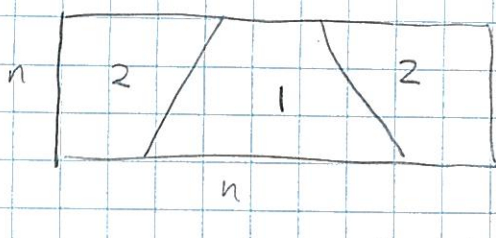
Notes on simultaneous vs. sequential space cuts ①

11/30/2010



$N = n^2$

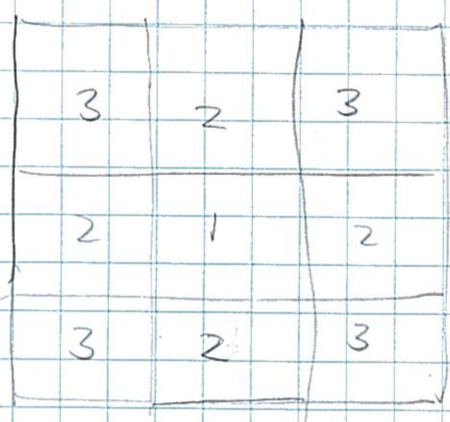
$$T_\infty(N) = 2\,T_\infty(N/3) + \Theta(1)$$
$$= \Theta(N^{\log_3 2})$$
$$\approx \Theta(N^{0.63})$$

$$A(N) = 2\,B(N/3) + \Theta(1)$$
$$B(N) = 3\,A(N/3) + \Theta(1)$$

$$A(n) = 6\,A(N/9) + \Theta(1)$$
$$= N^{\log_9 6}$$

$$\lg 6 = \frac{\lg 3 + \lg 2}{\lg 3 + \lg 3}$$

$$\Theta(N^{0.815})$$



2D + time

$$A_1(N) = 2\,A_2(N/3) + \Theta(1)$$
$$A_2(N) = 2\,B(N/3) + \Theta(1)$$
$$B(N) = 3\,A_1(N/3) + \Theta(1)$$

$$T_\infty(N) = 3\,T_\infty(N/9) + \Theta(1)$$
$$= \Theta(N^{1/2})$$

$$A_1(N) = 12\,A_1(N/27) + \Theta(1)$$
$$= \Theta(N^{\log_{27} 12})$$
$$\approx \Theta(N^{0.75})$$

d dimensions

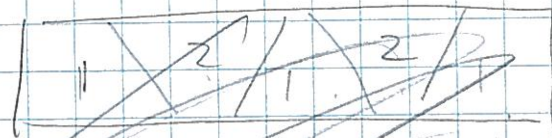
$$T_\infty(N) = (d+1)\,T_\infty(N/3^d) + \Theta(1)$$
$$= \Theta(N^{\log_{3^d}(d+1)})$$

$$A(N) = 3B(N/9) + \Theta(1)$$
$$B(N) = 3A(N/3) + \Theta(1)$$

$$A(N) = 9\,A(N/27) + \Theta(1)$$
$$= \Theta(N^{\log_{27} 9})$$
$$\approx \Theta(N^{0.667})$$

$$\frac{\lg(d+1)}{d \lg 3} = \frac{\log_3(d+1)}{d}$$

d	exponent
2	0.50
3	0.42
4	0.37



$$\cancel{T_\infty(N) = 2T_\infty(N/5) + \Theta(1)}$$
$$\cancel{= \Theta(N^{\log_5 2}) \approx \Theta(N}$$

One spatial at a time

$$A_1(N) = 2A_2(N/3) + \Theta(1)$$
$$\vdots$$
$$A_d(N) = 2B(N/3) + \Theta(1)$$
$$B(N) = 3A_1(N/3) + \Theta(1)$$
$$A_1(N) = 2^d \cdot 3 \cdot A_1(N/3^{d+1}) + \Theta(1)$$
$$= \Theta(N^{\log_{3^{d+1}} 2^d \cdot 3})$$

$\log_3 2 \approx 0.63$

$$\text{exp.} = \frac{\log(2^d \cdot 3)}{\log(3^{d+1})} = \frac{d\log 2 + \log 3}{(d+1)\log 3}$$
$$= \frac{d\log_3 2 + 1}{d+1}$$

d	exponent
1	.815
2	.754
3	.723
4	.705

Simultaneous spatial cuts

$$A(N) = (d+1)\,B(N/3^d) + \Theta(1)$$
$$B(N) = 3\,A(N/3) + \Theta(1)$$

$$A(N) = 3(d+1)\,A(N/3^{d+1}) + \Theta(1)$$
$$= \Theta\left(N^{\log_{3^{d+1}}(3(d+1))}\right)$$

$$\text{exp.} = \frac{\log(3(d+1))}{(d+1)\log 3}$$

d	exp.
1	0.815
2	.667
3	.565
4	.493