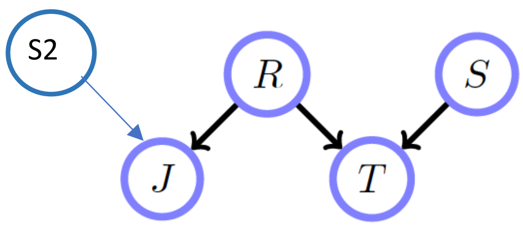
One morning Tracey leaves her house and realizes that her grass is wet. Is it due to overnight rain or did she forget to turn off the sprinkler last night? Next she notices that the grass of her neighbor, Jack, is also wet. This explains away to some extent the possibility that her sprinkler was left on, and she concludes therefore that it has probably been raining.

T - Tracey's grass (0 – dry; 1 – wet)
J - John's grass (0 – dry; 1 – wet)
S - Tracey's Sprinkler (0 – off; 1 – on)
R - Raining (0 – no rain; 1 – rains)
S2 - John's Sprinkler (0 – off; 1 – on) (Optional)



Case 1:
P(J, R, S, T) = P(R) P(S) P(J|R) P(T|R, S)
Case 2:
P(J, R, S, S2, T) = P(S2) P(R)P(S)P(J|S2, R)P(T|R, S)

(1) Define the remaining possible variables in this problem following the above example.
(2) Draw a Bayesian network model of these variables
(3) Write down your simplified joint probability according to your Bayesian Network
(4) What is the probability that Tracey's sprinkler was on last night? Write a probability equation.

P(S = 1|T = 1)
= P(S = 1, T = 1) / P(T=1)

Why do we need to convert the conditional probability in such way?
Answer: This is depends on how much probability information do we know after training. If we can only get the joint probability directly based on some background knowledge, then we have do this conversion based on Bayes rule.

$p(R = 1) = 0.2 \quad p(R=0) = 0.8$
$p(S = 1) = 0.1$
$p(J = 1|R = 1) = 1$, $p(J = 1|R = 0) = 0.2$ (because some other factors can cause John's grass wet)
$p(T = 1|R = 1, S = 0) = 1$
$p(T = 1|R = 1, S = 1) = 1$
$p(T = 1|R = 0, S = 1) = 0.9$
$p(T = 1|R = 0, S = 0) = 0$

$= \sum_{J,R} P(S=1, T=1, J, R) / \sum_{J,R,S} P(S, T=1, J, R)$
$= \sum_{J,R} P(R)\, P(S=1)\, P(J|R)\, P(T=1|R, S=1)$
$/ \sum_{J,R,S} P(R)\, P(S)\, P(J|R)\, P(T=1|R, S)$

$\sum_{J,R} P(R)\, P(S=1)\, P(J|R)\, P(T=1|R, S)$
$= \sum_{R} P(R)\, P(S=1)\, P(T=1|R, S)\, \cancel{\sum_{J} P(J|R)}$
$= P(S=1) \sum_{R} P(R)\, P(T=1|R, S)$
$= 0.1 * \{P(R=0)P(T=1|R=0, S=1) + P(R=1)P(T=1|R=1, S=1)\}$
$= 0.1 * \{0.8 * 0.9 + 0.2 * 1\}$
$= 0.1 * 0.92$
$= 0.092$

$\sum_{J,R,S} P(R)\, P(S)\, P(J|R)\, P(T=1|R, S)$
$=\sum_{R,S} P(R)\, P(S)\, P(T=1|R, S)$
$= P(R=0)P(S=0)P(T=1|R=0, S=0)$
$+ P(R=0)P(S=1)P(T=1|R=0, S=1)$
$+ P(R=1)P(S=0)P(T=1|R=1, S=0)$
$+ P(R=1)P(S=1)P(T=1|R=1, S=1)$

The answer: $P(S=1|T=1) = 0.3382$

$\sum_{J} P(J|R) = P(J=0|R) + P(J=1|R)$
$=(P(J=0|R=0) + P(J=1|R=0))P(R=0)$
$+(P(J=0|R=1) + P(J=1|R=1))P(R=1)$
$= 1.0 * 0.3 + 1.0 * 0.7$
$= 1.0$

$P(R=1) = 0.3$
$P(R=0) = 0.7$

Question: based on the above equations and probability table. Calculate what is the result of $P(S=1|T=1)$.

Marginalization:

$\sum_{A,B,C} P(A,B)P(B|C)P(C)P(B)$
$= \sum_{A,B} P(A,B)P(B) \sum_{C} P(B|C)P(C)$

$= \sum_{B,C} P(B|C)P(C)\ P(B) \sum_{A} P(A,B)$