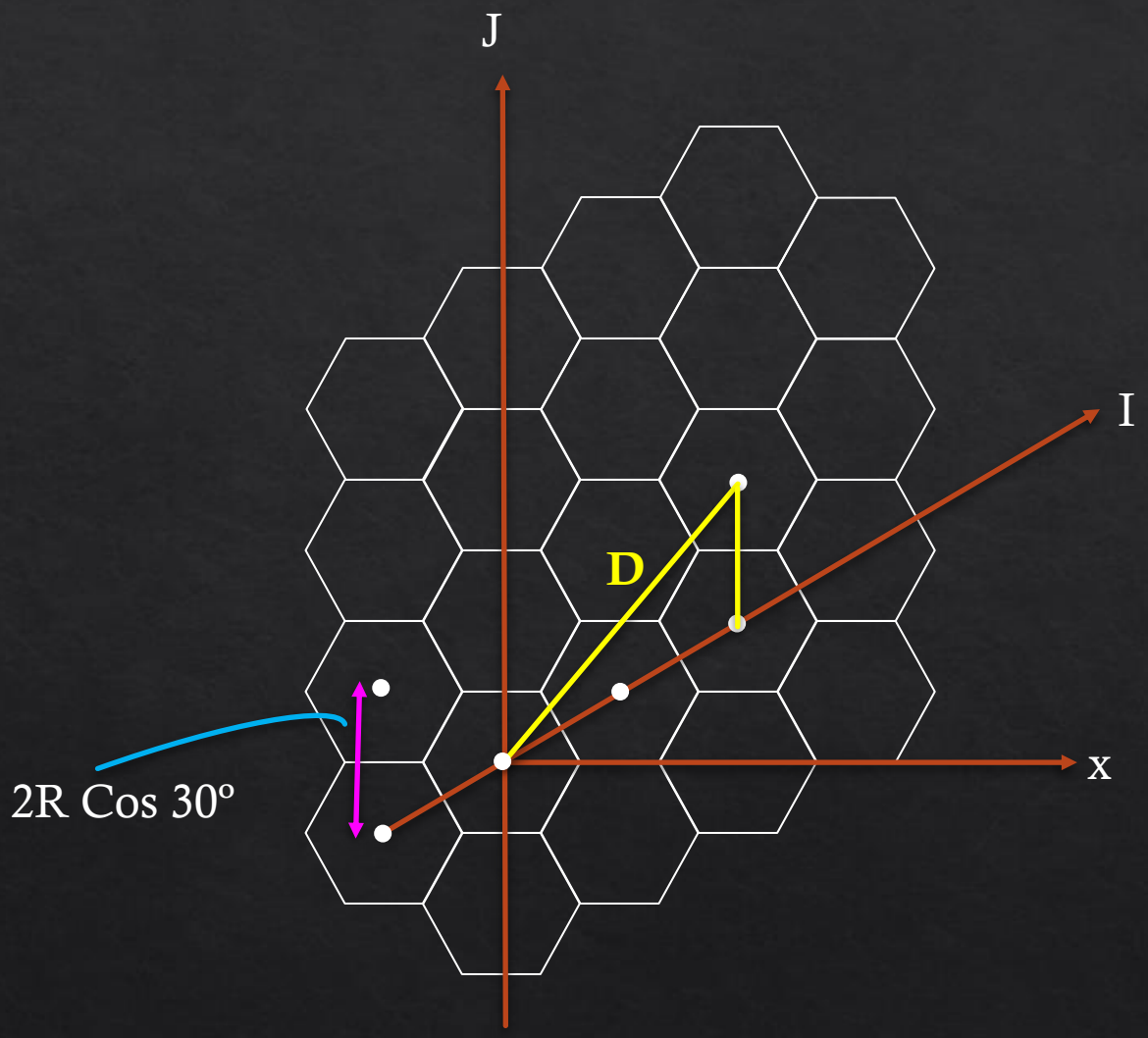
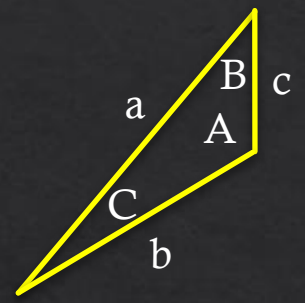
Proof of Co-channel Frequency Reuse Ratio



$N = i^2 + i\,j + j^2$ ---------- (1)



Cosine Rule

$a^2 = b^2 + c^2 - 2\,bc \cos(A)$

$d = 2\,R \cos 30^\circ = 2\,R \frac{\sqrt{3}}{2}$

$= \sqrt{3}\,R$ ---------- (2)

$D^2 = (i \times d)^2 + (j \times d)^2 - 2\,(i \times d) \times (j \times d) \cos 120^\circ$

$D^2 = i^2 d^2 + j^2 d^2 - 2\,i\,j\,d^2 \cos 120^\circ$

$D^2 = d^2 (i^2 + j^2 - 2\,i\,j\,(\frac{-1}{2}))$

$D^2 = d^2 (i^2 + i\,j + j^2)$

$D^2 = 3\,R^2 (i^2 + i\,j + j^2)$ (Substituting equation 2)

$D^2 = 3\,R^2\,N$

$D = \sqrt{3\,N}\;R$

$$q = \frac{D}{R} = \sqrt{3N}$$