The Hilbert Scheme of Curves and related Moduli Spaces of Reflexive Sheaves

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Abstract Let $H(d,g)$ be the Hilbert scheme of curves of degree d and arithmetic genus g in some projective space $\mathbf{P}$ and let $\mathbf{H_X}$ be the local Hilbert functor of flat deformations $X_S \subset \mathbf{P} \times S$, S a local artinian k-algebra, at a fixed curve X. An effective method of studying the local ring of $H(d,g)$ at (X) with respect to e.g. smoothness and dimension, is to look at other local deformation functors $\mathbf{D}$ over $\mathbf{H_X}$, $\mathbf{D} \to \mathbf{H_X}$, which allow a surjective tangent map $t_{\mathbf{D}} \to t_{\mathbf{H_X}} = H^0(N_X)$ and a corresponding injective map of obstruction spaces. We consider several such deformation functors $\mathbf{D}$ which determine $H(d,g)$ locally under various assumptions. In particular we look to the functor **(1)** of deformations of a pair (X,Y) where $X \subset Y$, **(2)** of graded deformations of the homogeneous coordinate ring of X and **(3)** of deformations of a pair (X,ξ) where ξ is an extension in the Serre-correspondence

$$\xi \ \ ; \quad 0 \to O_{\mathbf{P}} \to F \to I_X \otimes L \to 0 \ .$$

The latter allows us to prove results for the moduli space of stable reflexive sheaves of rank 2 on $\mathbf{P}$. Moreover by making some cup products explicit we get necessary and sufficient conditions for the unobstructedness of some classes of reflexive sheaves on $\mathbf{P^3}$.