線形代数1, 第5回の内容の理解度チェック (解答)

2024/5/16 担当：那須

$\boxed{1}$ 行列の基本変形を用いて, 連立1次方程式 $\begin{cases} x+y+z &= 12 \\ 3x-2y-2z &= 6 \\ 4x+3y-5z &= 2 \end{cases}$ を解け. (3点)

拡大係数行列 $\tilde{A}$ は,

$$\tilde{A} = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 3 & -2 & -2 & 6 \\ 4 & 3 & -5 & 2 \end{array}\right) \xrightarrow[③-4\times①]{②-3\times①} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 0 & -5 & -5 & -30 \\ 0 & -1 & -9 & -46 \end{array}\right)$$

$$\xrightarrow{②\times(-\frac{1}{5})} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 0 & 1 & 1 & 6 \\ 0 & -1 & -9 & -46 \end{array}\right) \xrightarrow[③+②]{①-②} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & -8 & -40 \end{array}\right)$$

$$\xrightarrow{③\times(-\frac{1}{8})} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 1 & 5 \end{array}\right) \xrightarrow{②-③} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

よって $x=6, y=1, z=5$.

$\boxed{2}$ 次の行列を簡約化せよ. (各1点)

(1) $\begin{pmatrix} 1 & -3 \\ -6 & -5 \end{pmatrix} \xrightarrow{②+6\times①} \begin{pmatrix} 1 & -3 \\ 0 & -23 \end{pmatrix} \xrightarrow{②\times(-1)/23} \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix} \xrightarrow{①+3\times②} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(2)
$$\begin{pmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 3 & -6 & 8 \end{pmatrix} \xrightarrow[③-3\times①]{②+2\times①} \begin{pmatrix} 1 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} \xrightarrow{③\times1/2} \begin{pmatrix} 1 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{①-2\times③} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{②\leftrightarrow③} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

(3)
$$\begin{pmatrix} 0 & 0 & 3 \\ 2 & 0 & 1 \\ -1 & 1 & -5 \end{pmatrix} \xrightarrow{①\leftrightarrow③} \begin{pmatrix} -1 & 1 & -5 \\ 2 & 0 & 1 \\ 0 & 0 & 3 \end{pmatrix} \xrightarrow{②+2\times①} \begin{pmatrix} -1 & 1 & -5 \\ 0 & 2 & -9 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\xrightarrow{③\times1/3} \begin{pmatrix} -1 & 1 & -5 \\ 0 & 2 & -9 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow[②+9\times③]{①+5\times③} \begin{pmatrix} -1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{②\times1/2} \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{①-②} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{①\times(-1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\boxed{3}$ 次の連立１次方程式を基本変形 (掃き出し法) を用いて解け．(各１点)

(1)
$$\begin{array}{c} \begin{array}{ccc} x & y & \end{array} \\ \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 3 & 2 & 5 \end{array}\right) \end{array} \xrightarrow{\textcircled{2}-3\times\textcircled{1}} \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -1 & -4 \end{array}\right) \xrightarrow{\textcircled{2}\times(-1)} \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 4 \end{array}\right) \xrightarrow{\textcircled{1}-\textcircled{2}} \left(\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 4 \end{array}\right)$$

よって $x=-1,\ y=4$

(2)
$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left(\begin{array}{ccc|c} 4 & -1 & 8 & 1 \\ 2 & 0 & 5 & 1 \\ 0 & 1 & 2 & 1 \end{array}\right) \end{array} \xrightarrow{\textcircled{1}\leftrightarrow\textcircled{2}} \left(\begin{array}{ccc|c} 2 & 0 & 5 & 1 \\ 4 & -1 & 8 & 1 \\ 0 & 1 & 2 & 1 \end{array}\right) \xrightarrow{\textcircled{2}-2\times\textcircled{1}} \left(\begin{array}{ccc|c} 2 & 0 & 5 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 1 & 2 & 1 \end{array}\right)$$

$$\xrightarrow{\textcircled{3}+\textcircled{2}} \left(\begin{array}{ccc|c} 2 & 0 & 5 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow[\textcircled{2}\times(-1)]{\textcircled{1}\times 1/2} \left(\begin{array}{ccc|c} 1 & 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$z=t$ とおけば，$\begin{cases} x=\dfrac{1-5t}{2} \\ y=1-2t \\ z=t \end{cases}$ $\quad$ (t は任意)

(3)
$$\begin{array}{c} \begin{array}{ccccc} x & y & z & w & \end{array} \\ \left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & -7 \\ -2 & -2 & 5 & -2 & 17 \end{array}\right) \end{array} \xrightarrow{\textcircled{2}+2\times\textcircled{1}} \left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & -7 \\ 0 & 0 & 1 & -2 & 3 \end{array}\right)$$

$$\xrightarrow{\textcircled{1}+2\times\textcircled{2}} \left(\begin{array}{cccc|c} 1 & 1 & 0 & -4 & -1 \\ 0 & 0 & 1 & -2 & 3 \end{array}\right)$$

y と w をパラメータにとり，それぞれ $y=t_1,\ w=t_2$ とおく．② より $z-2w=3$．よって $z=2t_2+3$．① より

$$x=-y+4w-1=-t_1+4t_2-1$$

故に

$$\begin{cases} x=-t_1+4t_2-1 \\ y=t_1 \\ z=2t_2+3 \\ w=t_2 \end{cases} \qquad (t_1, t_2 \text{ は任意})$$

[0]※この講義に関する情報はホームページを参照．http://fuji.ss.u-tokai.ac.jp/nasu/2024/la1.html