Year 1 Laboratory Manual
Introductory Experiments:
Stiff Pendulum

Department of Physics*

October 22, 2021

*You should reference this work as:'S. P. D. Mangles (Ed.) Year 1 Laboratory Manual Introductory Experiments: Stiff Pendulum Experiment', Imperial College London, 2021

1. Calculating g using a Stiff Pendulum

1.1 Introduction

In this experiment, we will be using a 'stiff' pendulum to calculate the gravitational acceleration, g, at the location of the laboratory. A stiff pendulum has the weight or 'bob' built onto the end of a stiff rod. This small change from the more familiar 'bob on a string' pendulum, has some interesting consequences as the mass of the rod cannot be neglected and so must be factored in to our calculations of the motion of the pendulum. Achieving an accurate and precise result will require you to accurately measure the period of oscillation of the pendulum and various properties of the pendulum and apparatus. If you choose a good strategy and make careful measurements it is possible to achieve an uncertainty of less than one percent. To choose a good strategy you will need to think about how the uncertainties in the various measurements combine to produce the final uncertainty in g. It may be beneficial to concentrate on reducing the uncertainty in some measurements more than others.

1.2 Physical Principles

The Stiff Pendulum

A diagram of the stiff pendulum used in this experiment is shown in Figure 1.1. The pendulum consists of a bob connected to a knife edge via a stiff rod. The knife edge is placed on a horizontal stage and an oscillation is set up by displacing the pendulum.

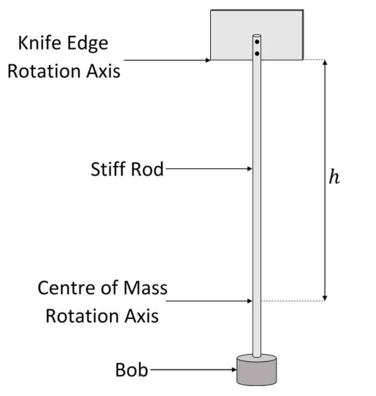

Figure 1.1: A diagram of the stiff pendulum used in this experiment.

For small oscillations the period of the stiff pendulum, T_S is independent of the amplitude and is given by:

$$T_S = \sqrt{\frac{4\pi^2 I}{Mgh}}, \tag{1.1}$$

where I is the *moment of inertia* of the pendulum about the knife edge, h is the distance between the knife edge and the centre of mass (CoM) of the pendulum and M is the mass of the pendulum. Rearranging this for g, we find

$$g = \frac{4\pi^2 I}{M h T_S^2} \,. \tag{1.2}$$

To measure g you must therefore measure the period of oscillation T, the pendulum mass, M, the pendulum length h and the moment of inertia, I.

Moment of Inertia and the Torsion Pendulum

In rotational motion the moment of inertia is the rotational analogue to mass. Just as inertial mass represents how "difficult" it is to push an object in a straight line (i.e. how much force is required for a given acceleration), the moment of inertia represents how "difficult" it is to change the rate of rotation of an object about a given axis. The moment of inertia depends on the shape of the object, and on the axis about which rotation is occurring.

For this experiment we will be using a *torsional* pendulum to determine the moment of inertia of the *stiff* pendulum. A torsional pendulum differs from both the simple and stiff pendulum in that, rather than an object *swinging* back and forth along an arc, the object twists a wire, which causes the object to *rotate* back and forth. The torsional pendulum you will use is shown in figure 1.2.

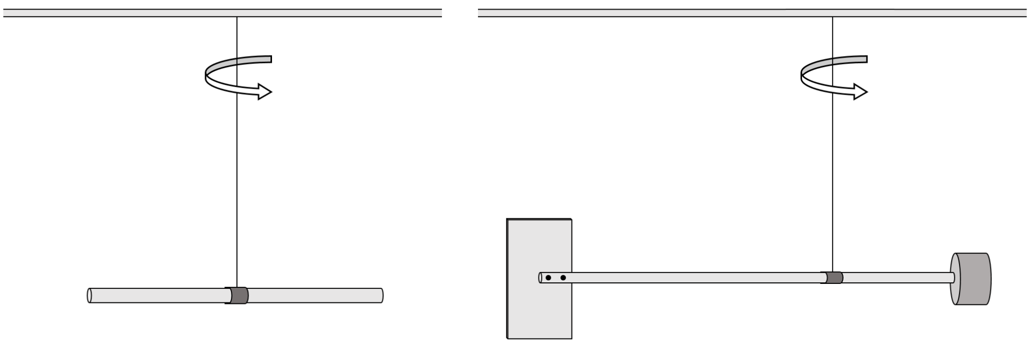

Figure 1.2: A sketch showing the set up for determining the moment of inertia of the stiff pendulum; a rod with a known moment of inertia and the stiff pendulum are both suspended from the same wire and made to oscillate (rotate). Their periods of oscillations are compared to find the moment of inertia of the pendulum.

The torsional pendulum you will use consist of an object suspended from a copper wire. By rotating the object through an angle θ, we twist the wire. The wire resists this twisting and applies a restoring torque (the rotational analogue of force), τ. This torque acts to return the wire to its untwisted state. For small angular displacements the torque is given by the rotational form of Hooke's Law:

$$\tau = -\kappa\theta \,, \tag{1.3}$$

where κ is a constant referred to as the torsion coefficient. When we release the twisted wire, the wire rotates towards equilibrium (the untwisted state), overshoots and then twists in the opposite direction. The pendulum thus twists back and forth with a period of T, given by

$$T = 2\pi\sqrt{\frac{I}{\kappa}}, \tag{1.4}$$

where I is the moment of inertia of the object about the point it is rotating.
Equation 1.4 shows that the period of the torsional pendulum depends on two quantities: the torsion coefficient and the moment of inertia of the object suspended from the wire.

By comparing the period of the torsional pendulum using an object with a known moment of inertia to the period measured using our stiff pendulum we can deduce the moment of inertia of the stiff pendulum.

$$\frac{T_P}{T_R} = \sqrt{\frac{I_P}{I_R}}, \tag{1.5}$$

where I_R is the moment of inertia of the rod. Rearranging for I_P

$$I_P = I_R\left(\frac{T_P}{T_R}\right)^2. \tag{1.6}$$

The moment of inertia of a thin cylindrical rod (i.e one whose length L is larger than it's radius R) through an axis through the centre of the rod is given by:

$$I_R = \frac{1}{12}M_R L^2, \tag{1.7}$$

where M_R is the mass of the rod.

If we measure the period of rotation for such a cylindrical rod, T_R and we measure the period of rotation of our stiff pendulum, T_P we then take the ratio T_P/T_R to deduce the stiff pendulum's moment of inertia.

The moment of inertia of an object depends on the axis of rotation. When measuring the stiff pendulum's period it will be balanced on its knife edge, and so it is the moment of inertia about this axis that we need to know to calculate g. However, you will find it very difficult (impossible?) to use the torsional pendulum to rotate the stiff pendulum about an axis that passes through the knife edge. Instead it is much easier to set it to rotate about its centre of mass and use something called *the parallel axis theorem* to calculate the moment of inertia of the stiff pendulum about the knife edge based on a measurement of its moment of inertia about its centre of mass.

The parallel axis theorem states that: "If we know the moment of inertia of a body about an axis which passes through its centre of mass, I_{CoM}, then the moment of inertia about any other parallel axis, $I_{||}$, is given by

$$I_{||} = I_{CoM} + Md^2, \tag{1.8}$$

where M is the mass of the body and d is the perpendicular distance between the CoM axis and the new parallel axis". This is illustrated for our pendulum in Figure 1.1. Here we see that our CoM axis and the axis about which the stiff pendulum swings (the knife edge) are parallel and are separated by h, thus, we can use the parallel axis theorem. Using Equation 1.8, we find

$$I = I_P + Mh^2. \tag{1.9}$$

As a result, by measuring I_P, M and h, we can calculate I. These can then be substituted into Equation 1.2 to determine g.

1.3 Experimental Procedure

To make the measurement of g you are supplied with the following apparatus at (or near) your bench:

- stiff pendulum
- cylindrical rod
- aluminium mounting plate
- G-clamp
- aluminium bar
- pipe clip
- aluminium prism
- 0.71 mm copper wire
- digital timer
- digital mass balance
- metre rule

To be able to calculate g using Equation 1.2, you will need to determine the following:

- h: the distance between the centre of mass of the pendulum and its knife edge
- M: the total mass of the pendulum
- I_R: the moment of inertia about the centre of mass of the reference rod
- I_P: the moment of inertia about the centre of mass of the stiff pendulum
- I: the moment of inertia of the pendulum about the knife edge
- T: the stiff pendulum period

Some of these quantities are measured directly, others are calculated based on other measurements.

You and your lab partner need to decide on the best strategy to take the measurements you will need to determine g. You should consider how the uncertainties in each of the quantities you measure directly affect the uncertainty in your final value of g and so try to minimise the uncertainty on your final result within the time available. Measuring one quantity to a high degree of accuracy may have a more significant effect on the final uncertainty than another.

Some points to consider:

- Take preliminary measurements to help you decide on the best method.
- When measuring the period of your various pendulums, is it better to measure one period many times, many periods once or many periods many times?
- When using the torsional pendulum how can you ensure that the wire is mounted in a consistent way on both the reference rod and the stiff pendulum?
- Plan your time: you should be able to get the data needed for a good result within the three hour lab session but if you spend too long on one measurement you may struggle to finish.
- Consider ways to reduce both systematic and random errors in your measurements.
- Make sure you keep a good record of the methods you use to make each measurement.

1.4 Data Analysis

Having measured all the necessary values in the lab, your next task is to calculate your measured value of g and its associated uncertainty. The equation that links the measured values is:

$$g = \frac{4\pi^2 I}{MhT^2} \tag{1.2}$$

First calculate the values and uncertainties for each of the quantities you measured, i.e.

- $h \pm \sigma_h$
- $M \pm \sigma_M$
- $I \pm \sigma_I$
- $T \pm \sigma_T$

Note that for quantities that you did not measure directly (e.g. I), you will need to work out how the uncertainties in the quantities that you actually measured affect the value.

Calculate your final value of g and it's *percentage* uncertainty. You will need to use the error propagation formulas covered in the Measurements and Uncertainties lecture and summarized in the Uncertainties Summary sheet (available on Blackboard).

It is of course very tempting to compare your final result to the 'known' value of g (i.e. 9.81 $\mathrm{m\,s^{-1}}$). For example, if your result is significantly different from this, it could indicate a systematic error in your measurements or an error in your calculation which you can go back and correct. However you should be aware that we cannot always do this in research experiments as the quantity might not be known, and by comparing to previously accepted results we can inadvertently introduce "experimenter's bias" (i.e. we are more likely to get a result that agrees with our expectation than an unexpected one). This can be dealt with by introducing 'blind' analysis techniques, and is particularly common in large collaborations in high energy physics or gravitational wave detection. One such technique involves unknown offsets to be added to data before analysis, with the offsets only being removed once the analysis has been completed.

Once you have calculated your final value for g and its uncertainty look over your analysis again– what could you have done differently to reduce your uncertainty? If possible try to quantify the effect of improving a measurement (e.g. if you measured 10 oscillation periods and had a 1% uncertainty, how much would your final answer improve if you had measured 1000 periods?)

Original Author: This experiment is based on the Stiff Pendulum experiment in the Cambridge Undergraduate Science degree. The lab script was adapted by Matthew Ward, Michaela Flegrova and Stuart Mangles