Year 1 Laboratory Manual
Imaging with Lenses

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November 8, 2021

*You should reference this work as:'S. P. D. Mangles (Ed.) Year 1 Laboratory Manual: Imaging with lenses experiment', Imperial College London, 2021

1. Investigation of imaging with lenses

1.1 Introduction

The vast majority of instruments that incorporate some optics have simple items such as lenses and mirrors. The function of these elements is usually to provide even, or controlled, illumination of an area (the reflector behind a torch bulb is perhaps one of the simplest examples of this) or to relay the image of an object to a specified plane or surface (e.g. a camera lens). In this series of experiments, you will study the imaging properties of lenses which can be understood using the concepts of geometrical optics. You will start by imaging with a single lens, testing the lens formulae that predict perfect images, free of any kind of defect or aberration. If you have time, you can examine the quality of images using a resolution test chart.

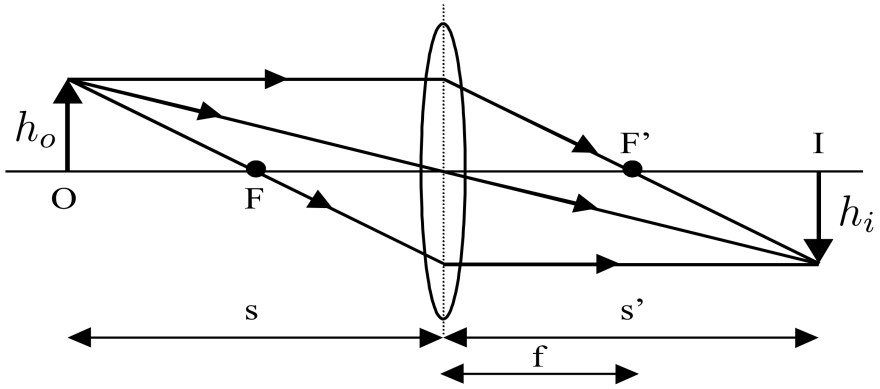

Figure 1.1: A lens with focal length f forms an image (I) of an object (O) where s is the object distance, h_o is the height of the object, s' is the image distance, and h_i is the height of the image.

1.2 Theory

Fig. 1.1 is a ray diagram illustrating how a lens forms an image. We call the line passing through the centre of the lens, normal (i.e. perpendicular) to the surface of the lens 'the principal axis' of the lens. This diagram is created using three rules for a converging lens.

1. Any ray incident on the lens that travels parallel to the principal axis will refract through the lens and travel through the focal point on the opposite side of the lens.
2. Any ray incident on the lens that is travels through the focal point on the way to the lens will refract through the lens and travel parallel to the principal axis on the other side.
3. Any ray incident on the lens that passes through the centre of the lens will continue in the same direction that it had when it entered the lens.

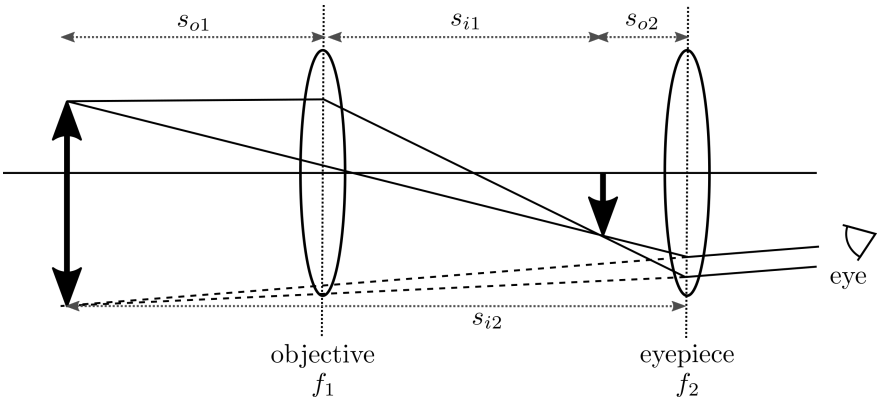

Figure 1.2: Illustration of a compound microscope which consists of two lenses, the objective with focal length f_1 and the eyepiece with focal length f_2. The first lens is placed a distance s_{o1} from the object and produces and image at s_{i1}. The second lens takes this image as its object, at a distance s_{o2} and produces the magnified image at s_{i2}.

The object distance s, the image distance s' are all measured from the centre of a thin converging lens. For small angles, the "thin lens formula" can be derived from this diagram:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \tag{1.1}$$

with the following sign convention:

- Light travels from left to right; s is positive (real) on the left, negative (virtual) on the right: s' is positive on the right, negative on the left: f is positive for a converging lens, negative for a diverging lens. (You should be aware that there are other sign conventions).

The *linear magnification* m is defined, in this sign convention, as

$$M = -\frac{s'}{s} = \frac{h_i}{h_o} \tag{1.2}$$

where the negative sign indicates that when a real image is formed it is inverted.

A single lens can only provide high-quality images for relatively small magnifications. Typically, lenses are combined together to create powerful and sophisticated imaging and magnification systems, including in instruments like microscopes and telescopes. One of the simplest examples of this is a compound microscope, as illustrated in Figure 1.2.

A compound microscope consists of two lenses, the objective which has focal length f_1 and the eyepiece with focal length f_2. The objective is placed $s_{o1} > f_1$ away from the object and forms a real, enlarged image at s_{i1}. The eyepiece acts as a magnifying glass and inspects the image at s_{o1}. The image formed by the first lens acts as the object for the second lens.

The total magnification for the compound microscope is

$$M = M_{\text{objective}} M_{\text{eyepiece}} = \left(\frac{-s_{i1}}{s_{o1}}\right)\left(\frac{-s_{i2}}{s_{o2}}\right) \tag{1.3}$$

In Experiment C, we will be looking at a variation of the compound microscope, which allows you to record a real image using a camera, rather than observing a virtual image with your eyes, as illustrated in Figure 1.2.

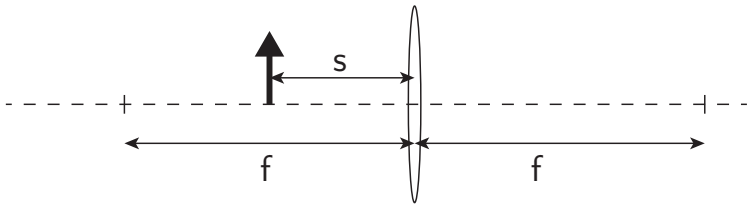

Figure 1.3: Copy this diagram into your lab book and draw a simple ray diagram using the three rules to show where the image forms if an object is placed within the the focal length of a converging lens. You may find it easier to draw by hand, taking a photo to insert the picture into your lab book.

As an additional note, a simple refracting telescope operates in a very similar way to the compound microscope, but with $s_{o1} \gg f_1$. If you are interested, you may want to look into the very first telescopes (from the early 1600s) developed by Hans Lippershey, Galileo Galilei, and Johannes Kepler.

Before you start your experiments do a couple of simple calculations, to help analyse what you might expect:

1. Let x be the distance from the lens to the object, let d be the total distance between the object and the image. Substitute x and d into the lens formula and rearrange so that you have a quadratic equation of the form $ax^2 + bx + c = 0$. What are a, b and c in terms of d and f? Solve this equation for real roots of x. Recall that the roots of a quadratic equation are real if $b^2 \geqslant 4ac$. What does this tell you that the minimum distance between the object and your screen (or detector) must be for you to be able to form an image? Try to sketch the graph of x versus d, where d is measured in units of f.
2. Using the definition for magnification M given above derive an expression for the object distance s in terms of the magnification and the focal length f. For a converging lens of focal length 100 mm where should you place the object if you want to produce a real image that is twice the size of the object ($M = -2$)? How far away from the lens will this image form?
3. If you put the object inside the focal length of a converging lens where does the image form? Draw a simple ray diagram based on figure 1.3
4. Sketch the ray diagram of Figure 1.1 where the converging lens is replaced with a diverging lens (which has a negative focal length, $f < 0$).

Discuss your answers with your demonstrator.

1.3 Experimental Procedure

List of equipment

- optical table with mounted rail
- Incandescent light source, post-mounted, with power supply. Maximum voltage is 5 V!
- Plano-convex lens, nominal focal length 100 mm
- Plano-convex lens, nominal focal length 160 mm
- Achromatic doublet lens, nominal focal length 160 mm
- A piece of white for viewing the image
- Set of object slides including a resolution test chart slide, a slide with an L on it, and a pinhole
- A CMOS* USB camera

EXPERIMENT A: Testing the thin lens formula and the magnification formula

The lens formula, Eq. 1.1, can be tested and used to find the focal length, by plotting the reciprocal of the image distance, $1/s'$, against the reciprocal of the object distance, $1/s$. The intercept of this graph on the $1/s'$ axis is the reciprocal of the focal length $(1/f)$.

We will also be looking at magnification, Eq. 1.2, in more detail by plotting the magnification of the image h_i/h_o versus s.

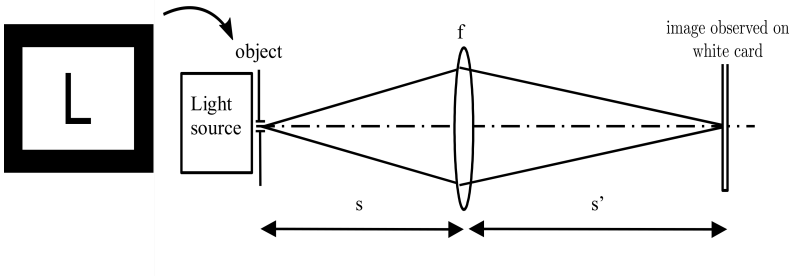

Figure 1.4: Experimental system for imaging with a positive lens.

Fig. 1.4 shows a simple set-up for imaging with a positive focal length lens. Switch on the light source — the voltage on the supply should be at voltage 5V (max.), remember there are two buttons to turn the power on! — and use the slide with the L printed on it as the object placed in the slot-holder in front of the light source. Use the plano-convex lens of nominal focal length 100 mm. Observe the image of the object slide on the white card screen by looking at the back of the screen and adjusting its position for sharpest image. It may be difficult to see the image until you are close to the image plane and you may need to centre the image on the screen by adjusting the height of the lens.

Preliminary observations

Set the distance between your lamp and the $f = 100$ mm lens to be $s = 120$ mm. Vary the position of the white card until you find the value of s' such that the image is focussed. What do you notice about the image? Is it larger or smaller? Is it inverted or the right-way up? Did you find it easy to find the *exact* location of the focal plane? Do you find you get different values

*CMOS stands for complementary metal-oxide-semiconductor. The camera has a total image area of 6.66 by 5.32mm (H x V), within which there are 1280 x 1024 pixels

if you move the card from far to near or when moving from near to far? How can you account for this in your measurements (you might find they are the same, depending on how good your eyesight is!)? Make some notes on your observations in your lab book.

Measurement procedure

The next step is to take some detailed measurements to test the thin lens and magnification formulas. Using a ruler, measure the s and s' values over as wide a range as the length of the bench allows. In addition, record the height of the image for each value of s and s'. Be sure to also measure the height of the object.

Plot a graph (e.g. using Python) of $1/s'$ versus $1/s$. Is your data consistent with the thin lens formula? From you graph (or otherwise) determine the focal length of the lens, f, together with its associated error.

Using the image height (and object height), plot h_i/h_o versus s and describe the trend. At what value of s does the maximum magnification occur? When does magnification become demagnification? How could you change the setup to increase the magnification using a single lens? Think about the limitations of magnifying an object using just a single lens.

EXPERIMENT B: Virtual images

In the Experiment A, it was possible to place a screen in a position where a clear, focussed image could be observed. These are real images.

In contrast, virtual images cannot be captured on a screen. However, they can still be observed by simply looking through the lens. This may seem strange at first, but you observe a virtual image every time you look in a mirror. In this example, both the light source and you (the object) exist in front of the mirror, but your image appears to be behind the mirror — this is a virtual image. Here you will take a few brief measurements, so that you become more familiar with virtual images formed by lenses.

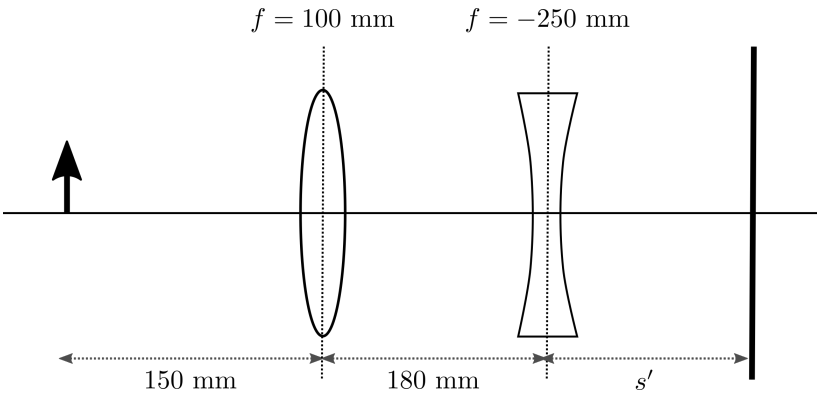

Figure 1.5: Experiment B setup.

Preliminary observations

Take the $\mathrm{f} = 100$ mm lens and move it closer to the light source, so that $s < \mathrm{f}$. Look through the lens towards the light source. What do you see? Is the image larger or smaller than the object? Is it inverted or the right way up? Make a comment in your lab book about what you see and draw a ray diagram of this setup.

Measurement procedure

Move the light source onto the optical rail so that you are able to adjust its position. Then set up the $\mathrm{f} = 100$ mm converging lens and the $\mathrm{f} = -250$ mm diverging lens as shown in Figure 1.5. Move your white card into place so that the image is focussed and record both s' and $\mathrm{h_i}$. Is the image the right way round or inverted? Is it smaller or bigger than the object?

Now remove the diverging lens from its holder, keeping the converging lens and the card *fixed*. *Move the lamp* until the image is once again focussed on the screen. Record the new values of s and $\mathrm{h_i}$.

Draw the ray diagram for these two setups, and explain what is going on.

EXPERIMENT C: Image resolution of a microscope

The setup shown in Figure 1.5 is operating as an alternative version of the compound microscope shown in Figure 1.2. However this setup allows us to record a real image using the CMOS camera. In this section we will investigate the magnifying power of the setup, and if you have time, how different the addition of more lenses can create powerful imaging systems.

Previously, we considered imaging with "thin" lenses and within a small-angle geometrical optics (ray) approximation. For real lenses a fuller treatment also needs to consider the physical optics (wave) picture.

There are two key reasons for loss of quality by imaging with a real lens:

1. The finite diameter D of the lens and
2. lens aberrations due to details of a real ("thick") lens.

Consider light from a point object (e.g. point object A in Fig. 1.6) that expands as a spherical wave (only the centre ray is illustrated in Fig. 1.6). Only the part of this wave that is within the lens diameter contributes to the image. The lens, therefore, acts as a limiting (circular) aperture that will diffract light as well as imaging it. The result is that the point object A is not imaged

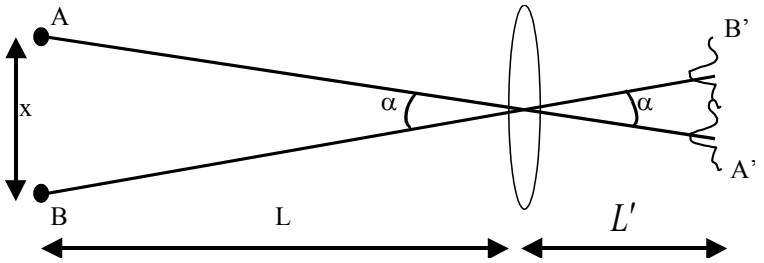

Figure 1.6: Resolution limit of a lens due to its finite diameter D (and aberrations)

to a point but to an 'image spread function' A'. For a 'perfect' lens with no aberrations the characteristic angular size of this image is:

$$\theta = 1.22\frac{\lambda}{D} \tag{1.4}$$

for light of wavelength λ. (The 1.22 is a geometrical factor due to the circular shape of the diffracting object). As a consequence, if two point objects (e.g. A and B) subtend an angle of $\alpha < \theta$, the two images will be merged and will not be resolved as two separate objects: they are said to be unresolved. The objects may be said to be just resolved when $\alpha = \theta$ (this is called the Rayleigh criterion). The minimum resolvable object size, x can be found by using the (small-angle) relationship $\alpha \approx x/L$. Eq. 1.4 is the "diffraction-limited" resolution of a lens, but lens aberrations further degrade and spread out the image, and hence lead to poorer resolution.

One of the slides provided is the three-bar chart (Fig. 1.7). Examine it visually. The figure shows group 0 and 1 with six 'elements' per group. Each 'element' consists of two target patterns of three lines at 90 deg to each other, the line-to-space ratio being equal. A line pair is one black line and one white line; Note that largest element in the even-numbered groups are not next to the other elements in the same group, they are in the bottom right corner. The pattern is repeated at a smaller scale in the centre of the figure up to group number 7. Only the largest groups are visible in figure 1.7 due to the limited resolution of the reproduction. The number of line pairs per millimetre equals 1.00 for the largest element in group 0 and increases by $\sqrt[6]{2} \approx 1.12$ per

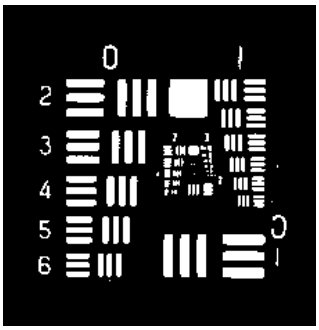

Figure 1.7: The pattern on the three-bar chart slide.

element. The table gives the line pairs per millimetre for all elements in groups 1 to 7 in the chart. Study the chart and try to identify the positions of the largest groups.

The purpose of this experiment is to give you some idea of the imaging quality and resolving power of lenses. In this experiment, you will measure imaging quality using a resolution test based on the *three bar chart*. If you can 'just-resolve' a particular element, then the resolution of your lens is the number of line pairs per millimetre, l_{pairs}, in that element. However we often want to talk about the smallest resolvable detail, x. The relationship between the smallest resolvable detail, x, and the resolution, l is $x = \frac{1}{2l_{pairs}}$.

Element	Group							
Number	0	1	2	3	4	5	6	7
1	1.00	2.00	4.00	8.00	16.0	32.0	64.0	128.
2	1.12	2.24	4.49	8.98	17.95	36.0	71.8	144.
3	1.26	2.52	5.04	10.1	20.16	40.3	80.6	161.
4	1.41	2.83	5.68	11.3	22.62	45.3	90.5	181.
5	1.59	3.17	6.35	12.7	25.39	50.8	102.	203.
6	1.78	3.56	7.13	14.3	28.51	57.0	114.	228.

The following experiments use this resolution chart to assess the resolution and quality of imaging systems.

Resolution of the eye

Put the resolution chart slide into the slide holder on the light source and sit in front of the source with the slide at approximately 250 mm, the near distance of the eye. Decide which is the finest group you can resolve (i.e. just see the bars on the target). You might wish to do two measurements, one for the vertical bars and a second for the horizontal bars. Your partner should also do this experiment. Take three or four independent readings each and find the average resolution, expressed in line pairs per millimetre. (If you wear glasses or contact lenses, you might want to try doing the experiment with and without these). Is your result consistent with Eq. 1.4? You will need to estimate the limiting diameter of your pupil and consider what the main wavelength is for the white light source.

Resolution of the microscope
Using the setup shown in Figure 1.5, replace the L slide with the resolution slide and record the image using the CMOS camera. This can be tricky, so you may want to align it with a different slide first and then swap to the resolution slide. What is the resolution of your setup, and does it align with Equation 1.4?

An Aside about Lens aberrations

A microscope in a laboratory (and a telescope in an observatory) will often consist of (a lot) more than just two lenses. This is due to aberrations that occur over the diameter of the lens.

A full discussion of lens aberration is beyond the scope of this script, however it is noted that two key types of aberration, *spherical and chromatic*, will be important in these cases. The lens formula of Eq. 1.1 assumes thin lenses and small angles.

Spherical aberration: Spherical aberration arises from 'errors' due to rays far from axis (and hence larger angles) not imaging to the same point as rays close to the axis (small angle, or paraxial, rays). Since refraction occurs at both lens surfaces, the strength of this aberration is more severe when a plano-convex lens is placed one way around compared to the other.

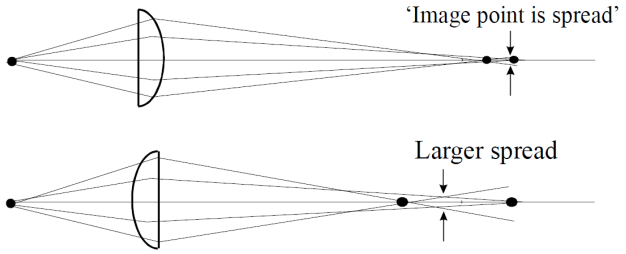

Figure 1.8: Spherical aberration.

Chromatic aberration: Chromatic aberration arises from the wavelength dependence of the refractive index (e.g. this produces the rainbow-like dispersion by a prism). Since the focal length of the lens depends on the refractive index different wavelengths will image at slightly different positions—and this can be quite severe for a white light source. For example, if $\mathrm{n}(\mathrm{blue}) > \mathrm{n}(\mathrm{red})$ then $\mathrm{f}(\mathrm{blue}) < \mathrm{f}(\mathrm{red})$.

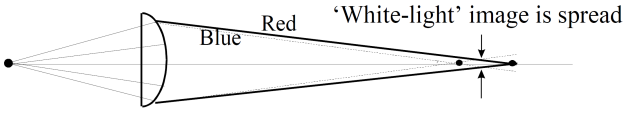

Figure 1.9: Chromatic aberration.

The doublet lens is made of two glass materials with refractive index properties that partially compensate for this aberration.

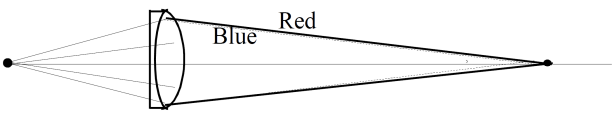

Figure 1.10: A doublet lens which can partially compensate for chromatic aberration.

Resolution of simple lenses (if you have time)
Using the three-bar resolution chart as an object, form an image of the chart directly onto the CMOS camera with the doublet lens provided (f = 160 mm), making sure its flatter surface is towards the object. The camera should be mounted on the adjustable saddle so that it can be moved easily in both the vertical and horizontal directions. The magnification required is about 4 (or more), giving an image of the central groups (2 and higher) that fills the detector. [You will need to use Eqs. 1.1 and 1.2]. It can be quite tricky to align and focus the system; in particular, make sure that the doublet is not twisted and that it is exactly perpendicular to the optical axis. It is also important to minimize stray light from desk lamps, etc.

When everything is aligned properly, you should be able to see targets in Group 6. Make careful observations of the appearance of the image in terms of both its resolution and contrast, for the following lenses:

a) Doublet, f $\approx$ 160 mm, flatter side towards object.
b) Doublet, f $\approx$ 160 mm, more curved side towards object.
c) Plano-convex singlet, f $\approx$ 160 mm, flat side towards object.
d) Plano-convex singlet, f $\approx$ 160 mm, curved side towards object.

Rank the four cases in order of image quality.