Year 1 Laboratory Manual
Pendulum

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1. The (not so) Simple Pendulum

The aim of this experiment is to explore some relatively complicated physics using a simple measurement device, the pendulum. There are two main topics: measuring the amplitude dependence of the pendulum's period, and measuring the mass dependence.

You will be using a digital camera to film the pendulum swinging and a provided Python code to analyse the video footage and determine the period of oscillation. It is a good idea to take the time to analyse your data as you go along—in particular it is important that you consider the uncertainties in your measurements.

1.1 Getting good video footage of your pendulum

For the Python code to work best you will want to film the pendulum against a contrasting background. We recommend filming it from above with a matt black sheet of paper under it. With the pendulum illuminated by the ceiling lights it will show up as a bright spot on a dark background.

Set up a simple pendulum using one of the brass bobs. Use a thread around 30 cm to 40 cm long clamped to one of the bars attached to a retort stand. The threaded rod the bob attaches to allows you to make fine adjustments to the length; it has a small hole at one of its ends which you should use to attach it to the string. Before you start filming, make sure you have an unobstructed view of the bob as it swings, and that the black background is along the whole swing path. It is also important to have the camera view parallel to the direction of swinging (i.e. the pendulum does not swing diagonally in the picture). Don't worry too much about the image being in focus – it doesn't make too much of a difference for the image processing. Once you have the right view of it, set the pendulum in motion and film it for about 5 seconds. Make sure your hands or any other tools you used to displace the pendulum are not in the recording.

1.2 Using the code for the experiment

You need to use this python code to analyse the video footage and determine the period of oscillation of your pendulum. Clicking on the hyperlink will direct you to the Google Colab Notebook. Users can view notebooks shared publicly without sign-in. In order to execute code, a Google account sign-in is required. Alternatively, you can find a copy of this notebook on Blackboard, and run it on your preferred navigator. However, be mindful, if you are using Spyder through the Anaconda Navigator, the required packages are not included in the basic installation but it is straightforward to add them.

The notebook is self-explanatory, so please read the description of each section, before running that cell of code. Please avoid running the whole code the first time you are using the code, as you need to understand the code and make required changes. You can run a cell in Google Colab by clicking on the arrow on top of the cell, or by using Ctrl+Enter shortcut.

This code should now be able to analyse any video recording of a swinging pendulum and output the X and Y coordinates as a function of frame number. However, before analysing a new recording you should make sure that the cropping range is correct for any new setup (even if you only move the camera or the background a little bit, it is always good to check again that you are cropping the correct part of the picture). It is also possible that you might need to adjust the threshold value for conversion to black and white. You can always use the first frame of a new video to check this.

Do not worry if you don't manage to work with the python code. If for any reason you do have trouble with the Python code then it is perfectly acceptable to record data manually (i.e. timing your pendulum with a stop watch).

Run the python code using your short video and plot the X and Y positions as a function of frame number on two separate plots. Is the outcome what you expect? Ideally your pendulum should be moving only in one direction, so the amplitude of the oscillation in either X or Y should be close to zero. Fit a sine function to your data to find the period of oscillation. The camera you are using is probably recording 30 frames per second, but you can check this by comparing the length of your video to the number of frames you analysed, or by checking in the properties of the video file. As you will be using the frame rate to measure the period of your pendulum, take a moment to think about how to determine or estimate the errors (random and systematic) associated with this. Discuss this with your demonstrator.

1.3 Experiments overview

A simple pendulum is an ideal way to demonstrate simple harmonic motion. Figure 1.1 shows the force on a simple pendulum of length l which makes an angle θ with the vertical. For a bob of mass m Newton's second law is

$$-ml\ddot{\theta} = mg\sin\theta\,. \tag{1.1}$$

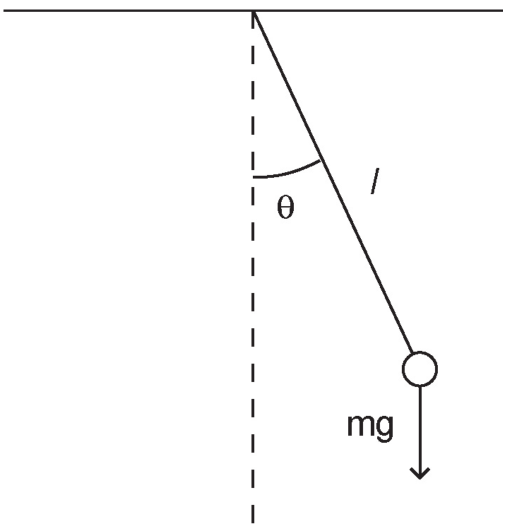

Figure 1.1: The simple pendulum.

We make the approximation $\sin\theta \approx \theta$, which leads to simple harmonic motion with angular frequency $\omega = \sqrt{g/l}$. It is easiest to measure the period T_0 of the pendulum, which is given by

$$T = 2\pi\sqrt{\frac{l}{g}}. \tag{1.2}$$

1.4 Experiment 1: Amplitude dependence

The expression for the period given by Eq. 1.2 has no dependence on the amplitude of the pendulum. In this experiment we are going to investigate the finite amplitude pendulum. Is there any change in period with amplitude?

Set up a simple pendulum using one of the brass bobs. Use a thread around 30 cm to 40 cm long clamped to one of the bars attached to a retort stand. The threaded rod the bob attaches to allows you to make fine adjustments to the length, but for this experiment the length should stay fixed.

Record a series of videos to measure the period of your pendulum after starting it with different amplitudes. You should decide a good strategy to measure the initial amplitude. How many periods should you record for to produce an acceptable uncertainty on the period. Aim for at least 1%, but 0.1% is not out of the question.

Make sure you save your movies and any analysis you do using the python code in a place you can access once you leave the lab.

1.5 Experiment 2: The equivalence principle

The mass m in Eq. 1.1 is the same on both sides of the equation, but on the left it is actually what we call the inertial mass and on the right it is the gravitational mass. There are good theoretical reasons to suppose they are the same, but their equivalence is certainly open to experimental test (and an active area of research). If we call these m_i and m_g, the period of a simple pendulum is given by the formula

$$T = 2\pi\sqrt{\frac{m_i}{m_g}\frac{l}{g}}. \tag{1.3}$$

Identically sized pendulum bobs made of tungsten, brass, aluminium and nylon are available.

By measuring the period using these different materials you can put a limit on the difference between gravitational and inertial mass. You should think carefully about systematic errors. In particular, you should try to keep the length constant by swapping the different bobs onto the same holder.

material	density (g cm^{-3})
Nylon	1.13
Aluminium	2.7
Brass	8.45
Tungsten	19.3

Table 1.1: Density of various material

1.6 Data Analysis

Experiment 1: Amplitude Dependence

The aim is to determine if the period changes as the amplitude increases. Using Python, make a plot of period vs. amplitude. Do you see any dependence on the amplitude of the pendulum? Is any difference in period significant compared to the uncertainties in your measurement?

You can assume that the period of a finite amplitude pendulum can be expressed as a correction to the small amplitude period, T_0, i.e. as:

$$T = T_0 \left(1 + \alpha\theta_0 + \beta\theta_0^2\right) , \tag{1.4}$$

where T_0 is the small amplitude period, θ_0 is the amplitude of the oscillation and α and β are constants to be determined. Using your plot of period vs amplitude, find α and β in the above equation and their associated uncertainties. Are α and β significantly different from zero?

Experiment 2:The equivalence principle

The aim here is to determine if the period changes if when the mass of the bob is changed.

If inertial mass and gravitational mass are the same thing then we would expect $m_g/m_i = 1$. However if they are not it is reasonable to hypothesise that the ratio depends on the density of the material,ρ. A simple way to interpret your data is to assume that $m_g/m_i = 1 + k\rho$, where k is a constant to be determined by experiment. The density of the materials used are shown in table 1.1.

You are interested in determining a limit on the size of k, which depends on your measurement uncertainties and systematics

These sort of experiments date all the way back to Newton. A modern interpretation is that you are measuring the equivalence principle for bodies with different ratios of protons to neutrons. These are usually called "Eötvös experiments", eg. S. Schlamminger et. al., Phys. Rev. Lett. 100, 041101 (2008). That work used a torsion balance to eliminate some of the systematic effects you undoubtedly have discovered.