

# Year 1 Laboratory Manual

## Grating Spectrometer

Department of Physics, Imperial College London\*

November 5, 2021

\*You should reference this work as: 'S. P. D. Mangles (Ed.) Year 1 Laboratory Manual: Grating Spectrometer Experiment', Imperial College London, 2021

# Contents

1	Measuring the Rydberg Constant with a diffraction grating	2
1.1	Introduction . . . . .	2
1.2	Theory . . . . .	2
1.3	Setting up the spectrometer . . . . .	4
1.4	The Rydberg constant . . . . .	5
1.5	Analysis . . . . .	7

# 1. Measuring the Rydberg Constant with a diffraction grating

## 1.1 Introduction

A spectrometer is an instrument used to study the spectrum of light. The light source can be anything from a cryogenically cooled crystal to a superhot plasma or a star, and “light” might be anything from the longest infrared all the way down to  $\gamma$ -rays. In the case of optical light the spectrum generally takes one of two forms. Hot solids, such as a tungsten lamp filament emit spectra where the intensity is a slowly varying function of wavelength - a “continuous spectra”. Hot or electrically excited gasses emit spectra in which the emission is concentrated at a few well defined wavelengths - called “emission line spectra”, from the appearance of the slit image of the spectrometer. In such a spectrum the ratio of intensities of spectral lines can tell you the temperature of the gas, while the line widths can give you the density of the emitting gas or, in an electrical discharge, the density of the free electrons in the discharge. Spectroscopy is a “non-invasive” technique and is one of the most powerful tools available to physicists studying plasmas, stars, flames, semiconductors, etc.

In its simplest form, a spectrometer uses a lens or mirror to produce a collimated beam of light and disperses it with a grating or a prism. The dispersed spectrum is then focused and viewed with an eyepiece or recorded with an electronic detector.

In this experiment you will measure the spectrum emitted by a hydrogen lamp and use your data to deduce the Rydberg constant.

## 1.2 Theory

A diffraction grating can be considered as a large number  $N$  of equally spaced very narrow slits, with slit separation  $d$ . For light of wavelength  $\lambda$  a principal maximum occurs when the phase difference between neighbouring rays is an integer multiple  $m$  of  $2\pi$ :

$$\text{phase difference} = \text{path difference} \times \frac{2\pi}{\lambda} = 2\pi m. \quad (1.1)$$

The path difference is  $d \sin \theta$ , thus

$$\frac{2\pi}{\lambda} d \sin \theta = 2\pi m$$

which means

$$d \sin \theta = m\lambda \quad (1.2)$$

$m$  is called the order number. This path difference is shown in the inset in Fig. 1.3. Fig. 1.1 shows a fringe pattern for the case  $N = 4$ .

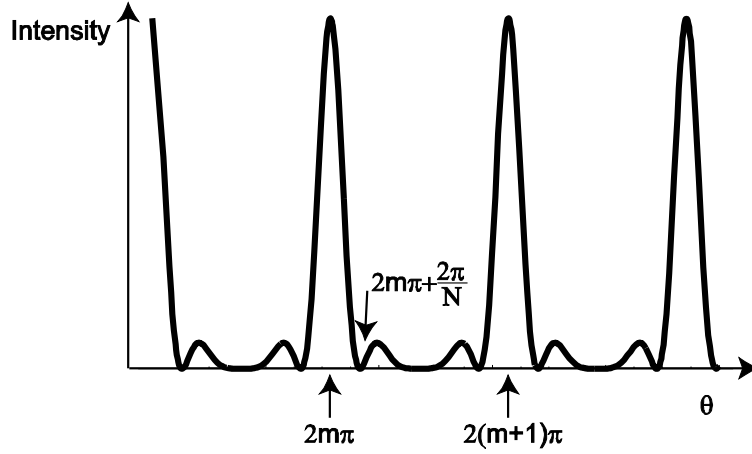


Figure 1.1: The diffraction pattern for  $N = 4$  slits, showing the principal maxima and minima.

With  $N$  slits illuminated, the first minimum occurs at a phase angle

$$\phi = 2\pi m + \frac{2\pi}{N}$$

We find the *resolving power* by considering the simple criteria that the minimum of one diffraction pattern occurs at the same position as the maximum of another pattern, as shown in Fig. 1.2. Light of a second wavelength  $\lambda_2 = \lambda - \Delta\lambda$  will produce its  $m^{\text{th}}$  principal maximum at a slightly

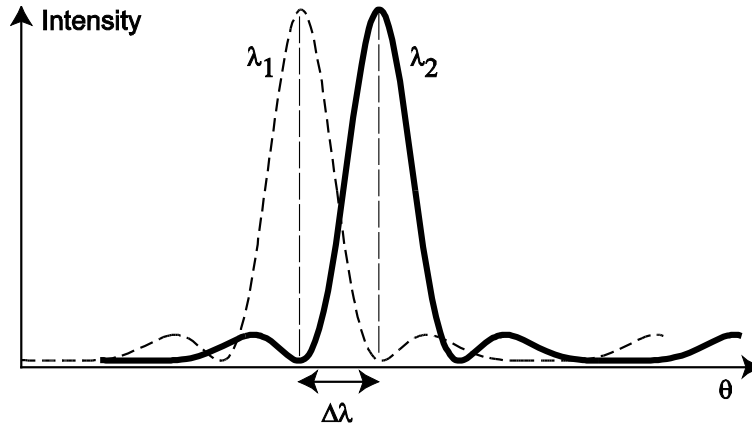


Figure 1.2: Diffraction patterns and resolution. The principal maximum of one pattern occurs at the same position as the first minimum of the other.

different angle  $\theta_2$  given by  $d \sin \theta_2 = m\lambda_2$ . It will have a minimum at angle  $\theta$  if

$$\frac{2\pi}{\lambda_2} d \sin \theta = 2\pi m + \frac{2\pi}{N}, \quad (1.3)$$

$$\text{that is, } d \sin \theta = m\lambda_2 + \frac{\lambda_2}{N}. \quad (1.4)$$

Taking Eqs. 1.2 and 1.4, we find

$$m\lambda = m\lambda_2 + \frac{\lambda_2}{N}.$$

We have defined  $\lambda - \lambda_2 = \Delta\lambda$ , where  $\Delta\lambda \ll \lambda$ . This means

$$\frac{\lambda}{\Delta\lambda} = mN. \quad (1.5)$$

At this separation the intensity at a point half way between two equal lines is about 20% less than the peak and the two lines can easily be seen to be separate.

### 1.3 Setting up the spectrometer

Fig. 1.3 shows the important parts of the spectrometer you will be using. Follow the sequence of operations below to align and focus the instrument.

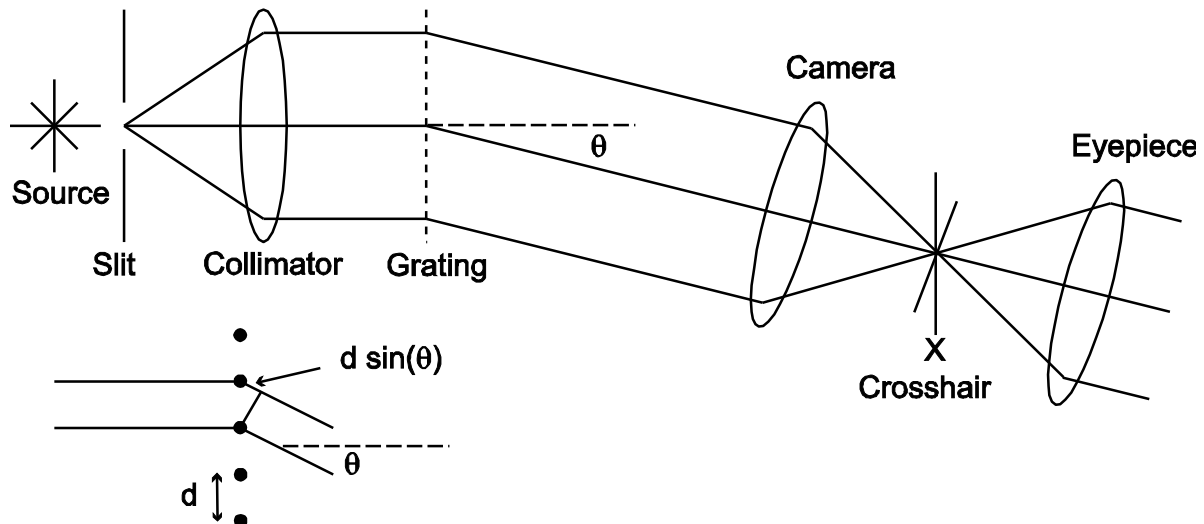


Figure 1.3: The spectrometer. Light from the source passes through a narrow slit and is made parallel by the collimating lens. It strikes the diffraction grating at normal incidence and the diffracted light is focused by the camera lens onto the crosshair "X". A magnified image is viewed using the eyepiece. The inset shows the path length difference  $d \sin \theta$ .

1. Focus the telescope (camera lens and eyepiece) so that a distant object is focused on the crosshair. To accomplish this, focus the telescope on a distant object in the lab. *Distant* in this context means 5 to 10 metres.
2. Rotate the telescope so that it is aligned with the axis of the collimator. Position the yellow sodium lamp close to the slit, so that it is illuminated. Without changing the camera focus adjust the focus of the collimator until a sharp image of the slit is focused at the crosshair. You might need to adjust the slit to be near its minimum width while carrying out this step. The crosshair is easier to use if set as an "X" rather than as a "+" and can be adjusted by rotating the lens tube that the eyepiece is inserted into.
3. Before going any further, investigate what each dial on the spectrometer controls and make a note of this in your lab book. It is easy to ruin the spectrometer calibration by using the wrong dial by mistake. Some of these dials will make both calibration and measurements far easier if you take the time to understand how they work now. Speak to your demonstrator to confirm your understanding of the dials at this point.
4. Align the camera and collimator so the crosshair is set on the slit image. Note the reading of the two scales on the spectrometer base. The main (lower) scale has half-degree increments and the vernier (upper) scale reads in arc minutes, or sixtieths of a degree. The appendix explains how to use a vernier scale.
5. Move the telescope through *precisely*  $90^\circ$  and clamp it in position. Put a mirror into the grating holder.
6. Turn the grating table until you see a reflected image of the entrance slit in the eyepiece; this indicates that the mirror surface is at  $45^\circ$  to the incident beam from the collimator.

- Adjust the height of this image using the adjustment screws under the grating table. Use the grating drive to set the mirror so the slit image is precisely at the crosshair.
7. Note the reading. Rotate the mirror through precisely  $45^\circ$ , using the scales, so that it is accurately set at  $0^\circ$  (normal) incidence to the incoming light from the slit.
  8. Switch the sodium lamp used for calibration with the hydrogen lamp provided. Note that the hydrogen lamp emission lines are at their brightest when used for short intervals and can burn out if left switched on for too long. A burnt out hydrogen lamp can take 20 minutes to cool down which will waste valuable experiment time, so only switch the lamp on when you intend to take measurements.
  9. Replace the mirror in the grating stage with one of the diffraction gratings provided (80 lines per mm or 300 lines per mm - try both and explain your choice in your lab book) and ensure that the hydrogen lamp is well aligned with the spectrometer slit by checking the intensity of the central maximum with the telescope. Make a note of the angle at which the central maximum occurs - this may have changed from the value you recorded earlier (step 4) as the grating stage (which moves with the vernier scale) has been rotated since this measurement was first taken.
  10. Move the telescope around to find the diffracted image of the slit, choosing one at a large angle of diffraction. If the grating rulings are not parallel to the rotation axis of the telescope, this image will be above or below the plane defined by the crosshair. If this is the case, rotate the grating in the plane of its surface (i.e. by lifting one side of the grating out of mount, not by rotating the grating stage) until the image is at the right height. Check by going to a large angle on the other side of the grating normal.

The spectrometer is now set up to make accurate, repeatable measurements. It is probably the most accurate instrument you will meet in the lab. In careful hands and using the average of a dozen or so measurements, it is capable of a precision of under three parts in ten thousand, or about  $\pm 0.2$  nm for yellow-orange light.

## 1.4 The Rydberg constant

Transition wavelengths in hydrogen are given by the remarkably simple formula

$$\frac{1}{\lambda} = -R_\infty \left( \frac{1}{n^2} - \frac{1}{p^2} \right) \quad (1.6)$$

where  $n$  and  $p$  are integers which represent the energy levels of the initial and final states of the atom, respectively.  $R_\infty$  is the Rydberg constant, which has the measured value  $10\,973\,731.6 \text{ m}^{-1}$ . It can be expressed in terms of more fundamental constants as

$$R_\infty = \frac{\alpha^2 m_e c}{2h},$$

where  $m_e$  is the mass of the electron,  $c$  the speed of light,  $h$  Planck's constant and  $\alpha$  the *fine structure constant*:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}.$$

Note that  $1/\alpha \cong 137.036$ . It is dimensionless.\*

The *Balmer* series in hydrogen is given by Eq. 1.6 with  $p = 2$  and  $n = 3, 4, 5 \dots$ . This is to say that the atom is dropping down from the  $n$ th energy level to the second ( $p = 2$ ) energy level,

---

\*see <http://physics.nist.gov/constants>

giving off a photon of a specific wavelength that depends on  $n$ . Balmer transition wavelengths are in the visible part of the spectrum. From Eq. 1.6, you can calculate the values of the wavelengths, and using a standard colour spectrum chart, determine the colours that you can expect the light to be visible as.

The aim of this part of the experiment is to use the spectrometer to measure the wavelengths via the grating equation (Eqn. 1.2) and to thereby deduce  $R_{\infty}$ , and its uncertainty.

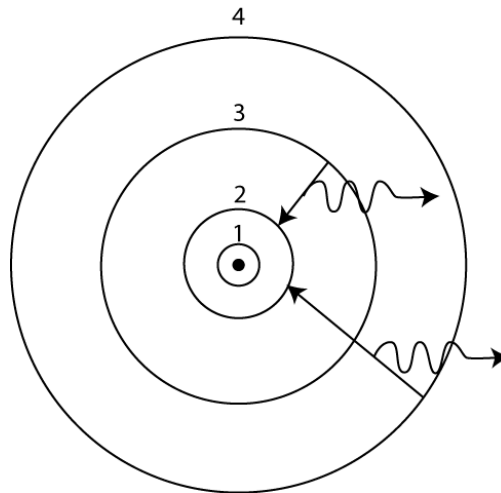


Figure 1.4: Representation of two Balmer transitions in a hydrogen atom: initial state  $n = 3$ , final state  $p = 2$ , and similarly for  $n = 4$ ,  $p = 2$ .

For each transition wavelength, measure  $\theta$  as a function of order  $m$ , considering both positive and negative values. Before recording data determine the optimal slit size for resolution and intensity. Do not change the slit size once you start taking data. Make sure that you estimate the uncertainty in your determination of the angle  $\theta$  at each order.

With practice you can make wavelength measurements quite quickly and thus build up a large database to help reduce statistical uncertainty. However, there are numerous possibilities for systematic error in this experiment. Some things to consider are the effects of spectrometer misalignment, defocus, lamp position, grating period, air pressure and humidity, background light etc. Some of these you can test experimentally, but all must be estimated using combinations of measurement, logic and well-justified assumptions. As always, the estimation of these uncertainties are just as important as the measurement of the central values.

#### Note on diffraction grating spacings

The provided grating has a number of lines per unit length which is different from the 80 lines per mm that is indicated. In fact it uses an old definition in units of 2001 lines per inch, which approximates to 78.8 lines per mm\*.

---

\*This discrepancy was found by a student, P. Goodman, through communication with the supplier of the equipment after a systematic error in the data was spotted.

## 1.5 Analysis

The aim of the analysis is to determine the Rydberg constant and its associated uncertainty from your measurements of the wavelength of different transitions.

First you will need to determine the wavelength and uncertainty for each transition line you observed. Eq. 1.2 implies a plot of  $\sin \theta$  vs.  $m$  will yield a straight line with a gradient equal to  $\lambda$ .

Once you have determined the wavelength of each transition line, you need to use eqn. 1.6 to estimate the Rydberg constant. For this you will need to ensure you have correctly identified the levels  $n$  and  $p$  involved in the transitions you have measured and calculate

$$x = \left( \frac{1}{n^2} - \frac{1}{p^2} \right)$$

and its associated uncertainty for each line you observed. A plot of  $x$  against  $1/\lambda$  should produce a straight line. Find the gradient of this and the associated uncertainty, and hence find a value for the Rydberg constant and its associated uncertainty.