Year 1 Laboratory Manual
Diffraction from Single and Double Slits

Department of Physics, Imperial College London*

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1. Investigation of diffraction and interference

1.1 Introduction

When two or more waves are superimposed, the resultant amplitude (and hence intensity) depends on the amplitudes and phases of the component waves. This process is known either as interference or diffraction, depending upon the geometry of the particular experiment. Usually when there is a small number of waves (e.g. light from two very narrow slits or two distinct optical paths) we talk of *interference* but if there is a continuum of waves (e.g. from light passing through a single slit of non-negligible width) then we talk of *diffraction*.

The aim this experiment is to observe diffraction patterns obtained when a laser illuminates various narrow slits and to compare these patterns with that predicted by the wave theory of light. We can then use these diffraction patterns to accurately determine the size and spacing of the slits.

Far Field Diffraction

Analysis of diffraction close to the diffracting object, known as 'near-field diffraction', can become quite mathematically complex. A simpler case is the so-called *far-field* diffraction regime in which the plane of observation is very far from the aperture (ideally at infinity)—as seen in Fig. 1.1a. Due to the physical size of optical bench this is not always practical and an equivalent effect is achieved by making all observations at the focal plane of a lens—as seen in Fig. 1.1b. From Fig. 1.1b it is noted that rays diffracted at angle θ are observed on the detector at a distance x from the lens axis where $\theta = x/f$ (for small angles expressed in radians).

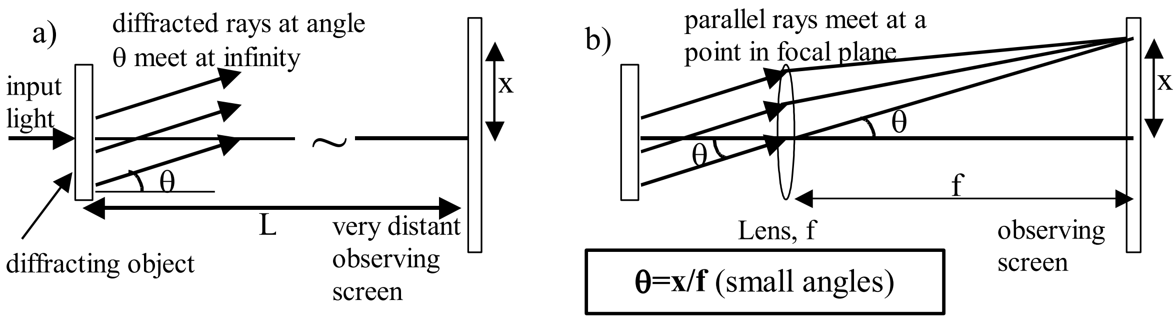

Figure 1.1: Input light beam (e.g. from laser) is diffracted into rays at new angles. The far-field diffraction pattern can be observed a) on a very distant detector (ideally at infinity) or b) in the focal plane of a lens.

1.2 Theory: Diffraction by a single slit

The theoretical derivation of the intensity of light in the "far-field" diffraction pattern of a single slit is found by summing the amplitude (of the electric field) of light from each elementary point on the slit. This procedure follows from Huygens' Principle. The intensity pattern is found by squaring the amplitude pattern. For a slit of width a, the intensity $I(x)$ of the diffraction pattern as a function of distance x in the observation plane (i.e. at the focal distance f from the lens, as shown in figure 1.1) is given by

$$I(x) = I_0 \left[\frac{\sin\left(\frac{\pi a x}{\lambda f}\right)}{\left(\frac{\pi a x}{\lambda f}\right)} \right]^2, \tag{1.1}$$

where I_0 is the intensity at the centre, λ is the wavelength. This expression is derived using the small angle approximation, i.e. where $\theta \approx x/f$. Note that the quantities a, x, λ and f in Eqn. 1.1 all have dimensions of length and should all be expressed in the same units, e.g. millimetres. Fig. 1.2 shows the single slit diffraction pattern.

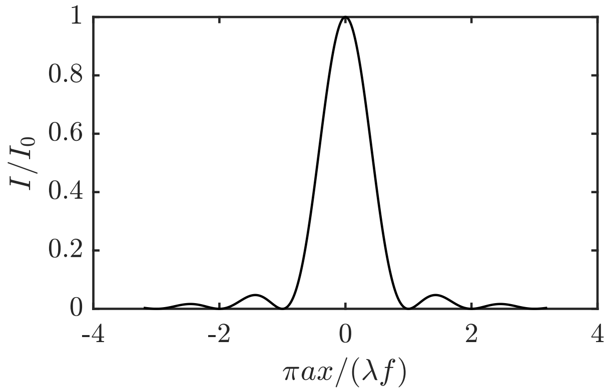

Figure 1.2: The single slit diffraction pattern of Eqn 1.1.

From Eqn. 1.1, the zeros of intensity occur at x-coordinates x_m given by

$$\left(\frac{\pi a x_m}{\lambda f}\right) = m\pi, \tag{1.2}$$

where $m = \pm 1, \pm 2, \pm 3, \ldots$. This can be rearranged as

$$x_m = m(\lambda f/a). \tag{1.3}$$

The spacing of the minima of the single slit diffraction pattern can therefore be used to measure of the slit width, a.

1.3 Theory: Double Slit Interference

Fig. 1.3 shows the geometry used to derive the form of the interference pattern produced by two *infinitely narrow* (i.e. idealised) slits. If the slits are infinitely narrow, then the difference between the paths S_2P and S_1P is (for a distant detector) given by

$$\delta = r_2 - r_1 \approx d \sin\theta \,. \tag{1.4}$$

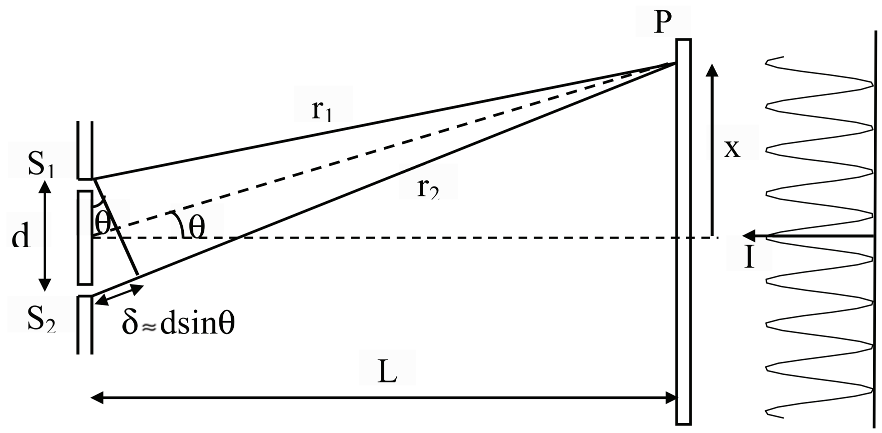

Figure 1.3: Geometry for double slit interference phenomenon: for large L, the path difference to point P is approximately $d \sin\theta$, where d is the slit separation.

Since the total light ampltiude at point P is found by summing the contributions from the two slits (with due account of the phases) the positions of the bright and dark fringes are given by constructive or destructive interference conditions:

$$\text{MAXIMA}: \quad d \sin\theta = m\lambda \tag{1.5}$$

$$\text{MINIMA}: \quad d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \tag{1.6}$$

where m= 0, ± 1, ± 2, ± 3, ...

Because you will be observing at the focal plane of a lens, we can relate the angle of observation, θ to the position of the bright and dark fringe, x, se the small angle approximation $\sin\theta \approx \theta = x/f$, therefore write the position of the bright and dark fringes as:

$$\text{MAXIMA}: \quad x_m = m\frac{\lambda f}{d} \tag{1.7}$$

$$\text{MINIMA}: \quad x_m = \left(m + \frac{1}{2}\right)\frac{\lambda f}{d} \,. \tag{1.8}$$

The *spacing* between fringes (between successive maxima or successive minima) is therefore $\lambda f/d$.

For slits of negligible width, the form of the intensity distribution of the interference pattern is, for small angles:

$$I(x) = 4I_0 \cos^2\left(\frac{\pi d x}{\lambda f}\right) \tag{1.9}$$

where I_0 is the intensity due to a single source and is assumed to be uniform across the detector.

When the slits have a finite width a, the intensity is equal to the product of the distribution of a single-slit (see Eq. 1.1) with the double slit diffraction pattern (Eq. 1.9) and may be written as:

$$I(x) = 4I_0 \left[\frac{\sin\left(\frac{\pi a x}{\lambda f}\right)}{\left(\frac{\pi a x}{\lambda f}\right)} \right]^2 \cos^2\left(\frac{\pi d x}{\lambda f}\right) \tag{1.10}$$

where a is the slit width and d is the separation of the slits. The pattern is sketched in Fig. 1.4 for the case where $d = 4a$.

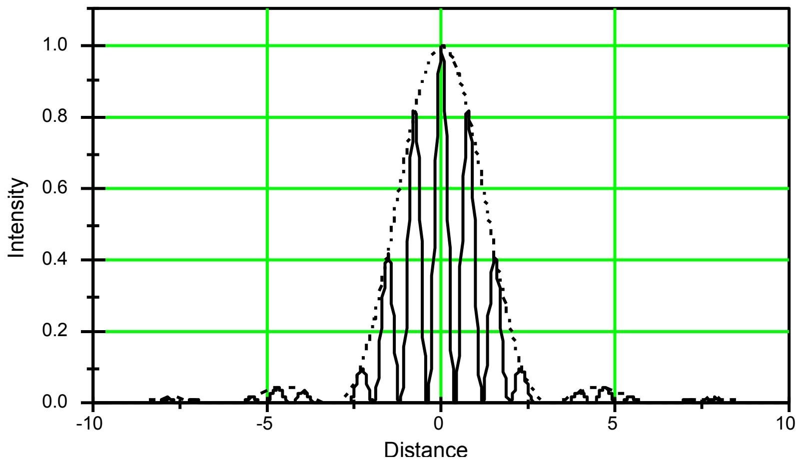

Figure 1.4: Example of a fringe pattern produced by double slit interference.

1.4 Experimental Procedure

Experiment A: Using diffraction to measure the physical dimensions of objects

In this part of the experiment you will first make measurements to determine the width of a single slit by measuring the position where the minima in the diffraction pattern occur (which are related to the slit width via equation 1.3). You will then measure the spacing of the fringes produced by a double slit and use this to determine the spacing of the slits (these are related by equation 1.8).

List of equipment

- optical table with mounted rail
- Diode laser and power supply, central wavelength $\lambda = 670 \pm 1\,\mathrm{nm}$.
- bi-convex lens, f = 500 mm,
- bi-convex lens, f = 1000 mm,
- plano convex lens, f = 160 mm
- set of various diffracting slits
- photodiode detector mounted on translation stage
- metre rule
- CMOS camera*
- polarizing sheet (useful for reducing intensity on the CMOS camera)

Fig. 1.5 shows the basic set-up for the experiments on interference and diffraction. In this experiments, the light source is a red diode laser. The basic diode laser inside the housing is an ultra-compact device and it is easy to forget that it is a laser which must be handled safely. Lasers can cause permanent damage to sight, and while this is a relatively low power laser you should still –NEVER LOOK DIRECTLY INTO A LASER— and NEVER POINT A LASER AT ANY OTHER PERSON.

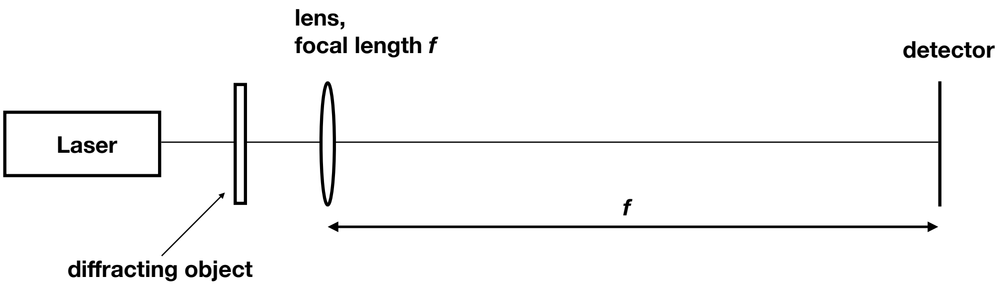

Figure 1.5: Basic arrangement for measurement of the intensity of interference and diffraction patterns. The object is placed close to the laser output and the detector is placed at the focal plane of the lens.

To measure the intensity of the diffraction pattern use the photodiode as the detector. The photodiode is mounted behind a narrow slit (width =100 μm) placed in front of it so that you can measure the intensity at one particular position. The photodiode voltage is linearly proportional to intensity of light on the diode. The slit/photodiode are provided as a single unit that attaches to a precision translation stage, you can use this to move the photodiode and measure the intensity of the diffraction pattern at different positions. The translation slide has a total range of 25 mm.

*CMOS stands for complementary metal-oxide-semiconductor. The camera has a total image area of 6.66 by 5.32mm (H x V), within which there are 1280 x 1024 pixels

You are supplied with a set of three lenses with different focal lengths. The size of the diffraction pattern will be different depending on which lens you use. Select a lens that will allow you to measure the position of the zeros using the diode and translation slide. You will either need to take some preliminary measurements or estimate the width of the slit to work out which is the most suitable lens. You should aim to measure the position of several minima. You should consider how you determine the uncertainty in the position of each minimum.

Now replace the single slit slide with the double slit slide, and record the position of the minima in the double slit diffraction pattern.

Experiment B: Investigating the Diffraction Pattern

The previous method used the position of the minima in the diffraction patterns to determine the physical dimensions of the slits. However, this is based on the assumption that the theoretical models used are an accurate description of the situation. In this part of the experiment you will record digital images of the diffraction patterns and compare these to the theoretical predictions.

- select the correct lens to enable the diffraction pattern to fit onto the CMOS detector
- place the CMOS camera the correct distance from the lens
- set the exposure time short enough to record an unsaturated image of the pattern. You may need to use the polarizer sheet to attenuate the intensity on the CMOS detector.
- record images of the single slit and double slit diffraction patterns. Make sure that you save the image in a location that you and your lab partner can access after you have left the lab (e.g. OneDrive).
- if you have time you could try measuring the diffraction pattern due to a similar sized object–such as a human hair.

1.5 Analysis

Experiment A: Using diffraction to measure the physical dimensions of objects

Recall that for a single slit of width a we expect the position of the minima to be governed by

$$x_m = m\frac{\lambda f}{a}\,.$$

Using this equation plot the position of the minima you recorded, x_m as a function of the order, m and hence determine the slit width and its associated uncertainty.

For a double slit of separation d we expect the position of the minima to be governed by

$$x_m = \left(m + \frac{1}{2}\right)\frac{\lambda f}{d}\,.$$

Using this equation plot the position of the minima you recorded, x_m as a function of the order, m and hence determine the slit separation and its associated uncertainty.

Experiment B: Investigating the Diffraction Pattern

Now examine the image(s) of the diffraction patterns you recorded of the single and double slit diffraction pattern. You can use the free software ImageJ to open the image and perform analysis of the image (you can also use a host of functions available for Python if you prefer). If you are using ImageJ here are some instructions:

- open the image file
- rotate the image so that the diffraction pattern is straight (using the Image > Transform > Rotate command)

- use the rectangle tool to draw a rectangle across the diffraction pattern.
- plot the profile of signal within this rectangle (using the Analyse > plot profile command). You may want to adjust the width of the rectangle you drew to improve the quality of the profile.
- In the profile window, select the "list" button to access the numerical values of your profile.
- save these numerical items (file > save as)
- load this numerical data into Python for analysis

Make plots of the recorded diffraction patterns and compare with the expected patterns based on equations 1.1 and 1.10. You could try to use the Python fitting methods you have learnt to determine the physical dimensions of the slits, or you could plot the diffraction patterns based on the physical parameters determined in experiment A. You should comment on any differences between the expected pattern and that which you actually observed.