Year 1 Laboratory Manual
Introductory Experiments
Planck Constant Experiment

Department of Physics*

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1. Determining Planck's Constant using LEDs

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1.1 Introduction

Planck's constant, h, is one of the most important constants in quantum physics. One key equation involving Planck's constant relates the frequency, ν, of a photon to its energy, E:

$$E = h\nu\,. \tag{1.1}$$

Since May 2019 Planck's constant has been *defined* to have the exact value, $h = 6.62607015 \times 10^{-34}$ J s, and is used to define the unit of mass, the kilogram.

The aim of this experiment is to show how using simple lab equipment we can make an accurate measurement of an important physical quantity *. The experiment is designed to help train you to minimise random and systematic errors so that you can obtain a measurement which is as accurate and precise as possible with the equipment provided. It also aims to train you in the use of key laboratory equipment, including the electronics breadboard which will be very useful throughout your lab career.

Light Emission in LEDs

In an LED, we inject electrons into the positive terminal of the LED using a power supply with a voltage V. For the LED to emit light, the electrons must have sufficient energy to be driven from the valence band to the conduction band of the semi conductor† (i.e. The energy of the electrons, E_e, must be at least equal to E_g). The energy, E_q, of a charged particle accelerated by a voltage V is given by

$$E_q = qV \tag{1.2}$$

*This experiment is based on multiple experiments described on the web and the apparatus based on that used by York University. The original authors of this lab script are Matthew Ward and Michaela Flegrova.

†please see the appendix in this section for some more details if you are interested

where q is the charge on the particle. Thus for an electron with charge e we have

$$E_e = eV. \tag{1.3}$$

If we set the power supply to our LED such that $E_e = eV > E_g$, electrons will be driven from the valence band to the conduction band and can then recombine, emitting photons of energy E_g.

IV Characteristics of LEDs

LEDs are non-ohmic conductors, meaning that they do not follow the $V \propto I$ relationship of ohmic conductors. The general current-voltage (IV) behaviour of LED's is shown in figure 1.1.

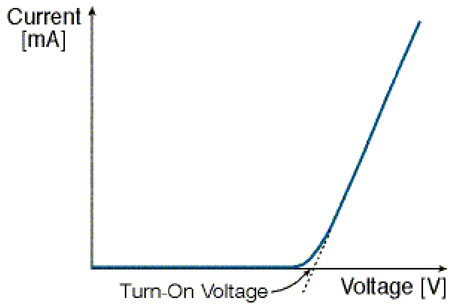

Figure 1.1: A graph of the general IV characteristics of an LED. Source: `https://web.phys.ksu.edu/vqm/laserweb/Labs/Labi-v/LAB-IV3.gif`

As shown in figure 1.1, the current passing through the LED is ≈ 0 A for voltages far below the so-called "turn-on" or "threshold voltage", V_T, of the LED. V_T represents the voltage at which $E_e = eV = E_g$ and the electrons have sufficient energy to be promoted to the conduction band to conduct current. As we approach V_T, current increases rapidly with voltage. This increase is initially non-linear, yet approaches linear behaviour at voltages greater than V_T. By extrapolating the near linear region of the graph to the origin, an estimate of V_T can be obtained.

Calculating Planck's Constant

How can this information be used to determine Planck's constant? We must relate the wavelength of the photons emitted to the threshold voltage. We know that at the threshold voltage

$$E_e = eV_T = E_g. \tag{1.4}$$

Using equation 1.1, we can relate E_g to the frequency of the photons emitted by the LED via

$$E_g = h\nu. \tag{1.5}$$

Substituting this into equation 1.4 we find

$$E_e = eV_T = h\nu = E_g \tag{1.6}$$

$$V_T = \frac{h}{e}\nu, \tag{1.7}$$

and given that a photon's wavelength and frequency are related by

$$\nu = \frac{c}{\lambda}, \tag{1.8}$$

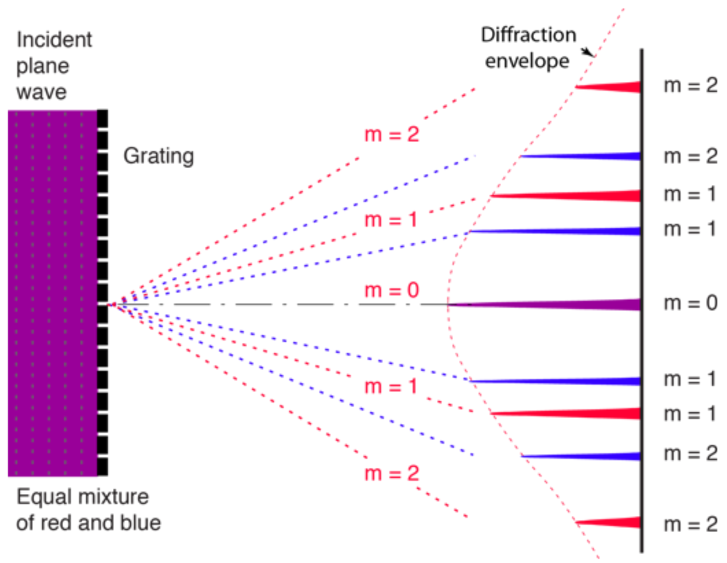

Figure 1.2: A diagram to illustrate the diffraction of light of multiple wavelength by different angles. Source: `http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/imgpho/DiffGrat.png`

we can relate the LED photon wavelength to its threshold voltage via

$$V_T = \frac{hc}{e}\frac{1}{\lambda}. \tag{1.9}$$

For a given LED, if the V_T and λ are measured, we can determine h using known values of c and e. In this experiment we will use six LEDs emitting in the visible spectrum, each with different λ and V_T. Using equation 1.9, we can see that plotting a graph of V_T against $1/\lambda$ for these LEDs should yield a straight line graph passing through the origin with gradient $(hc)/e$. The gradient, along with the known values of c and e, can thus be used to calculate h. (Consider: Why is using the gradient more accurate than using a single value?).

The Spectrometer

In this experiment, you will be using a digital spectrometer to measure λ for each LED. Whilst the detailed workings of the spectrometer will be covered in a later experiment, the general operating principles can be summarised as follows:

1. Light entering the spectrometer passes through a diffraction grating which causes incident light to be diffracted by an angle dependent on its wavelength—the longer the wavelength of the incident light, the greater the angle of diffraction. This is illustrated in figure 1.2.
2. The light leaving the diffraction grating is then incident on a CCD array. The same device is used in cameras and measures the intensity of light at each point on the array. Light of longer wavelengths which is diffracted through large angles is incident on the edges of the CCD, causing the CCD to measure a high light intensity at the edges. Conversely, light of shorter wavelengths, which has diffracted through smaller angles, is incident on the CCD closer to its centre, causing the CCD to measure greater light intensity at the centre of the CCD.
3. The intensity of light at different positions on the CCD can therefore be used to determine the wavelength of the incident light.

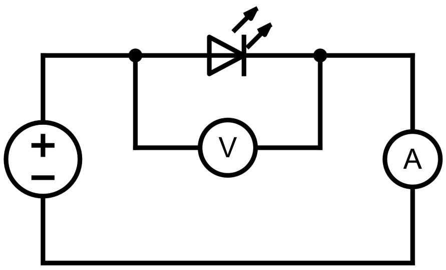

Figure 1.3: A diagram of the simple LED circuit used in this experiment.

1.2 Experimental Procedure

1. Building the LED circuit
2. Measuring and plotting the current-voltage characteristics of each LED to determine V_T
3. Measuring the wavelengths of the LEDs using a digital spectrometer
4. Plotting V_T vs $1/\lambda$ to determine h

While this is the order that this lab manual is written in, you can actually measure the LED wavelengths at any point. There is only a limited number of spectrometers, so it is recommended that every pair measures the wavelengths at a different stage of the experiment to avoid queues.

The Circuit

1. Using the breadboard, set up the circuit shown in figure 1.3 (refer to the equipment guides on Blackboard for further information). While this circuit is simple and you may think there is no need for a breadboard, it is important to get into the habit of using one. When building more complicated circuits, the breadboard is a good way of keeping the wiring neat and organised, and keeping track of what is connected to what. Don't forget to colour code your wiring (e.g. red for positive terminal, black for ground, blue for voltmeter, etc.)
2. LEDs are diodes and only allow current to flow through in one direction. The anode (the longer leg*) should be connected to the positive terminal of the power source. It is a good idea to label your LEDs, since their colour is not obvious when not connected to power source.
3. Before supplying power to the board, set the upper limit for the current to 30 mA (0.03 A). It is important to limit the current going through the LED to prevent it from burning. Once the current limit is set and you have supplied power to the board, you can start adjusting the voltage; the power source will not allow the voltage to exceed the value corresponding to 30 mA. You can read the instructions on how to use the power supply in the equipment guides.

*Another way to identify the anode/cathode is to find the flat edge on the LED's outer casing. The leg nearest this flat edge will be the cathode.

Determining the $I - V$ curve for the LEDs

To find the threshold voltage of each diode you should measure the current in the diode as a function of the applied voltage.

When you analyse your data, the threshold voltage will be found be extrapolating the linear region of the $I - V$ curve to the x-axis (see figure 1.1), so make sure you have enough data in the linear region to perform a good fit.

You should first take a preliminary set of data: identify the voltage at which you can see the diode start to emit light, and then check at voltage the current reaches the maximum 30 mA limit. Once you know the range, you can decide how many data points to take for each diode and the voltage increment you will need. Keep in mind that you need to map the $I - V$ curve for six diodes, so you don't want to spend too long taking the data for each diode.

It is good practice to plot your data as you go so that you can spot (and investigate) any unexpected behaviour. You do not need to determine the threshold voltage during the lab session–leave this till your data analysis session.

Determining the emission wavelength of the LEDs

The USB spectrometer can be used to quickly determine the wavelength, λ, of the LEDs. Using the spectrometer with the software "Thorlabs OSA":

1. Connect your diode into the breadboard provided and aim the optic fibre at it. Make sure not to touch the end of the fibre with your fingers!
2. Select "Repeat" from the "Sweep" menu on the program. This will continually display the spectrum of the light detected by the spectrometer.
3. Set the maximum and minimum limits of the intensity axis using the "Level" menu (What is the maximum intensity the spectrometer can detect?).
4. Aim the optic fibre at the LED to observe the peak wavelength. You should observe an approximately Gaussian spectral distribution. You are aiming for a smooth curve. If 'integration time' is set too short your signal can be very noisy, increasing it can significantly improve your signal. Check with your demonstrator when measuring the first diode to make sure you're getting the correct result.
5. To freeze the spectrum trace, select "Stop" in the "Sweep" menu. You can now perform a simple analysis of the spectrum without aiming the optic fibre at the LED.
6. To analyse spectrum, select "Peak Track" from the "Analysis" menu. This will display properties of the peak in your spectrum, such as the peak wavelength and the FWHM (Full Width at Half Maximum). The FWHM is the width of the peak at half its maximum intensity and can be used to estimate of the uncertainty associated with the LEDs emission wavelength.
7. Make a note of the peak wavelength and FWHM of your spectrum.
8. You may want to save your trace for further analysis or plotting later. Make sure you save your data somewhere you can access later (e.g. in OneDrive).

At the end of the lab you should have measured the IV curves for up to 6 diodes, as well as a measurement of the emission wavelength for each.

1.3 Data Analysis

Finding the Threshold Voltage

As you saw in the introduction, the threshold voltage V_T can be determined by extrapolating the linear region of the IV curve.

1. Plot the IV curve for each of your LEDs. Make sure you include error bars on your plot. You can do this in Excel or in Python using `matplotlib.pyplot.errorbar`, or even by hand.
2. Fit a straight line to the approximately linear region of the graph. Consider how you select which points to include in your linear fit.
   - If you are using Excel then you can calculate the linear fit and uncertainty using the LINEST function.
   - If you are analysing your data using Python, you can use the `scipy.polyfit` function to perform the fit.
   - If you are plotting by hand, use the min/max gradient method (i.e. the steepest and shallowest lines that go through your error bars and the mean (x,y) coordinate of your data.
3. Estimate the threshold voltage, V_T, below which the LED stops conducting.
4. How precise is this estimate? What affects the uncertainty? Make an estimate of the uncertainty associated with this value.
5. Determine V_T and the associated uncertainty for each LED. Make sure you add the graphs you have produced to your lab notebook.

Calculating Planck's Constant

1. Plot V_T as a function of $1/\lambda$. You can plot this using Python or Excel or by hand.
2. you should include error bars in your plot and add your plot into your Lab Note Book. The uncertainty on the emission wavelength is related to the width of the spectrum. We usually quote uncertainties in terms of the standard deviation, not the FWHM. To convert from the FWHM of a gaussian distribution to its standard deviation σ you can use the following formula:
$$\mathrm{FWHM} = 2\sqrt{2\ln 2}\sigma \approx 2.355\sigma. \tag{1.10}$$
   If you want to know more about where the formula comes from, you can find the full derivation and more information at `mathworld.wolfram.com/GaussianFunction.html`.
3. Find the best fit line for the data and find the gradient.
4. Determine the uncertainty associated with the value of the gradient.
5. Using equation 1.9 calculate the Planck's constant. What is the associated uncertainty? Comment on your result: Does the defined value of h lie within your error bars? If not can you explain why?

Appendix: Physical Principles

Semiconductor Band Theory

This will be covered in far more detail in your solid state physics course and in advanced electronics, but in short:
In the atoms of a gas, the electrons orbit the nucleus in a set of energy levels. As you know from atomic theory, photons can promote an electron into a higher energy level; and can be emitted by an electron falling from a higher energy level, into a lower level—providing there is space for it (i.e. it is quantum mechanically allowed). In a solid, the electron energy levels from adjacent atoms merge to form energy bands with gaps in between. Typically we consider two bands—the valence band, and the conduction band. In metals some electrons are in the conduction band and are free to move through the metal crystal, enabling electrical conduction. In insulators electrons are limited to the valence band.

In a semiconductor the gap between the valence and conduction bands E_g is relatively small and electrons can be promoted from the valence band to the conduction band (leaving holes in the valence band). The semiconductor material itself can be doped (i.e. have small amounts of other materials added to it) to produce regions of positive (p) material, with an excess of holes, and negative (n) material with an excess of electrons. Doping one part of material p and the adjacent part n, forms a PN junction. This is the heart of an electrical diode: current flows easily from the p-side, or anode, to the n-side, or cathode, but not in the reverse direction. Charge-carriers (electrons and holes) flow into the junction from electrodes with different voltages. This band structure is shown in Figure 1.4.

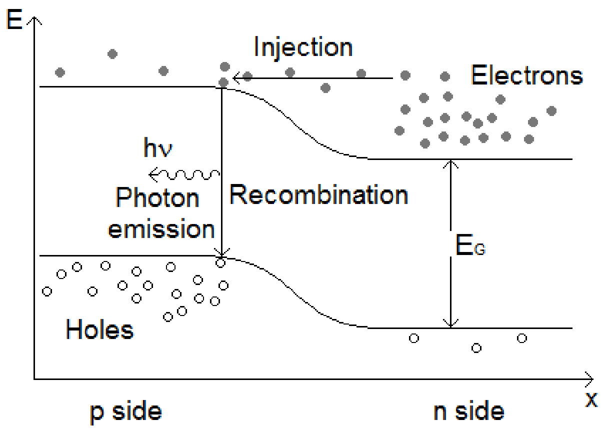

Figure 1.4: A graph detailing the basic band structure of an LED. Here the upper solid line on the graph represents the lowest energy of the conduction band and the lower solid line represents the highest energy of the valence band. Source: `https://www2.warwick.ac.uk/fac/sci/physics/current/postgraduate/regs/mpags/ex5/devices/led/`

When an electron meets a hole, it falls into a lower energy level. This can be through a non-radiative transition—but in a Light Emitting Diode (LED) the materials are chosen so that the transition from electron to hole emits a photon, with an energy equivalent to the band gap.

LEDs are very efficient at producing light, far more than a filament lamp where most energy is released as heat / infrared energy.

Behaviour of a Real Diode

While the current-voltage behaviour will follow a linear trend for voltages significantly higher than the threshold voltage, the relationship is actually exponential, described by

$$I(V) = AT^{\frac{5}{2}} exp\left(\frac{eV - E_g}{\eta k_B T}\right) \tag{1.11}$$

where A is a constant, T the absolute temperature, e the electron charge, V the applied voltage, E_g the energy band gap of the LED, k_B the Boltzmann constant, and η is a device-dependent parameter, usually in the range between 1 and 2.

As is obvious from the equation, there will be a small current flowing through the diode even for voltages close to zero, way below what we would consider the threshold voltage. This is because the electrons in LED have thermal energy at room temperature. In other words, even without any acceleration from applied voltage, a tiny fraction of electrons could have enough energy to cross the junction in the LED.

You could possibly choose to fit the I-V curve to the equation above, and identify E_g from the fit as the threshold voltage. But this is not the whole story. The LED will be heated as the current flows, i.e., T is a function of V.

For the purposes of this experiment, finding a straight best fit line for the linear region of the graph is sufficient to give you a good approximation for the value of the threshold voltage. You should, however, be aware that the real relationship is not that simple.

Original Author: This appendix was written by J. Lim