Year 1 Laboratory Manual
Measuring Capacitance

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1. Measuring capacitance using an oscilloscope

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1.1 Introduction

A capacitor is simply a pair of conductors that stores charge. The simplest configuration for a capacitor consists of two metal plates separated by a thin insulating layer.

The aim of this experiment is to measure the capacitance of a capacitor using a the oscilloscope.

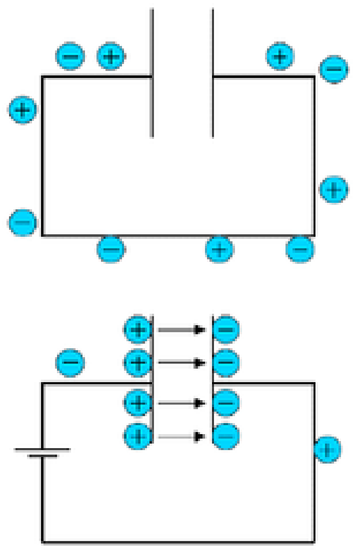

Figure 1.1: Charge distributions on an uncharged and charged capacitor.

Fig. 1.1 shows a capacitor with its plates connected by a wire (we call this "short-circuited") ensuring that the potential of both plates is the same. In this configuration free (electrons) and fixed (static positively charged metal atoms) charges are uniformly distributed throughout the circuit. If we add a battery supplying a voltage V, electrons are pulled from the positive terminal and pushed to the negative terminal. The charge distribution in the circuit now looks like the lower part of the diagram with an excess positive charge on one plate and excess negative charge on the other. This separation of charge across the insulating layer sets up an electric field in a direction opposite to that imposed by the battery trying to drive current around the circuit. When the two are equal there can be no further flow of electrons.

However the separation of charge this results in represents stored electrical energy. If we remove the battery and replace it with a short circuit, electrons will flow around the circuit again (albeit

Original Author: This experiment is based on previous experiments from Imperial's first year lab and has been updated by Laura Warwick in the opposite direction). This method of storing electrical energy is used in many applications, for example in a camera flash.

The quantity of charge delivered to the capacitor plates is given by $Q = CV$ where C is the value of the capacitor measured in *farads* (F). The farad is a very large unit, so typically a capacitor would have values of pF or nF.

The ability of capacitors to hold charge was discovered in the 18th century when early pioneers in electromagnetism inadvertently gave themselves electric shocks by touching a conductor that had been previously charged. In this experiment we will develop a quantitative and painless method for measuring capacitance.

If a charged capacitor is suddenly connected to a resistor, current will flow from one plate to another through resistor R. Apparatus for performing this is shown in Fig. 1.2.

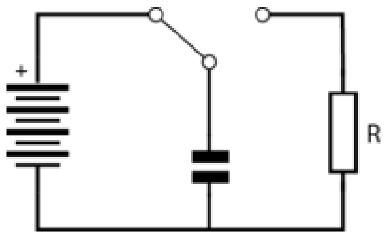

Figure 1.2: Circuit schematic for an apparatus which charges and discharges a capacitor.

The current that flows through resistor R will depend upon the voltage across the capacitor V. Recalling Ohm's law (V=IR) and the definition of capacitance above, we obtain:

$$I = \frac{Q}{RC}. \tag{1.1}$$

Recalling that current is the flow of charge per unit time we obtain the differential equation

$$\frac{dQ}{dt} = -\frac{Q}{RC}, \tag{1.2}$$

which if the capacitor has an initial charge Q_0, has the solution

$$Q = Q_0 \exp\left(\frac{-t}{RC}\right). \tag{1.3}$$

Therefore, we can expect an exponential discharge from the capacitor with a time constant of $1/RC$. Since $Q = CV$ if we can measure voltage against time accurately during the discharge, we can measure the capacitance. In this experiments you will compare two methods of measuring capacitance for two capacitors with difference values of capacitance.

1.2 Measuring the capacitance of a large capacitor using discharge decay

In the first experiment we will measure the capacitance of a electrolytic capacitor, commonly used in electronic circuits. To achieve this, we will use the set-up shown in Fig. 1.2. Electrolytic capacitors can be manufactured with relatively large values of capacitance, however they are polarised so must be connected the correct way round in a circuit.

1. Using the electronic breadboard, construct the circuit shown in Fig. 1.2 using the 1 mF capacitor provided, and a resistor of approximately 10 kΩ. Use the bench-top power supply to charge your capacitor.
2. Use a piece of wire to construct a three-way switch so you can charge the capacitor from the power supply then discharge it through the resistor.
3. Setup the scope to measure the voltage over the capacitor
4. Using your three-way switch, charge and discharge the capacitor whilst observing the response of the scope. Set up the time- and voltage-base of the oscilloscope to fit a complete decay on the oscilloscope screen.
5. Practice pausing the oscilloscope display until you have measured a full discharge.
6. Using the cursor measurements function record the voltage as a function of time at various points in the decay. In the data analysis you will try to fit an exponential decay to this data so make sure you record sufficient points.
7. If you have access to a USB stick, you can also save the complete voltage trace as a '.CSV' file for subsequent analysis.

1.3 Measuring the capacitance of a small capacitor using discharge decay

In this experiment, you will replicate the method above using a smaller ceramic capacitor and a 100 kΩ resistor.

Rather than physically switching the circuit between a voltage source and resistor, as in the previous experiment, in this experiment you will connect the capacitor and resistor in series and use the signal generator built into the scope to perform this task. By selecting a square wave which oscillates between 0 and 2 V the capacitor will charge when the voltage signal is high, and discharge through the resistor when the voltage signal drops to zero. (Consider: why didn't you use this method to measure the capacitance of the larger capacitor?)

1. Construct the circuit shown in Fig. 1.3b using the ceramic capacitor and 100 kΩ resistor. Take care to ensure that the oscilloscope is connected with the correct polarity. The negative terminals of the signal generator and oscilloscope are grounded, so must be connected with a common ground, as shown.
2. Set the signal generator function of the oscilloscope to produce a 2 V peak-peak square wave (i.e. an amplitude of 1 V). Set the offset so that the waveform starts at 0V and rises to +2 V (rather than the default +1V to −1V). Set the frequency to approximately 5 kHz and check the waveform using the oscilloscope. Your waveform should resemble that shown in Fig. 1.3a. (See the guide to the scope on Blackboard for more detailed instructions on the operation of the oscilloscope.)
3. On the oscilloscope you should observe the voltage rise during the charging phase, and decay during the discharge phase of the voltage signal generated by the signal generator.

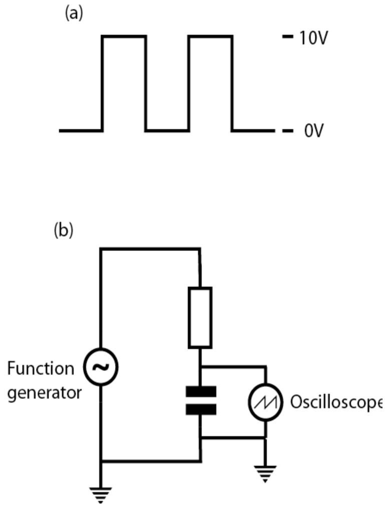

Figure 1.3: (a) Pulse shape for charging and discharging the capacitor. (b) Circuit diagram showing the measurement of voltage across the capacitor.

What effect does the frequency have on the trace that you observe?

4. Adjust the frequency of the signal generator until the capacitor fully charges and discharges with each cycle.
   What voltage does the capacitor charge up to? Is this what you expect based on the voltage set on the signal generator?
5. When you are satisfied with the trace you have measured, stop the scope running and make a record of the trace properties.

C. Measuring the capacitance of a small capacitor using phase shift

When the capacitance value of the capacitor under test becomes very small, we need to change our method of measuring capacitance. This is because other components in the circuit have their own resistance and capacitance which affects the measurements. In our case the scope behaves as a 1 MΩ resistor and 20 pC capacitor in parallel in the circuit. This is shown in Fig. 1.4. For large capacitor values, this is not a problem, since the capacitance being measured is significantly larger than that of the scope. However, this becomes a problem when the capacitor under test has a value that is comparable to that of the oscilloscope.

To accurately measure the capacitance of the capacitor in such a case, you must use an alternative method which accounts for the scope capacitance. We can achieve this by driving the capacitor using a sine wave, rather than a square wave.

The total capacitance of this circuit is given by

$$C_{\text{Total}} = \frac{1}{2\pi\nu X_C}, \tag{1.4}$$

where ν is the driving frequency of the signal generator and X_C is the total reactance of the capacitors in circuit. In AC electronics, reactance behaves similarly to resistance, but rather than opposing a current, reactance opposes changes in current, which can lead to attenuation and phase differences between input and output signals as you have observed already. The equation for reactance is in turn given by

$$X_C = \frac{V_x}{I_g \cos\phi}, \tag{1.5}$$

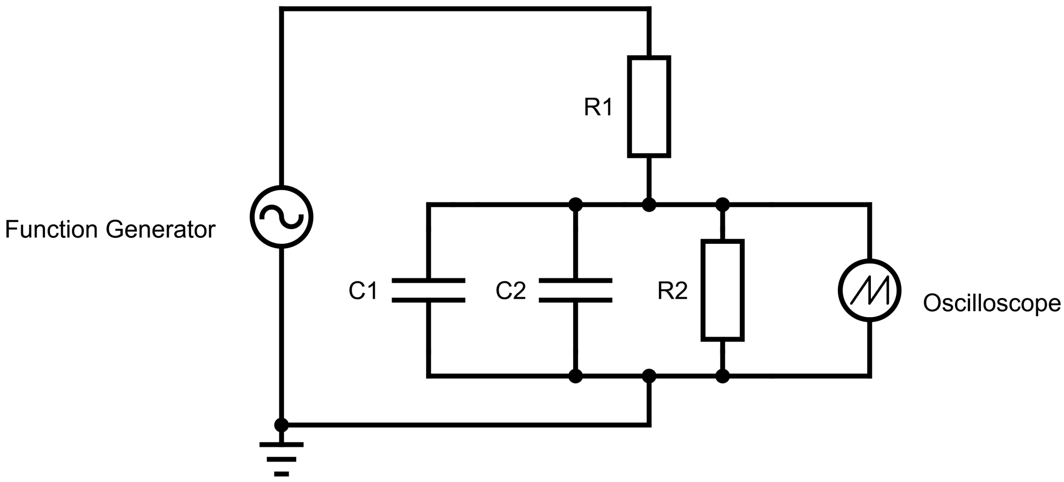

Figure 1.4: Circuit diagram for the measurement of a voltage across the capacitor including the effect of the oscilloscope. In this circuit, R1 and C1 are the resistor and capacitor you are using in the experiment. C2 and R2 represent the capacitance and resistance introduced by the oscilloscope.

where I_g is the current supplied by the signal generator, ϕ is the 'Loss angle' and V_x is the voltage amplitude across the capacitor. The loss angle, ϕ is given by the expression

$$\phi = \cos^{-1}\left(\frac{V_g \sin\alpha}{V_{R1}}\right), \tag{1.6}$$

where V_g is the voltage amplitude of the signal generator signal, α is the phase difference between the signal generator and capacitor signal and V_{R1} is the voltage drop across resistor R1. The current supplied by the signal generator,I_g is given by

$$I_g = \frac{V_{R1}}{R_1}, \tag{1.7}$$

where R_1 is the resistance of resistor R1. Finally, V_{R1} is given by

$$V_{R1} = \sqrt{(V_g \cos\alpha - V_x)^2 + (V_g \sin\alpha)^2}. \tag{1.8}$$

By measuring V_x, V_g and α, you can use the formulae above to calculate the *total* capacitance of the circuit. You should then be able to calculate the capacitance of the capacitor under investigation.

1. The circuit used for these measurements is the same as in the previous experiment however, the 100 kΩ resistor should be replaced with a 6.8 kΩ one.
2. Set the signal generator to drive the circuit with a 50 kHz sine wave at 2 V peak-to-peak.
3. Ensure you can see both the signal generated by the scope and the voltage across the capacitor on your screen. You should find that the amplitude of the capacitor signal is lower than the signal generator and the two signals are phase shifted relative to each other.
4. Take measurements of V_x, V_g and α. Consider how you will determine the uncertainty on each of these values.
5. If you have access to a USB stick, you can also save the complete voltage trace as a '.CSV' file for subsequent analysis.

1.4 Data Analysis

Measuring the capacitance of a large capacitor using discharge decay:

Using the measurements of voltage against time that you took last week, you will use Eqn. 1.3 to calculate capacitance.

1. Rearrange Eqn. 1.3 into a linear equation relating V and t with a gradient that depends on the value of C.
2. Import your data from Experiment 1.2 into Python - this could either be the values you recorded using the cursor measurements on the scope or from the data file(s) you saved as '.CSV' files.
3. Produce a plot of voltage against time and check the data looks as you expect.
4. Determine the value of V_0 from your plot. This can be at any point you chose. (Consider, where is the most appropriate point to take as V_0 and why?)
5. Plot your data and calculate the straight line fit of the data.
6. Calculate the capacitance of the capacitor and the associated uncertainty.
7. Comment on the result in your lab book. Do you think this method is appropriate for the capacitor you are testing?

Measuring the capacitance of a small capacitor using discharge decay"

The procedure for analysing the data in experiment 1.3 is essentially the same as that in experiment 1.2. If you are working with a '.CSV' file you will need to ensure that you correctly split your data into the charging/discharging cycles.

1. Calculate the capacitance of the capacitor and the associated uncertainty.
2. Comment on the result in your lab book. Do you think this method is appropriate for the capacitor you are testing?

Measuring the capacitance of a small capacitor using phase shift

1. Using the equations shown in the experimental procedure, calculate the capacitance of the small capacitor using the phase shift method.
2. Comment on the precision and accuracy of the phase shift method compared to discharge decay.