Module 1

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1 Fundamentals of Returns

Let P_t be the price of the asset.
The return on this asset is given by

$$R_{t,t+1} = \frac{P_{t+1} - P_t}{P_t}$$

The word "return" in this finance context, you should come up with this word.

2 Measures of Risk and Reward

The variance is given by

$$\sigma_R^2 = \frac{1}{N}(R_i - \bar{R})^2$$

where $\bar{R}$ is the arithmetic mean of the return.
And the standard diviation is computed by

$$\sigma_R = \sqrt{\frac{1}{N}(R_i - \bar{R})^2}$$

SInce you cannot compare daily data from monthly data, you need to use

$$\sigma_{ann} = \sigma_p \sqrt{p}$$

where p is the number of periods.
For example, to standaralize a daily data with a annual data,

$$\sigma_{ann} = \sigma_{daily} \sqrt{365}$$

To see the excess return, we see the **Sharp ratio**

$$\frac{R_p - R_f}{\sigma_p}$$

where R_p is a return of the portfolio and σ_p is a stdev of the portfolio

3 Measuring Max Drawdown

Instead of the risk measure, we use **Max drawdown**

This is the value of the maximum loss from the previous high to a subsequent low.

In order to compute the drawdowns, we will take these steps:

1. Construct a wealth index
2. Look at the prior peak at any point in time The drawdown is literally the difference between the prior peak and the current value
3. We can plot the drawdown by seeing how long does it tak to recover from the drawdowns

The value heavily depends on the frequwncy of observations. In general, the drawdown of daily series data is greater than the one of monthly series data.

4 Deviations from Normality

We assume that a asset return is **Normally** distributed.

It is called "The Gaussian Assumption"

But somtimes they larger changes which contradict to this assumption.

The skewness is given by

$$\frac{\mathbb{E}\left[(R-\mathbb{E}[R])^3\right]}{[Var(R)]^{\frac{3}{2}}}$$

It is basically the measure of a symmetry. If the data is inclined in the right side, the skewness will be positive and vice versa.

The kurtosis s given by

$$\frac{\mathbb{E}\left[(R-\mathbb{E}[R])^4\right]}{[Var(R)]^2}$$

It is a measure of a possibility of extreme events.

For the Gaussian distribution, it takes 0 for skewness and 3 for kurtosis.

But in a reality, asset returns are not normally distributed.

5 Downside risk measure

Deviation is not always a bad signal. Only downside deviation is the risk of the asset.

To capture this downside risk, we have a **Semi-deviation**.

This is the sub-sample of below-average or below-zero returns which is given by

$$\sigma_{semi} = \sqrt{\frac{1}{N}\Sigma_{R_t \leq \bar{R}}(R_t - \bar{R})^2}$$

where N is the number of returns that fall below the mean.
Additionlayy, there is a concept which is called as **Value at Risk(VaR)**.
To think of it, we have to set the things below

1. A specified confidence level (Percentage)
2. A specified holding period

The VaR is the absolute value of worst return in the specified period. Suppose the confidence level is n. We need to exclude tha last (1-n)% from the returns for calculation.
Also, there is **Conditional VaR** which is given by

$$\text{CVaR} = -\mathbb{E}[R|R \leq -VaR] = \frac{-\int_{-\infty}^{-VaR} x f_R(x) dx}{F_R(-VaR)}$$

This means the expected loss beyond VaR. Pay attention to that this CVaR only returns positive number.

6 Estimating VaR

There are mainly 4 typrs of calulating VaR.

1. Historical(non-parametric)
2. Variance-Covariance(Parametric Gaussian)
3. Parametric non Gaussian
4. Cornish-Fisher (Semi-parametric)

For the first one, we use the histrical data. The pro is that they don't assume, however, the accuracy heavily depends on the sample periods.
For the second one, by assuming the Gaussian for the returns, and estimate the parameters. One of the method is Gaussian VaR. We think of Z_α as the α-quantile of the standard normal distribution. For example, $Z_\alpha = -1.65$at 5%
We just need to think of a Z_α which satisfy

$$\int_{-\infty}^{Z_\alpha} \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-x^2}{2\sigma^2} dx = \alpha$$

$$\therefore \mathbb{P}\left(\frac{R-\mu_P}{\sigma_P} \leq Z_\alpha\right) = \alpha$$

$$\therefore \mathbb{P}\left(R \leq \mu_P + \sigma_\alpha Z_\alpha\right) = \alpha$$

and we can get:

$$VaR_\alpha =^(\mu + Z_\alpha \sigma)$$

For the third one, there are a lot of methodology for this.

When you take a parametiric way, you will have a model-risk. So that we take a Cornish-Fisher VaR, which is a part of semi-parametric approach. According to the Cornish-Fisher expansion,

$$\tilde{Z}_\alpha = Z_\alpha + \frac{1}{6}(Z_\alpha^2 - 1)S + \frac{1}{24}(Z_\alpha^3 - 3Z_\alpha)(K-3) - \frac{1}{36}(2Z_\alpha^3 - 5Z_\alpha)S^2$$

where $\tilde{Z_\alpha}$ have a α-quantile of non-Gaussian distribution, S is a skewness and K is a kurtosis. In many cases, the form of

$$Var_{mod}(1-\alpha) = -(\mu + \tilde{Z_\alpha}\sigma)$$

is taken.