Beyond of diversification/ Module 3

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1 Limits of diversification

The benefit of the diversification is to eliminate the idiosyncratic risks. However, there is a limit of diversification.
Under the situation like the financial crisis, every asset in the market goes down and the effect of the diversification is likely to decline. Improving the portfolio doesn't work well.
The hedging is the only effective against the downside risk, which means that they cannnot treat the upside risk.
However, in terms of the insurance, we need to consider the upside risk as well as the downside risk. Insurance is a kind of "Dynamic hedging", which means that we need to adjust the allocation of the assets accoding to the market situation.

2 An introduction to CPPI

The CPPI procedure allows for the construction of convex payoffs.
This procedure dynamically allocates total assets to a risky asset and a safe asset.
Let Floor(F) be the protection floor. This floor is the minimun of value that you can accept the loss.
The Cushion(C) is given by

$$\mathrm{CPPI} - F$$

and the potion of investment for the risky asset is given by

$$\mathrm{Multiplier(M)} \times C$$

For example, $whenM = 3$ and the floor is 80%, the initial investment in the risky asset is

$$3 \times (\$100 - \$80) = \$60$$

However, if there is a gap between the trdable dates and the huge decline in the price happens

within the period, it is impossible to adjust the portfolio. This is called as **Gap risk**. In other words, the loss of the risky asset relative to the safe asset exceeds $\frac{1}{M}$,
AT the same time, let's introduce the maximum drawdown constraint.
The max drawdown constraint is

$$V_t > \alpha M_t$$

where V_t is the value of the portfolio at time t, M_t is the maximum value of the portfolio between time 0 and t and α is the maximum acceptable drawdown.
The maximum drawdown flow is a continuously increasing function.
The next extension is a perfomance CAP.
Let F_t be the Floor, T_t be the threshold, C_t be the CAP and A_t be the value of the asset at the time t.
Here, we should have a strategy below

$$E_t = \begin{cases} m(A_t - F_t) & F_t \leq A_t \leq T_t \\ m(C_t - A_t) & T_t \leq A_t \leq C_t \end{cases}$$

where E_t is the equity. You just need to adjust your investment depending on the A_t.
The threshold T_t is given by

$$T_t = \frac{F_t + C_t}{2}$$

which is called as **Sommoth-pasting** condition.

3 Simulating asset returns with random walks

The return can be modelled as

$$\begin{cases} \frac{dS_t}{S_t} = (r + \sigma\lambda)dt + \sigma dW_t \\ \frac{dB_t}{B_t} = rdt \iff B_t = B_0 e^{rt} \end{cases}$$

where S_t is the price of the stock, B_t is the value of cash you hold, r is the risk-free rate, σ is the volatility and λ is the sharpe ratio.
And the mean μ is given by

$$\mu = r + \sigma\lambda$$

and the W_t is the **Brownian Motion**.
This Brownian Motion is a random walk in continuous time. This one is introduced by Louis Bachelier.
In other words, the return process S_t satisfies

$$\frac{S_{t+dt} - S_t}{S_t} = (r + \sigma\lambda)dt + \sigma\sqrt{dt}\xi_t$$

where $\xi_t \sim \mathcal{N}(0, 1)$

4 Monte Carlo Simulation

In general, every coefficient will change as time passes.By adding time r to each coefficient,

$$\frac{dS_t}{S_t} = \left(r_t + \sqrt{V_t}\lambda_t^S\right) dt + \sqrt{V_t} dW_t^S$$

For example, model can be like this:

$$dr_t = a(b - r_t)d_t + \sigma_r dW_t^r$$

$$dV_t = \alpha(\bar{V}_t - V_t)dt + \sigma_V \sqrt{V_t} dW_t^V$$