Introduction to Asset-Lianility Management

2026/1/1

1 From Asset Management to Asset-Liability Management

There was a PENSION FUND CRISIS. The S&P 500 DB Pension Plans fell enormouls between Dec 1999 and May 2003. At the Dex 1999, it had a net surplus of $ 239 billion, and at the end of the period the surplus turned into a net deficit of $ 252 billion. It was in the end of Tech bubble. NASDAQ index also declined grately in that period.
The simmilar thing happend again in 2008.
In asset-liability management, the asset value against the liability value matters. In other words,

$$F_t = \frac{A_t}{L_t}$$

And surplus is given by a difference between assets and liability values:

$$S_t = A_t - L_t$$

2 Liability hedging portfolios

The investers concern mainly about an unexpected increase in the present value of thier liabilities.
A possible solution is called Liability-Hedging portfolios(LHR) or Goal-Hedging portfolio. There should be a certain cashflow that the liability will take, so that we need to try to make the similar cashflow by adjusting our portfolio.
With respect to the goal, a standard vond is not safe because it only makes a temporary huge cashflow in one period and the payoff of the bond change dramatically depending on the other factors. The portfolio should be like a retirement bond.
In reality, convinient bond which precisely fit the cashflow unlikely to exist so that we use **Duration Matching**. Since duration is the figure of sensitivity against inflation or interest rate change

3 Liability-driven investing (LDI)

In order to have a better performance, the one have to expose hisself in to the "Risk". The hedging is the method to control the risk appropriately to prepare for an unexpected shocks. Performance-seeking portfolio (PSP) and Liability Hedging Portfolio (LHR) are the one of the strategies which try to tame risk.
The optimal allocation strategy is giben by

$$\max_{w} \mathbb{E}\left[u\left(\frac{A_T}{L_T}\right)\right]$$

$$\therefore w^* = \frac{\lambda_{\text{PSP}}}{\gamma\sigma_{\text{PSP}}} w^{\text{PSP}} + \beta_{\text{L, LHP}}(1 - \frac{1}{\gamma}) w^{\text{LHP}}$$

where A_T is an asset value, L_T is a liability value and u is an utility function
In the LDI, they also have **Greeks**

1. λ_{PSP} is an sharpe ratio
2. β_{LHP} is the beta, sensitivity, of liability
3. σ_{PSP} is a volatility of the PSP strategy
4. γ is a risk-aversion