

$$\begin{pmatrix} u_o \\ u'_o \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{R}\tan\beta_2 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & R\sin\theta \\ -\frac{1}{R}\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{R}\tan\beta_1 & 1 \end{pmatrix} \begin{pmatrix} u_i \\ u'_i \end{pmatrix} \quad (1)$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{1}{R}\tan\beta_2 & 1 \end{pmatrix} \begin{pmatrix} u \\ u' \end{pmatrix} \quad (2)$$

$$\begin{pmatrix} u \\ u' \end{pmatrix} = \begin{pmatrix} \cos\theta & R\sin\theta \\ -\frac{1}{R}\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{R}\tan\beta_1 & 1 \end{pmatrix} \begin{pmatrix} u_i \\ u'_i \end{pmatrix} \quad (3)$$

$$\begin{pmatrix} u \\ u' \end{pmatrix} = \begin{pmatrix} \cos\theta & R\sin\theta \\ -\frac{1}{R}\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{R}\tan\beta_1 & 1 \end{pmatrix} \begin{pmatrix} u_i \\ u'_i \end{pmatrix} + \begin{pmatrix} \Delta u \\ \Delta u' \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} u_o \\ u'_o \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{R}\tan\beta_2 & 1 \end{pmatrix} \begin{pmatrix} u \\ u' \end{pmatrix} + \begin{pmatrix} \Delta u \\ \Delta u' \end{pmatrix} \quad (5)$$

$$\begin{pmatrix} z_o \\ z'_o \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-1}{R}\tan\beta_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-1}{R}\tan\beta_1 & 1 \end{pmatrix} \begin{pmatrix} z_i \\ z'_i \end{pmatrix} \quad (6)$$

$$\begin{pmatrix} z \\ z' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-1}{R}\tan\beta_1 & 1 \end{pmatrix} \begin{pmatrix} z_i \\ z'_i \end{pmatrix} \quad (7)$$

$$u = u(s) \quad (8)$$

$$z = z(s) \quad (9)$$

$$u' = \frac{du}{ds} \quad (10)$$

$$z' = \frac{dz}{ds} \quad (11)$$

$$\begin{pmatrix} \Delta u \\ \Delta u' \end{pmatrix} = \begin{pmatrix} \sqrt{2a^2 \cos^2 \theta + 2a \cos \theta \sqrt{R^2 - 2aR + a^2 \cos^2 \theta} + R^2 - 2aR} - R \\ \frac{-4a^2 \cos \theta \sin \theta - 2a \sin \theta \sqrt{R^2 - 2aR + a^2 \cos^2 \theta} - \frac{2a^3 \cos^2 \theta \sin \theta}{\sqrt{R^2 - 2aR + a^2 \cos^2 \theta}}}{2R \sqrt{2a^2 \cos^2 \theta + 2a \cos \theta \sqrt{R^2 - 2aR + a^2 \cos^2 \theta} + R^2 - 2aR}} \end{pmatrix} \quad (12)$$

$$\Delta u = \sqrt{2a^2 \cos^2 \theta + 2a \cos \theta \sqrt{R^2 - 2aR + a^2 \cos^2 \theta} + R^2 - 2aR} - R \quad (13)$$

$$\Delta u' = \frac{-4a^2 \cos \theta \sin \theta - 2a \sin \theta \sqrt{R^2 - 2aR + a^2 \cos^2 \theta} - \frac{2a^3 \cos^2 \theta \sin \theta}{\sqrt{R^2 - 2aR + a^2 \cos^2 \theta}}}{2R \sqrt{2a^2 \cos^2 \theta + 2a \cos \theta \sqrt{R^2 - 2aR + a^2 \cos^2 \theta} + R^2 - 2aR}} \quad (14)$$

$$a = R \sqrt{\frac{E - \Delta E}{E}} \quad (15)$$