Exponential Distribution Simulation

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Overview

In this report I investigate the exponential distribution in R and compare it with the central limit theorem.

Simulation

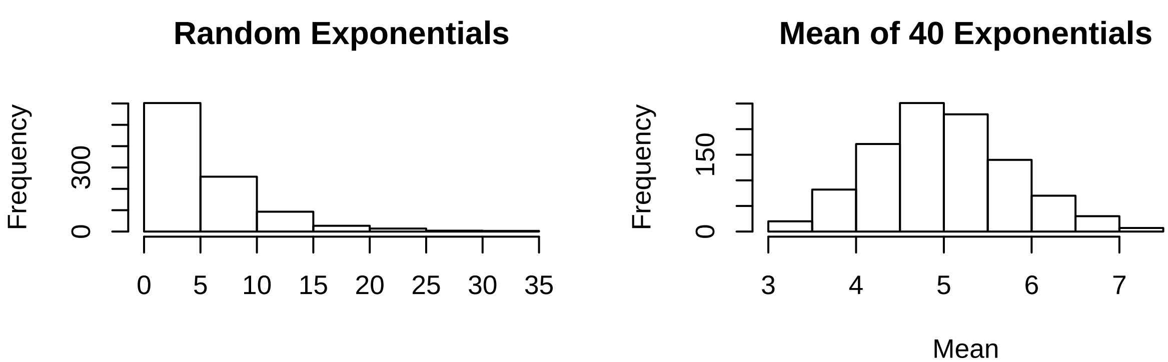
Theoretically it is known that the mean of an exponential distribution is $1/\lambda$ and the standard deviation is $1/\lambda$. For the purposes of my analysis I set $\lambda = 0.2$. I also set the seed for reproducibility. I gather values taken from the exponential distribution a thousand times and call it `randomexp`. Then I determine the distribution of averages of 40 exponentials by performing a thousand simulations and call it `mns`.

```
lambda = 0.2
set.seed(1)
randomexp = rexp(1000, lambda)
mns = NULL
for (i in 1 : 1000) mns = c(mns, mean(rexp(40, lambda)))
```

Distribution

The distribution of one thousand random exponentials and the average of 40 exponentials for a thousand simulations are visually shown in the histogram below to the left and right respectively.

```
par(mfrow=c(2,2))
hist(randomexp, xlab = "", main = "Random Exponentials")
hist(mns, xlab = "Mean", main = "Mean of 40 Exponentials")
```



Note that the distribution of a large collection of random exponentials (left) just reproduces the original exponential distribution. But the mean of 40 exponentials (right) looks more like a Gaussian distribution, which we can tell from its bell-shaped curve and symmetry about the mean.

Simulated exponential distribution

First examining the random exponential distribution `randomexp` (left histogram), 5.1565134 is the mean of the exponential distribution given by `mean(randomexp)`, and 24.4658326 is the variance of the exponential distribution given by `var(randomexp)`. Both of these values are in good agreement with the expected behavior of an exponential distribution with $\lambda = 0.2$ ($1/\lambda = 5$ and $1/\lambda^2 = 25$ respectively).

Comparison of sample and theoretical mean and variance

Next, by examining the distribution of the average of 40 exponentials `mns` (right histogram), the mean is theoretically expected to be $1/\lambda = \mathbf{5}$, which is in good agreement with the simulation mean of **4.988882** given by `mean(mns)`. The variance of the mean distribution is theoretically expected to be $1/\lambda^2/40 = \mathbf{0.625}$ for the average of 40 exponentials, so this is also in good agreement with the simulation variance of **0.6119066** given by `var(mns)`.

Conclusion

The mean of the exponential distribution can be well-described by a Gaussian distribution centered about $1/\lambda$ with variance σ^2/N where $\sigma = 1/\lambda$, in good agreement with the central limit theorem.