Pricing and Optimization in Shared Vehicle Systems

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Before Proceeding

Context

Sharing Economy

- Sharing under-utilized assets
- More resources. Promising in terms of general equilibrium
- Prices should equate supply and demand
- Specifically, ride apps

Therefore

Problem

- Inefficiency (limited supply and demand heterogenity)
- Why do we need a better algorithm? (Uber)
- Externalities (this paper)

Model as

- Infinite-horizon control problem, Closed queuing network models, Markov chains, Geographic effects of supply and demand. Prices as the decision variable. Non-concave problem that require use approximation.

Contribution

- Approximation framework that works well
- State-independent prices are optimal

Alternative Models

1. Non-static arrival dates (Garg and Hamid, 2022)
2. Pickup frictions (what is the best way to geographically match drivers?) (Özkan and Amy, 2022)
3. Impatience (Aveklouris et al., 2024)

Setup

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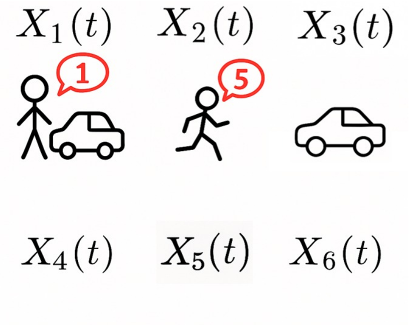
Approximation Framework

Conclusion

- m units, n nodes with people appearing with ϕ_{ij} Poisson rate, $F_{ij}(\cdot)$ Value distribution, p_{ij} price, $q_{ij} = 1 - F_{ij}(q_{ij})$ inverse demand function

- m units (cars), n nodes (locations), ϕ_{ij} Poisson rate (average rate of events), $F_{ij}(\cdot)$ value distribution (how much does this ride value to me?), $p_{ij} = F_{ij}^{-1}(1 - q_{ij})$ price (decision variable), $q_{ij} = 1 - F_{ij}(q_{ij})$ inverse demand function (how many accept)
- $\mathbf{X}(t)$ Vector of units at each node i, instantaneous transition $\mathbf{X} \to X - e_i + e_j$, state space $S_{n,m} = \{(x_1, \dots, x_n) \in \mathbb{N}_0^n | \sum_i x_i = m\}$, Steady-state distribution $\pi(\mathbf{x}) \geq 0 \ \forall \mathbf{x} \in S_{n,m}$


$X_1(t)$
$X_2(t)$
$X_3(t)$
1
5
$X_4(t)$
$X_5(t)$
$X_6(t)$

Max what?

Max what?

- Throughput: total rate of rides in the system $I_{ij}^{T}(p_{ij}) = 1$
- Social Welfare $I_{ij}^{W}(p_{ij}) = \mathbb{E}_{V\sim F_{ij}}[V|V \geq p_{ij}]$
- Revenue: $I_{ij}^{r}(p_{ij}) = p_{ij}$

$$R_{ij}(q_{ij}) := q_{ij}I_{ij}(q_{ij})$$

Define $I_{ij}(q_{ij}) \equiv I_{ij}(F_{ij}^{-1}(1 - q_{ij}))$ and reward curves $R_{ij}(q_{ij}) := q_{ij}I_{ij}(q_{ij})$, assume concave. Steady-state rate of reward accumulation

$$O_{BJ_m}(\mathbf{q}) = \sum_{\mathbf{x}\in S_{n,m}} \pi(\mathbf{x}) \left(\sum_{i,j} \phi_{ij} q_{ij}(\mathbf{x}) I_{ij}(q_{ij}(\mathbf{x})) \right)$$

State-independent policies: point-to-point prices $\{p_{ij}\}$ (do not react to the state of system) and $X_i(t) > 0$ constant and independent of state of network
Gordon-Newell network: continuous time Markov chain on states $\mathbf{x} \in S_{n,m}$; for any state $\mathbf{x}$, $\forall i, j \in [n]$, the chain transitions from $\mathbf{x}$ to $\mathbf{x} - e_i + e_j$ at rate $P_{ij}\mu_i \mathbf{1}_{x_i(t)>0}$, $\mu_i > 0$ service rate at node i and $P \geq 0$ routing probabilities such that $\sum_j P_{ij} = 1$

- State-independent prices $\rightarrow$ stationary distribution of states $\rightarrow$ steady-state availability $A_{i,m} \equiv \sum_{\mathbf{x} \in S_{n,m}} \pi(\mathbf{x}) \mathbf{1}_{x_i > 0}$

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Difficulties

Diff. 1 State-dependent pricing (exponential number)

Diff. 2 Nonconvex problems. Work on convex relaxation

Steady-state availability of units at node i

$$A_{i,m} \equiv \sum_{\mathbf{x} \in S_{n,m}} \pi(\mathbf{x}) \mathbf{1}_{x_i > 0}$$

Focus on **state independent pricing policy**.

$$O_{BJ_m} = \sum_{i,j} A_{i,m}(\mathbf{q})\phi_{ij}q_{ij}I_{ij}(q_{ij})$$

$f_{ij,m}(\mathbf{q}) = A_{i,m}(\mathbf{q})\phi_{ij}q_{ij}$ = steady-state rate flows.
Theoretical quantiles $q_{ij} = 1 - F_{ij}(q_{ij})$ and effective quantiles $\hat{q}_{ij} = \frac{f_{ij,m}(\mathbf{q})}{\phi_{ij}} = A_{i,m}(\mathbf{q})q_{ij}$
Elevated objective function (easier to deal with)

$$\hat{O_{BJ}}(\mathbf{q}) = \sum_{i,j} \phi_{ij}q_{ij}I_{ij}(q_{ij}) = \sum_{i,j} \phi_{ij}R_{ij}(q_{ij})$$

Proposition (Demand Bounding)

Under any state-dependent policy, the steady-state rate of flows obeys the demand bounding property $A_{i,m}(\mathbf{q})\phi_{ij}q_{ij} \leq \phi_{ij}$

Proposition (Supply Circulation)

Under any state-dependent policy and finite m-unit system,

$$\sum_k f_{ki,m}(\mathbf{q}) = \sum_j f_{ij,m}(\mathbf{q}) \forall i$$

Steady-state flow getting in and out are the same.

Algorithm 1: Computing State-Independent Prices (Easy)

Inputs: Arrival rates ϕ_{ij}, value distributions F_{ij}, reward curves R_{ij}

1. Solve the following optimization problem to obtain $\{\tilde{q}_{ij}\}$:

$$\max_{\{q_{ij}\}} \sum_{(i,j)} \phi_{ij} R_{ij}(q_{ij})$$

subject to:

$$\sum_{k} \phi_{ki} q_{ki} = \sum_{j} \phi_{ij} q_{ij} \ \forall i \quad \text{(Balanced demand)}$$

$$q_{ij} \in [0,1] \quad \forall (i,j)$$

2. Compute state-independent prices:

$$\tilde{p}_{ij} = F_{ij}^{-1}(1 - \tilde{q}_{ij})$$

and output the corresponding quantiles $\tilde{q}_{ij}$.

Approximation Guarantee

Elevated flow relaxation upper bounds the true objective of any state-dependent pricing policy.

Lemma

For any state-independent pricing policy $\mathbf{q}$ *balanced demand, then the value of the elevated objective function is equal to the value of objective function in the infinite-unit system*

$$O_{BJ_\infty} = \hat{O}_{BJ}(\mathbf{q})$$

Lemma

State-independent pricing policy $\mathbf{q}$, *the maximum availability in the m-unit system is at least* $\frac{m}{m+n-1}$*:*

$$A_m(\mathbf{q}) \geq \frac{m}{m+n-1}$$

Theorem

Any objective function O_{BJ_m}*. Let* $\tilde{\mathbf{q}}$ *be the pricing policy returned by Algorithm 1 and* O_{PT_m} *be the value of the objective function for the optimal state-dependent pricing policy in the m-unit system. Then*

$$O_{BJ_m}(\tilde{\mathbf{q}}) \geq \frac{m}{m+n-1} O_{PT_m}$$

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Extensions

Showing that Elevated flow relaxation works in a plethora of scenarios

Theorem (Approx guarantee for supply redirection)

Any O_{BJ_m} with concave reward curves R_{ij}. A3 gives $\tilde{\mathbf{q}}, \tilde{\mathbf{r}}$, and O_{PT_m} value of the objective of the optimal

$$O_{BJ_m}(\tilde{\mathbf{q}}, \tilde{\mathbf{r}}) \geq \frac{m}{m+n-1} O_{PT_m}$$

1. Travel Times
2. Demand Redirection
3. Supply Redirection

Conclusion

- Closed queuing networks for externalities
- Design feature: enforcing flow conservation yields near-optimal policies.
- Does not capture endogenous driver decisions
- Main contribution: elevated flow relaxation: can be applied to other settings.
- Theoretical framework to deal with hard problem
- Q: what about bounded above prices? That satisfy triangle inequality? (I don't see that as a problem)
- State-independent prices: strong performance!

What I would like to add

1. Are those normative or descriptive?
2. (Uber) uses Reinforcement Learning
3. No closed form of optimal policy and intuition
4. POV: analyzing stochastic system, not maximizing it
5. Driver's Preferences: not clear where they wanna go and risk tolerance: Brasil
6. No competition of the platforms
7. Cumulative list of non-satisfied customers
8. m, n, ϕ_{ij} can be endogenous

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2. Garg, Nikhil, and Hamid Nazerzadeh. "Driver surge pricing." Management Science 68.5 (2022): 3219-3235.
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