Causal Diagrams for empirical research

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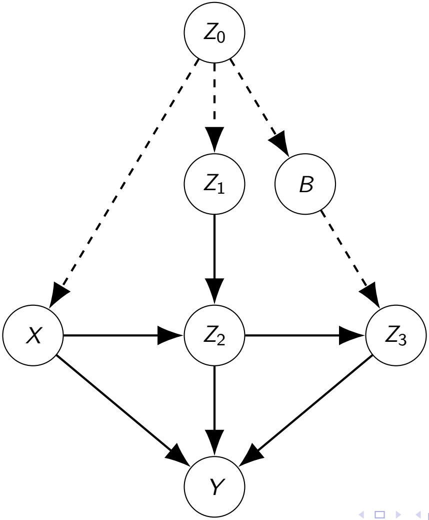
Before Proceeding

The Power of Changing Earth

X : Soil Fumigants
Y : Crop Yields
Z : Eelworm population. Z_0 : last year's (unknown), Z_1 : before treatment, Z_2 : after treatment, Z_3 : end of the season,
B :Population of birds

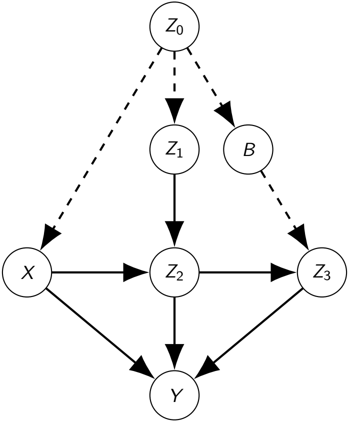
- Unobserved and observed variables

The System



1. Find: total effect of X on Y given the distribution
2. $P(y|do(x)) \neq P(y|x)$

$$X_i = f_i(pa_i, \varepsilon_i)$$



$$Z_0 = f_0(\varepsilon_0), Z_1 = f_1(Z_0, \varepsilon_1), Z_2 = f(X, Z_1, \varepsilon_2), Z_3 = f_3(B, Z_2, \varepsilon_3)$$

$$X = f_X(Z_0, \varepsilon_X), Y = f_Y(X, Z_2, Z_3, \varepsilon_Y), B = f_B(Z_0, \varepsilon_B),$$

Notation

d-separation: X, Y, Z disjoint subsets of nodes. $(X \perp\!\!\!\perp Y|Z)_G \Leftrightarrow Z$ blocks every path from a node in X to a node in Y

Z **blocks** path p if:

1. Chain ($X \rightarrow w \rightarrow Y$, $w \in Z$), Fork ($X \leftarrow w \rightarrow Y, w \in Z$)
2. Collider ($X \rightarrow w \leftarrow Y, w \notin Z$, Descendents of $w \notin Z$)

Identifiability: Causal effect of X on Y if $P(y|do(x))$ can be computed uniquely from any positive distribution of the observed variables that is compatible with G.

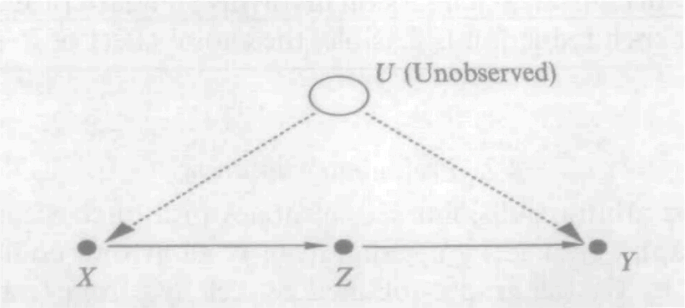


Fig. 3. A diagram representing the front-door criterion.

- Let X, Y be disjoint sets of variables. The **causal effect** is $P(y|do(x)) : X \to \mathcal{P}(Y)$. For each realisation x of X, the function $P(y|do(x))$ gives the probability of $Y = y$ induced on deleting from the model $X_i = f_i(pa_i, \varepsilon_i), \forall i$ all equations corresponding to variables in X and substituting x for X in the remainder

For atomic interventions $X_i = x_i'$

$$P(x_1, \ldots, x_n | do(x_i')) = \begin{cases} \frac{P(x_1, \ldots, x_n)}{P(x_i | pa_i)} \\ 0 \text{ if } x_i \neq x_i' \end{cases}$$

- Back-door criterion: Z relative to variables (X_i, X_j) if
  1. No node in Z is a descendant of X_i
  2. $\forall$ path p between X_i and X_j with an arrow into X_i, Z blocks p

Theorem (Back-door Identifiability)

If a set of variables Z satisfies the back-door criterion relative to (X, Y), then the causal effect of X on Y is identifiable and is given by

$$P(y|do(x)) = \sum_z P(y|x, z)P(z)$$

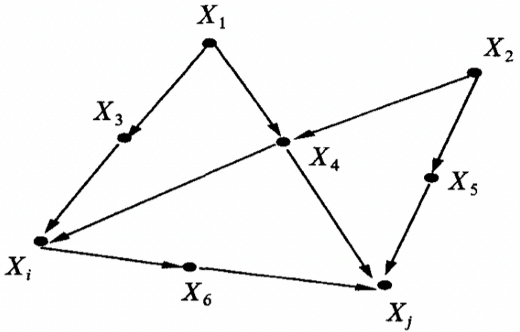


Fig. 2. A diagram representing the back-door criterion; adjusting for variables $\{X_3, X_4\}$ or $\{X_4, X_5\}$ yields a consistent estimate of $\mathrm{pr}(x_j | \check{x}_i)$.

Theorem (Front-Door Criterion)

Z satisfies relative to subsets (X, Y) :

1. *(Interception)* $\forall$ *path p between X and Y,* $\exists w \in Z\ X \rightarrow w \rightarrow Y$
2. *(Absence of Back-door) between X and Z*
3. *(Back-door Block) All back-door paths between Z and Y is blocked by X*

Then, the causal effect of X on Y is ***identifiable***

$$P(y|do(x)) = \sum_{x} P(z|x) \sum_{x'} P(y|x', z)P(x')$$

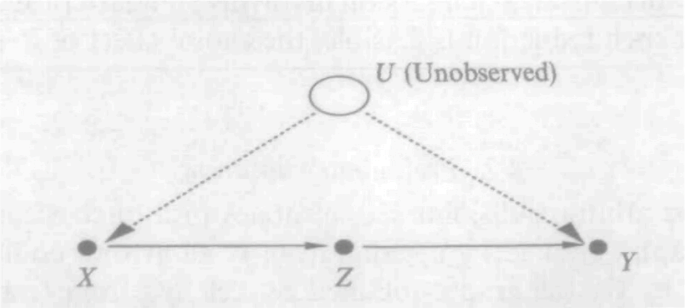


Fig. 3. A diagram representing the front-door criterion.

Figure: Caption

Notation

- $G_{\bar{X}} = G$ with all arrows pointing to nodes in X deleted
- $G_{\underline{X}} = G$ with all arrows emanating from X deleted

Theorem (Manipulation Rules)

Let G DAG and $P(\cdot)$ the induced probability distribution. For any disjoint subsets of variables X, Y, Z and W we have:

Rule 1 (insertion/deletion of observations):

$$P(y|do(x), z, w) = P(y|do(x), w) \quad \text{if } (Y \perp\!\!\!\perp Z|X, W)_{G_{\bar{X}}}.$$

Rule 2 (action/observation exchange):

$$P(y|do(x), do(z), w) = P(y|do(x), z, w) \quad \text{if } (Y \perp\!\!\!\perp Z|X, W)_{G_{\bar{X}\underline{Z}}}.$$

Rule 3 (insertion/deletion of actions):

$$P(y|do(x), do(z), w) = P(y|do(x), w) \quad \text{if } (Y \perp\!\!\!\perp Z|X, W)_{G_{\overline{X}, \overline{Z(W)}}},$$

where $Z(W)$ is the set of Z-nodes that are not ancestors of any W-node in $G_{\overline{X}}$.

Rule 1: d-separation as conditional independence
Rule 2: External intervention set $Z = z$ same as passive observation
Rule 3: Adding/deleting interventions

Corollary (Reduction)

A causal effect $q = P(y_1, \ldots, y_k | do(x_1), \ldots, do(x_m))$ *is identifiable in G if* $\exists$ *finite sequence of transformations (using rules 1,2,3), which reduces q into a standard probability expression involving observed quantities.*

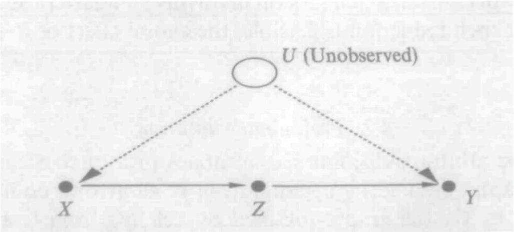


Fig. 3. A diagram representing the front-door criterion.

$$P(y \mid do(x))?$$

1. $P(z \mid do(x))$

$$P(z \mid do(x)) = P(z \mid x)$$

2. $P(y \mid do(z))$

$$P(y \mid do(z)) = \sum_x P(y \mid x, do(z)) P(x \mid do(z))$$

$$P(x \mid do(z)) = P(x)$$

$$P(y \mid do(z)) = \sum_x P(y \mid x, z) P(x)$$

3. $P(y \mid do(x))$

$$P(y \mid do(x)) = \sum_z P(y \mid z, do(x)) P(z \mid do(x))$$

$$P(y \mid z, do(x)) = P(y \mid do(z), do(x)) = P(y \mid do(z))$$

$$P(y \mid do(x)) = \sum_z P(z \mid x) \sum_{x'} P(y \mid x', z) P(x')$$

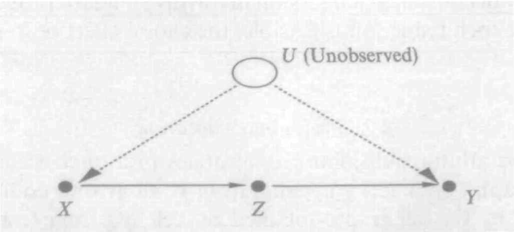


Fig. 3. A diagram representing the front-door criterion.

Surrogate Experiments

$$P(y|do(x))?$$

X : Cholesterol Y : Heart Disease Z : Subject's diet (surrogate variable)

Transform the problem to something that has $do(z)$ only.

- No direct effect of diet on heart conditions
- No confounding between cholesterol (X) and heart condition (Y) (unless by observable variable)

Limitation of Nonparametric

Confounding arc between X and Y: $U \to X, U \to Y, X \to Y$

$$Y = bX + \gamma U + \varepsilon_1$$

$$X = \alpha Y + \delta U + \varepsilon_2$$

$$b := \frac{\partial E(Y|do(x))}{\partial x} \text{ not identifiable}$$

$$\text{If } \exists U \to X, \text{ then } b = \frac{E(Y|u)}{E(X|u)}$$

In nonparametric models, adding more variables never help identification

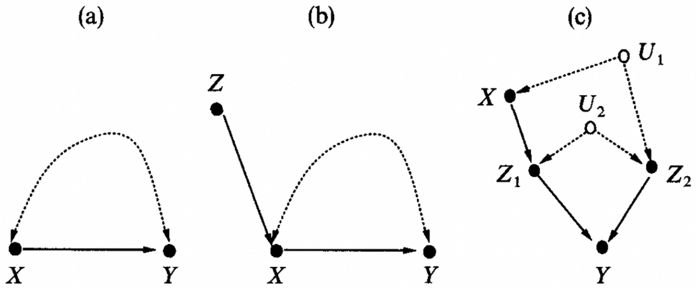


Fig. 5. (a) A bow-pattern: a confounding arc embracing a causal link $X \rightarrow Y$, thus preventing the identification of $\mathrm{pr}(y \mid \check{x})$ even in the presence of an instrumental variable Z, as in (b). (c) A bow-less graph still prohibiting the identification of $\mathrm{pr}(y \mid \check{x})$.

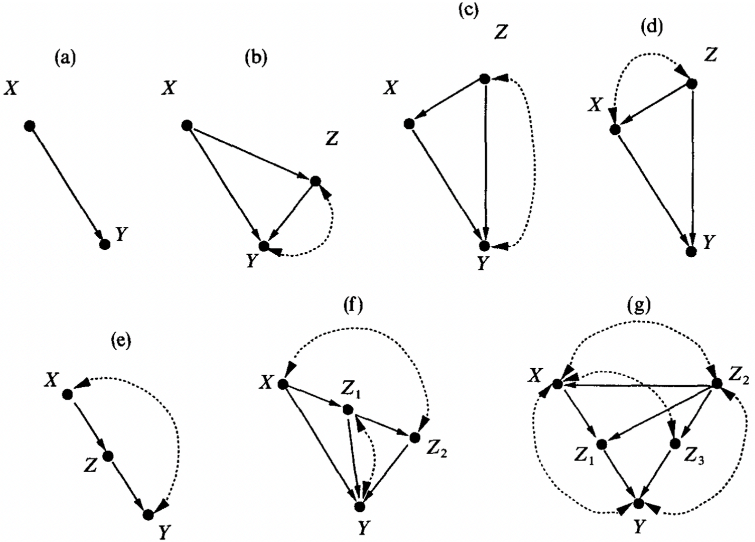


Fig. 6. Typical models in which the effect of X on Y is identifiable. Dashed arcs

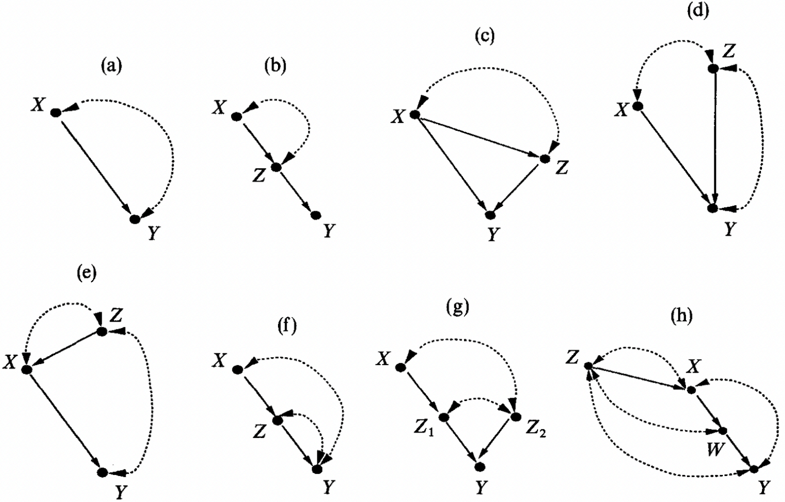


Fig. 7. Typical models in which $\mathrm{pr}(y|\check{x})$ is not identifiable.

Conclusion

- New language to describe intervention
- Take-home: Intuitive use of DAGs and reduction of effects of intervention to normal probabilities
- Limitation: based on causal assumptions in the proposed DAG which cannot be tested, generality of nonparametric model

Inherently Subjective

- Causality influenced by language (Framing effects)
- Must be conceived in the mind first
- Subjective Causality I: Rareness of "if" make us see the "then", not only the order of things (Kolmogorov complexity) (Alexander and Gilboa, 2023)
- Subjective Causality II: can elicit subjective G and $f_i(\cdot), P(\cdot), u(\cdot)$ if preferences follow certain axioms (Halpern and Piermonte, 2024)
- Subjective everything
- No notion of model ambiguity
- Meta: Lucas' critique, sociological stance, prescription vs prediction
- Behavioral Causal Inference (Spiegler, 2023)

Lore

- Judea Pearl: AGI
- Sample size of 1 + conjectural theory vs problem of probability of historic events (Gilboa, 2009)
- Bayesian is part of statistics, comes from the joint, cannot come up with questions outside of statistics
- Fact-free learning: procedure to find causal (because it is given), AI cannot do it
- $P(drink|\text{alcoholic parents})$

References I

1. Alexander, Yotam, and Itzhak Gilboa. "Subjective causality." Revue économique 74.4 (2023): 619-633.
2. Halpern, Joseph Y., and Evan Piermont. "Subjective Causality." arXiv preprint arXiv:2401.10937 (2024).
3. Spiegler, Ran. "Behavioral causal inference." arXiv preprint arXiv:2305.18916 (2023).
4. Gilboa, Itzhak. Theory of decision under uncertainty. No. 45. Cambridge university press, 2009.

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