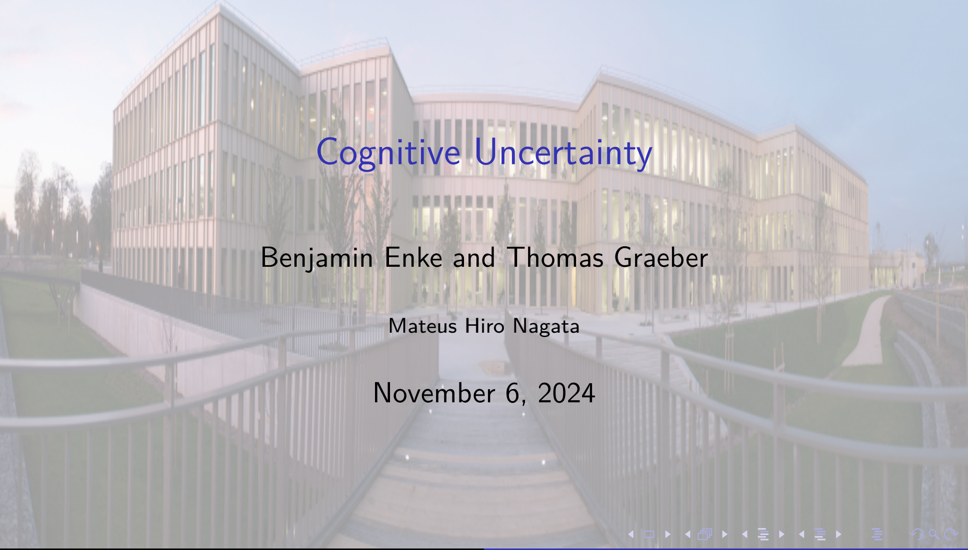

Cognitive Uncertainty

Benjamin Enke and Thomas Graeber

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November 6, 2024

In One Frame

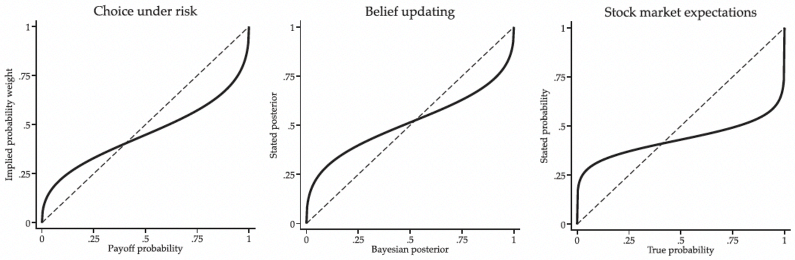


FIGURE I

Decisions as Functions of Objective Probabilities

Cognitive Uncertainty (CU)

Cognitive Uncertainty: subjective uncertainty over decision.

1. Direct theoretical interpretation of awareness of noise
2. Can be tweaked to different domains
3. Captures imperfect decision making and its awareness
4. Quick, simple, costless to elicit
5. CU strongly correlated with decision variability in repetitions of the same decision problem, which is a key choice signature of cognitive noise.
6. Boundedly rational origin of deviations from optimal behavior

Setup

Utility $u(\cdot)$ given by decision a that depends on the objective probability p.

1. **Risky Choice**: CE of lottery $(y, p; 0)$. Normalize $u(y) = 1$. EU max action $a^* = u^{-1}(p)$.
2. **Belief Formation**: state $\in \{R, B\}$, prior $b = P(R)$, signal $\in \{H, L\}$, discriminative power $h = P(H|R) = P(L|B)$, posterior $p \equiv P(R|H) = P(B|L) = \frac{bh}{bh+(1-b)(1-h)}$. Action $a^* = p$.
3. **Economic Forecast**: some macroeconomic variable X. What is the perceived $p = P(X \leq x)$? Action $a^* = p$.

Does the DM

1. Maximize expected utility?
2. Update according to Bayes' rule?
3. Forecast well macroeconomic variables?

Bayesian Cognitive Noise

Assumption: imprecise estimation of $a^*(p)$.

- Cognitive signal S with precision N and $E[S] = a^*(p)$
- Bayesian DM's decision a^0 given by posterior mean

$$a^0 = E[a^*|S = s] = \lambda(N)s + [1 - \lambda(N)]d$$

$$E[a^0] = \lambda(N)a^*(p) + [1 - \lambda(N)]d$$

- $d =$ cognitive default decision
- Anchoring-and-adjustment heuristic
- Neo-additive weighting function

In One Frame

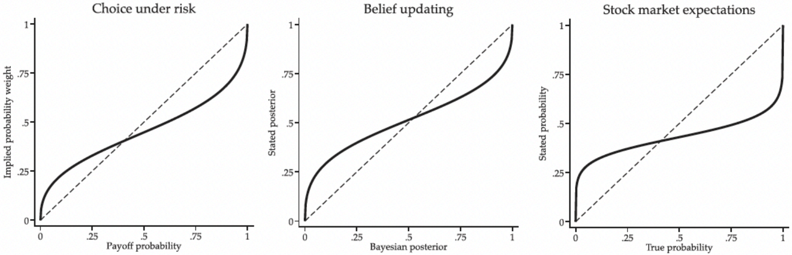


FIGURE I

Decisions as Functions of Objective Probabilities

Empirical Implementation

Cognitive noise is unobservable but the subjective uncertainty about the optimal decision, which we denote cognitive uncertainty (CU), is. Let κ be of arbitrary length

$$p_{CU} = P(|[a^*|S = s] - a^0| > \kappa)$$

- $a^*|S = s$ perceived posterior distribution about maximizing decision, conditional on having received cognitive signal s
- CU as a proxy for the magnitude of cognitive noise λ.

Experimental Design

Choice Under Risk: This task elicits the certainty equivalents (CE) for six binary prospects of the form $(y, p; 0)$, where $y \in \{15, 16, \ldots, 25\}$ and $p \in \{0.01, 0.05, 0.1, 0.25, 0.35, 0.50, 0.65, 0.75, 0.90, 0.95, 0.99\}$.

- **Risk A**: BDM, **Risk B**: Choice lists, with similar results.
- Q: $CE = u^{-1}(pu(y))$? Or $CE_{norm} = \frac{CE}{y}$?

Belief Updating

- Bags A, B contain 100 balls. Each ball is either red or blue.
- Base rate $b =$ chance of selecting bag A $= P(A)$.
- Discriminative power $d = P(red|A) = P(blue|B) \in \{65, 75, 90\}$.
- Computer selects $M \in \{1, 3, 5\}$ balls with replacement.
- Q: $P(A|balls)$?

Economic Forecasts

The S&P 500 is an American stock market index that includes 500 of the largest companies based in the US. Jon invested $100 in the S&P 500 today. *What is the percent chance* that the value of his investment will be less than y in one year from now?

- Random draw of $y \in \{62, 77, 90, 100, 112, 123, 127, 131, 134\}$
- **Experiment A**: probabilistic forecasts of S&P 500 performance.
- **Experiment B**: inflation rates / GDP
- No financial incentives
- Q: $P(\text{S\&P500} \leq y)$?

Measuring Cognitive Uncertainty

Choice Under Risk:
Your decision on the previous screen indicated that you value this lottery as much as receiving \$$x$ with certainty. How certain are you that you actually value this lottery somewhere between getting \$$(x-0.50)$ and \$$(x+0.50)$?

- Q: $P(a^* \in [x-0.5, x+0.5])$?

Belief Updating:

- *Optimal guess* is explained: law of probability to compute statistically correct statement of the probability that either bag was drawn, based on Bayes' rule.
- This optimal guess based only on what the subject knows. Then asks again:

Your decision on the previous screen indicates that you believe there is a $x\%$ change that Bag A was selected. How certain are you that the optimal guess is somewhere between $(x-1)\%$ and $(x+1)\%$?

- Q: $P(a^* \in [x-1\%, x+1\%])$?

Economic Forecasts

On the previous screen, that you think there is an x% chance that a \$100 investment into the S&P500 today will be worth less than \$y in one year. How certain are you that the statistically optimal guess is somewhere between $(x-1)\%, (x+1)\%$?

- Q: given a y, let $x = P_{subjective}(S\&P500 \leq y)$. $P(a^* \in [x-1\%, x+1\%])$?
- Instead of awareness of ignorance, measures perceived ability to remember and employ information.
- Skeptical: one does not need to know the exact return in a year, but probability of profit, the problem is complex, that knowledge is not necessarily useful, difference from past and future returns.

Potential origins of CU

- **Decision under Risk**: imperfect perception, inability to deal with utils and probabilities, ignorance of true preferences.
- **Belief Updating**: ignorance of update rule or difficulty of implementing it.
- **Survey Expectations**: imperfect retrieval of knowledge, or incorrect update rule.
- Ad hoc self conscious measure

Incentives

- In general, **Choice under Risk** and **Belief Updating** were incentivized by performance
- No financial incentive in the CU elicitation: 1) Lessen cognitive burden 2) Costs for future research on CU measures
- Validate the measure by non-trivial correlations (especially across-trial same problems)

Complexity Manipulations

1. **Complex Numbers**: base rates, as mathematical expressions "Get \$ 20 with probability $\left(\frac{7\times6}{2} - 11\right)$ %. In Risk A and Beliefs A, 6 baseline and 6 complex number tasks
2. **Compound Problems**: if baseline gives p% chance of getting \$20, then the compound equivalent is \$20 with probability $p' \sim U\{p - 0.05, \ldots, p + 0.05\}$. In belief update: baseline $h, b = 50\%$ then the corresponding compound problem has $h' \sim U\{h - 0.1, \ldots, h + 0.1\}$. The Bayesian posterior is identical. In Risk B and Beliefs B, 6 baseline and 6 compound problem tasks

- Caveat: it may increase the uncertainty on the setup and not measure correctly the DM's ability

Task 7 of 12

Decision Screen (1/2)

50 out of 100 balls are **red**. If one is drawn: Get **$25**.

5 out of 100 balls are **blue**. If one is drawn: Get **$0**.

5 out of 100 balls are **green**. If one is drawn: Get **$0**.

5 out of 100 balls are **orange**. If one is drawn: Get **$0**.

5 out of 100 balls are **brown**. If one is drawn: Get **$0**.

5 out of 100 balls are pink. If one is drawn: Get **$0**.

5 out of 100 balls are **black**. If one is drawn: Get **$0**.

5 out of 100 balls are **gold**. If one is drawn: Get **$0**.

5 out of 100 balls are **gray**. If one is drawn: Get **$0**.

10 out of 100 balls are **purple**. If one is drawn: Get **$0**.

Which certain payment is worth as much to you as this lottery?

Getting $ ____ with certainty is worth as much to me as this lottery.

Next

Variation

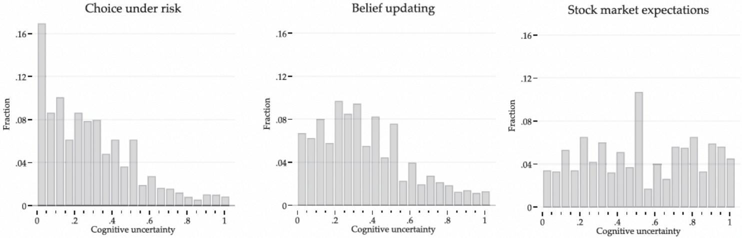


FIGURE II

Histograms of Cognitive Uncertainty in the Baseline Tasks in *Risk A* ($N = 4{,}524$), *Beliefs A* ($N = 4{,}590$), and Stock Market Expectations ($N = 1{,}000$).

Stability

- Majority of decisions include positive and heterogeneous CU.

Unincentivized CU picks real variation or noise?

- Participant fixed effects: 51%-54%. Lottery choice and belief updating (where multiple decisions per subject). Consistent in a given domain.
- **Within-subject test-retest correlation**: highly correlated $r = 0.7$ in Risk and $r = 0.68$ in Beliefs
- **Cross-domain stability**. ρ(CU in choice, CU in stock market expectations) $= 0.19$, ρ(CU in beliefs, CU in stock markets) $= 0.35$ with $p < 0.01$.

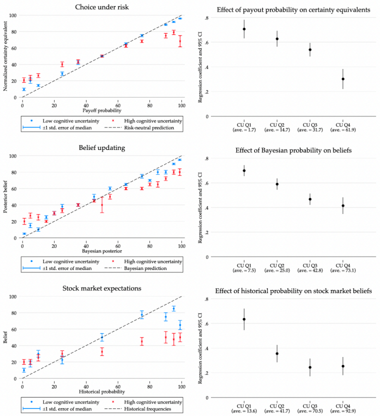

Choice under risk
Normalized certainty equivalent
Payoff probability
Low cognitive uncertainty
High cognitive uncertainty
±1 std. error of median
Risk-neutral prediction
Effect of payout probability on certainty equivalents
Regression coefficient and 95% CI
CU Q1 (ave. = 1.7)
CU Q2 (ave. = 14.7)
CU Q3 (ave. = 31.7)
CU Q4 (ave. = 61.9)
Belief updating
Posterior belief
Bayesian posterior
Bayesian prediction
Effect of Bayesian probability on beliefs
CU Q1 (ave. = 7.5)
CU Q2 (ave. = 25.0)
CU Q3 (ave. = 42.8)
CU Q4 (ave. = 73.1)
Stock market expectations
Belief
Historical probability
Historical frequencies
Effect of historical probability on stock market beliefs
CU Q1 (ave. = 13.6)
CU Q2 (ave. = 41.7)
CU Q3 (ave. = 70.5)
CU Q4 (ave. = 92.9)

FIGURE III

- $\uparrow$ CU, $\uparrow$ compression
- Bounded rationality $\Rightarrow$ Probability-dependent risk preferences
- "Extreme belief aversion" due to cognitive noise and not to preferences
- Market expectations: CU predicts overestimation of small and underestimate big probabilities

Regression Evidence

$$CE_{norm} = \alpha + \beta_1 p + \beta_2 p \times p_{cu} + \beta_3 p_{cu} + \sum \beta_k control_k$$

- Predictions: 1) interaction coefficient β_2 is negative (likelihood insensitivity), 2) cognitive uncertainty term is positive (possibility effect)
- CU strongly related to the regression intercept (as predicted). Does not mean higher average probability weighting.
- Certainty effect, possibility effect and the fourfold pattern of risk attitudes explained by the model.

TABLE III

COGNITIVE UNCERTAINTY AND LIKELIHOOD INSENSITIVITY IN *Risk A*

	Dependent variable: Normalized certainty equivalent					
	Full sample		$p < 50\%$		$p \geqslant 50\%$	
	(1)	(2)	(3)	(4)	(5)	(6)
Payout probability	0.73***	0.73***	0.56***	0.55***	0.60***	0.59***
	(0.03)	(0.03)	(0.06)	(0.05)	(0.04)	(0.04)
Payout probability × Cognitive uncertainty	– 0.67***	– 0.67***				
	(0.08)	(0.08)				
Cognitive uncertainty	25.1***	22.7***	15.0***	11.0**	– 26.3***	– 27.0***
	(6.18)	(6.02)	(5.64)	(5.48)	(3.51)	(3.60)
Constant	19.7***	31.5***	22.3***	39.4***	30.7***	38.1***
	(2.35)	(4.38)	(2.35)	(6.09)	(3.23)	(4.66)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	4,524	4,524	2,035	2,035	2,489	2,489
R^2	0.49	0.50	0.10	0.15	0.32	0.32

Notes. OLS estimates; robust standard errors (in parentheses) are clustered at the subject level. Demographic controls include age, gender, college education, and performance on a Raven matrices test. * $p < .10$, ** $p < .05$, *** $p < .01$.

- **Belief Updating**: $\uparrow$ CU $\downarrow$ Sensitivity of beliefs to variation in objective probabilities and $\uparrow$ intercept. Neo-additive.
- **Grether Regressions**: if Bayesian

$$\underbrace{ln\left(\frac{p}{1-p}\right)}_{\text{log posterior odds}} = \underbrace{ln\left(\frac{b}{1-b}\right)}_{\text{log prior odds}} + \underbrace{ln\left(\frac{h}{1-h}\right)}_{\text{log likelihood ratio}}$$

- Estimate following model and find that CU leads to base rate neglect, likelihood insensitivity and extreme belief aversion.

$$\text{log post} = \alpha + \beta_1 \text{ log prior} + \beta_2\, p_{cu} + \beta_3 \text{ log prior} \times p_{cu} + \beta_4 \text{ log like} + \beta_5 \text{ log like} \times p_{cu} + \sum_k \beta_k \, \text{controls}_k$$

TABLE IV

COGNITIVE UNCERTAINTY AND BELIEF UPDATING IN *Beliefs A*

	Dependent variable:					
	Posterior belief		ln [Posterior odds]			
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.71***	0.71***				
	(0.02)	(0.02)				
Bayesian posterior × Cognitive uncertainty	−0.43***	−0.43***				
	(0.06)	(0.06)				
Cognitive uncertainty	12.8***	12.7***	−0.47***	−0.49***	−0.48***	−0.49***
	(3.54)	(3.53)	(0.14)	(0.14)	(0.13)	(0.14)
ln [Bayesian odds]			0.55***	0.55***		
			(0.02)	(0.02)		
ln [Bayesian odds] × Cognitive uncertainty			−0.42***	−0.42***		
			(0.07)	(0.07)		
log[Prior odds]					0.69***	0.69***
					(0.03)	(0.03)
log[Likelihood ratio]					0.37***	0.37***
					(0.03)	(0.03)
ln [Prior odds] × Cognitive uncertainty					−0.52***	−0.52***
					(0.10)	(0.10)
ln [Likelihood ratio] × Cognitive uncertainty					−0.21***	−0.21***
					(0.07)	(0.07)
Constant	19.5***	18.7***	0.23***	0.29**	0.24***	0.28**
	(1.53)	(2.22)	(0.06)	(0.12)	(0.06)	(0.11)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	4,602	4,602	4,602	4,602	4,602	4,602
R^2	0.49	0.49	0.45	0.45	0.48	0.48

Notes. OLS estimates; robust standard errors (in parentheses) are clustered at the subject level. To avoid a mechanical loss of observations resulting from the log odds definition, the log posterior odds in columns (3)–(6) are computed by replacing stated posterior beliefs of 100% and 0% by 99% and 1%, respectively. The results are virtually identical without this replacement. Demographic controls include age, gender, college education, and performance on a Raven matrices test. * $p < .10$, ** $p < .05$, *** $p < .01$.

Sample Size Effects

- If Bayesian, posterior is same for 1 draw of a blue and 2 of blue and 1 red.
- However, subjects often update based on sample proportions, and Bayesian on sample differences.
- Alternative: CU increases in sample size. Easier to form beliefs based on one blue ball than based on two blue balls and one red ball. So, use sample difference when sample proportions when sample is smaller and sample difference when it's bigger.

Measurement Error

Measurement error in our framework implies

1. If error correlated (inattentive people may state more CU but also be more insensitive to probabilities) then CU has no predictive power in the positive CU sample.
2. Coefficient attenuation. Solution: instrument out measurement error through repeated elicitations. Feasible since every subject completed at least two decisions twice. CU highly correlated across repetitions of the same decision problem. Results almost identical. Also, CU has strong predictive power. ME is not a major concern.

Magnitude of cognitive noise (N) was given. **Hypothesis**: $\uparrow$ complexity of decision problem $\Rightarrow \uparrow$ noise.

- Complexity (hence CU) *causes* (not just correlates to) the compression effect
- **Complex numbers**: $\uparrow$ CU by 45% in risky choice and 48% in belief updating; **compounding** $\uparrow$ 23% in risky choice and 33% in belief updating.

Causal Link

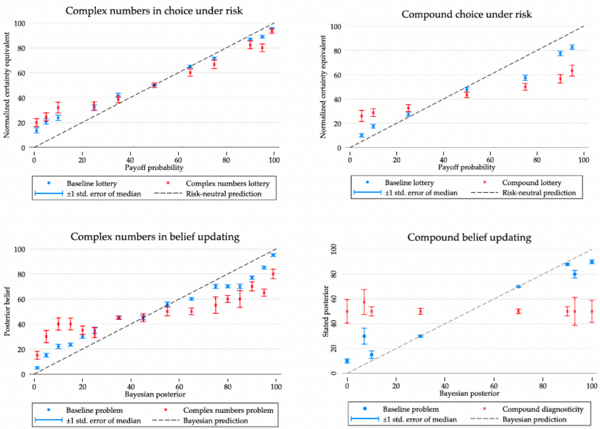
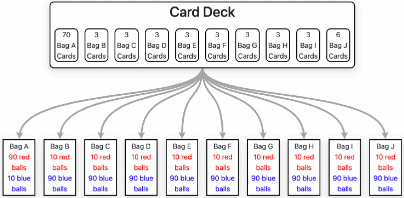


FIGURE V

Task 7 of 12

Decision Screen (1/2)



The computer randomly selected a card from the deck and then randomly drew the following balls from the selected bag:

Decision 1: Your guess

Given that these balls were drawn, **how likely** do you think it is that Bag A (as opposed any of the Bags B, C etc. up to J.) has been selected (in %)?

I believe it is ____ % likely that Bag A was selected.

Submit your guess

Model: DM's decision a^0 given by

$$a^0 = \underbrace{\max\{1 - \gamma p_{CU}, 0\}}_{\lambda} \cdot a^*(p) + \underbrace{\min\{\gamma p_{CU}, 1\}}_{1-\lambda} \cdot d + \epsilon.$$

- λ is a function of the unobserved cognitive noise, which we heuristically approximate by $\lambda = \max\{1 - \gamma p_{CU}; 0\}$.
- γ is a nuisance parameter to be estimated
- For choice, assume CRRA utility $a^*(p) = p^{\frac{1}{\alpha}}$
- Restricted model: set $p_{CU} = 0$
- p_{CU} is observed, γ, d are to be estimated, ϵ is a disturbance term.

- Nonlinear least squares. Leverage individual-level variation in CU but estimate a single average d for the population. Also benchmark on a restricted model: $p_{CU} = 0$.
- Estimated (aggregate) $d = 0.52$ for beliefs and $d = 0.43$ for choice (RDU).
- Significantly lower Akaike Information Criterion (AIC) in every model that includes CU.

TABLE V

ESTIMATES OF CENTRAL TENDENCY EFFECT ACROSS EXPERIMENTS

	Risk A		*Beliefs A*		*Risk B*		*Beliefs B*	
	Restr. (1)	CU (2)	Restr. (3)	CU (4)	Restr. (5)	CU (6)	Restr. (7)	CU (8)
$\hat{d}$	NA	0.43	NA	0.52	NA	0.40	NA	0.52
AIC	18,958	18,477	211	−936	7,996	7,707	211	−935

Notes. Estimates of different versions of equation (4). Columns (1), (3), (5), and (7): set $\gamma = 1$ and $p_{CU} = 0$. All estimated standard errors (computed based on clustering at the subject level) are smaller than 0.02. AIC, Akaike information criterion.

Extension

S-Shaped response functions

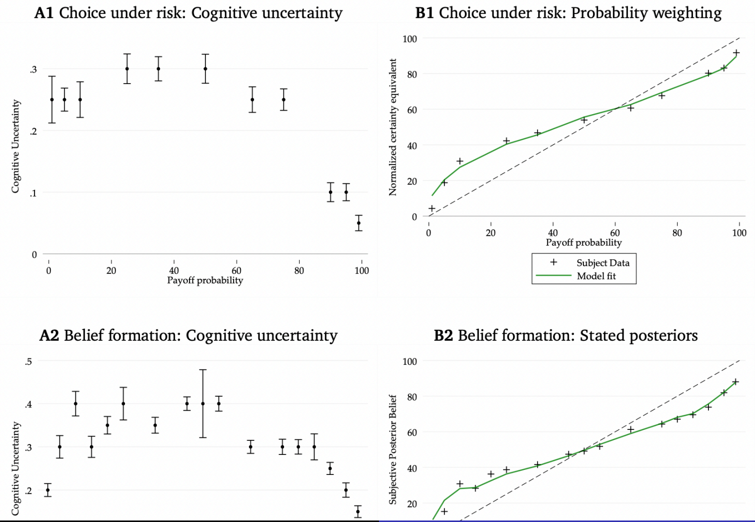
- More cognitive uncertainty on the extremities
- Posterior close to 50% $\Rightarrow$ low CU

Limitations:

- No theory for cognitive default (salience of a choice, misleading, intuition)
- S-shapedness disappear if choices come from memory (rather than from stated probabilities)
- No theory on the generation of the cognitive noise (time pressure, experience, prior beliefs, attention)

Implications:

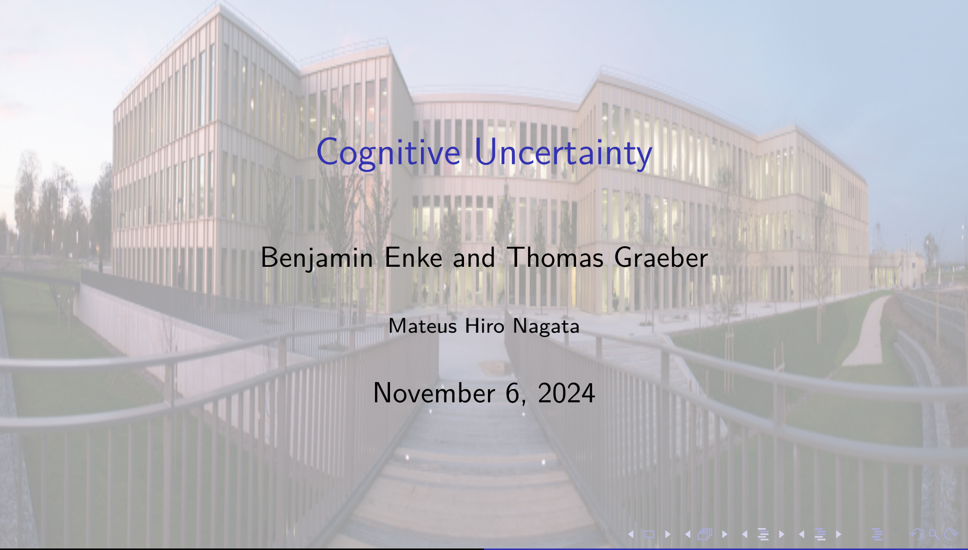
- Demographic differences in expectations could reflect heterogeneity in cognitive noise rather than true beliefs.


A1 Choice under risk: Cognitive uncertainty
Cognitive Uncertainty
Payoff probability
B1 Choice under risk: Probability weighting
Normalized certainty equivalent
Payoff probability
Subject Data
Model fit
A2 Belief formation: Cognitive uncertainty
Cognitive Uncertainty
B2 Belief formation: Stated posteriors
Subjective Posterior Belief

In summary

Model: DM has an imprecise estimation of the best decision a^*, with a bias towards an intermediate value d. If a problem is more complex, one has more cognitive uncertainty p_{CU}, which aggravates this compression effect. There is more CU on the edges, resulting in the S-shaped distortions of the probabilities.

- Maybe can discuss the "as-if" motivation part of the model, but hard to argue against the empirical/introspective existence of subjective cognitive uncertainty and that the estimations of the effects of the measures in decisions are reasonable.



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