# Causal Diagrams for empirical research

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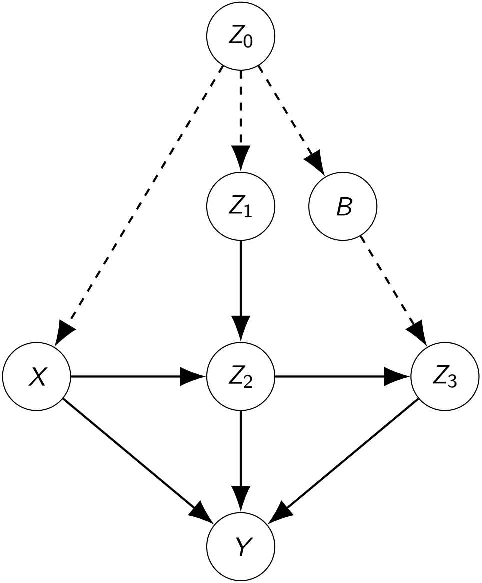
# Before proceeding

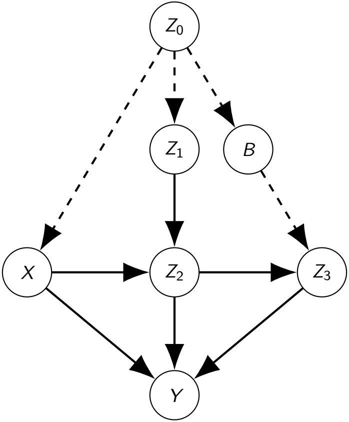

$Z_0$
$Z_1$
$B$
$X$
$Z_2$
$Z_3$
$Y$

$X$ : Soil Fumigants
$Y$ : Crop Yields
$Z$ : Eelworm population. $Z_0$ : last year's (unknown), $Z_1$ : before treatment, $Z_2$ : after treatment, $Z_3$ : end of the season,
$B$ :Population of birds

- Unobserved and observed variables

1. Find: total effect of $X$ on $Y$ given the distribution
2. $P(y|do(x)) \neq P(y|x)$

$$X_i = f_i(pa_i, \varepsilon_i)$$




$$Z_0 = f_0(\varepsilon_0), Z_1 = f_1(Z_0, \varepsilon_1), Z_2 = f(X, Z_1, \varepsilon_2), Z_3 = f_3(B, Z_2, \varepsilon_3)$$

$$X = f_X(Z_0, \varepsilon_X), Y = f_Y(X, Z_2, Z_3, \varepsilon_Y), B = f_B(Z_0, \varepsilon_B),$$

# Notation

**d-separation**: $X, Y, Z$ disjoint subsets of nodes. $(X \perp\!\!\!\perp Y|Z)_G \Leftrightarrow Z$ blocks every path from a node in $X$ to a node in $Y$

$Z$ **blocks** path $p$ if:

1. Chain $(X \rightarrow w \rightarrow Y,\ w \in Z)$, Fork $(X \leftarrow w \rightarrow Y, w \in Z)$
2. Collider $(X \rightarrow w \leftarrow Y, w \notin Z$, Descendents of $w \notin Z)$

**Identifiability**: Causal effect of $X$ on $Y$ if $P(y|do(x))$ can be computed uniquely from any positive distribution of the observed variables that is compatible with $G$.

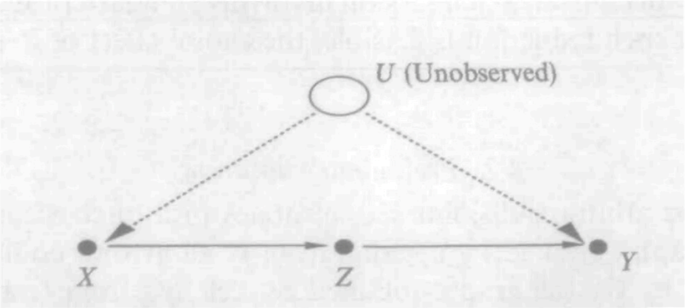



Fig. 3. A diagram representing the front-door criterion.

- Let $X, Y$ be disjoint sets of variables. The **causal effect** is $P(y|do(x)) : X \to \mathcal{P}(Y)$. For each realisation $x$ of $X$, $P(y|do(x))$ gives the probability of $Y = y$ induced on deleting from the model $X_i = f_i(pa_i, \varepsilon_i), \forall i$ all equations corresponding to variables in $X$ and substituting $x$ for $X$ in the remainder

For atomic interventions $X_i = x_i'$

$$P(x_1, \ldots, x_n|do(x_i')) = \begin{cases} \frac{P(x_1,\ldots,x_n)}{P(x_i|pa_i)} \\ 0 \text{ if } x_i \neq x_i' \end{cases}$$

- Back-door criterion: $Z$ relative to variables $(X_i, X_j)$ if
  1. No node in $Z$ is a descendant of $X_i$
  2. $\forall$ path $p$ between $X_i$ and $X_j$ with an arrow into $X_i$, $Z$ blocks $p$

### Theorem (Back-door Identifiability)

*If a set of variables $Z$ satisfies the back-door criterion relative to $(X, Y)$, then the causal effect of $X$ on $Y$ is identifiable and is given by*

$$P(y|do(x)) = \sum_z P(y|x, z)P(z)$$

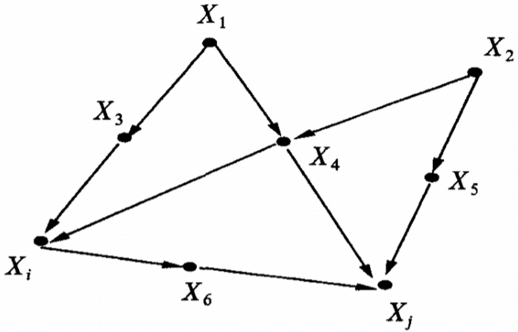



Fig. 2. A diagram representing the back-door criterion; adjusting for variables $\{X_3, X_4\}$ or $\{X_4, X_5\}$ yields a consistent estimate of $\mathrm{pr}(x_j \mid \check{x}_i)$.

## Theorem (Front-Door Criterion)

*$Z$ satisfies relative to subsets $(X, Y)$ :*

1. *(Interception) $\forall$ path $p$ between $X$ and $Y$, $\exists w \in Z$ $X \rightarrow w \rightarrow Y$*
2. *(Absence of Back-door) between $X$ and $Z$*
3. *(Back-door Block) All back-door paths between $Z$ and $Y$ is blocked by $X$*

*Then, the causal effect of $X$ on $Y$ is **identifiable***

$$P(y|do(x)) = \sum_{x} P(z|x) \sum_{x'} P(y|x', z)P(x')$$

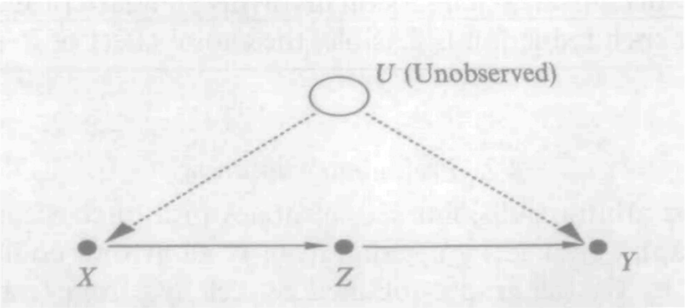



Fig. 3. A diagram representing the front-door criterion.

Figure: Caption

# Notation

- $G_{\overline{X}} = G$ with all arrows pointing to nodes in $X$ deleted
- $G_{\underline{X}} = G$ with all arrows emanating from $X$ deleted

## Theorem (Manipulation Rules)

*Let G DAG and $P(\cdot)$ the induced probability distribution. For any disjoint subsets of variables $X, Y, Z$ and $W$ we have:*

***Rule 1 (insertion/deletion of observations):***

$$P(y|do(x), z, w) = P(y|do(x), w) \quad \text{if } (Y \perp\!\!\!\perp Z|X, W)_{G_{\bar{X}}}.$$

***Rule 2 (action/observation exchange):***

$$P(y|do(x), do(z), w) = P(y|do(x), z, w) \quad \text{if } (Y \perp\!\!\!\perp Z|X, W)_{G_{\bar{X}\underline{Z}}}.$$

***Rule 3 (insertion/deletion of actions):***

$$P(y|do(x), do(z), w) = P(y|do(x), w) \quad \text{if } (Y \perp\!\!\!\perp Z|X, W)_{G_{\bar{X},\bar{Z(W)}}},$$

*where $Z(W)$ is the set of Z-nodes that are not ancestors of any W-node in Gx.*

Rule 1: d-separation as conditional independence
Rule 2: External intervention set $Z = z$ same as passive observation
Rule 3: Adding/deleting interventions

### Corollary (Reduction)

*A causal effect* $q = P(y_1, \ldots, y_k | do(x_1), \ldots, do(x_m))$ *is identifiable in G if* $\exists$ *finite sequence of transformations (using rules 1,2,3), which reduces q into a standard probability expression involving observed quantities.*

Causal Inference by Surrogate Experiments

Graphical Tests of Identifiability

Visual Learner Oasis

Conclusion
References

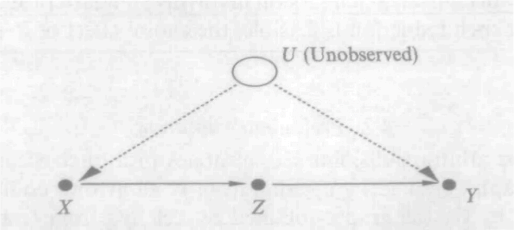



Fig. 3. A diagram representing the front-door criterion.

$$P(y \mid do(x))?$$

1. $P(z \mid do(x))$

$$P(z \mid do(x)) = P(z \mid x)$$

2. $P(y \mid do(z))$

$$P(y \mid do(z)) = \sum_x P(y \mid x, do(z))P(x \mid do(z))$$
$$P(x \mid do(z)) = P(x)$$
$$P(y \mid do(z)) = \sum_x P(y \mid x, z)P(x)$$

3. $P(y \mid do(x))$

$$P(y \mid do(x)) = \sum_z P(y \mid z, do(x))P(z \mid do(x))$$
$$P(y \mid z, do(x)) = P(y \mid do(z), do(x)) = P(y \mid do(z))$$
$$P(y \mid do(x)) = \sum_z P(z \mid x) \sum_{x'} P(y \mid x', z)P(x')$$

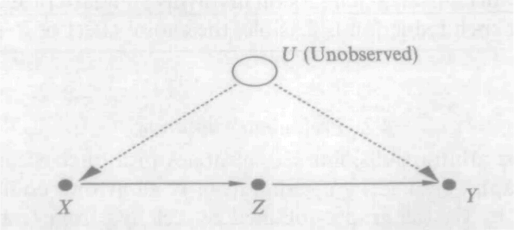



Fig. 3. A diagram representing the front-door criterion.

# Surrogate Experiments

$$P(y|do(x))?$$

$X$ : Cholesterol $Y$ : Heart Disease $Z$ : Subject's diet (surrogate variable)
Transform the problem to something that has $do(z)$ only.

- No direct effect of diet on heart conditions
- No confounding between cholesterol ($X$) and heart condition ($Y$) (unless by observable variable)

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# Limitation of Nonparametric

Confounding arc between $X$ and $Y$: $U \to X, U \to Y, X \to Y$

$$Y = bX + \gamma U + \varepsilon_1$$

$$X = \alpha Y + \delta U + \varepsilon_2$$

$$b := \frac{\partial E(Y|do(x))}{\partial x} \text{ not identifiable}$$

$$\text{If } \exists U \to X, \text{ then } b = \frac{E(Y|u)}{E(X|u)}$$

In nonparametric models, adding more variables never help identification

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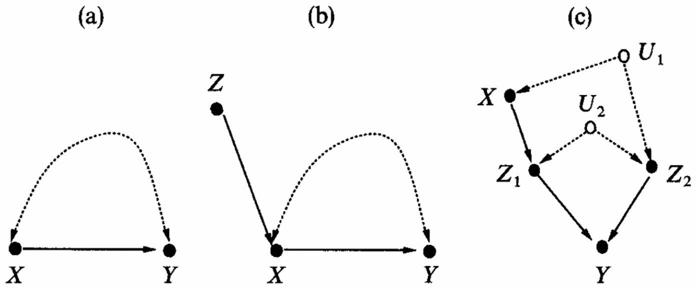


Fig. 5. (a) A bow-pattern: a confounding arc embracing a causal link $X \rightarrow Y$, thus preventing the identification of $\mathrm{pr}(y \mid \check{x})$ even in the presence of an instrumental variable $Z$, as in (b). (c) A bow-less graph still prohibiting the identification of $\mathrm{pr}(y \mid \check{x})$.

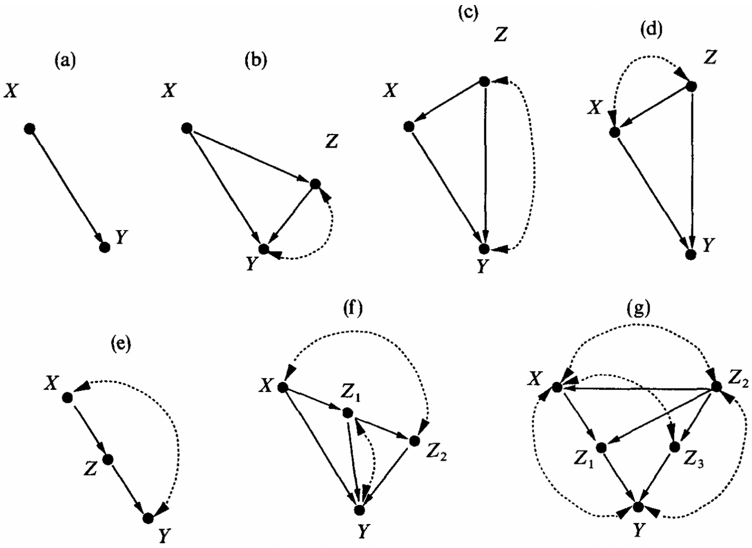


Fig. 6. Typical models in which the effect of $X$ on Y is identifiable. Dashed arcs represent confounding paths, and $Z$ represents observed covariates.

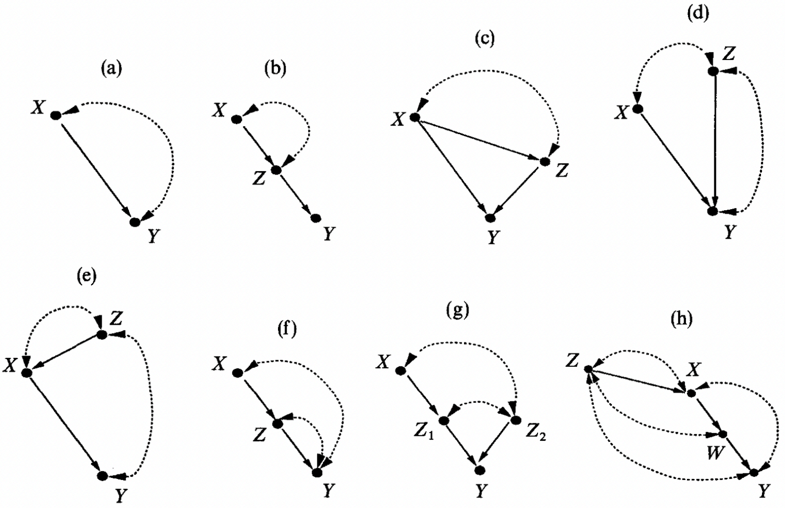


Fig. 7. Typical models in which $\mathrm{pr}(y \mid \check{x})$ is not identifiable.

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# Conclusion

- New language to describe intervention
- Intuitive use of DAGs
- Limitation: based on causal assumptions in the proposed DAG which cannot be tested, generality of nonparametric model

# Further

- Causality influenced by language (Framing effects)
- Must be conceived in the mind first
- Subjective Causality I: Rareness of "if" make us see the "then", not only the order of things (Kolmogorov complexity) (Alexander and Gilboa, 2023)
- Subjective Causality II: can elicit subjective $G$ and $f_i(\cdot), P(\cdot), u(\cdot)$ if preferences follow certain axioms (Halpern and Piermonte, 2024)
- Subjective everything
- No notion of model ambiguity
- Meta: Lucas' critique, sociological stance, prescription vs prediction
- Behavioral Causal Inference (Spiegler, 2023)

# Lore

- Judea Pearl: AGI
- Sample size of 1 + conjectural theory vs problem of probability of historic events
- Bayesian is part of statistics, comes from the joint, cannot come up with questions outside of statistics
- Free-fact learning: procedure to find causal (because it is given), AI cannot do it
- $P(drink|\text{alcoholic parents})$

# References I

1. Alexander, Yotam, and Itzhak Gilboa. "Subjective causality." Revue économique 74.4 (2023): 619-633.
2. Halpern, Joseph Y., and Evan Piermont. "Subjective Causality." arXiv preprint arXiv:2401.10937 (2024).
3. Spiegler, Ran. "Behavioral causal inference." arXiv preprint arXiv:2305.18916 (2023).

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