Notes for master's thesis

Foundations & applications of generalised symmetries

大阪大学大学院理学研究科物理学専攻　大和 寛尚

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[1]

1 Foundations

1.1 Higher-form global symmetries

1.1.1 Symmetry as topological operators

Statement 1.1. Symmetry generators are topological operators.

We have Ward-Takahashi identity (in differential forms).

$$\langle \mathrm{d}{*j}(x)\mathscr{O}(y)\rangle = -i\delta^{(D)}(x-y)\,\langle \delta\mathscr{O}(y)\rangle\,. \tag{1.1}$$

Noether charge is defined as follows.

$$Q_\Sigma = \int_\Sigma *j(x), \tag{1.2}$$

where Σ is a equal-time hypersurface on the spacetime manifold M.[2] Integrating the both sides of this equation on the manifold Ω that has the hypersurface Σ as its boundary, we get

$$\langle Q_\Sigma \mathscr{O}(x)\rangle = -i\,\mathrm{Link}(P,\Omega)\,\langle \delta\mathscr{O}(x)\rangle\,, \tag{1.3}$$

where P is defined as $P = \left\{(y^0,\cdots,y^{D-1})\right\}$. We can see that $\langle Q_\Sigma \mathscr{O}(x)\rangle$ is toplological in the sense that it is invariant under the deformation of the manifold Ω that does not cross the point P.

[1] The sections with asterisk " * " are materials that is not directly related to generalised symmetries but supplemental.

[2]

This formulation leads us to the following observation.

Observation 1.1. The Ward-Takahashi identity we have derived

$$\langle Q_\Sigma \mathscr{O}(x)\rangle = -i\,\mathrm{Link}(P, \Omega)\,\langle \delta\mathscr{O}(x)\rangle \tag{1.4}$$

provides a new perspective in generalising symmetries. We may put this relation as the starting point of the discussion of symmetries and generalising them by substituting P, Ω with some manifolds X, Y such that $\dim X + \dim Y = D$. And naturally, after the generalisation, the operator $\mathscr{O}(x)$ is not supported at a point anymore but supported on a manifold X.

$$\langle Q_\Sigma \mathscr{O}(X)\rangle = -i\,\mathrm{Link}(X, Y)\,\langle \delta\mathscr{O}(X)\rangle\,. \tag{1.5}$$

1.1.2 Definition of p - form symmetries

Now, let us put the observation as the definition of symmetries (and we call this symmetry a higher-form symmetry).

Definition$^{\text{(ph)}}$ 1.1. We define a p - form symmetry as follows. We say that there exists an p - form symmetry in the system in consider if there exists an operator Q_Σ that is supported on a $D-p-1$ dimensional manifold Σ and it satisfies Ward-Takahashi identity

$$\langle Q_\Sigma \mathscr{O}(X)\rangle = -i\,\mathrm{Link}(X, Y)\,\langle \delta\mathscr{O}(X)\rangle\,, \quad \Sigma = \partial Y. \tag{1.6}$$

We call this operator Q_Σ a symmetry operator.

1.1.3 Exmaples of p - form symmetries

<u>$D = 4$ dimensional $U(1)$ gauge theory</u> We consider an action

$$S[a] = -\frac{1}{2e^2}\int_M f \wedge * f, \quad f = \mathrm{d}a, \tag{1.7}$$

where a is the $U(1)$ gauge field. Considering the trivial relation $\mathrm{d}^2 = 0$ and the equation of motion, we get

$$\mathrm{d}{*f} = 0, \quad \mathrm{d}f = 0. \tag{1.8}$$

Now, let us define new operators as follows.

$$Q_\Sigma^{(\mathrm{e})} = \int_\Sigma *f, \quad Q_\Sigma^{(\mathrm{m})} = \int_\Sigma f. \tag{1.9}$$

Let us stick to $Q_\Sigma^{(\mathrm{e})}$ for a while. We first prove that $Q_\Sigma^{(\mathrm{e})}$ is a symmetry operator.

2 Applications