## Notes for master's thesis

# Foundations & applications of generalised symmetries

# 大阪大学大学院理学研究科物理学専攻 大和 寬尚

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#### 1 Foundations

## 1.1 Conventional symmetries \*

Most parts of this section are adopted from the matirials below.

- [1] M. E. Peskin and D. V. Schroeder, "An Introduction to quantum field theory"
- [2] M. Srednicki, "Quantum field theory"
- [3] P. R. S. Gomes, "An introduction to higher-form symmetries", Section 3

## 1.1.1 Schwinger-Dyson equation & conversation laws

 $\textbf{Proposition}^{(\mathrm{ph})} \ \textbf{1.1.} \ \text{We have following relation, which is well-known as Schwinger-Dyson equation.}$ 

$$\left\langle \frac{\partial S[\varphi]}{\partial \varphi(x)} \varphi(x_1) \cdots \varphi(x_n) \right\rangle = i \sum_{i=1}^n \left\langle \varphi(x_1) \cdots \delta^{(D)}(x - x_i) \cdots \varphi(x_n) \right\rangle. \tag{1.1}$$

*Proof.* First of all, we prove the general relation

$$\int \mathscr{D}\varphi \, \frac{\delta}{\delta\varphi(x)} \Big( F[\varphi] e^{iS[\varphi]} \Big) = 0. \tag{1.2}$$

<sup>&</sup>lt;sup>1</sup>The sections with asterisk "\*" are materials that is not directly related to generalised symmetries but supplemental.

We start from the obvious relation

$$\int \mathscr{D}\varphi \, F[\varphi]e^{iS[\varphi]} = \int \mathscr{D}\varphi' \, F[\varphi']e^{iS[\varphi']},\tag{1.3}$$

which is just a renaming of a dummy variable of the integration. Now we perform a change of variables  $\varphi'(x) \longmapsto \varphi(x) + \varepsilon(x)$ . Then we have

$$\int \mathcal{D}\varphi \, F[\varphi] e^{iS[\varphi]} = \int \mathcal{D}\varphi \, F[\varphi + \varepsilon] e^{iS[\varphi + \varepsilon]} \\
= \int \mathcal{D}\varphi \, \left\{ F[\varphi] + \int \mathrm{d}^D x \, \frac{\delta F[\varphi]}{\delta \varphi(x)} \varepsilon(x) \right\} \exp \left\{ iS[\varphi] + i \int \mathrm{d}^D x \, \frac{\delta S[\varphi]}{\delta \varphi(x)} \varepsilon(x) \right\} \\
= \int \mathcal{D}\varphi \, \left\{ F[\varphi] + \int \mathrm{d}^D x \, \frac{\delta F[\varphi]}{\delta \varphi(x)} \varepsilon(x) \right\} \left\{ 1 + i \int \mathrm{d}^D x \, \frac{\delta S[\varphi]}{\delta \varphi(x)} \varepsilon(x) \right\} e^{iS[\varphi]} \\
= \int \mathcal{D}\varphi \, F[\varphi] e^{iS[\varphi]} + \int \mathcal{D}\varphi \, \int \mathrm{d}^D x \, \left\{ \frac{\delta F[\varphi]}{\delta \varphi(x)} e^{iS[\varphi]} + F[\varphi] i \frac{\delta S[\varphi]}{\delta \varphi(x)} e^{iS[\varphi]} \right\} \varepsilon(x), \quad \forall \varepsilon(x), \quad (1.4)$$

which leads us to the identity

$$\int \mathscr{D}\varphi \left\{ \frac{\delta F[\varphi]}{\delta \varphi(x)} e^{iS[\varphi]} + F[\varphi] i \frac{\delta S[\varphi]}{\delta \varphi(x)} e^{iS[\varphi]} \right\} = 0. \tag{1.5}$$

Now, by substituting  $F[\varphi]$  with  $\varphi(x_1)\cdots\varphi(x_n)$  we get

$$i\int \mathscr{D}\varphi \,\frac{\delta S[\varphi]}{\delta \varphi(x)}\varphi(x_1)\cdots\varphi(x_n)e^{iS[\varphi]} = -\sum_{i=1}^n \int \mathscr{D}\varphi \,\varphi(x_1)\cdots\delta^{(D)}(x-x_i)\cdots\varphi(x_n)e^{iS[\varphi]},\tag{1.6}$$

which we simply denote

$$\left\langle \frac{\delta S[\varphi]}{\delta \varphi(x)} \varphi(x_1) \cdots \varphi(x_n) \right\rangle = i \sum_{i=1}^n \left\langle \varphi(x_1) \cdots \delta^{(D)}(x - x_i) \cdots \varphi(x_n) \right\rangle. \tag{1.7}$$

Remark 1.1. Schwinger-Dyson equation is the generalisation of Ward-Takahashi identity.

**Proposition**<sup>(ph)</sup> **1.2.** We have following relation, which is a correlation function representation of the conversation law of Noether current.

$$\langle \{\partial_{\mu} j^{\mu}(x)\} \varphi_{a_1}(x_1) \cdots \varphi_{a_n}(x_n) \rangle = -i \sum_{i=0}^{n} \left\langle \varphi_{a_1}(x_1) \cdots \delta^{(D)}(x - x_i) \delta \varphi_{a_i}(x_i) \cdots \varphi_{a_n}(x_n) \right\rangle. \tag{1.8}$$

*Proof.* Now, we consider the transformation

$$\varphi_a(x) \longmapsto \varphi_a'(x) = \varphi_a(x) + \delta_{\varepsilon(x)}\varphi_a(x) = \varphi_a(x) + \varepsilon(x)g_a(x).$$
 (1.9)

We first prove the relation

$$\delta_{\varepsilon(x)}S[\varphi] = -\int d^D x \,\varepsilon(x)\partial_\mu \left\{ \sum_{a=1}^N g_a(x) \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a(x))} - K^\mu[\varphi] \right\} = -\int d^D x \,\varepsilon(x)\partial_\mu j^\mu(x) \tag{1.10}$$

where we defined  $j^{\mu}(x)$  as

$$j^{\mu}(x) = \sum_{a=1}^{N} g_a(x) \frac{\partial \mathcal{L}}{\partial(\partial_{\mu} \varphi_a(x))} - K^{\mu}[\varphi]. \tag{1.11}$$

The direct calculation of  $\delta_{\varepsilon(x)}S[\varphi]$  is as follows.

$$\delta_{\varepsilon(x)}S[\varphi] = \int d^{D}x \, \delta_{\varepsilon(x)} \mathscr{L}[\varphi, \partial_{\mu}\varphi] = \int d^{D}x \, \{\mathscr{L}[\varphi(x) + \varepsilon(x)\varphi(x), \partial_{\mu}\{\varphi(x) + \varepsilon(x)g(x)\}] - \mathscr{L}[\varphi(x), \partial_{\mu}\varphi(x)]\}$$

$$= \int d^{D}x \, \{\mathscr{L}[\varphi(x) + \varepsilon(x)\varphi(x), \partial_{\mu}\varphi(x) + \partial_{\mu}\varepsilon(x)g(x) + \varepsilon(x)\partial_{\mu}g(x)] - \mathscr{L}[\varphi(x), \partial_{\mu}\varphi(x)]\}$$

$$= \int d^{D}x \, \left\{\varepsilon(x) \sum_{a=1}^{N} g_{a}(x) \frac{\partial \mathscr{L}}{\partial \varphi_{a}(x)} + \varepsilon(x) \sum_{a=1}^{N} \partial_{\mu}g_{a}(x) \frac{\partial \mathscr{L}}{\partial (\partial_{\mu}\varphi_{a}(x))} + \partial_{\mu}\varepsilon(x) \sum_{a=1}^{N} g_{a}(x) \frac{\partial \mathscr{L}}{\partial (\partial_{\mu}\varphi_{a}(x))}\right\}$$

$$= \int d^{D}x \, \varepsilon(x) \partial_{\mu}K^{\mu}(x) + \int d^{D}x \, \partial_{\mu}\varepsilon(x) \sum_{a=1}^{N} g_{a}(x) \frac{\partial \mathscr{L}}{\partial (\partial_{\mu}\varphi_{a}(x))}$$

$$(1.12)$$

Here, we assumed

$$\sum_{a=1}^{N} g_a(x) \frac{\partial \mathcal{L}}{\partial \varphi_a(x)} + \sum_{a=1}^{N} \partial_{\mu} g_a(x) \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \varphi_a(x))} = \partial_{\mu} K^{\mu}[\varphi(x)], \quad {}^{\exists} K^{\mu}[\varphi(x)]. \tag{1.13}$$

Integrating the second term by parts, we get

$$\delta_{\varepsilon(x)}S[\varphi] = \int d^D x \,\varepsilon(x)\partial_\mu K^\mu(x) - \int d^D x \,\varepsilon(x)\partial_\mu \left\{ g_a(x) \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a(x))} \right\} = -\int d^D x \,\varepsilon(x)\partial_\mu j^\mu(x). \tag{1.14}$$

Now, we move on to the proof of the relation in consider. We start from the trivial relation

$$\int \left(\prod_{a=1}^{N} \mathscr{D}\varphi_{a}\right) \varphi_{a_{1}}(x_{1}) \cdots \varphi_{a_{n}}(x_{n}) e^{iS[\varphi]} = \int \left(\prod_{a=1}^{N} \mathscr{D}\varphi'_{a}\right) \varphi'_{a_{1}}(x_{1}) \cdots \varphi'_{a_{n}}(x_{n}) e^{iS[\varphi]}. \tag{1.15}$$

Here, we perform the change of variable  $\varphi_a'(x) = \varphi_a(x) + \varepsilon(x)g_a(x)$  and we get

$$\int \left(\prod_{a=1}^{N} \mathscr{D}\varphi_{a}\right) \varphi_{a_{1}}(x_{1}) \cdots \varphi_{a_{n}}(x_{n}) e^{iS[\varphi]} = \int \left(\prod_{a=1}^{N} \mathscr{D}\varphi_{a}\right) \varphi_{a_{1}}(x_{1}) \cdots \varphi_{a_{n}}(x_{n}) e^{iS[\varphi]} 
+ i \int \left(\prod_{a=1}^{N} \mathscr{D}\varphi_{a}\right) \varphi_{a_{1}}(x_{1}) \cdots \varphi_{a_{n}}(x_{n}) \left\{-\int d^{D}x \,\varepsilon(x) \partial_{\mu} j^{\mu}(x)\right\} e^{iS[\varphi]} 
+ \sum_{i=1}^{n} \int \left(\prod_{a=1}^{N} \mathscr{D}\varphi_{a}\right) \varphi_{a_{1}}(x_{1}) \cdots \varepsilon(x_{i}) g_{a_{i}}(x_{i}) \cdots \varphi_{a_{n}}(x_{n}) e^{iS[\varphi]},$$
(1.16)

which leads us to

$$-i \int \left( \prod_{a=1}^{N} \mathscr{D}\varphi_{a} \right) \varphi_{a_{1}}(x_{1}) \cdots \varphi_{a_{n}}(x_{n}) \left\{ \int d^{D}x \, \varepsilon(x) \partial_{\mu} j^{\mu}(x) \right\} e^{iS[\varphi]}$$

$$= \sum_{i=1}^{n} \int \left( \prod_{a=1}^{N} \mathscr{D}\varphi_{a} \right) \varphi_{a_{1}}(x_{1}) \cdots \varepsilon(x_{i}) g_{a_{i}}(x_{i}) \cdots \varphi_{a_{n}}(x_{n}) e^{iS[\varphi]} = 0. \tag{1.17}$$

Now, we substitute  $\varepsilon(x_i)$  in RHS with a trivial relation  $\varepsilon(x_i) = \int d^D x \, \varepsilon(x) \delta^{(D)}(x - x_i)$ , we get

$$-i \int d^{D}x \,\varepsilon(x) \int \left(\prod_{a=1}^{N} \mathscr{D}\varphi_{a}\right) \varphi_{a_{1}}(x_{1}) \cdots \varphi_{a_{n}}(x_{n}) \{\partial_{\mu}j^{\mu}(x)\} e^{iS[\varphi]}$$

$$= \int d^{D}x \,\varepsilon(x) \sum_{i=1}^{n} \int \left(\prod_{a=1}^{N} \mathscr{D}\varphi_{a}\right) \varphi_{a_{1}}(x_{1}) \cdots \delta^{(D)}(x-x_{i}) g_{a_{i}}(x_{i}) \cdots \varphi_{a_{n}}(x_{n}) e^{iS[\varphi]}, \quad \forall \varepsilon(x). \quad (1.18)$$

Then we get

$$i\int \left(\prod_{a=1}^{N} \mathscr{D}\varphi_{a}\right) \varphi_{a_{1}}(x_{1}) \cdots \varphi_{a_{n}}(x_{n}) \{\partial_{\mu}j^{\mu}(x)\} e^{iS[\varphi]} = \sum_{i=1}^{n} \int \left(\prod_{a=1}^{N} \mathscr{D}\varphi_{a}\right) \varphi_{a_{1}}(x_{1}) \cdots \delta^{(D)}(x-x_{i}) g_{a_{i}}(x_{i}) \cdots \varphi_{a_{n}}(x_{n}) e^{iS[\varphi]},$$

$$(1.19)$$

which means that

$$\langle \varphi_{a_1}(x_1) \cdots \varphi_{a_n}(x_n) \{ \partial_{\mu} j^{\mu}(x) \} \rangle = -i \sum_{i=0}^{n} \left\langle \varphi_{a_1}(x_1) \cdots \delta^{(D)}(x - x_i) g_{a_i}(x_i) \cdots \varphi_{a_n}(x_n) \right\rangle. \tag{1.20}$$

#### 1.1.2 Reformulation in the language of differential forms

Fact<sup>(ph)</sup> 1.1. Mathematically speaking, Noether current  $j^{\mu}(x)$  are coefficients of a vector field on the spacetime manifold  $\mathcal{M}$ , which is a section of the tangent bundle  $T\mathcal{M}$ . And equivaletly  $j_{\mu}(x)$  are coefficients of a covector field of a section of the cotangent bundle  $T^*\mathcal{M}$ . Let us stick to the representation on cotangent bundle in this notes, then we can write

$$j = j_{\mu}(x)\mathrm{d}x^{\mu}.\tag{1.21}$$

And obviously, we have the Hodge dual of it.

$$*j = \frac{1}{(D-1)!} j_{\mu_0}(x) \epsilon^{\mu_0}{}_{\mu_1 \dots \mu_{D-1}} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_{D-1}}.$$
 (1.22)

**Proposition**<sup>(ph)</sup> 1.3. The conversation law  $\partial_{\mu}j^{\mu}(x) = 0$  can be written as

$$d * j = 0. ag{1.23}$$

*Proof.* We will see the equivalence of the two representations  $\partial_{\mu}j^{\mu}=\partial^{\mu}j_{\mu}=0$  and d\*j=0. We start from

$$d * j = \frac{1}{(D-1)!} \partial_{\alpha} j_{\mu_0}(x) \epsilon^{\mu_0}{}_{\mu_1 \cdots \mu_{D-1}} dx^{\alpha} \wedge dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{D-1}} = \frac{1}{(D-1)!} \partial_{\alpha} j_{\mu_0}(x) \epsilon^{\mu_0}{}_{\mu_1 \cdots \mu_{D-1}} \epsilon^{\alpha \mu_1 \cdots \mu_{D-1}}$$
$$= (-1)^{D-1} \partial_{\alpha} j_{\mu_0}(x) \eta^{\mu_0 \alpha} dx^0 \wedge \cdots \wedge dx^{D-1} = (-1)^{D-1} \partial^{\mu} j_{\mu}(x) dx^0 \wedge \cdots dx^{D-1}. \tag{1.24}$$

Now, if we assume d \* j = 0, obviously  $\partial_{\mu} j^{\mu}(x) = 0$  because  $dx^0 \wedge \cdots \wedge dx^{D-1} \neq 0$ . Also if we assume  $\partial_{\mu} j^{\mu}(x) = 0$ , obviously d \* j = 0 from the relation we just derived.

Remark 1.2. In the Euclidean theory, which we will be using so often in this note, we simply have

$$d * j = \partial_{\mu} j^{\mu}(x) dx^{0} \wedge \dots \wedge dx^{D-1}.$$
(1.25)

**Proposition**<sup>(ph)</sup> **1.4.** The definition of Noether charge  $Q = \int d^{D-1}x j_0(x)$  is translated to differential forms as follows.

$$Q = \int_{\Sigma} *j, \tag{1.26}$$

where,  $\Sigma$  is the spatial submanifold of the spacetime manifold  $\mathcal{M}$ .

*Proof.* We start by rewriting the integral.

$$Q = \int_{\Sigma} *j = \int_{\Sigma} \frac{1}{(D-1)!} j_{\mu_0}(x) \epsilon^{\mu_0}{}_{\mu_1 \cdots \mu_{D-1}} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{D-1}}$$

$$= \int_{\Sigma} \frac{1}{(D-1)!} j_{\mu_0}(x) \epsilon^{\mu_0}{}_{\mu_1 \cdots \mu_{D-1}} \epsilon^{0\mu_1 \cdots \mu_{D-1}} dx^1 \wedge \cdots \wedge dx^{D-1} = \int_{\Sigma} j_{\mu_0}(x) (-1)^{D-1} \eta^{\mu_0 0} dx^1 \wedge \cdots \wedge dx^{D-1}.$$
(1.27)

Obviously, (1.27) = 0 for  $\mu_0 \neq 0$ . Then we have

$$Q = Q(\Sigma) = (-1)^{D-1} \int_{\Sigma} j_0(x) \, dx^1 \wedge \dots \wedge dx^{D-1},$$
 (1.28)

which has the same meaning as  $\int d^{D-1}x j_0(x)$ .

**Proposition**(ph) 1.5. The conversation law in a correlation function representation is interpreted as follows.

$$\langle (\mathbf{d} * j)\varphi_{a_1}(x_1)\cdots\varphi_{a_n}(x_n)\rangle = -i\sum_{i=0}^n \left\langle \varphi_{a_1}(x_1)\cdots\delta^{(D)}(x-x_i)g_{a_i}(x_i)\cdots\varphi_{a_n}(x_n)\right\rangle dx^0 \wedge \cdots \wedge dx^{D-1}.$$
(1.29)

*Proof.* By simply putting  $dx^0 \wedge \cdots \wedge dx^{D-1}$  on both sides of (1.8), we get (1.29).

**Definition**<sup>(ph)</sup> **1.1.** We say that, in the case of conventional symmetries, the operator  $\mathcal{O}(\mathcal{M})$  defined on some manifold  $\mathcal{M}$  is topological if  $\langle \mathcal{O}(\mathcal{M})\varphi(x_1)\cdots\varphi(x_n)\rangle$  is invariant under the deformation of  $\mathcal{M}$ .

**Proposition**(ph) **1.6.** We have the following relation.

$$\langle Q(\Sigma)\varphi_{a_1}(x_1)\cdots\varphi_{a_n}(x_n)\rangle = -i\sum_{i=0}^n \operatorname{Link}(\Sigma,x_i) \langle \varphi_{a_1}(x_1)\cdots g_{a_i}(x_i)\cdots\varphi_{a_n}(x_n)\rangle, \qquad (1.30)$$

where we defined the link number as

$$\operatorname{Link}(\Sigma, x_i) = \int_{\Omega_{\Sigma}} dx^0 \wedge \cdots dx^{D-1} \delta^{(D)}(x - x_i), \quad \partial \Omega_{\Sigma} = \Sigma.$$
 (1.31)

*Proof.* We just integrate the both sides of (1.29) in terms of x on  $\Omega_{\Sigma}$ .

LHS of (1.29) = 
$$\int_{\Omega_{\Sigma}} \langle (d * j) \varphi_{a_{1}}(x_{1}) \cdots \varphi_{a_{n}}(x_{n}) \rangle$$

$$= \left\langle \int_{\Omega_{\Sigma}} (d * j) \varphi_{a_{1}}(x_{1}) \cdots \varphi_{a_{n}}(x_{n}) \right\rangle \xrightarrow{\text{Stokes's theorem}} \left\langle \int_{\Sigma} * j \varphi_{a_{1}}(x_{1}) \cdots \varphi_{a_{n}}(x_{n}) \right\rangle$$

$$\stackrel{(1.27)}{=} \left\langle \int_{\Sigma} Q(\Sigma) \varphi_{a_{1}}(x_{1}) \cdots \varphi_{a_{n}}(x_{n}) \right\rangle \tag{1.32}$$

$$\text{RHS of (1.29)} = -i \sum_{i=0}^{n} \int_{\Omega_{\Sigma}} \delta^{(D)}(x - x_{i}) \left\langle \varphi_{a_{1}}(x_{1}) \cdots g_{a_{i}}(x_{i}) \cdots \varphi_{a_{n}}(x_{n}) \right\rangle dx^{0} \wedge \cdots \wedge dx^{D-1}$$

$$= -i \sum_{i=0}^{n} \int_{\Omega_{\Sigma}} \delta^{(D)}(x - x_{i}) \left\langle \varphi_{a_{1}}(x_{1}) \cdots g_{a_{i}}(x_{i}) \cdots \varphi_{a_{n}}(x_{n}) \right\rangle dx^{0} \wedge \cdots \wedge dx^{D-1}$$

$$= -i \sum_{i=0}^{n} \operatorname{Link}(\Sigma, x_{i}) \left\langle \varphi_{a_{1}}(x_{1}) \cdots g_{a_{i}}(x_{i}) \cdots \varphi_{a_{n}}(x_{n}) \right\rangle \tag{1.33}$$

This proves the proposition.

<sup>&</sup>lt;sup>2</sup>Here, we ignore the factor  $(-1)^{D-1}$  because it does not affect the physical invariant.

**Remark 1.3.** The link number (1.31) is topological under the deformation that does not cross the point  $x_i$ .

For simplicity, we will be considering n=1 case only for the present. (1.29) and (1.30) are

$$\langle \{ d * j(x) \} \varphi(y) \rangle = -i\delta^{(D)}(x - y) \langle g(y) \rangle dx^{0} \wedge \dots \wedge dx^{D-1}, \quad \langle Q(\Sigma)\varphi(y) \rangle = -i\operatorname{Link}(\Sigma, y) \langle g(y) \rangle. \tag{1.34}$$

**Theorem**<sup>(ph)</sup> **1.1.** Noether charge is topological under the deformation  $\Omega_{\Sigma} \longmapsto \Omega'_{\Sigma} = \Omega_{\Sigma} \cup \Omega_{0}$  such that y does not belong to  $\Omega_{0}$ . This is the topological expression of the conversation law of Noether charge.

*Proof.* Under the defirmation  $\Omega_{\Sigma} \longmapsto \Omega'_{\Sigma} = \Omega_{\Sigma} \cup \Omega_{0}$ , Nother charge transforms as follows.

$$\langle Q(\Sigma + \partial \Omega_0)\varphi(y)\rangle = \int_{\Omega_\Sigma \cup \Omega_0} \langle \{d * j(x)\}\varphi(y)\rangle = \underbrace{\int_{\Omega_\Sigma} \langle \{d * j(x)\}\varphi(y)\rangle}_{\langle Q(\Sigma)\varphi(y)\rangle} + \int_{\Omega_0} \langle \{d * j(x)\}\varphi(y)\rangle. \tag{1.35}$$

Here, since  $\Omega_0$  does not include the point y and d\*j(x)=0, the second term vanishes. Then we get

$$\langle Q(\Sigma + \partial \Omega_0)\varphi(y)\rangle = \langle Q(\Sigma)\varphi(y)\rangle. \tag{1.36}$$

This proves the theorem.

**Proposition**<sup>(ph)</sup> 1.7. We have the following relation.

$$\langle U(\Sigma)\varphi(y)\rangle = R\langle \varphi(y)\rangle$$
 (1.37)

Here, we defined

$$U(\Sigma) = e^{i\epsilon Q}, \quad R = e^{i\epsilon T},$$
 (1.38)

where  $\epsilon$  is the arbitary real number and T is the representation on the vector space spanned by the field  $\varphi(y)$  of the generator of the symmetry we are considering.

**Remark 1.4.**  $Q(\Sigma)$  is the representation on the Hilbert space of the generator of the symmetry.

#### 1.1.3 New perspective on symmetries

**Observation**(ph) 1.1. It may be possible to generalise symmetries as

symmetry generator = topological operator. 
$$(1.39)$$

#### 1.2 Definition of higher-form symmetries

Most parts of this section are adopted from the materials below.

[3] P. R. S. Gomes, "An introduction to higher-form symmetries"

# 2 Applications

# 2.1 Ginzburg-Landau theory \*

## References

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- [2] M. Srednicki, Cambridge University Press, 2007, ISBN 978-0-521-86449-7, 978-0-511-26720-8 doi:10.1017/CBO9780511813917
- [3] P. R. S. Gomes, SciPost Phys. Lect. Notes **74** (2023), 1 doi:10.21468/SciPostPhysLectNotes.74 [arXiv:2303.01817 [hep-th]].