Dynamic Models, New Gains from Trade*

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Abstract

We propose a class of dynamic trade models, which includes dynamic Krugman, Arkolakis and Melitz-Pareto models, which satisfy an Arkolakis, Costinot, and Rodríguez-Clare (2012)-type formula in their steady state. In these models, the elasticity relevant to compute steady state welfare gains is composed of a short run and a long run component, which implies that estimation frameworks need to take these dynamics into account. The short-run component is the commonly estimated trade elasticity and the long run component contains a parameter governing the speed of adjustment of trade flows to a change in trade costs. Additionally, the mapping between the elasticity of trade flows to tariffs and non-tariff trade barriers differs from that in standard models. Quantitatively, the steady state formula predicts welfare gains broadly similar to calculations that explicitly compute the transition path. Using recent elasticity estimates of the short and long run components from Boehm, Levchenko, and Pandalai-Nayar (2023), this class of models predict welfare gains substantially greater than prior calculations based on static models.

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1. Introduction

Dynamic trade models are increasingly used to study the adjustment of economies to shocks over time. These models differ from the conventional static models of trade in several salient ways, including providing an explicit notion of sluggish adjustment to trade cost shocks. In contrast to the widely known gains from trade results in many static trade models (Arkolakis, Costinot, and Rodríguez-Clare, 2012) (ACR), however, we do not yet have a clear understanding of whether similar simple formulas apply in conventional dynamic trade models. Further, if they do, we do not yet know whether adding explicit dynamics to trade models quantitatively affects the implied gains from trade.

In this paper, we show that a large class of conventional dynamic trade models admit a simple formula for the dynamic gains from trade. Similar to the widely used static ACR formula, the dynamic gains from trade are a function of the change in domestic absorption shares and a trade elasticity. In contrast to ACR, the relevant trade elasticity is the long-run trade elasticity with respect to non-tariff trade barriers, that captures the steady state response of trade flows to a change in non-tariff trade barriers. We show that in explicitly dynamic trade models, this elasticity is a function of the short-run (standard) trade elasticity and a parameter governing the speed of adjustment of trade flows to a change in trade costs. Estimating the elasticity relevant for the dynamic gains from trade therefore requires atleast two estimates of the trade elasticity at different horizons.

The class of dynamic models we study include dynamic versions of the Krugman (XXX), Arkolakis and Melitz-Pareto models. We begin by illustrating the result in a dynamic Krugman model. Describe theoretical approach, how formula is derived

We then derive a proposition that clarifies the sufficient conditions that dynamic trade models must satisfy to have gains from trade represented by our formula. The assumptions are: XXXXXXX. Dynamic versions of commonly used models including Arkolakis and Melitz-Pareto meet this conditions, and so dynamic gains from trade in these models can be computed with appropriate elasticity estimates.

(2) Need for multiple horizon elasticity estimates to calculate dynamic gft (2b) tariff vs non tariff elasticities, estimates in the literature

Using the mapping between tariff and non-tariff elasticities and the trade elasticity estimates by Boehm, Levchenko, and Pandalai-Nayar (2023), we quantify dynamic welfare gains from trade using our formula. We contrast the implied gains from trade to those calculated by explicitly computing the transition path in a dynamic Krugman model. The formula delivers gains from trade that are very close to those implied by computing the exact model, which suggests it is a reasonable approximation of dynamic gains from trade that can be computed at very low cost.

In a second quantitative exercise, we compare the welfare gains from trade implied by our formula to those implied by static trade models using the ACR formula and the same trade elasticity. The dynamic trade models imply substantially larger welfare gains from trade.

(3) extensions

2. Warmup: Welfare Gains in a Dynamic Krugman Model

This section derives the gains from trade formula in the simplest possible setup: a dynamic version of the Krugman (1980) model. It serves to introduce the notation maintained throughout the paper, and to demonstrate what features are essential for the result to go through.

2.1 Model Setup

Consider a dynamic economy with J countries indexed by i and j, and discrete time indexed by t. Each country is populated by a representative consumer who consumes C_{jt} and inelastically supplies labor L_j.

Households. Taking prices as given, consumers in country j maximize

$$\max_{\{C_{jt}\}} \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\gamma}}{1-\gamma}$$

subject to the budget constraint

$$P_{jt} C_{jt} + \frac{B_{jt}}{1 + r_{jt}^n} = w_{jt} L_j + \Pi_{jt} + R_{jt}^g + B_{jt-1}, \tag{2.1}$$

and a no-Ponzi game condition. Here, P_{jt} is the consumption price index in country j, B_{jt} are bond holdings, r_{jt}^n is the nominal interest rate, w_{jt} the nominal wage, Π_{jt} aggregate profits, and R_{jt}^g are government revenues rebated to the household. The parameters β and γ denote the household's discount factor and the coefficient of relative risk aversion, respectively. We assume that firms producing in country j are exclusively owned by the consumer in j, and hence the consumer receives all profits as income.[1]

Optimal behavior implies that consumption follows the Euler equation

$$\left(1 + r_{jt}\right) \beta C_{jt+1}^{-\gamma} = C_{jt}^{-\gamma},$$

where $1 + r_{jt} = \left(1 + r_{jt}^n\right) \frac{P_{jt}}{P_{jt+1}}$ is the real interest rate in country j.

The consumption bundle C_{jt} is a CES aggregate of quantities $q_{ijt}(\omega)$ supplied by firms indexed by ω, from all countries i serving market j:

$$C_{jt} = \left(\sum_i \int_{\Omega_{ijt}} q_{ijt}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}.$$

Ω_{ijt} denotes the endogenous set of varieties produced in country i and available for purchase in

[1] All the results go through if we instead assume that the home consumers receive a constant fraction of aggregate profits.

country j and σ is the demand elasticity. Demand for each variety ω and the ideal price index satisfy:

$$q_{ijt}(\omega) = C_{jt}\left(\frac{p^c_{ijt}(\omega)}{P_{jt}}\right)^{-\sigma}, \tag{2.2}$$

$$P_{jt} = \left(\sum_i \int_{\Omega_{ijt}} \left(p^c_{ijt}(\omega)\right)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}},$$

where $p^c_{ijt}(\omega)$ is the price faced by the consumer in country j.

Firms. Firms are monopolistically competitive, face a downward-sloping demand curve given by (2.2), and take the ideal price index as given. The production function is linear in labor. Shipments from country i to j are subject to iceberg transport costs κ_{ijt}, so that

$$q_{ijt}(\omega) = \frac{1}{\kappa_{ijt}} l_{ijt}(\omega),$$

where $l_{ijt}(\omega)$ is the firm's labor input for producing for market j. The marginal cost of serving market j is therefore $\kappa_{ijt} w_{it}$. Profit-maximizing firms charge a constant markup over marginal cost:

$$p^x_{ijt}(\omega) = \frac{\sigma}{\sigma-1}\kappa_{ijt} w_{it},$$

where $p^x_{ijt}(\omega)$ is the price received by the exporter. As a result, per-period profits are a constant fraction of firm revenue:

$$\pi_{ijt}(\omega) = \frac{1}{\sigma} p^x_{ijt}(\omega)\, q_{ijt}(\omega) = \frac{1}{\sigma} x_{ijt}(\omega). \tag{2.3}$$

Entry. Every period there is a unit mass of potential firms that can enter market j from i. Entry is subject to a stochastic sunk cost of $\xi^s_{ijt}(\omega)$, which is denominated in units of labor. A firm ω from i that pays the sunk costs in period t sells to j from $t+1$ until it exits. Exit is random and occurs with probability δ. The value of exporting is therefore

$$v_{ijt}(\omega) = \frac{1}{1+r^n_{it}}\left(\pi_{ijt+1}(\omega) + (1-\delta)\, v_{ijt+1}(\omega)\right). \tag{2.4}$$

A potential entrant enters if the value of exporting exceeds the sunk cost of entry. The marginal firm's sunk costs $\bar{\xi}^s_{ijt}$ satisfy

$$v_{ijt}(\omega) = w_i \bar{\xi}^s_{ijt}(\omega). \tag{2.5}$$

Since all operating firms in the model are identical, we will drop the firm subscript ω going forward.

Denote by n_{ijt} the mass of exporters from i to j. Its law of motion is

$$n_{ijt} = (1-\delta)\, n_{ijt-1} + G\left(\bar{\xi}^s_{ijt-1}\right),$$

where G denotes the cumulative distribution function of ξ^s_{ijt}.

Tariffs, Aggregates, and Market Clearing. Let τ_{ijt} denote *gross* ad valorem tariffs.[2] Then the prices paid by the consumers and prices received by the exporters satisfy the relationship $p^c_{ijt}(\omega) = \tau_{ijt} p^x_{ijt}(\omega)$, and the government collects $(\tau_{ijt} - 1) p^x_{ijt}(\omega)$ revenue per unit sold.

Total exports from i to j non-inclusive of tariff payments are:

$$X_{ijt} = \int_{\Omega_{ijt}} x_{ijt}(\omega)\, d\omega = n_{ijt} x_{ijt}. \tag{2.6}$$

The tariff revenue of government j is

$$R^g_{jt} = \sum_i \left(\tau_{ijt} - 1\right) X_{ijt}.$$

Profits in country j are

$$\Pi_{jt} = \sum_{i=1}^{n} \int_{\Omega_{jit}} \pi_{jit}(\omega)\, d\omega - \sum_{i=1}^{n} \int_{\Omega^e_{jit}} w_{jt} \xi^s_{jit}(\omega)\, d\omega. \tag{2.7}$$

where $\Omega^e_{jit} = \left\{\omega \in [0,1] : \bar{\xi}^s_{jit} \geq \xi^s_{jit}(\omega)\right\}$ is the set of entrants. Trade is balanced, so that in all countries j and periods t

$$w_{jt} L_j + \Pi_{jt} + R^g_{jt} = \sum_{i=1}^{n} X_{jit}. \tag{2.8}$$

Trade balance trivially implies that all bond positions are zero: $B_{jt} = 0$. We include the bond in the households' optimization problems only to pin down the interest rates.

2.2 Steady State Welfare Gains from Trade

In this subsection, we abstract from tariff revenues: $\tau_{ijt} = 1$ for all i and j, implying that $R^g_{jt} = 0$. The steady state objects are denoted by suppressing the time subscripts. From the budget constraint (2.1), real consumption is:

$$C_j = \frac{w_j L_j + \Pi_j}{P_j}. \tag{2.9}$$

We will denote the gross proportional gains from trade as the ratio of real consumption under the current trade regime relative to autarky:

$$GFT = \frac{C_j}{C^{AUT}_j}.$$

[2] In this notation, a 5% ad valorem tariff implies $\tau_{ijt} = 1.05$.

In the tradition following Eaton and Kortum (2002) and Arkolakis, Costinot, and Rodríguez-Clare (2012), we seek to express (2.9) as a function of the domestic trade share and exogenous parameters. We start with the standard step that the domestic trade share is:

$$\lambda_{jj} \equiv \frac{n_{jj} x_{jj}}{Y_j} = \frac{n_{jj} \left(\frac{\sigma}{\sigma-1} w_j\right)^{1-\sigma}}{P_j^{1-\sigma}}, \tag{2.10}$$

where $Y_j \equiv P_j C_j$ is total expenditure. Solving this expression for the price index and combining the result with equation (2.9) implies that real consumption satisfies:

$$C_j \propto \frac{w_j L_j + \Pi_j}{w_j \lambda_{jj}^{\frac{1}{\sigma-1}} n_{jj}^{-\frac{1}{\sigma-1}}}. \tag{2.11}$$

From here, we proceed to show that (i) aggregate profits are a constant fraction of the labor income; and that (ii) the mass of domestic firms n_{jj} is a power function of λ_{jj}. To compute profits and the mass of entrants, we must make a distributional assumption on the sunk costs of entry. We assume that the sunk costs are drawn from an inverse Pareto distribution:

$$G\left(\xi^s\right) = \left(b\xi^s\right)^{\chi},$$

defined over the domain $0 < \xi^s \leq \frac{1}{b}$ for given parameters $\chi, b > 0$. We assume throughout that b is sufficiently small to ensure that not all potential entrants find it worthwhile to enter in any given period ($\bar{\xi}^s_{ijt} < \frac{1}{b}$ for all t). Under this assumption the steady state mass of firms becomes

$$n_{ji} = \frac{1}{\delta} \left(b \bar{\xi}^s_{ji}\right)^{\chi}. \tag{2.12}$$

Since $1 + r_j = 1/\beta$ in the steady state, the value of selling to i is:

$$v_{ji} = \frac{\beta}{1 - \beta(1-\delta)} \pi_{ji} = \frac{\beta}{1 - \beta(1-\delta)} \frac{1}{\sigma} x_{ji},$$

and the threshold sunk cost of entry is:

$$\bar{\xi}^s_{ji} = \frac{\beta}{1 - \beta(1-\delta)} \frac{1}{\sigma} \frac{x_{ji}}{w_j}. \tag{2.13}$$

Combining (2.7), (2.8), (2.12), and (2.13) leads to the desired result that total profits are a constant multiple of labor income:

$$\Pi_j = \frac{\frac{1}{\sigma}\left(1 - \frac{\chi}{\chi+1} \frac{\beta}{1-\beta(1-\delta)} \delta\right)}{1 - \frac{1}{\sigma}\left(1 - \frac{\chi}{\chi+1} \frac{\beta}{1-\beta(1-\delta)} \delta\right)} w_j L_j. \tag{2.14}$$

Finally, starting with the expression for steady state n_{jj} in (2.12), and combining it with (2.13), (2.14), and the expression for domestic sales x_{jj} leads to:

$$n_{jj} \propto \lambda_{jj}^{\frac{\chi}{1+\chi}}. \tag{2.15}$$

Combining (2.11) with (2.14) and (2.15) yields the desired result that real consumption is proportional to the domestic trade share:

$$C_j \propto \lambda_{jj}^{-\frac{1}{\sigma-1}} n_{jj}^{\frac{1}{\sigma-1}} = \lambda_{jj}^{-\frac{1}{(\sigma-1)(1+\chi)}}. \tag{2.16}$$

Since in autarky $\lambda_{jj} = 1$, (2.16) is also the gains from trade.

Note the difference with the ACR formula for the "classic" Krugman model, $\lambda_{jj}^{\frac{1}{\sigma-1}}$, which would obtain in a static setting in which n_{jj} is either exogenously fixed or constant across equilibria. Compared to the classic case, the gains from trade are moderated because international trade leads to the reduction in domestic varieties. The log change in real consumption following a change in the domestic trade share can be written as:

$$\begin{aligned} d \ln C_j &= -\frac{1}{\sigma-1} d \ln \lambda_{jj} + \frac{1}{\sigma-1} \frac{d \ln n_{jj}}{d \ln \lambda_{jj}} d \ln \lambda_{jj} \\ &= \underbrace{-\frac{1}{\sigma-1} d \ln \lambda_{jj}}_{\text{Gain from foreign varieties}} + \underbrace{\frac{1}{\sigma-1} \frac{\chi}{1+\chi} d \ln \lambda_{jj}}_{\text{Loss of domestic varieties}}. \end{aligned}$$

The first term is the usual direct effect of the change in the interior trade share, interpreted as the utility gains from the availability of foreign goods. It increases with trade openness (recall that an increase in trade openness is a fall in λ_{jj}). The second term is the utility reduction from the loss of domestic varieties, as an increase in trade openness unambiguously lowers n_{jj}. It contributes negatively to the gains from trade. In this case, however, the net gain from openness is positive.

The long-run trade elasticity. An important part of the appeal of the ACR result is that the exponent on the domestic trade share is the inverse of the trade elasticity. We now show that the dynamic gains from trade share this feature. Recall that bilateral trade flows are given by (2.6). The long-run trade elasticity with respect to iceberg trade costs therefore has the following components:

$$\frac{\partial \ln X_{ij}}{\partial \ln \kappa_{ij}} = \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} + \frac{\partial \ln n_{ij}}{\partial \ln \kappa_{ij}} \tag{2.17}$$

It is immediate that the $\frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} = 1 - \sigma$, as usual. From (2.12) and (2.13), $\frac{\partial \ln n_{ij}}{\partial \ln \kappa_{ij}} = \chi(1-\sigma)$. Together, the long-run trade elasticity is

$$\frac{\partial \ln X_{ij}}{\partial \ln \kappa_{ij}} = (1+\chi)(1-\sigma),$$

and thus the gains from trade formula (2.16) features the inverse of the trade elasticity. Note that as in ACR and elsewhere in the literature, this is a partial elasticity, that ignores the general-equilibrium changes in expenditures, wages, and prices.

3. General Result

Proposition 3.1. *Consider a class of dynamic models, which satisfy the following three conditions in their steady state:*

A.1 For all countries j, trade is balanced (expenditure = revenue)

$$Y_j = w_j L_j + \Pi_j,$$

where $Y_j = C_j P_j$.

A.2 For all countries j, profits are a constant share of GDP

$$\frac{\Pi_j}{Y_j} = const$$

A.3 For all country pairs (i, j) trade flows satisfy

$$X_{ij} = n_{ij} x_{ij} \tag{3.1}$$

where

$$x_{ij} \propto Y_j \left(\kappa_{ij} \frac{w_i}{P_j} \right)^{-\varepsilon_\kappa^0} \tag{3.2}$$

$$n_{ij} \propto \left(\frac{x_{ij}}{w_i} \right)^{\chi} \tag{3.3}$$

for some constants ε_κ^0 and χ.

Then

$$C_j \propto \left(\lambda_{jj} \right)^{-\frac{1}{(1+\chi)\varepsilon_\kappa^0}} \tag{3.4}$$

where $\lambda_{jj} = \frac{X_{jj}}{Y_j}$, and $-(1+\chi)\varepsilon_\kappa^0$ is the long-run elasticity of trade flows with respect to iceberg trade costs.

Proof. See Appendix A. □

Note that since $\lambda_{jj} = 1$ in autarky, (3.4) is also the gross proportional gains from trade in steady state. Assumptions A.1 and A.2 are identical to R1 and R2 in ACR. Assumption A.3 puts structure on supply and demand. Condition (3.1) stipulates that total exports from i to j can be written as a product of some generic mass n_{ij} and sales per unit mass of sellers x_{ij}. This in and of itself is without loss of generality, as we can in principle always express total exports as some (average) sales per firm/variety/HS code/etc times the total number/mass of those units. The rest of A.3 puts structure x_{ij} and n_{ij}. Condition (3.2) states that demand per unit mass is CES. It essentially corresponds to ACR's R3.

Finally, assumption (3.3) is new. It puts a specific structure on how entry occurs. Qualitatively, it is intuitive: entry increases in the ratio of sales to unit costs. Section 2, in particular equations (2.3) and (2.5), illustrate how this can arise: the value of exporting scales with per period profits, which are in turn proportional to sales (numerator). Fixed costs are paid in terms of domestic labor (denominator). However, the proposition require more than that: it requires that entry is exponential in this ratio. This places a restriction on the nature of the entry decision. Section 2 shows that the Pareto distribution satisfies this restriction.

3.1 Mapping from specific models

Section 2 shows that the dynamic Krugman model satisfies the conditions of Proposition 3.1. We now go through two more commonly used dynamic models: the customer base model and the Melitz-Pareto model.

Customer base model. In the customer base model (e.g. Arkolakis, 2010; Drozd and Nosal, 2012; Gourio and Rudanko, 2014; Fitzgerald, Haller, and Yedid-Levi, 2023), firms gradually build up the mass of customers they serve. Let there be a country i representative firm that faces downward-sloping demand (2.2) per unit mass of customers in country j. As above, its profits per unit mass of customers are given by (2.3). Let n_{ijt} be the mass customers that the firm serves. This mass depreciates at rate δ and can be built up by investment a_{ijt}, that acts with a one-period lag. Thus, the customer mass evolves according to:

$$n_{ijt} = (1-\delta)\, n_{ijt-1} + a_{ijt-1}. \tag{3.5}$$

Investment has a cost $f(a_{ijt})$. The firm chooses the path of customer base investment to maximize the NPV of profits:

$$\max_{\{a_{ijt+s}\}} \sum_{s=0}^{\infty} m^{n}_{it,t+s} \left[n_{ijt+s}\pi_{ijt+s} - w_{it} f\left(a_{ijt+s}\right)\right] \tag{3.6}$$

subject to (3.5), where $m^{n}_{it,t+s}$ is the firm's discount factor. The first-order conditions of this problem can be manipulated to yield:

$$w_{it} f'\left(a_{ijt}\right) = v_{ijt} \tag{3.7}$$

$$v_{ijt} = \frac{1}{1+r^{n}_{it}} \left(\pi_{ijt+1} + (1-\delta)\, v_{ijt+1}\right), \tag{3.8}$$

where we assumed that the discount factor of the firm coincides with that of the representative consumer. Let the cost of accessing customers be given by the following functional form:

$$f\left(a_{ijt}\right)=\frac{\chi}{(1+\chi)\,\zeta}\left(a_{ijt}\right)^{\frac{1}{\chi}+1}. \tag{3.9}$$

Then, in steady state:

$$n_{ij}=\frac{1}{\delta}a_{ij}=\left(\zeta\frac{v_{ij}}{w_i}\right)^{\chi}. \tag{3.10}$$

In turn, combining (2.3) and (3.8) yields the proportionality of v_{ij} to x_{ij}, verifying assumption A.3 in Proposition 3.1.

To see that Assumption A.2 is satisfied, note that aggregate profits can be written as:

$$\Pi_i=\sum_j\left(\frac{1}{\sigma}n_{ij}x_{ij}-w_i\frac{\chi}{(1+\chi)\,\zeta}\left(a_{ij}\right)^{\frac{1}{\chi}+1}\right). \tag{3.11}$$

Since a_{ij} is proportional to n_{ij}, and $a_{ij}^{\frac{1}{\chi}}$ is proportional to x_{ij}/w_i (see 3.10 for both), $\left(a_{ijt}\right)^{\frac{1}{\chi}+1}$ is proportional to $n_{ij}x_{ij}$, and w_i cancels out in the consumer base cost term.

The deeper microfoundation, and thus the interpretation of some equilibrium quantities (e.g. n_{ij}) or parameters (e.g. χ) are different from the Krugman model. However, this model is isomorphic to Krugman in its predictions for aggregate trade flows, and the functional forms of the trade elasticities.

Melitz-Pareto. The dynamic Melitz (2003) model differs from the Krugman model in Section 2 in 2 ways. First, firms are heterogeneous in productivity, denoted $\varphi(\omega)$. Continuing to assume constant Dixit-Stiglitz markups, the firm ω's price becomes:

$$p_{ijt}^{x}\left(\omega\right)=\frac{\sigma}{\sigma-1}\frac{\kappa_{ijt}w_{it}}{\varphi(\omega)}. \tag{3.12}$$

We assume that $\varphi(\omega)$ is distributed Pareto:

$$F\left(\varphi\right)=1-\left(\frac{\varphi_L}{\varphi}\right)^{\theta}. \tag{3.13}$$

Second, in addition to needing to pay one-time sunk costs of entering a market, the firm in i needs to pay a per-period fixed cost ξ denominated in units of i's labor in order to serve market j. As in Section 2, each firm must pay a stochastic sunk cost $\xi_{ijt}^{s}\left(\omega\right)$ to enter market j, drawn from an inverse Pareto distribution. Paying this sunk cost also reveals to the firm its productivity for serving market j. Thus, the entry decision is made based on expected profits.

Not all firms that draw a sunk cost will end up exporting. The marginal firm earns variable profits that just cover the per-period fixed costs: $\frac{1}{\sigma}x_{ijt}\left(\omega\right)=w_{it}\xi$. Combining (2.2) and (3.12) (and noting that without tariffs $p^x_{ijt}(\omega) = p^c_{ijt}(\omega)$) leads to the productivity cutoff for selling from i to j:

$$\varphi^m_{ijt} = \frac{\sigma}{\sigma-1}\kappa_{ijt}w_{it}\left(\frac{\sigma w_{it}\xi}{C_{jt}\left(P_{jt}\right)^{\sigma}}\right)^{\frac{1}{\sigma-1}}. \tag{3.14}$$

Total sales from i to j are:

$$\begin{aligned} X_{ijt} &= \int x_{ijt}(\omega)\,d\omega \\ &= n_{ijt}\int_{\varphi^m_{ijt}}^{\infty} x_{ijt}(\varphi)\,dF(\varphi) \\ &= n_{ijt}\underbrace{C_{jt}\left(P_{jt}\right)^{\sigma}\left(\left(\frac{\theta\varphi_L^{\theta}}{\theta-(\sigma-1)}\right)^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}\frac{\kappa_{ijt}w_{it}}{\left(\varphi^m_{ijt}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}}\right)^{1-\sigma}}_{x_{ijt}}, \end{aligned} \tag{3.15}$$

where the last line comes from applying the Pareto distribution. Relative to the Krugman model, there is the extra complication that the average sales per firm are affected by entry/exit of the marginal firms – movements in φ^m_{ijt}. Combining (3.14) and (3.15) leads to the following expression for φ^m_{ijt}:

$$\varphi^m_{ijt} = \left(\frac{\theta\varphi_L^{\theta}\xi\sigma}{\theta-(\sigma-1)}\frac{w_{it}}{x_{ijt}}\right)^{\frac{1}{\theta}}. \tag{3.16}$$

In turn, combining (3.15) and (3.16) produces the following expression for x_{ijt}:

$$x_{ijt} \propto \left(\frac{Y_{jt}}{w_{it}}\right)^{\frac{\theta-(\sigma-1)}{\sigma-1}} Y_{jt}\left(\frac{\kappa_{ijt}w_{it}}{P_{jt}}\right)^{-\theta}. \tag{3.17}$$

If A.2 holds, then the ratio Y_{it}/w_{it} is constant and this expression for representative sales conforms to A.3 in Proposition 3.1. We will show below that A.2 holds.

At the time sunk costs are paid, the expected profits are:

$$E\left[\pi_{ijt}(\omega)\right] = \frac{1}{\sigma}x_{ijt}(\tilde{\varphi}) - w_{it}\xi\left(\frac{\varphi_L}{\varphi^m_{ijt}}\right)^{\theta}. \tag{3.18}$$

Combining with (3.16) leads to the familiar results that expected profits are a constant fraction of expected sales: $E\left[\pi_{ijt}(\omega)\right] = \frac{\sigma-1}{\theta}\frac{x_{ijt}}{\sigma}$. Since (2.12) and (2.13) hold unchanged in the Melitz model (with the qualification that here, x_{ij} is expected, rather than representative firm, sales), they lead to (3.3), and Assumption A.3 is satisfied.

To see that A.2 is satisfied, note that the steady state profits to coutry i firms from selling to j are:

$$\Pi_{ij} = \frac{\sigma-1}{\theta}\frac{1}{\sigma}n_{ij}x_{ij} - w_i \int_0^{\bar{\xi}^s{}_{ij}} \xi^s g\left(\xi^s\right) d\xi^s \tag{3.19}$$
$$= \frac{\sigma-1}{\theta}\frac{1}{\sigma}n_{ij}x_{ij} - \frac{\chi}{\chi+1}\beta\frac{\sigma-1}{\theta\sigma}x_{ij}n_{ij} \tag{3.20}$$
$$= \left(1-\beta\frac{\chi}{\chi+1}\right)\frac{\sigma-1}{\theta\sigma}X_{ij}, \tag{3.21}$$

where the second line uses the distributional assumpion on the sunk costs ξ^s. Summing across destinations and imposing trade balance delivers Assumption A.2.

We obtain the familiar result that the elasticity x_{ij} with respect to trade costs is no longer the elasticity of substitution, but the Pareto curvature parameter. Relative to the Krugman model, following a change in trade costs, average sales per unit mass will change both because of the intensive margin (all firms' sales change) and the extensive margin (marginal firms entering/exiting). As in **?** and ACR, when it comes to x_{ij}, those two margins net effect is captured by θ.

Differently from those static models, and along the lines of the Krugman model in Section 2, the gains from trade are conditioned not just by θ, but also by the curvature of the sunk costs χ, due to the adjustment of the mass of firms n_{ij}. Thus, the Melitz extension retains the intuitions laid out in Section 2.

3.2 Generalization to Tariffs

Often, trade elasticities are estimated using variation in tariffs. To build up towards measurement and quantification, we state a generalization of Proposition 3.2 to a case with tariffs.

Proposition 3.2. *Consider a class of dynamic models, which satisfy the following three conditions in their steady state:*

A.1′ For all countries j, trade is balanced (expenditure = revenue)

$$Y_j = w_j L_j + \Pi_j + R_j^g,$$

where $Y_j = C_j P_j$ and $R_j^g = \sum_i \left(\tau_{ij}-1\right) X_{ij}$.

A.2′ For all countries j, profits are a constant share of labor income

$$\frac{\Pi_j}{w_j L_j} = const$$

A.3′ For all country pairs (i, j) trade flows satisfy

$$X_{ij} = n_{ij} x_{ij}$$

where

$$x_{ij} \propto \frac{1}{\tau_{ij}} Y_j \left(\tau_{ij} \kappa_{ij} \frac{w_i}{P_j} \right)^{-\varepsilon_\kappa^0} \tag{3.22}$$

$$n_{ij} \propto \left(\frac{x_{ij}}{w_i} \right)^{\chi} \tag{3.23}$$

for some constants ε_κ^0 and χ.

Then

$$C_j \propto \lambda_{jj}^{-\frac{1}{\varepsilon_\kappa^0 (1+\chi)}} \left(1 - \frac{R_j^g}{Y_j} \right)^{-\left(1 + \frac{\chi}{1+\chi} \frac{1}{\varepsilon_\kappa^0}\right)}, \tag{3.24}$$

and $-(1+\chi)\varepsilon_\kappa^0$ is the long-run elasticity of trade flows with respect to iceberg trade costs.

Proof. See Appendix A. □

Note that since $\lambda_{jj} = 1$ and $R_j^g = 0$ in autarky, (3.24) is also the gross proportional gains from trade in steady state. Because tariffs generate revenue, (3.24) differs from (3.4) by the multiplicative factor that is a function of one minus the tariff revenue share in final expenditure. This multiplicative factor is greater than 1 as long as tariff revenue is positive. Thus, it amplifies the gains from trade relative to the no-tariff formula, conditional on the same λ_{jj}.

In static models, this tariff adjustment to the ACR formula was first stated by Felbermayr, Jung, and Larch (2015). We show that it operates in a similar way in a dynamic setting. As in Felbermayr, Jung, and Larch (2015), the exponent on the tariff adjustment term cannot be recovered from the long-run trade elasticity on its own. We show below how to recover this exponent from estimates of short- and long-run trade elasticities.

The data requirements for computing (3.24) are low. In addition to the domestic trade share, all it additionally requires is the total tariff revenue as share of GDP. This information is often available in national accounts. For the quantification below, we will require bilateral ad valorem tariff rates. Thus, it will be convenient to state the following alternative functional form for this adjustment factor:

$$1 - \frac{R_j^g}{Y_j} = \sum_i \frac{1}{\tau_{ij}} \lambda_{ij},$$

where $\lambda_{ij} \equiv \frac{\tau_{ij} X_{ij}}{Y_j}$ is the tariff-inclusive expenditure shares on goods from i.

4. Measurement and Quantification

4.1 Measurement: Trade Elasticities

As in ACR, the gains from trade in this class of dynamic models is a function of the domestic absorption share and exogenous parameters. Propositions 3.1-3.2 state that the domestic share is exponentiated with the inverse of the long-run trade elasticity. In a dynamic model, the long-run trade elasticity is a function of different structural parameters than the "trade elasticity" in static models. We now show that this has important implications for how this long-run elasticity can be recovered from the data.

The exponent in the gains from trade formula is the inverse of the long-run elasticity of trade with respect to iceberg trade costs κ_{ij}:

$$\frac{d\ln X_{ij}}{d\ln \kappa_{ij}} = \frac{d\ln n_{ij}}{d\ln \kappa_{ij}} + \frac{d\ln x_{ij}}{d\ln \kappa_{ij}} = -\varepsilon_{\kappa}^{0}(1+\chi). \tag{4.1}$$

Though a few papers have used shipping cost data to compute the trade elasticity (e.g. Hummels, 2001; Adão, Costinot, and Donaldson, 2017), the large majority of existing trade elasticity estimates use tariffs. However, the trade elasticity with respect to tariffs differs from that with respect to iceberg costs. The tariff elasticity is:

$$\frac{d\ln X_{ij}}{d\ln \tau_{ij}} = \frac{d\ln n_{ij}}{d\ln \tau_{ij}} + \frac{d\ln x_{ij}}{d\ln \tau_{ij}} = \left(-\varepsilon_{\kappa}^{0}-1\right)(1+\chi), \tag{4.2}$$

The two elasticities differ because iceberg costs are reflected in the border price, whereas tariffs are not. Most (though not all) of the literature that estimates trade elasticities recognizes this distinction, and either adjusts the tariff elasticity by 1 to recover the iceberg cost elasticity, or uses trade flows inclusive of tariff payments in estimation. In a dynamic setting, however, these simple adjustments do not work, requiring another strategy to recover the iceberg elasticity.[3]

Fortunately, we show that it is possible to use tariff elasticity estimates at different horizons to reconstruct the long-run elasticity that is needed for the gains from trade formula. The key is to use estimates of both short- and long-run tariff elasticities to separately pin down ε_{κ}^{0} and $1+\chi$.

We will call the "short run" a time period over which x_{ij} can adjust but n_{ij} cannot. This is consistent with the model laid out in Section 2, in which n_{ij} only starts adjusting with a one-period lag. The short-run tariff elasticity is then:

$$\frac{d\ln X_{ijt}}{d\ln \tau_{ijt}} = \frac{d\ln x_{ijt}}{d\ln \tau_{ijt}} = -\varepsilon_{\kappa}^{0}-1. \tag{4.3}$$

It is immediate that with both the short- and long-run tariff elasticities (4.2)-(4.3) in hand, one can recover both ε_{κ}^{0} and $1+\chi$, and reconstruct the object needed for the welfare gains formula, (4.1). This is the strategy we will pursue in the quantification that follows.

[3]For example, in a static Armington model the tariff elasticity is $-\sigma$, while the iceberg cost elasticity is $-(\sigma-1)$. So simply adding 1 to the tariff elasticity would recover the iceberg elasticity. It is immediate from (4.1)-(4.2) that this won't work in the dynamic setting. It is also easy to verify that the long-run tariff elasticity of tariff-inclusive trade flows $\tau_{ij}n_{ij}x_{ij}$ also does not recover the needed object (4.1).

4.2 Calibration

To compute the gains from trade using (2.16) requires estimates of the trade elasticity in the short and the long run, as discussed above, in addition to the domestic absorption share λ_{jj}. Further, when ad-valorem tariffs are non-zero, tariff revenue enters the gains from trade formula, and the domestic absorption share is computed using tariff-inclusive trade as a share of tariff-inclusive output. We use trade elasticity estimates from Boehm, Levchenko, and Pandalai-Nayar (2023). Their short-run tariff elasticity estimate implies $\sigma = XX$, and their long-run tariff elasticity estimate implies $\chi = XX$. To compute tariff revenue, we multiply the bilateral, product-level applied tariffs obtained from TRAINS with bilateral trade flows from BACI for 2006. Our sample includes XX countries and a rest-of-the-world. To obtain an effective tariff rate for the whole economy, which includes services trade flows, we divide this bilateral tariff revenue by total bilateral trade for 2006, obtained from the OECD inter-country input output tables. This assumes that services trade is subject to a 0% tariff rate. We validate our tariff revenue measures by comparing with tariff revenue obtained from the World Bank for 2006. As the World Bank tariff revenue data are provided in local currency, we convert them to US dollars using an annual exchange rate obtained from the same source. Appendix figure XX illustrates that our baseline tariff revenue measures are very similar to those obtained from the World Bank. Finally, we construct tariff-adjusted domestic absorption shares using the effective economy-wide bilateral tariff rates.

4.3 Welfare gains from trade in dynamic models

4.4 Comparison between formula and full model

non-linear with transition path.

linearized model.

5. Conclusion

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Appendix

A. Theory Appendix

Proof of Proposition 3.1. From A.1 and A.2, real consumption is proportional to the real wage:

$$C_j \propto \frac{w_j}{P_j}. \tag{A.1}$$

From A.3, the price index

$$P_j \propto w_j \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0}} n_{jj}^{-\frac{1}{\varepsilon_\kappa^0}}. \tag{A.2}$$

From A.3, the mass of firms

$$n_{jj} \propto \lambda_{jj}^{\frac{\chi}{1+\chi}}, \tag{A.3}$$

where we also used A.1. Putting (A.1)-(A.3) together yields the first result.

To derive the last claim, note that:

$$\frac{\partial \ln X_{ij}}{\partial \ln \kappa_{ij}} = \frac{\partial \ln n_{ij}}{\partial \ln x_{ij}} \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} + \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}}.$$

It is immediate from 3. that $\frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} = -\varepsilon_\kappa^0$, and $\frac{\partial \ln n_{ij}}{\partial \ln x_{ij}} = \chi$, which gives the result.

Proof of Proposition 3.2. From A.1′,

$$Y_j = \left(1 - \frac{R_j^g}{Y_j}\right)^{-1} \left(w_j L_j + \Pi_j\right)$$

From A.2′,

$$C_j \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-1} \frac{w_j}{P_j} \tag{A.4}$$

From A.3′,

$$\frac{w_j}{P_j} = \lambda_{jj}^{-\frac{1}{\varepsilon_\kappa^0}} n_{jj}^{\frac{1}{\varepsilon_\kappa^0}} \tag{A.5}$$

Also from A.3′,

$$n_{jj} \propto \lambda_{jj}^{\frac{\chi}{1+\chi}} \left(1 - \frac{R_j^g}{Y_j}\right)^{-\frac{\chi}{1+\chi}}. \tag{A.6}$$

Where we also used A.1′. Putting (A.4)-(A.6) together yields the first result. The proof of the claim about the trade elasticity is identical to Proposition 3.1.