A Baseline RBC Model

David Murakami

17$^{\text{th}}$ April, 2024

Ramsey social planner problem

Consider a Hansen-style RBC model.Final good firms use the production function $Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}$. The representative household utility function is $u(C_t, L_t) = \ln C_t - \chi L^{1+\nu^{-1}}/(1+\nu^{-1})$. Set $\alpha = 0.3$, $\nu = 2$, $\chi = 4.5$, $\beta = 0.99$, $\delta = 0.025$, $\rho_a = 0.95$, and $\sigma_a = 0.01$.

As there are no distortions, we can solve the model from the perspective of the Ramsey social planner who aims to

$$\max_{\{C_t, L_t, K_t\}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\ln C_{t+i} - \chi \frac{L_{t+i}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right),$$

subject to

$$Y_t = C_t + I_t, \tag{1}$$

$$K_t = I_t + (1-\delta) K_{t-1}, \tag{2}$$

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{3}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t^a, \quad \varepsilon_t^a \overset{IID}{\sim} \mathcal{N}(0, \sigma_a^2). \tag{4}$$

Solving the problem yields the following first-order conditions:

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}} \left(\alpha \frac{Y_{t+1}}{K_t} + 1 - \delta \right) \right],$$

$$\chi L_t^{\frac{1}{\nu}} = \frac{(1-\alpha) Y_t}{C_t L_t}.$$

Define the marginal value of an additional unit of capital in period $t+1$ as

$$R_{t+1} = \alpha \frac{Y_{t+1}}{K_t} + 1 - \delta, \tag{5}$$

and define the marginal product of labour as the real wage,

$$W_t = (1-\alpha)\frac{Y_t}{L_t} \tag{6}$$

Use these to write the FOCs as:

$$\frac{1}{C_t} = \beta \mathbb{E}_t\left[\frac{R_{t+1}}{C_{t+1}}\right], \tag{7}$$

$$W_t = \chi L_t^{\frac{1}{\nu}} C_t. \tag{8}$$

Centralised equilibrium. The equilibrium is a set of prices, R_t and W_t; allocations, Y_t, C_t, L_t, I_t, and K_t; and productivity, A_t, which satisfy eight equations, (1)-(8).

Steady state. Steady state quantities are found by first using the fact that $A = 1$, and from (5) and (7) we have:

$$\begin{aligned} \frac{1}{\beta} &= \alpha\left(\frac{K}{L}\right)^{\alpha-1} + 1 - \delta \\ \implies \frac{K}{L} &= \left(\frac{\alpha}{\beta^{-1} - 1 + \delta}\right)^{\frac{1}{1-\alpha}}. \end{aligned}$$

Then express W in (6) as:

$$W = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}.$$

From (2) we know:

$$I = \delta K.$$

Then, combine this with (1) and (3) to get the consumption-labour ratio:

$$\frac{C}{L} = \left(\frac{K}{L}\right)^{\alpha} - \delta\left(\frac{K}{L}\right).$$

Next, turn to the intratemporal Euler equation (8), and multiply both sides by L:

$$\chi L^{\frac{1+\nu}{\nu}} = \frac{(1-\alpha)L}{C}\left(\frac{K}{L}\right)^{\alpha},$$

and then rearrange to get C/L:

$$\frac{C}{L} = \frac{(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}}{\chi L^{\frac{1+\nu}{\nu}}}.$$

Set this equal to the previous expression for C/L to solve for L:

$$L = \left\{ \frac{(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}}{\chi\left[\left(\frac{K}{L}\right)^{\alpha} - \delta\frac{K}{L}\right]} \right\}^{\frac{\nu}{1+\nu}}.$$

With L in hand, turn back to the capital-labour ratio to obtain K:

$$K = \left(\frac{\alpha}{\beta^{-1} - 1 + \delta}\right)^{\frac{1}{1-\alpha}} L.$$

Thus, I and Y can easily be computed, after which C can obtained easily using the aggregate resource constraint (1).