Credit Chains

Kiyotaki and Moore (1997, mimeo)
presented by David Murakami

4th April 2023

Research question

How do shocks propogate through a network of firms who borrow from, and lend to, each other?

Roadmap of talk

1. Introduction
2. Basic model
   - I will not cover the section on "postponement".
3. Stochastic model with insurance

Introduction

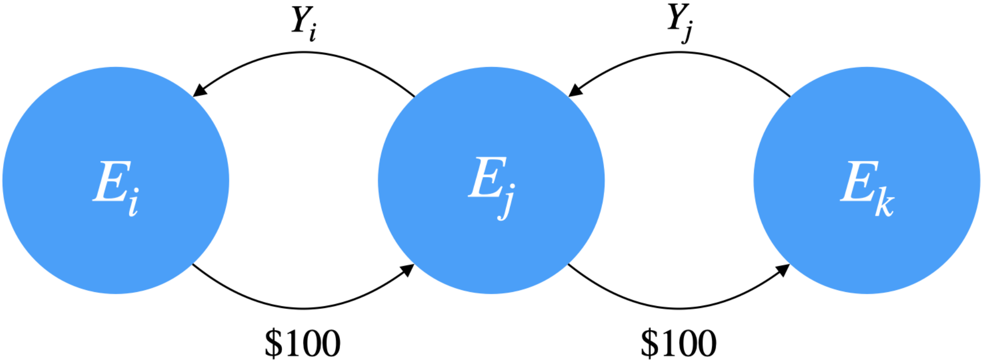
The framework and environment is as follows:

- Many firms owned by entrepreneurs who are financially constrained.
- Presence of wealthy, liquid investors.
- Entrepreneurs can borrow from suppliers – investors and other entrepreneurs – as suppliers have leverage over entrepreneurs.
- Supply contracts have to involve an element of lending, or else supplier will lost to other suppliers.
    - An entrepreneur has to lend to other entrepreneurs even if they are short on liquidity.
    - Balance sheet has financial assets (accounts receivable) and liabilities (accounts payable).
- Entrepreneur cannot net out gross positions in order to shed default risk.

Example: line of credit

- Suppose entrepreneur E_i orders 100 units of input Y_i^n from E_j at \$1 each in period t.
- E_i owes \$100 to E_j and delivery is due in $t+1$. E_i expects to have \$100 in cash to E_j.
- E_j has also ordered 100 units from E_k. E_j has no cash but anticipates to use the \$100 from E_i to pay E_k.
- Money is paid on the date at which Y^n are delivered.

Example: line of credit



Real life example: Apple Mac Pro (2013)

New York Times

Apple is unlikely to bring its manufacturing closer to home. A tiny screw illustrates why.

In 2012, Apple's chief executive, Timothy D. Cook, went on prime-time television to announce that Apple would make a Mac computer in the United States. It would be the first Apple product in years to be manufactured by American workers, and the top-of-the-line Mac Pro would come with an unusual inscription: "Assembled in USA."

But when Apple began making the $3,000 computer in Austin, Tex., it struggled to find enough screws, according to three people who worked on the project and spoke on the condition of anonymity because of confidentiality agreements.

In China, Apple relied on factories that can produce vast quantities of custom screws on short notice. In Texas, where they say everything is bigger, it turned out the screw suppliers were not.

Tests of new versions of the computer were hamstrung because a 20-employee machine shop that Apple's manufacturing contractor was relying on could produce at most 1,000 screws a day.

Real life example: Apple Mac Pro (2013)

Figure 1: Source: iFixit

Basic model

- Three period economy.
- Many entrepreneurs: E_i, $i \in [0, 1]$.
- Investors: D ("deep pockets").
- Initial endowment: M_i and $\bar{M}$ (large).
- Labour: N_i and $\bar{N}$ (large).

Storage and production

- Agents are risk neutral, and do not discount the future.
- Investors have access to a safe storage technology:

$$\bar{Y}_{t+1} = R^* \bar{N}_t,$$

  where $R^* = 1$.
- Entrepreneurs have access to a short-term technology (storage) and a long-term technology (production).
  - Storage pays $R > 1$ on stored goods. Accessible in all periods.

Storage and production

- Production is a two-stage process.
- In period t, E_i places an order on intermediate input good, Y_i^n, from another entrepreneur or investor.
- E_i cannot use own labour to make this input.
- Y_i^n is delivered in $t+1$. Then E_i produces α goods in $t+2$
- Y_i^n is specific to E_i's production; of little value to supplier. Supplier will liquidate excess stock at price $\phi < 1$ in $t+1$ (constant returns; instantaneous).

Contracting

- Any entrepreneur is free to place an order with any other entrepreneur or investor for intermediate products to be supplied in $t+1$.
- No counter-trade.
- Limited enforcement of contracts.
- Supply contracts incomplete.
- Equal treatment in default.
- Entrepreneur cannot borrow against a promise to supply.

Implicit debt/supply contract

In period t:

- E_i agrees with supplier a contract which stipulates λ_i, fraction of Y_i^n that will be liquidated by supplier in $t+1$.
- E_i makes a down-payment of $Q_i Y_i^n$, where Q_i is the down-payment price.

In period $t+1$:

- E_i pays $P_i(1-\lambda_i)Y_i^n$ goods for the delivery of $(1-\lambda_i)Y_i^n$.
- E_i's supplier liquidates $\lambda_i Y_i^n$ goods at price ϕ_i.
- Revenue of supplier is:

$$Q_i Y_i^n + P_i Y_i^n - (P_i - \phi_i)\lambda_i Y_i^n, \quad \phi_i < P_i \leq 1. \tag{1}$$

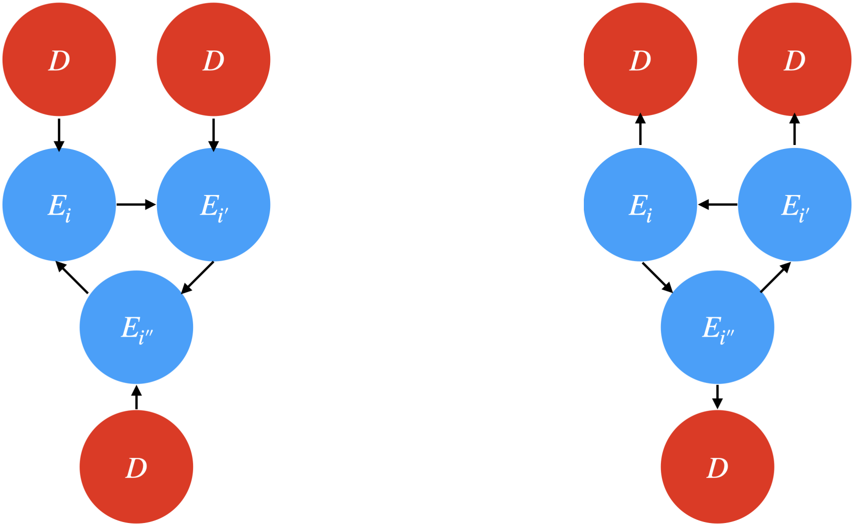
Implicit debt/supply contract

- λ_i is determined by E_i's goods holdings at date $t+1$, and Q clears market in t.
- "No arbitrage condition" for investor:

$$1 = Q + P - (P - \phi)\lambda_i, \tag{2}$$

  where $P \leq 1 \implies Q \geq 0$.
- In eqm, $AD > AS$ for Y^n by entrepreneurs; D supplies difference.
- E_i will have same λ_i to all suppliers. So contract with either D or $E_{i'}$ is the same.

Canonical network



(a) Supply chain

(b) Credit chain

No default equilibrium

No default at date $t+1$: need sufficient funds flowing in and out.

- Suppose there are no shocks, then E_i will not default (costly and wasteful).
- $\lambda_i = 0 \implies Q = 1 - P$.
- By symmetry, $\lambda_{i'} = 0$ and so $Q_{i'} = Q_i = Q = 1 - P$.

In period t:

- E_i's plans expenditure of PY_i^n in t.
- Planned income:
  - Short-term investment, Y_i, with a return of R undertaken in t.
  - Payment from customers/debtors. E_i can produce N_i goods. Anticipates PN_i.
- Flow of funds:

$$(1-P)Y_i^n + Y_i = M_i + (1-P)N_i. \tag{3}$$

Period $t+1$ flow of funds:

$$PY_i^n \leq RY_i + PN_i. \tag{4}$$

No default equilibrium – balanced investment

- If inequality in (4) is strict: surplus goods which will lower return in $t+2$.
- If equality then this is the balanced investment strategy. E_i consumes αY_i^n in $t+2$.
- Long-term return needs to be sufficient high: $\alpha > R^2$.
- Using (3) and (4) to solve for Y_i^n and Y_i:

$$Y_i^n = N_i + \frac{RM_i}{P + R(1-P)}, \tag{5}$$

$$Y_i = \frac{PM_i}{P + R(1-P)}. \tag{6}$$

- E_i consumption in $t+2$ is αY_i^n:

$$C_i = \alpha N_i + \frac{\alpha RM_i}{P + R(1-P)}. \tag{7}$$

Unexpected shock

- Suppose that a shock manifests in period $t+1$: $\hat{R} < R$.
- All entrepreneurs must default. Can only afford $\hat{Y}_i^n < Y_i^n$ and $\hat{Y}_{i'}^n < Y_{i'}^n$. By symmetry: $\hat{Y}_i^n = \hat{Y}_{i'}^n$.
- Entrepreneurs default on a pro-rata basis:

$$\lambda_i = \frac{Y_i^n - \hat{Y}_i^n}{Y_i^n}, \quad \forall i.$$

- From (1), E_i loses $(P - \phi)\lambda Y_{i'}^n$ revenue in $t+1$.

Unexpected shock

- E_i's flow of funds constraint in $t+1$:

$$P\hat{Y}_i^n = \hat{R}Y_i + P\frac{\hat{Y}_{i'}^n}{Y_{i'}^n}N_i + \phi\left(1 - \frac{\hat{Y}_{i'}^n}{Y_{i'}^n}\right)N_i \tag{8}$$

- Credit chain $\implies$ multiplier effect comprised of a direct effect and an indirect effect.

Unexpected shock: numerical example

- Suppose: $\alpha = 1.8$, $R = 1.2$, $\phi = 0.5$, $P = 1$, $M_i = 5$, $N_i = 24$, and $\hat{R} = 1.14$ (5% shock).
- Assuming symmetry: $Y_i^n = Y_{i'}^n = Y_n$.
- Rearrange (8) to get:

$$\hat{Y}^n = \frac{Y^n(\hat{R}Y + \phi N)}{Y^n + N(\phi - 1)} = 29.5.$$

- i.e., Y^n and C declines by -1.67%. Then use (5) to calculate net worth decline of 1%.
- No credit chain: Decline in net worth with (7) $\implies$ C declines by -1%.

Key multiplier mechanism

- Indirect shock is over and above direct productivity shock, $(R - \hat{R})Y$.
- E_i defaults on their suppliers, some of whom are entrepreneurs.
- These entrepreneurs default more on their suppliers until E_i is defaulted on.
- Credit chain amplifies the shock.
- Pareto improvement:
  - Entrepreneurs can charge each other γP, $\gamma < 1$.
  - In effect, they could pass their custom-made goods around the triangle for free.
- Root cause of inefficiency is lack of liquidity and coordination.

Stochastic model: setup

- Agents have rational expectations.
- Two possible states in $t+1$: boom w.p. $1-\pi$ and bust w.p. π.
  - Boom: E receives return of R.
  - Bust: Fraction θ of E receives $\underline{R} < R$; $1-\theta$ receives $\bar{R} > R$.
- No other shocks.
- Recession may be induced by a mean-preserving spread,

$$\theta \underline{R} + (1-\theta)\bar{R} = R,$$

due to chain reaction.

Stochastic model: insurance

- In period $t+1$ we cannot verify if E_i has productivity $\bar{R}$ or $\underline{R}$.
- Thus, E_i cannot insurance against individual return.
- However, E_i can insurance against aggregate observable state.

Ideal insurance would be:

- Agree to pay out in $t+1$ if one particular aggregate state occurs; receive payment in the other.
- Infeasible due to limited enforcement restriction.

Stochastic model: insurance

Typical insurance policy on offer: pay in advance

- In t, E_i purchases insurance and is paid according to aggregate state in $t+1$.
- Sub-optimal: takes away resources from investment.
- D sets aside funds at t as security. Rate of return on insurance is $R^* = 1$.
- E_i pays Z in t to get $\frac{Z}{\pi}$ in recession.

Stochastic model: insurance

- In t, E_i must choose $\{Y_i^n, Y_i, Z_i\}$.
- $E_{i'}$ chooses $\{Y_{i'}^n, Y_{i'}, Z_{i'}\}$.
- But in a symmetric equilibrium: $\{Y_i^n, Y_i, Z_i\} = \{Y^n, Y, Z\}, \forall i$.

Stochastic model: no insurance equilibrium

- Suppose that $Z_i = Z_{i'} = 0$, then following the flow of funds constraint (4):

$$PY_i^n = RY_i + PN_i. \tag{9}$$

- Boom: No fall in accounts received.
- Bust: Unproductive entrepreneurs will default. Suppose E_i is unproductive:
  - Now let $\hat{Y}_i^n$ be amount of intermediate input that E_i can afford.
  - E_i's flow of funds constraint in $t+1$ is:

$$P\hat{Y}_i^n = \underline{R}Y_i + PN_i - (P - \phi)\theta \left(\frac{Y_{i'}^n - \hat{Y}_{i'}^n}{Y_{i'}^n} \right) N_i, \tag{10}$$

from (8).

Stochastic model: equilibrium

- Then suppose that E_i draws high productivity in a bust.
- Want equilibrium where E_i has spare funds at $t+1$; $\bar{R}Y_i$ makes up for loss in accounts received:

$$PY_i^n < \bar{R}Y_i + PN_i - (P-\phi)\theta\left(\frac{Y_{i'}^n - \hat{Y}_{i'}^n}{Y_{i'}^n}\right)N_i. \tag{11}$$

- E_i can then reinvest in $t+1$ to earn R in $t+2$.

Stochastic model: equilibrium

Now, turn to period t:

- E_i will default w.p. $\pi\theta$ in $t+1$ and only take delivery of fraction $\hat{Y}_i^n / Y_i^n$ of order.
- Must make down payment of Q to satisfy:

$$1 = Q + P - (P - \phi)\pi\theta \left(\frac{Y_i^n - \hat{Y}_i^n}{Y_i^n} \right), \tag{12}$$

  which follows from equilibrium condition (2).
- E_i's flow of funds constraint is thus:

$$QY_i^n + Y_i = M_i + QN_i, \tag{13}$$

  corresponding to (3).

Stochastic model: equilibrium

E_i's expected consumption in $t+2$ is:

$$\begin{aligned} C_i &= (1-\pi)\alpha Y_i^n + \pi\theta\alpha \hat{Y}_i^n \\ &\quad + \pi(1-\theta)\left\{\alpha Y_i^n + R\left[\bar{R}Y_i + PN_i - (P-\phi)\theta\left(\frac{Y_{i'}^n - \hat{Y}_{i'}^n}{Y_{i'}^n}\right)N_i - PY_i^n\right]\right\}, \end{aligned} \tag{14}$$

where the RHS features agent's consumption following boom; consumption following a bust and agent is unproductive; and consumption following a bust and agent is productive.

Stochastic model: equilibrium

A symmetric equilibrium is where price, Q, and quantities, Y^n, Y, $\hat{Y}^n$, and C, solve a system of equations given by (9)-(14). This is a unique equilibrium if:

$$\underbrace{\alpha R}_{\text{marginal opp. cost}} > \underbrace{\underbrace{\theta \alpha R(1-\phi)}_{\text{drop in } Q} + \underbrace{\theta\alpha}_{\text{gain from production}}}_{\text{w/ insurance default is lower}} + \underbrace{(1-\theta)R}_{\text{invest insurance payout if productive}}, \tag{15}$$

for small $\underline{R} - R$ and $\bar{R} - R$.

Stochastic model: equilibrium

Comparative statics of (15):

- Condition holds tighter as $\alpha \uparrow$, $R \uparrow$, or $\phi \uparrow$, or $\theta \downarrow$.
- Rise in α pr R dominate and pushes up the opportunity cost of paying insurance premium in t.
- As $\phi \to P$, insurance does not cause Q to fall by much.
- If $\theta \downarrow$, the pr. of being productive in a bust, insurance doesn't bring much benefit.

Final remarks

- Simple and analytically tractable framework to show systematic risk: that a small temporary shock to liquidity may cause a large chain reaction.
- The longer the chain or network, and if liquidity is inadequate, the larger the disruption.
- Inability to precommit not to default and leverage relationships is what leads to credit chains.
- Even if insurance is available, agents may choose to not undertake it as its opportunity cost is too high.
- Personal opinion: I'm a big fan of Kiyotaki's work – it's only a matter of time until him and Moore win the Nobel.