CBDCs, Financial Inclusion, and Optimal Monetary Policy*

David Murakami† Ivan Shchapov‡ Ganesh Viswanath-Natraj§

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Abstract

In this paper we study the interaction between monetary policy and financial inclusion in an economy that introduces a central bank digital currency (CBDC). Using a New Keynesian two-agent framework with banked and unbanked households, we show that CBDCs provide a more efficient savings device for the unbanked to smooth consumption, increasing welfare. A Ramsey optimal policy exercise reveals that the CBDC rate is set at a constant spread to the policy rate. We observe a policy trade-off: a higher CBDC rate benefits the unbanked, but disintermediates banks and reduces welfare of banked households. Taken together, our findings highlight the role of tailoring CBDC design based on the level of financial inclusion in an economy.

Keywords: central bank digital currency, financial inclusion, optimal monetary policy, Taylor rules, welfare

JEL Classifications: E420, E440, E520, E580

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†University of Milan and University of Pavia. Email: david.murakami@unimi.it

‡Institut Polytechnique de Paris, CREST. Email: ivan.shchapov@ip-paris.fr

§University of Warwick, Warwick Business School. Email: ganesh.viswanath-natraj@wbs.ac.uk

1 Introduction

Central bank digital currency (CBDC) are digital tokens, similar to a cryptocurrency, issued by a central bank. Central banks are actively studying the potential adoption of CBDCs, and notable examples include Sweden's E-Krona and China's Digital Currency Electronic Payment. In this emerging macroeconomics literature there is a focus on the macroeconomic effects (Barrdear and Kumhof 2022; Benigno, Schilling, and Uhlig 2022; Ferrari Minesso, Mehl, and Stracca 2022; Assenmacher, Bitter, and Ristiniemi 2023; Burlon et al. 2022; Kumhof et al. 2021; Davoodalhosseini 2022), and implications for banking and financial stability (Brunnermeier and Niepelt 2019; Niepelt 2020; Fernández-Villaverde et al. 2021; Chiu et al. 2023; Keister and Sanches 2021; Skeie 2019; Whited, Wu, and Xiao 2022; Ahnert et al. 2023). While CBDCs present obvious advantages – improving cross-border payments, and facilitating fiscal transfers – there are still many unresolved issues in their design. For example, do CBDCs increase welfare of the unbanked through financial inclusion? Do CBDCs attenuate or amplify monetary policy transmission channels? How should the interest rate on the CBDC be set?

In answering these questions, our paper focuses on CBDC design and in particular the financial inclusion effects of introducing a digital currency.[1] To motivate our financial inclusion channel, we start with a simple two period, two-agent endowment economy with a representative banked household (BHH) and unbanked household (UHH). The unbanked use money subject to a cash-in-advance constraint while the banked have access to deposits.[2] This framework is tractable and allows us to derive expressions for the ratio

1. For more detail on the taxonomy of CBDC designs we refer readers to Auer and Böhme (2020). They discuss many aspects of CBDC design, such as whether the CBDC uses a distributed ledger technology (DLT), is account or token based or wholesale or retail. In this paper we focus solely on retail CBDCs.

2. The BHH and UHH can be thought of as Ricardian and non-Ricardian households, respectively, as is typical in the two-agent New Keynesian literature. See, for example, Debortoli and Galí (2017).

of lifetime consumption between the banked and unbanked households. Our analysis shows that without a digital currency, the BHH generally has higher lifetime consumption compared to the UHH, especially when facing negative income shocks, as the BHH can use deposits to smooth consumption. We then introduce an interest bearing digital currency that can be used by the UHH as an alternative to money. Digital currency provides a more efficient savings mechanism compared to money balances, and improves the resilience of the UHH to negative income shocks. This enhances welfare through a financial inclusion channel.

In the next part of the paper, we setup a two-agent New Keynesian model to include production and endogenous labor supply, monopolistic pricing of firms, a financial intermediary that lends to firms that use capital in production, and monetary policy set by a central bank. This setup allows us to consider aspects of optimal CBDC design. Using this model, we simulate the economy with respect to productivity shocks, cost-push inflationary shocks, and monetary policy shocks.

First, we address the question on whether a CBDC attenuates or amplifies the transmission of monetary policy. Our results suggest that monetary policy transmission, which we define as how well it stabilizes business cycle fluctuations, is amplified upon introducing a CBDC. The analysis assumes the central bank implements a standard Taylor rule, and the CBDC's interest rate tracks the deposit rate. In response to a raise in the policy rate, inflation, asset prices, and consumption decline. The rise in deposit rates leads to reduced bank net worth, asset prices, and bank equity, along with a decrease in the deposit base due to higher funding costs, through the financial accelerator effect. However, when a CBDC is introduced, the most notable difference lies in the response of unbanked household (UHH) consumption. With access to the CBDC as a savings tool, these households can actively counter the shock by reducing their savings, thereby attenuating the decline in UHH consumption. This moderates the overall impact on consumption and output, leading to a quicker dissipation of the shocks compared to an economy without a CBDC. In contrast, for banked households, there is little change in consumption response to monetary policy shocks upon introducing a CBDC This is because they already have access to optimal savings devices like deposits and do not significantly adjust their CBDC holdings in response to the shock. Consequently, the bank's balance sheet, net worth, and capital remain similar to the scenario without a CBDC.

Second, we use our model to evaluate the welfare effects of the introduction of the CBDC relative to an economy with no CBDC. Similar to our endowment economy, we analyze the effects of introducing a CBDC. Unbanked households benefit from access to the CBDC, as it enables them to smooth consumption with respect to macroeconomic shocks more effectively. In contrast, banked households are worse off with access to a CBDC, as it is an imperfect substitute to bank deposits and incurs management costs and due to general equilibrium effects. Furthermore, banked household consumption is lower due to a disintermediation channel. The shift of deposits to CBDC means the bank now intermediates a lower deposit base, and this has effects of reducing equilibrium lending, capital and steady state consumption for the banked. In summary, the welfare benefits of introducing a CBDC is highest for economies with a high unbanked population share and a corresponding low level of financial inclusion.

Third, we analyze elements of CBDC design, in particular the optimal path of interest rates on the central bank and CBDC rate. We conduct a Ramsey policy exercise to evaluate the path of monetary policy that maximizes welfare of households. The policy instruments include both the central bank rate on household deposits and the CBDC deposits rate. The social planner maximizes a weighted average of banked and unbanked household utility subject to the two policy instruments. This is a non-parametric estimation of interest rates that deviates from more traditional methods of interest-rate setting such as the Taylor rule. Our framework allows us to test alternative regimes for the CBDC monetary policy implementation, such as whether the CBDC rate should be adjustable or fixed. The optimal policy results show that when CBDC deposits are a near substitute to regular deposits, it is optimal for the CBDC rate to track a constant spread with the deposit rate.

Finally, we show how CBDC rates that are a spread above or below deposit rates have distributional implications on welfare. Our welfare results show that unbanked households are better off when CBDC rates are higher than the deposit rate. In contrast, banked households are worse off as the CBDC rate is higher than the deposit rate. Explaining these findings, we note that the unbanked benefit through the savings channel, where CBDC deposits receive a higher rate of interest. Alternatively, banked households are worse off when CBDC rates are high because of the disintermediation channel: as CBDC rates increase, an increase in the holdings of CBDCs leads to the deposit base of bank balance sheets to shrink. This in turn leads to a lower equilibrium levels of bank lending, capital and consumption of banked households. Our analysis sheds light upon an important trade-off to consider when setting an interest rate on CBDC: for economies with high levels of financial inclusion, it is optimal to set CBDC rates lower than the policy rate. In contrast, it may be beneficial to set higher CBDC rates in economies with low financial inclusion to incentivize unbanked households to use CBDC as a consumption smoothing device.

The remainder of the paper is structured as follows. In Section 1.1 we summarize the contributions of our paper to the related literature. In Section 2 we outline the baseline endowment economy to clarify our intuition, and examine the welfare implications of introducing a CBDC. In Section 3, we introduce a two-agent New Keynesian (TANK) model with a banking sector. Section 4 examines the effect of introducing a CBDC on monetary policy, includes optimal policy exercises for when a social planner can set interest rates on deposits and digital currencies, and examines the welfare implications of alternative rules for targeting the interest rate on the CBDC. Section 5 concludes the paper.

1.1 Related Literature

Our work relates to an emerging literature on CBDCs. The first strand of literature is on the implications for financial stability (Brunnermeier and Niepelt 2019; Niepelt 2020; Fernández-Villaverde et al. 2021; Agur, Ari, and Dell'Ariccia 2022; Andolfatto 2021; Chiu et al. 2023; Keister and Sanches 2021; Benigno 2019; Skeie 2019; Ramadiah, Galbiati, and Soramäki 2021; Hemingway 2022; Kim and Kwon 2022; Whited, Wu, and Xiao 2022; Ahnert et al. 2023; Keister and Monnet 2022), the role of privacy and microeconomics of payments in CBDCs (Ahnert et al. 2023; Liu, Reshidi, and Rivadeneyra 2023) and the competition between CBDC and alternative payment systems (Cong and Mayer 2021; Chiu et al. 2023; Ahnert et al. 2023). Financial stability considerations include studying the competition between bank deposits and CBDCs. For example, Keister and Sanches (2021) determine conditions in which the private sector is disintermediated with CBDC leading to welfare losses. Chiu et al. (2023) study the role of CBDCs when banks have market power, and show the introduction of CBDCs can lead to increased competition among banks, an increase in deposit rates and lending raising welfare. In contrast to these papers, our study focuses on the benefits of CBDCs in a two-agent framework. By studying households that do not have access to a financial asset, we focus on the financial inclusion benefits of a retail CBDC. The novelty of our framework in this literature is to include an additional set of households (the unbanked) that do not have access to domestic banking channels. Critically, the unbanked only have access to digital currency as a medium of exchange and savings vehicle. Within this literature we are the first paper to evaluate the welfare benefits of retail CBDC designs.

The next strand of literature is on understanding the macroeconomic effects of introducing a CBDC. There is an emerging literature that deals with the closed economy (Barrdear and Kumhof 2022; Assenmacher, Bitter, and Ristiniemi 2023; Burlon et al. 2022) and open economy implications (George, Xie, and Alba 2020; Ikeda 2020; Benigno, Schilling, and Uhlig 2022; Kumhof et al. 2021). Closed economy considerations include a discussion of optimal monetary policy and transmission effects Burlon et al. (2022), Davoodalhosseini (2022), and Das et al. (2023), the use of CBDC in a monetarist framework Assenmacher, Bitter, and Ristiniemi (2023), and the introduction of CBDC on output and the ability to stabilize business cycle fluctuations Barrdear and Kumhof (2022). Open economy considerations include Benigno, Schilling, and Uhlig (2022) which derives an equilibrium result of synchronization of interest rates across the two countries in which users are indifferent between holding the global crypto currency and the domestic currency. Ferrari Minesso, Mehl, and Stracca (2022) use a two country framework and find productivity spillovers are amplified in the presence of a CBDC. Lastly, we contribute to a literature on understanding the financial inclusion benefits of introducing a CBDC. On the empirical front, a number of studies have shown, using both survey and financial data (Ozili 2022, 2023; Banerjee and Sinha 2023; Chen et al. 2022), that there is a potential for CBDCs to address financial inclusion in emerging economies such as India and Nigeria, which have a large unbanked population and increasing reliance on digital payments and private payment providers. There is also related work on theoretical models of financial inclusion in an economy with competition between different types of payments, and the potential for disintermediation of bank deposits (Tan 2023a, 2023b; Wang and Hu 2022).

Our contribution is the analysis of macroeconomic effects of a CBDC in a setup that allows us to examine the interaction between financial inclusion and monetary policy.

2 Simple Endowment Economy

To highlight the key mechanisms through which digital currency can improve welfare, we consider a simplified two-agent model, where an agent can be of type $i = \{h, u\}$. In this setup, the banked household (BHH; $i = h$) has access to a first-best risk-free savings device (D), while the unbanked household (UHH; $i = u$) can save in money balances (M).[3] Each of the agents lives for two periods, receives an initial endowment (y) in the first period, and maximizes lifetime utility,

$$u^i = \ln c_1^i + \beta \ln c_2^i,$$

subject to a set of budget constraints for each period.

No digital currency. For the banked, they face the following budget constraints:

$$c_1^h + D = y, \tag{1a}$$

$$c_2^h = RD + \epsilon, \tag{1b}$$

3. We abstract from inflation in this simple setup as we do not discuss considerations in setting interest rates. We include inflation in our New Keynesian framework in Section 3 where we study optimal monetary policy.

where $R > 1$ is the return on D, and ϵ is a shock that impacts resources in the second period. Conversely, the unbanked face a set of budget constraints and a cash-in-advance (CIA) constraint on their consumption in the second period, and so their constraints are:

$$c_1^u + M = y, \tag{2a}$$

$$c_2^u \leq M + \epsilon, \tag{2b}$$

$$\alpha_M c_2^u \leq M, \tag{2c}$$

where $\alpha_M \in (0, 1]$ is the fraction of consumption that is subject to the cash-in-advance constraint. It is similar to the inverse of the velocity of money. In what follows, we assume $\alpha_M = 1$ for tractability.[4]

Solving for optimal consumption in periods 1 and 2[5] for both households yields the following lifetime consumption ratio:

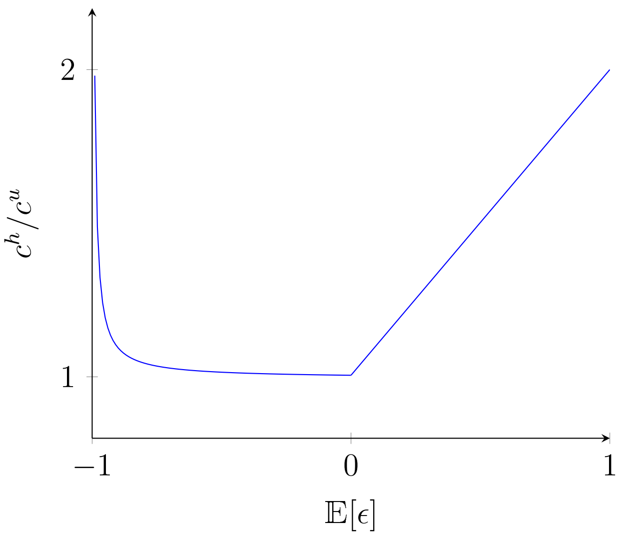
$$\frac{c_1^h + c_2^h}{c_1^u + c_2^u} = \begin{cases} \frac{\frac{2}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right)}{y + \mathbb{E}[\epsilon]} & \text{if } \mathbb{E}[\epsilon] < 0, \\ \frac{\frac{2}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right)}{y} & \text{if } \mathbb{E}[\epsilon] \geq 0 \end{cases} \tag{3}$$

Figure 1 plots the consumption ratio (3) with respect to the expected value of the shock.[6] As the Figure illustrates, the BHH have higher lifetime consumption than the UHH. These consumption gains are increasing in the magnitude of the income shock. Deposits of the BHH are countercyclical with respect to the income shock: the banked save in anticipation of a negative income shock, and reduce savings in anticipation of positive income shocks, enabling them to better smooth consumption. In contrast, the unbanked

4. We check that the results for consumption are qualitatively similar for different values of $\alpha_M < 1$.
5. See Appendix A.1 for details.
6. The expected value of the income shock can be written as $\mathbb{E}[\epsilon] = 1 - 2p$, where p is the probability of a negative realization of the shock. Therefore for $p \in [0, 1]$, the range of our income shock is $[-1, 1]$.

do not have access to an interest-bearing consumption smoothing device. This suggests that they are more adversely exposed by negative income shocks. For positive income shocks ($\mathbb{E}[\epsilon] > 0$), we note that the UHH cannot increase consumption in period 2 as they are bounded by the cash-in-advance constraint. Therefore the ratio of lifetime consumption of the banked to unbanked generates a linear relationship with respect to positive expected income shocks.

Figure 1: **Consumption ratios: Banked to unbanked without digital currencies**



Note: Vertical axis: lifetime consumption ratios of the banked relative to the unbanked households. Horizontal axis: Period 2 resource shock. For calibration, $\beta = 0.99$ and $y = 1$ and $R = \frac{1}{\beta}$.

With digital currency. Now assume that the unbanked have access to digital currency (DC) which is an interest bearing savings device that pays out $R^{DC} \leq R$ upon maturity. Their set of budget constraints are now:[7]

$$c_1^u + M + DC = y, \tag{4a}$$

7. Technically, one can extend the access of DC to the banked household. But so long as the returns to D dominate the returns on DC, the banked will choose to hold no digital currency.

$$c_2^u \leq R^{DC} DC + M + \epsilon, \tag{4b}$$

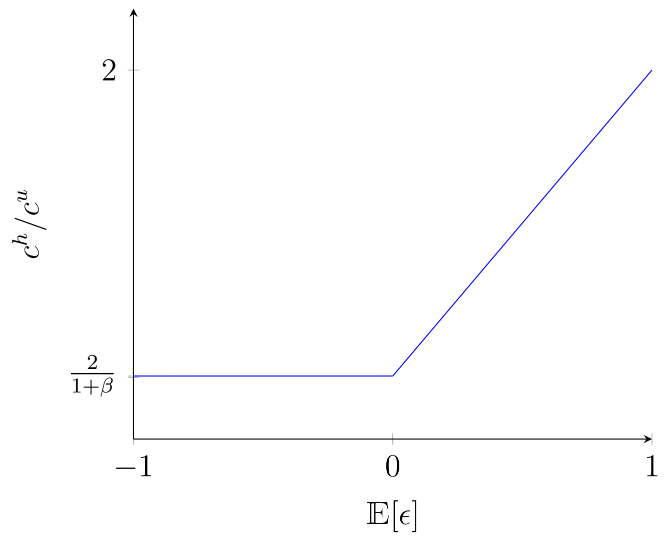
$$\alpha_M c_2^u \leq M, \tag{4c}$$

We can analyze the implications of the introduction of digital currency for household lifetime consumption. Repeating the previous exercise, we solve for optimal consumption quantities for the households to express the consumption of the banked to unbanked with digital currencies:

$$\frac{c_1^h + c_2^h}{c_{1,DC}^u + c_{2,DC}^u} = \begin{cases} \frac{2}{1+\beta} & \text{if } \mathbb{E}[\epsilon] < 0, \\ \frac{\frac{2}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right)}{y} & \text{if } \mathbb{E}[\epsilon] \geq 0 \end{cases} \tag{5}$$

Figure 2 plots the consumption ratio (5) with respect to the expected value of the income shock. Introducing digital currency makes the unbanked more resilient with respect to the anticipation of a negative income shock – particularly for large expected negative shocks – as they now have access to a savings device and can better smooth consumption than with just holding money balances. The ratio of lifetime consumption between the two sets of households is constant with respect to expected negative income shocks ($\mathbb{E}[\epsilon] < 0$). However, for positive anticipated income shocks the digital currency cannot improve the welfare of the UHH. This is due to two factors: (i) the UHH's consumption in period 2 is limited by the CIA constraint, and (ii) we do not allow the unbanked to take a short-position on DC (we require $DC \geq 0$).

Figure 2: **Consumption ratios: Banked to unbanked with digital currencies**



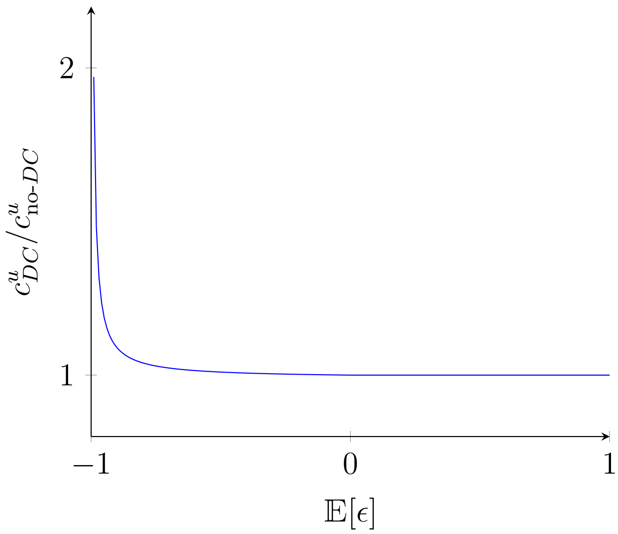
Note: Vertical axis: lifetime consumption ratios of the banked relative to the unbanked households after introducing a digital currency. Horizontal axis: Period 2 resource shock. For calibration, $\beta = 0.99$ and $y = 1$ and $R_{DC} = \frac{1}{\beta}$

Therefore, the ratio of lifetime consumption of the banked to unbanked is identical to the regime with no digital currency in Figure 1 for positive anticipated income shocks; but the unbanked are better off for the case of a large negative anticipated shock. This can be seen by plotting the ratio of lifetime consumption of the unbanked household under the two regimes – with and without digital currencies, illustrated in Figure 3.

$$\frac{c^u_{1,DC} + c^u_{2,DC}}{c^u_{1,no-DC} + c^u_{2,no-DC}} = \begin{cases} \frac{\frac{2}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R_{DC}}\right)}{y+\mathbb{E}[\epsilon]} & \text{if } \mathbb{E}[\epsilon] < 0, \\ 1 \quad \text{if } \mathbb{E}[\epsilon] \geq 0. \end{cases} \tag{6}$$

In summary, our analysis highlights one channel of welfare improvement associated with introduction of digital currency. If the digital currency is interest bearing, it is a more efficient savings device than money. It allows the unbanked to engage in more efficient consumption smoothing, particularly providing better insurance against anticipated negative income shocks. While our simple model sheds light on the role of financial inclusion, this framework is limited as we cannot study: (i) the role of monetary policy, and (ii) whether it is optimal for the interest rate on digital currency to track movements in the deposit rate. We now turn to these policy questions in Section 3 by embedding the two-agent framework in a New Keynesian model with digital currency access to unbanked households.

Figure 3: **Consumption ratios: Unbanked with and without digital currencies**



Note: Vertical axis: lifetime consumption ratios. Horizontal axis: Period 2 resource shock. For calibration, $\beta = 0.99$ and $y = 1$ and $R_{DC} = 1/\beta$.

3 Two-Agent New Keynesian Model with Central Bank Digital Currency

In this section, we present a two-agent New Keynesian (TANK) model as in Bilbiie (2018), Bilbiie and Ragot (2021), and Debortoli and Galí (2017, 2022). Notably, our model features a banking sector accompanied with credit frictions (Gertler and Karadi 2011; Gertler and Kiyotaki 2010). In this framework, a fixed fraction of the banked household are bankers, which allows us to maintain a representative setup of the household sector. Banked households hold claims on CBDC and deposits. Deposits are denominated in fiat currency and held at banks. Banked households may also directly invest in firms by purchasing equity holdings. Banks convert deposits into credit, facilitating loans to firms who acquire capital for the means of production, as in Gertler and Kiyotaki (2010, 2015). Unbanked households are still limited to money holdings and CBDCs.

3.1 Production

The supply side of the economy is standard. Final goods are produced by perfectly competitive firms that use labor and capital to produce their output.[8] They also have access to bank loans, and conditional on being able to take out a loan, they do not face any financial frictions. These firms pay back the crediting banks in full via profits. Meanwhile, capital goods are produced by perfectly competitive firms, which are owned by the collective household.

Capital good firms. We assume that capital goods are produced by perfectly competitive firms, and that the aggregate capital stock grows according to the following law of motion:

$$K_t = I_t + (1 - \delta)K_{t-1}, \tag{7}$$

where I_t is investment and $\delta \in (0, 1)$ is the depreciation rate.

The objective of the capital good producing firm is to choose I_t to maximize revenue, $Q_t I_t$. Thus, the representative capital good producing firm's objective function is:

$$\max_{I_t} Q_t I_t - I_t - \Phi\left(\frac{I_t}{I}\right) I_t,$$

8. We relegate the discussion of final good firms to the Appendix A.2.1 as it is standard.

where $\Phi(\cdot)$ are investment adjustment costs similar to Christiano, Eichenbaum, and Evans (2005), and are defined as:

$$\Phi\left(\frac{I_t}{I}\right) = \frac{\kappa_I}{2}\left(\frac{I_t}{I} - 1\right)^2,$$

with $\Phi(1) = \Phi'(1) = 0$ and $\Phi''(\cdot) > 0$. Differentiating the objective function with respect to I_t/I gives the FOC:

$$Q_t = 1 + \Phi\left(\frac{I_t}{I}\right) + \left(\frac{I_t}{I}\right)\Phi'\left(\frac{I_t}{I}\right). \tag{8}$$

Intermediate goods producers. The continuum of intermediate good producers are normalized to have a mass of unity. A typical intermediate firm i produces output according to a constant returns to scale technology in capital and labor with a common productivity shock:

$$Y_t(i) = A_t K_{t-1}(i)^{\alpha} L_t(i)^{1-\alpha}.$$

The problem for the i-th firm is to minimize costs,

$$\min_{K_{t-1}(i), L_t(i)} z_t^k K_{t-1}(i) + w_t L_t(i),$$

subject to their production constraint:

$$A_t K_{t-1}(i)^{\alpha} L_t(i)^{1-\alpha} \geq Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t.$$

This yields the minimized unit cost of production:[9]

$$MC_t = \frac{1}{A_t}\left(\frac{z_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}. \tag{10}$$

9. Cost minimization implies:

$$\frac{z_t^k K_{t-1}}{w_t L_t} = \frac{\alpha}{1-\alpha}. \tag{9}$$

The price-setting problem of firm i is set up à la Rotemberg (1982) where firm i maximizes the net present value of profits,

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda^h_{t,t+s} \left\{ \left(\frac{P_{t+s}(i)}{P_{t+s}}(1-\tau) - MC_{t+s} \right) Y_{t+s}(i) - \frac{\kappa}{2} \left(\frac{P_{t+s}(i)}{P_{t-1+s}(i)} - 1 \right)^2 Y_{t+s} \right\} \right],$$

by optimally choosing $P_t(i)$, and where κ denotes a price adjustment cost parameter for the firms.[10] Differentiating the above expression with respect to $P_t(i)$ yields the following FOC:

$$\begin{aligned}
\kappa \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right) \frac{Y_t}{P_{t-1}(i)} = {} & \frac{1-\tau}{P_t} \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \\
& + \kappa \mathbb{E}_t \left[\Lambda^h_{t,t+1} \left(\frac{P_{t+1}(i)}{P_t(i)} - 1 \right) \frac{P_{t+1}(i)}{P_t(i)^2} Y_{t+1} \right] \\
& - \epsilon \left(\frac{P_t(i)}{P_t}(1-\tau) - MC_t \right) \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon-1} \frac{Y_t}{P_t}.
\end{aligned}$$

Evaluating at the symmetric equilibrium where intermediate firms optimally price their output at $P_t(i) = P_t, \forall i$, allows us to write an expression for the evolution of inflation:

$$\pi_t(\pi_t - 1) = \frac{\epsilon - 1}{\kappa} \left(\mathcal{M}_t MC_t + \tau - 1 \right) + \mathbb{E}_t \left[\Lambda^h_{t,t+1} (\pi_{t+1} - 1) \pi_{t+1} \frac{Y_{t+1}}{Y_t} \right], \qquad (11)$$

where $\mathcal{M}_t$ is the representative intermediate firm's markup.[11]

10. We calibrate κ to the following:

$$\kappa = \frac{\epsilon \theta}{(1-\theta)(1-\beta\theta)},$$

where θ is the probability of firm i being unable to optimally adjust its price in any given period as in a model with Calvo (1983) pricing. For further details please refer to Appendix A.2.2.

11. In the deterministic steady state the markup is

$$\mathcal{M} = \frac{\epsilon - 1}{\epsilon}.$$

Also, under the symmetric equilibrium we can express output as:

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{12}$$

where it follows that:

$$K_{t-1} = \int_0^1 K_{t-1}(i)di, \quad L_t = \int_0^1 L_t(i)di.$$

As noted above, there is a distortion arising from monopolistic competition among intermediate firms. We assume that there is a lump-sum subsidy to offset this distortion, τ. From Equation (11), we see that the policymaker chooses a subsidy such that the markup over marginal cost is offset in the deterministic steady state:[12]

$$\tau = -\frac{1}{\epsilon - 1}$$

which guarantees a non-distorted steady-state. Hereinafter, we abstract from distorted steady states and only consider the efficient steady state. Our choice to model nominal rigidity following Rotemberg pricing should not alter our welfare analysis in Section 4. As noted by Nisticò (2007) and Ascari and Rossi (2012), up to a second order approximation and provided that the steady state is efficient, models under both Calvo and Rotemberg pricing imply the same welfare costs of inflation. Therefore, a welfare-maximizing social planner would prescribe the same optimal policy across the two regimes.

3.2 Households and Workers

The representative household contains a continuum of individuals, normalized to 1, each of which are of type $i \in \{h, u\}$, following the setup in Murakami and Viswanath-Natraj (2021). Bankers and banked workers ($i = h$) share a perfect insurance scheme, such that

12. Note that this assumes that steady state inflation is net-zero, i.e., $\pi = 1$.

they each consume the same amount of real output. However, unbanked workers ($i = u$) are not part of this insurance scheme, and so their consumption volumes are different from bankers and workers. Similar to before in Section 2, we define Γ_h as the proportion of the BHH and bankers, and the UHH are of proportion $\Gamma_u = 1 - \Gamma_h$.

We endogenize labor supply decisions on the part of households, and so the BHH maximizes the present value discounted sum of utility:[13]

$$\mathbb{V}_t^h = \max_{\{C_{t+s}^h, L_{t+s}^h, D_{t+s}, K_{t+s}^h, DC_{t+s}^h\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Xi_{t+s} \ln\left(C_{t+s}^h - \zeta_0^h \frac{(L_{t+s}^h)^{1+\zeta}}{1+\zeta}\right), \tag{13}$$

subject to their period budget constraint:

$$\begin{aligned} &C_t^h + D_t + Q_t K_t^h + \chi_t^h + DC_t^h + \chi_t^{DC,h} + T_t^h \\ &\qquad = w_t L_t^h + \Pi_t + (z_t^k + (1-\delta)Q_t)K_{t-1}^h + \frac{R_{t-1}D_{t-1} + R_{t-1}^{DC} DC_{t-1}^h}{\pi_t}, \end{aligned} \tag{14}$$

where w_t are real wages, $L_t^i, i \in \{h, u\}$, is labor supply, ζ is the inverse-Frisch elasticity of labor supply, ζ_0^i is a relative labor supply parameter, K_t^h are equity holdings in firms by the BHH, χ_t^h are the costs of equity acquisitions incurred by the BHH, $\chi_t^{DC,i}$ are digital currency management costs,[14] T_t^i are lump-sum taxes, Q_t is the price of equity/capital, and Π_t are distribution of profits due to the ownership of banks and firms. There is a

13. We make use of Greenwood–Hercowitz–Huffman preferences for both the BHH and UHH to eliminate the income effect on an agent's labor supply decision. Additionally, it allows us to develop a tractable analytical solution for the model steady state.

14. The digital currency management costs for household of type i are:

$$\chi_t^{DC,i} = \frac{\varkappa^{DC}}{2}\left(\frac{DC_t^i}{\widetilde{DC}^i}\right)^2, \quad i \in \{h, u\},$$

where $\widetilde{DC}^i$ are target digital currency balances, calibrated in the baseline case such that aggregate holding of digital currencies is one-third of output. Alternatively, we could assume a non-pecuniary motive for holding digital currency that would manifest as an additional term of the same form in the household utility function. This setup would imply the same first-order conditions.

shock to agents' preferences, Ξ_t, and it is given by:

$$\Xi_{t+s} = \begin{cases} \xi_1 \xi_2 ... \xi_s & \text{for } s \geq 1, \\ 1 & \text{for } s = 0, \end{cases}$$

where ξ_t is a preference (demand) shock. We also note that $\Lambda^h_{t,t+s}$ is the BHH stochastic discount factor (SDF):

$$\Lambda^h_{t,t+s} \equiv \beta^s \mathbb{E}_t \left(\frac{\xi_{t+s} \lambda^h_{t+s}}{\lambda^h_t} \right), \tag{15}$$

where λ^h_t is the marginal utility of consumption for the BHH.

One distinction between banked workers and bankers purchasing equity in firms is the assumption that the worker pays an efficiency cost, χ^h_t, when they adjust their equity holdings. We assume the following functional form for χ^h_t:

$$\chi^h_t = \frac{\varkappa^h}{2} \left(\frac{K^h_t}{K_t} \right)^2 \Gamma_h K_t. \tag{16}$$

Meanwhile, the UHH maximizes the present discounted sum of per-period utilities given by:

$$\mathbb{V}^u_t = \max_{\{C^u_{t+s}, L^u_{t+s}, M_{t+s}, DC^u_{t+s}\}^\infty_{s=0}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Xi_{t+s} \ln \left(C^u_t - \zeta^u_0 \frac{(L^u_t)^{1+\zeta}}{1+\zeta} \right), \tag{17}$$

subject to its budget constraint,

$$C^u_t + M_t + \chi^M_t + DC^u_t + \chi^{DC,u}_t + T^u_t = w_t L^u_t + \frac{M_{t-1} + R^{DC}_{t-1} DC^u_{t-1}}{\pi_t}, \tag{18}$$

and the CIA constraint,

$$\alpha_M C^u_t \leq \frac{M_{t-1}}{\pi_t}. \tag{19}$$

3.3 Banks

Bankers are indexed on the continuum $j \in [0,1]$. Among the population of bankers, each j-th banker owns and operates their own bank which has a continuation probability given by σ_b. A banker will facilitate financial services between households and firms by providing loans to firms in the form of equity, k_t^b, funded by deposits, d_t, and their own net worth, n_t.

As is standard in the literature, bankers face a balance sheet constraint:

$$Q_t k_t^b = d_t + n_t, \tag{20}$$

and a flow of funds constraint:

$$n_t = [z_t^k + (1-\delta)Q_t]k_{t-1}^b - \frac{R_{t-1}}{\pi_t} d_{t-1}, \tag{21}$$

where net worth is the difference between gross return on assets and liabilities. Note that for the case of a new banker, the net worth is the startup fund given by the collective household by fraction γ_b:

$$n_t = \gamma_b [z_t^k + (1-\delta)Q_t]k_{t-1}.$$

The objective of a banker is to maximize franchise value, $\mathbb{V}_t^b$, which is the expected present discount value of terminal wealth:

$$\mathbb{V}_t^b = \mathbb{E}_t \left[\sum_{s=1}^{\infty} \Lambda_{t,t+s}^h \sigma_b^{s-1}(1-\sigma_b) n_{t+s} \right]. \tag{22}$$

A financial friction in line with Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) is used to limit the banker's ability to raise funds from depositors, whereby the banker faces a moral hazard problem: the banker can either abscond with the funds they have raised from depositors, or the banker can operate honestly and pay out their obligations. Absconding is costly, however, and so the banker can only divert a fraction $\theta^b > 0$ of assets they have accumulated.[15] Thus, bankers face the following incentive compatibility constraint:

$$\mathbb{V}_t^b \geq \theta^b Q_t k_t^b. \tag{23}$$

The problem of the banker consists of maximizing (22) subject to the balance sheet constraint (20), the evolution of net worth (21), and the incentive compatibility constraint (23).

Since $\mathbb{V}_t^b$ is the franchise value of the bank, which we can interpret as a "market value", we can divide $\mathbb{V}_t^b$ by the bank's net worth to obtain a Tobin's Q ratio for the bank denoted by ψ_t:

$$\psi_t \equiv \frac{\mathbb{V}_t^b}{n_t} = \mathbb{E}_t \left[\Lambda_{t,t+1}^h (1 - \sigma_b + \sigma_b \psi_{t+1}) \frac{n_{t+1}}{n_t} \right]. \tag{24}$$

We define ϕ_t as the maximum feasible asset to net worth ratio, or, rather, the leverage ratio of a bank:

$$\phi_t = \frac{Q_t k_t^b}{n_t}. \tag{25}$$

Additionally, if we define $\Omega_{t,t+1}$ as the stochastic discount factor of the banker, μ_t as the excess return on capital over fiat currency deposits, and υ_t as the marginal cost of deposits, we can write the banker's problem as the following:

$$\psi_t = \max_{\phi_t} \left\{ \mu_t \phi_t + \upsilon_t \right\}, \tag{26}$$

subject to

$$\psi_t \geq \theta^b \phi_t.$$

15. It is assumed that the depositors act rationally and that no rational depositor will supply funds to the bank if they clearly have an incentive to abscond.

Solving this problem yields:

$$\psi_t = \theta^b \phi_t, \tag{27}$$

$$\phi_t = \frac{\upsilon_t}{\theta^b - \mu_t}, \tag{28}$$

where:

$$\mu_t = \mathbb{E}_t \left[\Omega_{t,t+1} \left\{ \frac{z^k_{t+1} + (1-\delta)Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}} \right\} \right], \tag{29}$$

$$\upsilon_t = \mathbb{E}_t \left[\Omega_{t,t+1} \frac{R_t}{\pi_{t+1}} \right], \tag{30}$$

$$\Omega_{t,t+1} = \Lambda^h_{t,t+1}(1 - \sigma_b + \sigma_b \psi_{t+1}). \tag{31}$$

For the complete solution of the banker, please refer to Appendix A.2.4 and A.2.5.

3.4 Fiscal and Monetary Policy

We assume that the government operates a balanced budget:

$$\frac{R^{DC}_{t-1}}{\pi_t} DC_{t-1} + \frac{M_{t-1}}{\pi_t} = \tau Y_t + \Gamma_h T^h_t + \Gamma_u T^u_t + DC_t + M_t, \tag{32}$$

where it levies taxes to cover the producer subsidy to address the distortions arising from monopolistic competition, money balances, and digital currencies. Our budget constraint allows for money and digital currency to be a liability of the central bank, and is consistent with other studies that model the issuance of CBDC (Barrdear and Kumhof 2022; Kumhof et al. 2021), and is similar to direct retail CBDC schemes such as the Bank of England's proposal on the digital pound.[16]

16. For more information on the digital pound we refer readers to https://www.bankofengland.co.uk/paper/2023/the-digital-pound-consultation-paper. Under this scheme, digital pounds are issued similar to bank-notes, however it is through non-bank payment providers. This is different to two-tiered systems in which the CBDC issued by the central bank is distributed by commercial banks, and is discussed in Tan (2023a).

Meanwhile, the central bank is assumed to operate an inertial Taylor rule for the nominal interest rate:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_Y}\right]^{1-\rho_R} \exp(\varepsilon_t^R). \tag{33}$$

Additionally, we assume that the central bank sets the nominal return on digital currency one-for-one in line with the nominal interest rate on deposits:

$$R_t^{DC} = R_t. \tag{34}$$

We explore the implications of alternative rules on model dynamics and welfare in Section 4.

3.5 Market Equilibrium

Aggregate consumption, labor supply, and digital currency holdings by the BHH and UHH are given as:

$$C_t = \Gamma_h C_t^h + \Gamma_u C_t^u, \tag{35}$$

$$L_t = \Gamma_h L_t^h + \Gamma_u L_t^u, \tag{36}$$

$$DC_t = \Gamma_h DC_t^h + \Gamma_u DC_t^u. \tag{37}$$

The aggregate resource constraint of the economy is:

$$Y_t = C_t + \left[1 + \Phi\left(\frac{I_t}{I}\right)\right] I_t + \frac{\kappa}{2}(\pi_t - 1)^2 Y_t + \Gamma_h(\chi_t^h + \chi_t^{DC,h}) + \Gamma_u(\chi_t^M + \chi_t^{DC,u}), \tag{38}$$

with aggregate capital being given by:

$$K_t = \Gamma_h(K_t^h + K_t^b). \tag{39}$$

Aggregate net worth of the bank is given by:

$$N_t = \sigma_b \left[(z_t^k + (1-\delta)Q_t)K_{t-1}^b - \frac{R_{t-1}}{\pi_t} D_{t-1} \right] + \gamma_b (z_t^k + (1-\delta)Q_t) \frac{K_{t-1}}{\Gamma_h}, \tag{40}$$

and the aggregate balance sheet of the bank is given by the following equations:

$$Q_t K_t^b = \phi_t N_t, \tag{41}$$

$$Q_t K_t^b = D_t + N_t. \tag{42}$$

Finally, the stationary AR(1) processes for TFP, markup, and preference shocks are given by:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A, \tag{43}$$

$$\mathcal{M}_t = (1-\rho_M)\mathcal{M} + \rho_M \mathcal{M}_{t-1} + \varepsilon_t^M, \tag{44}$$

$$\ln \xi_t = \rho_\xi \ln \xi_{t-1} + \varepsilon_t^\xi \tag{45}$$

A competitive equilibrium is a set of seven prices, $\{ MC_t, R_t, R_t^{DC}, \pi_t, Q_t, w_t, z_t^k \}$, 19 quantity variables, $\{ C_t, C_t^h, C_t^u, D_t, DC_t, DC_t^h, DC_t^u, I_t, K_t, K_t^b, K_t^h, L_t, L_t^h, L_t^u, M_t, N_t, T_t^h, T_t^u, Y_t \}$, four bank variables, $\{ \psi_t, \phi_t, \mu_t, \upsilon_t \}$, and three exogenous variables, $\{ A_t, \xi_t, \mathcal{M}_t \}$, that satisfies 33 equations. For a complete list of the equilibrium conditions please refer to Appendix A.2.6. Parameter values are provided in Table 1, and a brief description of our parameterization strategy is provided below in Section 3.6. Steady state solutions are provided in Appendix A.2.7 for the baseline TANK model, and numerical model steady state values can be found in Table 2.

3.6 Model Parameterization and Steady State Values

We set model parameters, which are found in standard New Keynesian models, in line with the literature. See, for example, Galí (2015), Walsh (2010), and Woodford (2003).

Model parameters that are not standard, particularly the bank parameters, are set based on Aoki, Benigno, and Kiyotaki (2016). For example, a banker's survival rate, σ_b, is chosen so that the annual dividend payout is a share of $4 \times (1 - \sigma_b) = 0.24$ of net worth. The banker absconding ratio, θ^b; the banker management cost of digital currencies, $\varkappa^b$; and the fraction of total assets inherited by new bankers, γ^b, are chosen so that in steady state the bank leverage ratio is approximately 4 and that the share of equity financed by bank finance is approximately 0.70. Furthermore, parameters pertaining to adjustment costs of money balances, ϕ_M, and of CBDCs, $\varkappa^{DC}$, are calibrated such that digital currency is more easily adjustable than money balances and deposits are the first-best transactions and savings vehicle. Our results are robust to different calibrations of these parameters as long as $0 < \varkappa^{DC} < \phi_M$. We calibrate $\widetilde{DC}$ such that CBDC to output ratio is approximately one third, which is similar to the baseline calibration in (Barrdear and Kumhof 2022; Kumhof et al. 2021), and implies the ratio of CBDC to the sum of CBDC and deposits of approximately 14%, similar to Assenmacher, Bitter, and Ristiniemi (2023).

Finally, we set the parameters pertaining to monetary policy, namely the sensitivity of nominal interest rates to inflation, ϕ_π, the sensitivity of nominal interest rates to the output gap, ϕ_Y, and the interest rate smoothing parameter, ρ_R, in line with Guerrieri and Iacoviello (2015). We assume the persistence of our exogenous AR(1) processes to be 0.85 per quarter. Standard deviations of shocks are set to be 1% per quarter, unless otherwise stated – for instance, innovations to the interest rate are 1% annualized. We assume no cross correlation of our shocks.

Table 1: **Parameter values**

θ^b	0.399	Banker absconding ratio
σ_b	0.940	Survival probability
γ^b	0.005	Fraction of total assets inherited by new banks
$\varkappa^b$	0.022	Management cost for DC
$\widetilde{DC}$	0.230	Steady state DC holdings
β	0.990	Discount rate
ζ	0.333	Inverse-Frisch elasticity
ζ_0^h	3.050	Labor supply capacity
$\varkappa^h$	0.020	Cost parameter of direct finance
Γ_h	0.500	Proportion of BHH
α_M	1	Inverse velocity of money
ϕ_M	0.010	Money adjustment cost parameter
$\varkappa^{DC}$	0.001	Digital currency adjustment cost parameter
α	0.333	Capital share of output
δ	0.025	Depreciation rate
ϵ	10	Elasticity of demand
κ_I	0.167	Investment adjustment cost
θ	0.750	Calvo parameter
τ	0.111	Producer subsidy
$\mathcal{M}$	1.111	Markup
ϕ_π	1.500	Taylor rule inflation coefficient
ϕ_Y	0.100	Taylor rule output coefficient
ρ_b	0.850	AR(1) coefficient for demand shock
ρ_A	0.850	AR(1) coefficient for TFP shock
ρ_M	0.850	AR(1) coefficient for markup shock
ρ_R	0.550	Taylor rule persistence

Table 2: **Baseline TANK steady state values**

π	1	Gross inflation (annual)
R^k	1.066	Gross return on capital (net of depreciation; annual)
R	1.041	Gross nominal interest rate (annual)
Q	1	Price of capital
C/Y	0.798	Consumption to Output
C^h/C	0.613	BHH consumption to aggregate consumption
C^u/C	0.389	UHH consumption to aggregate consumption
DC/Y	0.3329	Aggregate DC holdings to output
I/Y	0.203	Investment to output
K/Y	8.127	Capital to output

4 Dynamics and Welfare Implications

4.1 Impulse Responses to a Monetary Policy shock

Figure 4 presents impulse responses to a 1% (annualized) monetary policy shock with the Taylor rule (33) and $R_t = R_t^{DC}$. Additionally, we plot impulse responses for two alternative regimes: an economy with CBDCs as described in Section 3 (red dashed line) and an economy with no CBDCs (blue line).

With the exception of inflation, prices respond differently to the monetary policy shock under the different regimes – rates on deposits and CBDCs have a smaller decline, falling to just under 4% with CBDCs compared to below 3.9% without CBDCs, and share the same "V"-shaped time path. This is most likely due to the initial sharper decline in output of the no-CBDC economy, stemming from the fact that the unbanked cut consumption drastically by approximately 3% when they only have access to money as a savings vehicle. Interestingly, equity prices in the baseline CBDC economy initially decline by approximately 0.06%; yet, in the no-CBDC economy, equity prices decline by less than 0.02% upon impact, but quickly rise to above 0.02%, featuring a hump-shaped response as prices revert to steady state.

To understand the different paths of equity prices, first inspect the response of wages and labor supply of the banked household. Despite the initial disinflation easing the CIA constraint of the unbanked household, this changes from period 2 onwards. For the unbanked, as the value of real money balances declines, the CIA constraint increases in severity, and thus the unbanked household supplies less labor[17]. In response, firms must attract more labor from the banked workers by offering a higher wage. Due to GHH preferences, labor supply decisions of the banked follow that of wages. Thus, the small rebound in real wages from period 2 onwards leads to the banked household supplying more labor and in turn receiving more income to fund consumption. The large decline of deposit rates in the no-CBDC economy disincentivizes the banked household from saving in deposits, however this is partially offset by the increase in wage income. In fact, note that deposits decline by a larger magnitude in the CBDC economy where there is no increase in wage income on the part of the banked household. With a smaller decline in deposits, banks are better able to provide capital to firms and equity prices actually increase between periods 1 and 4.

Despite the banking sector performing better in an economy without CBDCs – for example, note that μ_t is lower in a no-CBDC economy – through general equilibrium effects, the lack of an adequate savings vehicle on the part of unbanked households and the costs that arise from the CIA constraint, leads to the real economy underperforming compared to the economy with CBDCs.

With the provision of a CBDC, the key difference is the response of unbanked consumption and labor. Access to a savings device through the CBDC allows these households to buffer against the shock by reducing their savings (the decline in DC^u), dampening the decline in their consumption. This mutes the aggregate response of consumption

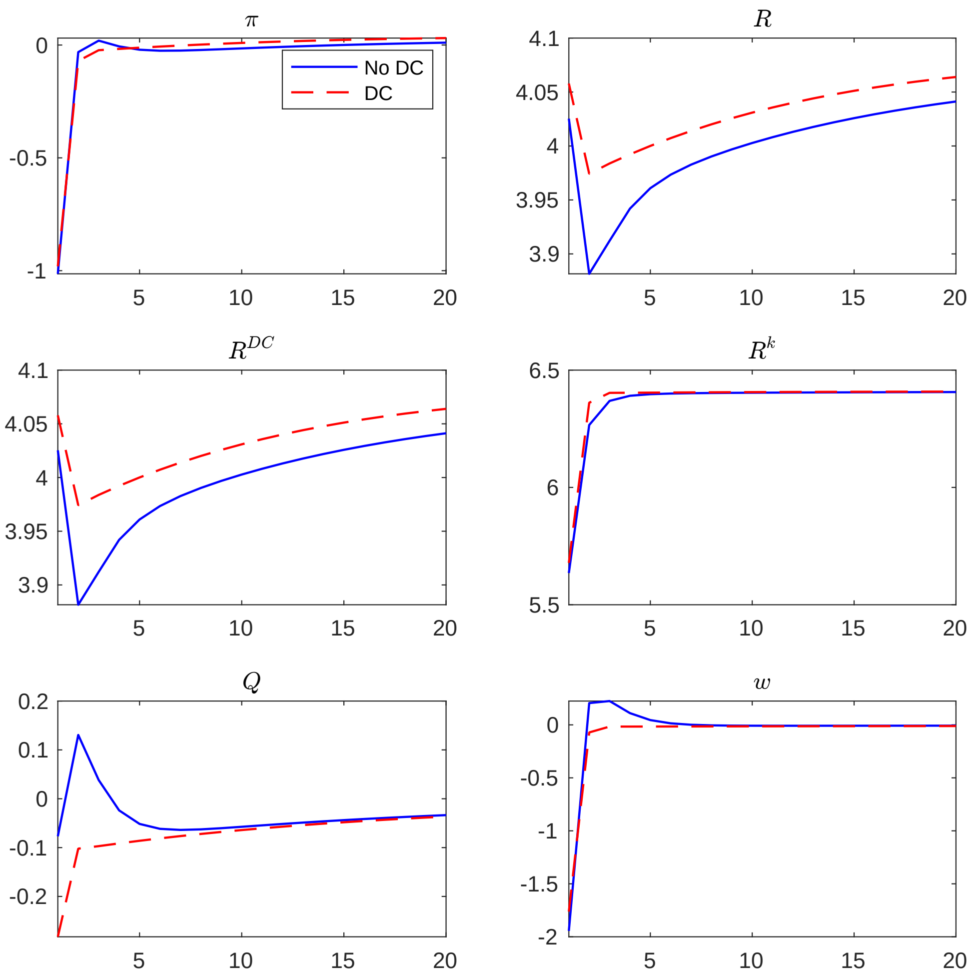
17. See (74) and (75) in the Appendix.

and output, and the effects of shocks dissipate quicker than in an economy without CB-DCs. For the banked household, in contrast, there is little difference in the response of consumption to monetary policy shocks upon introducing a digital currency. This is because they have access to a first best savings device, deposits, and do not adjust their holdings of digital currency in response to the shock. In summary, monetary policy transmission to aggregate consumption, output, and pass-through to inflation is amplified with the introduction of the CBDC.[18]

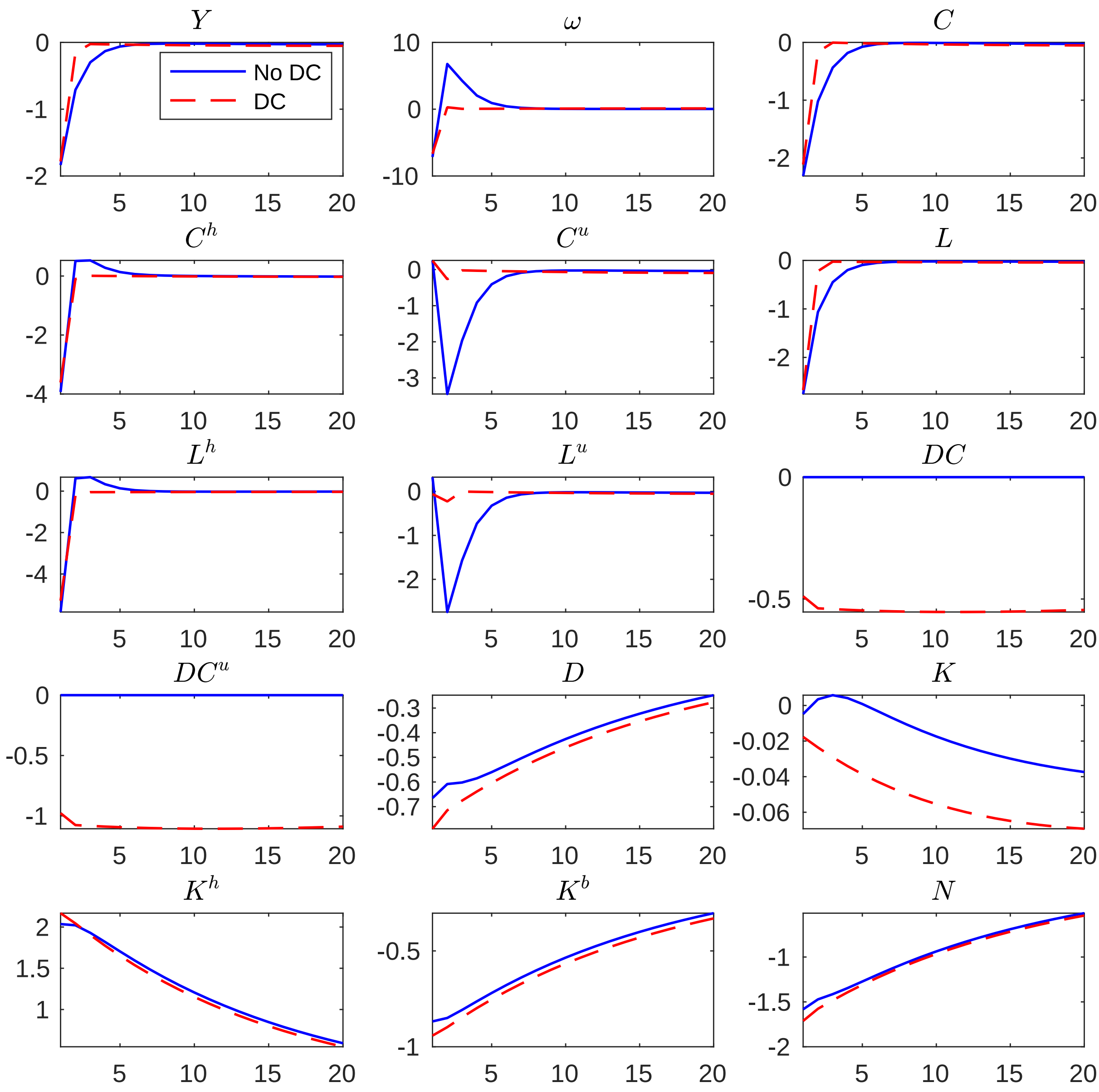
18. IRFs to a fundamental TFP, cost-push, and demand shock are provided in Appendix A.2.8.

Figure 4: **IRFs to a 1% annualized monetary policy shock**

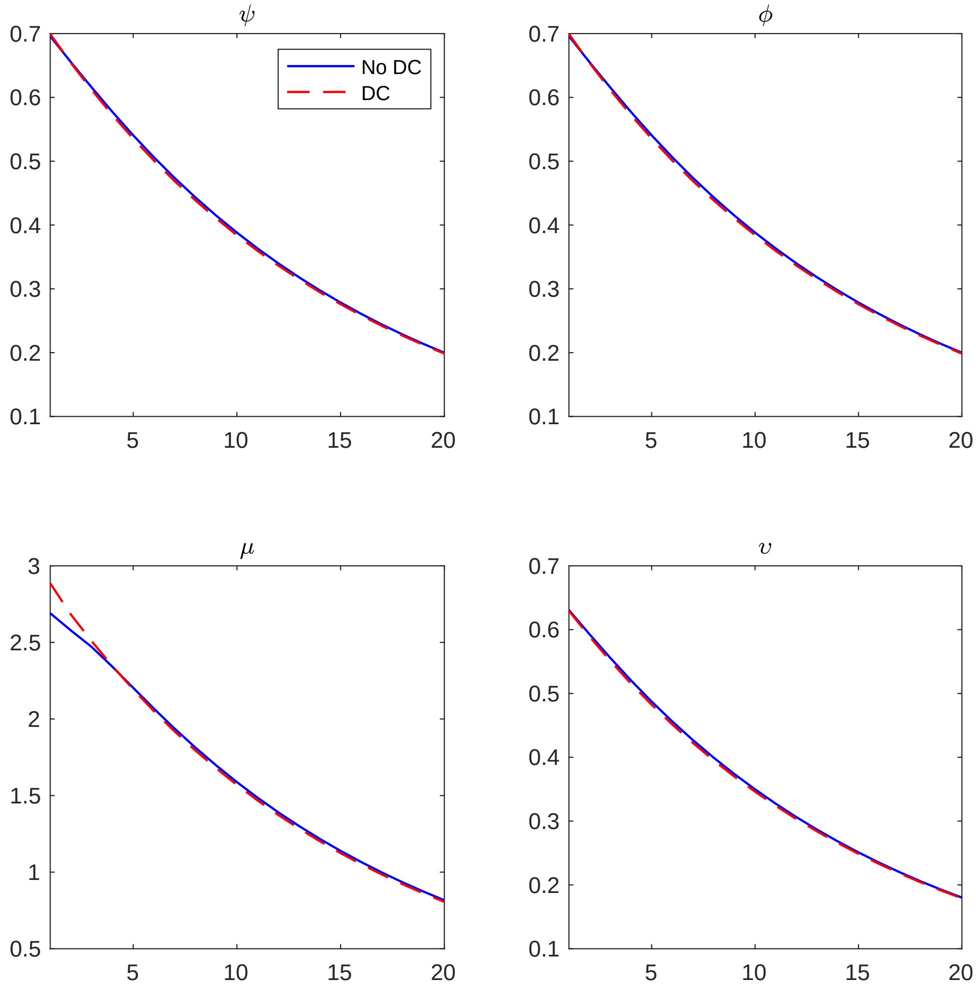
(a) **Prices**



(b) **Quantities**



(c) **Bank variables**

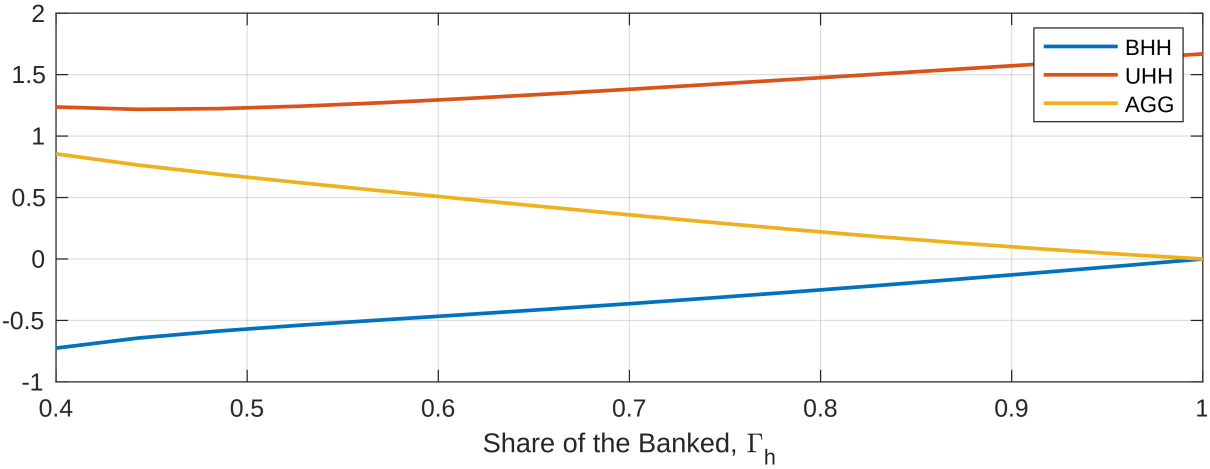


Note: Figure plots impulse responses of model variables with respect to a 1% annualized innovation to the Nominal Interest Rate. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R), Digital Currency Returns (R^{DC}), and Gross Returns to Capital (R^k) which are expressed as annualized net rates.

4.2 Welfare Effects of Introducing a CBDC

Figure 5 evaluates the welfare effects of introducing a CBDC, when the economy is subject to TFP, cost-push, demand, and monetary policy shocks. We find that unbanked experience welfare gains when CBDC is adopted. This is due to CBDC offering a rate of remuneration and it being a more efficient savings device than money balances, allowing the unbanked to better insure against adverse shocks.

Figure 5: **Welfare comparison (CBDC regime %ch. over no-CBDC regime)**



Note: Figure plots welfare for BHH, UHH and aggregate households as a function for the share of the banked population, Γ_h. The welfare is calculated as a per cent change from the regime with no digital currency.

Turning to the banked households, we find that they experience net negative welfare benefits after introduction of the CBDC. To explain this, we note that the banked household face management costs in holding a CBDC relative to bank deposits, and therefore do not gain directly from access to a CBDC as they already have bank deposits – which are a first best transaction and savings device. Second, banked households experience net negative welfare losses from a disintermediation channel: as their holdings of CBDC increase, the bank loses deposit funding (Keister and Sanches 2021). The decline in bank funding leads to lower intermediation, amplifying the response of capital and production through bank balance sheets via a financial accelerator mechanism (Bernanke, Gertler, and Gilchrist 1999; Kiyotaki and Moore 1997, 2019). Both of these channels can explain why banked households experience net negative welfare losses relative to an economy with no CBDC. Turning to aggregate welfare, we observe net welfare benefits are highest when the economy is primarily unbanked. As the ex-ante proportion of the unbanked population declines, the welfare benefits of introducing CBDC tend to zero which suggests a stronger use case of CBDCs in emerging markets with lower degrees of financial inclusion.

4.3 CBDC Design and Optimal Monetary Policy

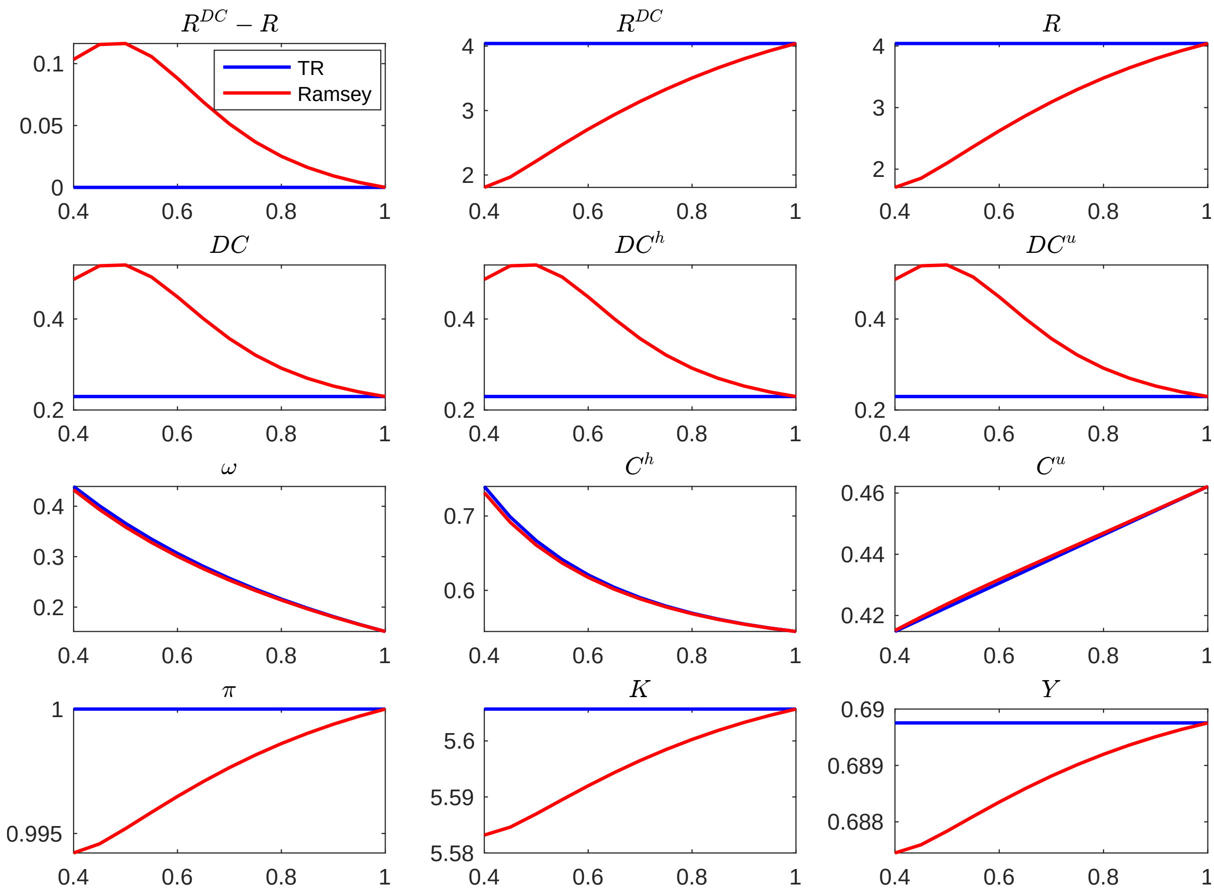
We now explore the implications for optimal policy, assuming that a policymaker has access to two instruments in order to maximize welfare: nominal interest rates on deposits, R, and nominal interest rates on digital currency, R^{DC}. More formally, let us state the problem for the welfare maximizing policymaker as:

$$\max_{\{R_{t+s}, R^{DC}_{t+s}\}_{s=0}^{\infty}} \mathbb{V}_t = \Gamma_h \mathbb{V}_t^h + \Gamma_u \mathbb{V}_t^u, \tag{46}$$

subject to the entire set of structural equations as set out in Sections 3.1-3.5. As CBDC and deposits are imperfect substitutes, the instruments available to the policymaker are not collinear, allowing us to conduct the optimal policy exercise.[19]

19. We argue that R^{DC} is different to R as a Ramsey-instrument in two distinct ways. First, DC are a sub-optimal savings and consumption smoothing instruments compared to D due to the presence of convex adjustment costs. Secondly, the existence of DC in the economy potentially induces disintermediation. Thus, R^{DC} is set to balance positive welfare effects that DC brings in for the unbanked against the broader effects of bank deposit disintermediation.

Figure 6: **Steady state values and financial inclusion**



Note: Vertical axis indicates absolute values of variables in steady state, except for π, R, and R^{DC}: these variables are represented as annualized rates. The horizontal axes are values of financial inclusion parameter Γ_h.

The steady state values implied by the solution of the social planner problem are shown in Figure 6. The choice of instruments by the Ramsey policymaker implies a steady state that is generally different to the one under the baseline configuration with a Taylor rule. The presence of unbanked households subject to a CIA constraint leads the social planner to pick a deflationary steady state. This is a result well covered in, for example, Chari, Christiano, and Kehoe (1991) and Schmitt-Grohé and Uribe (2010). Deflation is, however, costly through inefficient price adjustments; thus the policymaker induces a relatively low level of deflation. As the share of unbanked households converges to zero (greater financial inclusion), the model becomes a standard representative agent setup and the optimal net inflation rate converges to zero, $\pi \to 1$. Moreover, for relatively low ex-ante financial inclusion, the social planner picks higher values of steady state CBDC holdings by picking a higher spread between R^{DC} and R. This is due to the fact that while maximizing the aggregate welfare of the economy as in (46), the social planner wishes to redistribute resources from the otherwise wealthier banked household to the unbanked household, and is able to do so only via interest-bearing CBDC holdings.

Figure 7 shows the decomposition of welfare gains associated with both the introduction of the CBDC and optimal monetary policy. For different levels of the banked population share, we decompose welfare improvements associated with the transition from the no-CBDC economy and a standard Taylor rule to the economy with CBDCs and a Ramsey-optimal monetary policy (two instruments). The model economy here is subject to TFP, markup and preference shocks. These welfare gains are associated with: (i) the introduction of CBDCs, (ii) optimal conventional monetary policy, and (iii) optimal R_t^{DC} setting.[20]

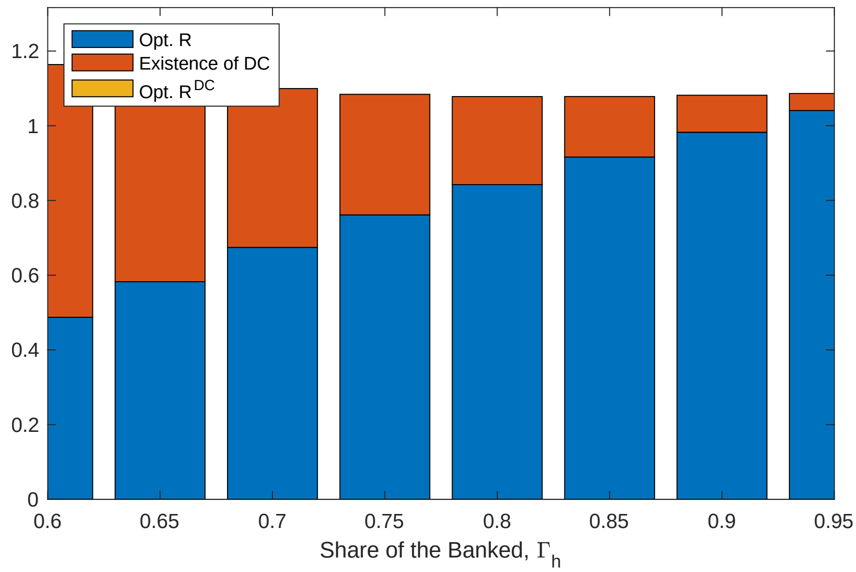
We observe that for the economy with low financial inclusion, welfare improvements are mainly associated with the introduction of a CBDC. As the share of the banked population Γ_h increases, we observe that the welfare benefits associated with the provision of CBDC go to zero. Secondly, we see that as financial inclusion increases, the importance of setting R in accordance to optimal monetary policy for welfare increases. In response

20. We compare welfare under the three policy changes to the baseline Taylor-rule regime and no CBDCs. The welfare improvements associated with each regime change do not include cross effects, which are small in magnitude. We approximate all the models around the Ramsey-optimal steady state to ensure that welfare rankings are not spurious, following Benigno and Woodford (2012). This implies steady-state deflation and a spread between R^{DC} and R.

to TFP, cost-push, and preference shocks, optimal monetary policy is similar to a rule in which the rate on CBDC tracks the policy rate adjusted for a constant spread, $\mathcal{S}^{DC} = R_t^{DC} - R_t$. Deviating from this rule results in negligible welfare improvements and is an order of numerical approximation error.[21]

Taken together, our optimal policy exercise suggests that the gains from introducing a CBDC diminish as financial inclusion increases, consistent with our welfare analysis in Section 4.2. The key finding is that welfare is maximized when, conditional on R_t being set optimally, the optimal level of R_t^{DC} is given when the Ramsey policymaker sets a spread, $\mathcal{S}^{DC} = R_t^{DC} - R_t$, between the return on CBDCs and deposits.

Figure 7: **Welfare improvement decomposition**



Note: Vertical axis indicates percent increase in welfare compared to baseline specification without digital currency access.

21. For brevity, we provide IRFs to a TFP, cost-push, and preference shocks in Appendix A.2.8 for two alternative regimes: (i) an economy with a central bank that applies a Taylor rule on D and a constant spread on DC, and (ii) a two-instrument Ramsey policymaker that maximizes household welfare. Note that for this set of IRFs, the model is approximated about the Ramsey-optimal steady state.

4.4 Welfare Implications of Constant Spread Rules

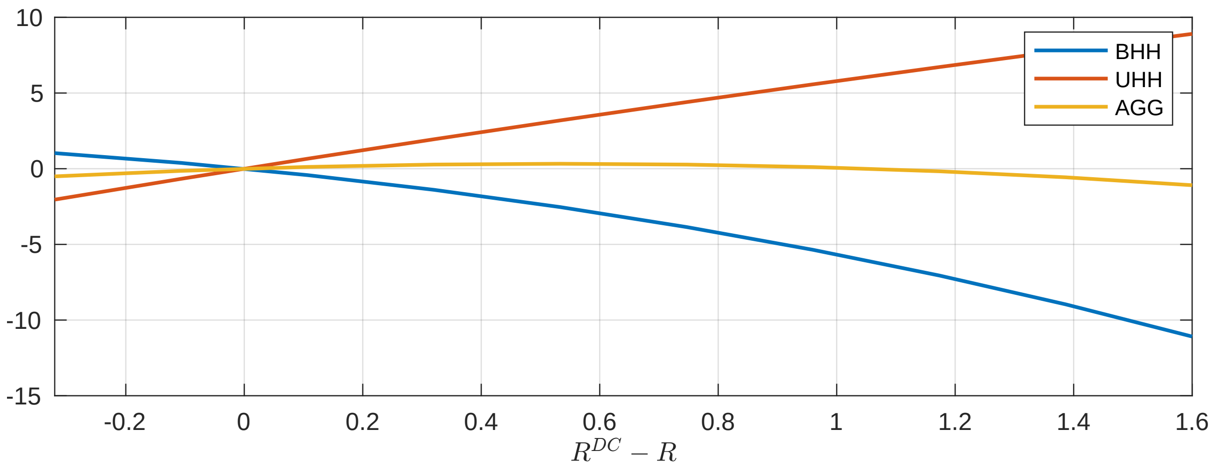
In this section, we explore the effect of different levels of $\mathcal{S}^{DC}$ on welfare. We hold constant the baseline degree of financial inclusion ($\Gamma_h = 0.5$), and the economy is subject to TFP, cost-push, and preference shocks.[22] Figure 8 plots the relative welfare gains and losses of agents in a CBDC-equipped economy against a benchmark zero spread between the CBDC and policy rate ($\mathcal{S}^{DC} = 0$). The spread is quoted in annualized percent levels. Our welfare results show that the unbanked are better off when CBDC rates are higher than the deposit rate. Explaining these findings, we note that the unbanked benefit through the savings channel, where CBDC deposits receive a higher rate of interest.

In contrast, the banked are worse off as the CBDC rate is higher than the deposit rate through the disintermediation channel. As CBDC rates increase, larger balances of CBDC holdings leads to a substitution away from holding bank deposits. Therefore the deposit base of bank balance sheets shrinks. This in turn leads to a lower equilibrium levels of bank lending, capital, and consumption of banked households. Our analysis shows that setting the optimal spread between the rate on deposits and CBDCs depends on the level of financial inclusion. All else equal, our model suggests that economies with lower levels of financial inclusion and a higher share of the unbanked population should find it optimal to set a higher spread of digital currency rates to deposit rates. In contrast, developed economies with a predominantly banked population should set rates on the CBDC lower than the policy rate. This is consistent with discussions on how CBDC in developed economies can result in crowding out bank deposits as in Keister and Sanches (2021), and policy discussions on the Bank of England's digital pound project

22. To be clear, this exercise is conducted when monetary policy is conducted according to the Taylor rule (33).

which argues that the rate of remuneration should be zero.[23]

Figure 8: **CBDC economy welfare comparison (% ch.)**



Note: Figure plots relative welfare gains for BHH, UHH, and aggregate households as a function of the spread between the policy rate and the CBDC rate. Note that $\Gamma_h = 0.5$.

5 Conclusion

In this paper we focus on the interaction of financial inclusion, introduction of a CBDC, and monetary policy. We address a number of research questions. What are the welfare implications of a retail CBDC, and does it address financial inclusion of the unbanked? Does the strength and role of stability of monetary policy transmission increase or decrease after CBDC adoption? What are optimal interest rate rules in an economy with a CBDC?

To motivate our analysis, we use a simple economic model featuring two-agent house-

23. The Bank of England's consultation on the digital pound states that it will not offer a rate of remuneration on the pound. See: https://www.bankofengland.co.uk/news/2023/february/hm-treasury-and-boe-consider-plans-for-a-digital-pound. The rationale as indicated in the proposal is due to concerns that an interest bearing pound can result in deposit outflow. This is still contested by some of the literature which argues that CBDC can raise deposit competition if there is monopolistic competition in the banking sector, see for example Chiu et al. (2023).

holds. Unbanked households face a cash-in-advance constraint and do not have access to a savings device, while banked households have access to deposits. Without a digital currency, our analysis shows that banked households enjoy higher lifetime consumption, particularly when agents anticipate negative income shocks, as they can rely on deposits to smooth consumption. However, the introduction of an interest-bearing digital currency provides the unbanked with a more efficient savings tool, enhancing their resilience to negative shocks and promoting financial inclusion.

In the second part of our paper, we extend the model to incorporate production, labor supply, sticky prices, financial intermediaries, and monetary policy. This New Keynesian framework allows us to explore optimal monetary policy rules and the role of financial intermediaries in CBDC transmission effects. We simulate the economy's responses to various shocks, primarily focusing on monetary policy shocks.

We find that the introduction of a CBDC amplifies the transmission of monetary policy. In response to a monetary policy tightening, inflation, asset prices, and consumption decrease. The rise in deposit rates results in reduced bank net worth, asset prices, and bank equity, along with a decrease in the deposit base. However, the unbanked household consumption response to monetary shocks is notably different with a CBDC. Access to the digital currency enables the unbanked to counter the shock by reducing their savings, mitigating the decline in consumption and leading to a faster transition to the steady state. In contrast, the banked household consumption response remains largely unchanged with the introduction of a CBDC.

Furthermore, we evaluate the welfare effects of CBDC introduction relative to an economy without CBDCs. The unbanked benefit from improved consumption smoothing, while banked households are worse off due to management costs of their financial portfolios, and the disintermediation of banks due to a shrinking deposit base, leading to lower consumption. This welfare trade-off suggests that economies with a high share of unbanked households and low financial inclusion stand to gain the most from CBDC adoption.

We also analyze aspects of CBDC design, including the optimal path of interest rates on both the central bank and digital currency deposits. Our results indicate that when CBDC deposits are close substitutes to bank deposits, it is optimal for the CBDC rate to track movements in the deposit rate albeit with a spread. We highlight the distributional implications of welfare based on interest rate rules which preserve a constant spread between the CBDC rate and the deposit rate. We show that unbanked households benefit when CBDC rates are at a positive spread with respect to the rate on deposits, while banked households are worse off. This finding has policy implications, as central banks need to consider the level of financial inclusion to determine whether a retail CBDC should be interest bearing, and how it should be priced with respect to the deposit rate.

In summary, our paper contributes to the design and macroeconomic effects of CBDCs, emphasizing their potential to enhance financial inclusion and their implications for monetary policy and household welfare. Our analysis underscores the importance of tailoring CBDC policies to the degree of financial inclusion in the economy.

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A Appendix

A.1 Simple Endowment Economy

No digital currency. Assuming that the banked and unbanked face constraints (1) and (2), respectively, and make the simplifying assumption that:

$$\epsilon = \begin{cases} -1 & \text{w.p.} \quad p, \\ 1 & \text{w.p.} \quad 1-p, \end{cases}$$

where $p \in (0,1)$.

Solving the BHH problem for optimal consumption across periods yields

$$c_1^h = \frac{1}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right), \tag{47}$$

$$c_2^h = \frac{\beta}{1+\beta}R\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right), \tag{48}$$

$$D = \frac{\beta}{1+\beta}y - \frac{\mathbb{E}[\epsilon]}{(1+\beta)R}, \tag{49}$$

with the standard consumption Euler equation:

$$c_2^h = \beta R c_1^h.$$

As expected, c_1^h and c_2^h are decreasing in p, while D is increasing in p, highlighting the role of consumption smoothing for the banked household.

For the UHH, it is clear that the CIA constraint is not binding if $p > \frac{1}{2}$, which yields the following solutions:

$$c_1^u = \frac{1}{1+\beta}\left(y + \mathbb{E}[\epsilon]\right), \tag{50}$$

$$c_2^u = \frac{\beta}{1+\beta}\left(y + \mathbb{E}[\epsilon]\right), \tag{51}$$

$$M = \frac{\beta}{1+\beta}y - \frac{\mathbb{E}[\epsilon]}{1+\beta}, \tag{52}$$

and where their Euler equation is:

$$c_1^u = \beta c_2^u.$$

In the case where $p < \frac{1}{2}$ we have:

$$c_1^u = \frac{1}{1+\beta} y, \tag{53}$$

$$c_2^u = \frac{\beta}{1+\beta} y, \tag{54}$$

$$M = \alpha_M c_2^u. \tag{55}$$

With digital currency. The banked problem remains the same as without digital currency. The unbanked now face constraints in (4), and solving their problem yields the following FOCs:

$$\frac{1}{c_1^u} = \lambda_1,$$

$$\frac{1}{c_2^u} = \lambda_2 + \alpha_M \mu,$$

$$\lambda_1 = \beta \lambda_2 R^{DC},$$

$$\lambda_1 = \beta \lambda_2 + \beta \mu,$$

where λ_t is the period-t marginal utility of consumption and μ is the CIA constraint Lagrangian multiplier. Rearrange the above FOCs, and combine with the fact that for $R^{DC} > 1$ (4c) binds with equality, to get:

$$\frac{1}{c_2^u} = \lambda_2 \left[1 + \alpha_M (R^{DC} - 1)\right],$$

$$c_2^u = \mathcal{S} c_1^u,$$

$$M = \alpha_M c_2^u,$$

where $\mathcal{S} = \beta R^{DC} / \left[1 + \alpha_M (R^{DC} - 1)\right]$ is the marginal rate of transformation of c_1^u and c_2^u – the discounted return on deferring consumption using M and DC. Then write the optimal quantities for the unbanked as:

$$c_1^u = \frac{1}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R^{DC}}\right), \tag{56}$$

$$c_2^u = \frac{\mathcal{S}}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R^{DC}}\right), \tag{57}$$

$$M = \alpha_M c_2^u, \tag{58}$$

$$DC = \frac{\mathcal{S}(1-\alpha_M)}{(1+\beta)R^{DC}}\left(y + \frac{\mathbb{E}[\epsilon]}{R^{DC}}\right) - \frac{\mathbb{E}[\epsilon]}{R^{DC}}. \tag{59}$$

There is a second case to the problem of the unbanked: when the second period budget constraint does not bind with equality but the CIA does. This yields the following expressions for consumption and digital currency holdings:

$$c_1^u = \frac{\alpha_M}{\alpha_M + \beta} y, \tag{60}$$

$$c_2^u = \frac{\beta}{\alpha_M + \beta} y, \tag{61}$$

$$DC = 0. \tag{62}$$

To understand the two cases, assume for simplicity that $\alpha_M = 1$. This means that (59) simplifies to

$$DC = -\frac{\mathbb{E}[\varepsilon]}{R^{DC}} \tag{63}$$

Since there is a non-negativity constraint on DC, it would imply that the above expression yields a positive balance of DC if and only if $p > \frac{1}{2}$. In other words, if the expected value of the income shock is negative, then an unbanked household will attempt to save in DC in order to fund its consumption in the second period. If the expected value of the income shock is positive, then the unbanked household would attempt to take a short position to increase period 2 consumption – which would violate the non-negativity constraint we placed on DC. Hence, in the simplifying case where $\alpha_M = 1$, expected lifetime consumption of the unbanked with and without DC is given by

$$c^u_{\text{w/ }DC} = \begin{cases} y - \frac{1}{R^{DC}} & \text{w.p. } p, \\ y & \text{w.p. } 1-p, \end{cases} \tag{64}$$

$$c^u_{\text{w/o }DC} = \begin{cases} y - 1 & \text{w.p. } p, \\ y & \text{w.p. } 1-p. \end{cases} \tag{65}$$

A.2 TANK model with Central Bank Digital Currency

A.2.1 Final Good Firms

There is a representative competitive final good producing firm which aggregates a continuum of differentiated intermediate inputs according to a Dixit-Stiglitz aggregator:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{66}$$

Final good firms maximize their profits by selecting how much of each intermediate good to purchase, and so their problem is:

$$\max_{Y_t(i)} P_t Y_t - \int_0^1 P_t Y_t(i) di.$$

Solving for the FOC for a typical intermediate good i is:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon} Y_t. \tag{67}$$

The relative demand for intermediate good i is dependent of i's relative price with ϵ, the price elasticity of demand, and is proportional to aggregate output, Y_t.

From Blanchard and Kiyotaki (1987), we can derive a price index for the aggregate economy:

$$P_t Y_t \equiv \int_0^1 P_t(i) Y_t(i) di.$$

Then, plugging in the demand for good i from (67) we have:

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}.$$

A.2.2 The New Keynesian Phillips Curve

If we log linearize Equation (11) about the non-inflationary steady state, we yield the NKPC. First start by totally differentiating (11):

$$(2\pi - 1) d\pi_t = \frac{(\epsilon - 1)\mathcal{M}}{\kappa} dMC_t + \frac{(\epsilon - 1)MC}{\kappa} d\mathcal{M}_t + \beta(2\pi - 1)\mathbb{E}_t d\pi_{t+1},$$

where $\pi = \frac{\mathcal{M}}{1-\tau} MC = 1$. Substitute these values in and assume that $dMC_t = MC_t - MC$ to get the log-linearized NKPC:

$$\hat{\pi}_t = \frac{(\epsilon - 1)(1 - \tau)}{\kappa} \hat{MC}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} + \hat{u}_t, \tag{68}$$

where hatted variables denote log-deviations from steady state values, $\hat{u}_t = \frac{(\epsilon-1)(1-\tau)}{\kappa} \hat{\mathcal{M}}_t$ is a cost-push shock, and where we calibrate κ to a standard value as in Blanchard and Galí (2007):

$$\kappa = \frac{\epsilon \theta}{(1 - \theta)(1 - \beta \theta)}.$$

A.2.3 Household Optimization Problem

The FOCs to the BHH problem are:

$$\lambda_t^h = \frac{1}{C_t^h - \zeta_0^h \frac{(L_t^h)^{1+\zeta}}{1+\zeta}}, \tag{69}$$

$$w_t = \zeta_0^h (L_t^h)^{\zeta}, \tag{70}$$

$$1 = \mathbb{E}_t \Lambda^h_{t,t+1} \frac{R_t}{\pi_{t+1}}, \tag{71}$$

$$1 = \mathbb{E}_t \Lambda^h_{t,t+1} \left(\frac{z^k_{t+1} + (1-\delta) Q_{t+1}}{Q_t + \varkappa^h \Gamma_h \left(\frac{K^h_t}{K_t} \right)} \right), \tag{72}$$

$$1 + \varkappa^{DC} \frac{DC^h_t}{\widetilde{DC}^h} = \mathbb{E}_t \Lambda^h_{t,t+1} \frac{R^{DC}_t}{\pi_{t+1}}. \tag{73}$$

The FOCs to the UHH problem are:

$$\lambda^u_t + \alpha_M \mu^u_t = \frac{1}{C^u_t - \zeta^u_0 \frac{(L^u_t)^{1+\zeta}}{1+\zeta}}, \tag{74}$$

$$\lambda^u_t w_t = \frac{\zeta^u_0}{C^u_t - \zeta^u_0 \frac{(L^u_t)^{1+\zeta}}{1+\zeta}} (L^u_t)^\zeta, \tag{75}$$

$$\lambda^u_t \left[1 + \phi_M (M_t - M) \right] = \beta \mathbb{E}_t \xi_{t+1} \left[\frac{\lambda^u_{t+1} + \mu^u_{t+1}}{\pi_{t+1}} \right], \tag{76}$$

$$1 + \varkappa^{DC} \frac{DC^u_t}{\widetilde{DC}^u} = \beta \mathbb{E}_t \xi_{t+1} \frac{\lambda^u_{t+1}}{\lambda^u_t} \frac{R^{DC}_t}{\pi_{t+1}}. \tag{77}$$

A.2.4 Rewriting the Banker's Problem

To setup the problem of the banker as in Section 3.3, first iterate the banker's flow of funds constraint (21) forward by one period, and then divide through by n_t to yield:

$$\frac{n_{t+1}}{n_t} = \frac{\left(z^k_{t+1} + (1-\delta) Q_{t+1} \right)}{Q_t} \frac{Q_t k^b_t}{n_t} - \frac{R_t}{\pi_{t+1}} \frac{d_t}{n_t}.$$

Rearrange the balance sheet constraint (20) to yield the following:

$$\frac{d_t}{n_t} = \phi_t - 1.$$

Substitute this value for d_t/n_t into the expression for n_{t+1}/n_t, and we get:

$$\frac{n_{t+1}}{n_t} = \left(\frac{z^k_{t+1} + (1-\delta) Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}} \right) \phi_t + \mathbb{E}_t \frac{R_t}{\pi_{t+1}}.$$

Substituting this expression into (24), yields the following:

$$\psi_t = \mathbb{E}_t \Lambda^h_{t,t+1}(1 - \sigma_b + \sigma_b \psi_{t+1}) \left[\left(\frac{z^k_{t+1} + (1-\delta)Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}} \right) \phi_t + \frac{R_t}{\pi_{t+1}} \right]$$
$$= \mu_t \phi_t + \upsilon_t,$$

which is (26) in the text.

A.2.5 Solving the Banker's Problem

With $\{\mu_t\} > 0$, the banker's incentive compatibility constraint binds with equality, and so we can write the Lagrangian as:

$$\mathcal{L} = \mu_t \phi_t + \upsilon_t + \lambda_t(\psi_t - \theta^b \phi_t),$$

where λ_t is the Lagrangian multiplier. The FOCs are:

$$(1 + \lambda_t)\mu_t = \lambda_t \theta^b, \tag{78}$$

$$\psi_t = \theta^b \phi_t. \tag{79}$$

Substitute (79) into the banker's objective function to yield:

$$\phi_t = \frac{\upsilon_t}{\theta^b - \mu_t}, \tag{80}$$

which is (28) in the text.

A.2.6 Full Set of Equilibrium Conditions

Households.

$$w_t = \zeta^h_0 L^h_t \tag{81}$$

$$1 = \mathbb{E}_t \Lambda^h_{t,t+1} \frac{R_t}{\pi_{t+1}} \tag{82}$$

$$1 = \mathbb{E}_t \Lambda^h_{t,t+1} \frac{z^k_{t+1} + (1-\delta) Q_{t+1}}{Q_t + \varkappa^h \Gamma_h \left(\frac{K^h_t}{K_t}\right)} \tag{83}$$

$$1 + \varkappa^{DC} \frac{DC^h_t}{\widetilde{DC}^h} = \mathbb{E}_t \Lambda^h_{t,t+1} \frac{R^{DC}_t}{\pi_{t+1}} \tag{84}$$

$$C^u_t + M_t + \chi^M_t + DC^u_t + \chi^{DC,u}_t + T^u_t = w_t L^u_t + \frac{M_{t-1}}{\pi_t} + \frac{R^{DC}_{t-1}}{\pi_t} DC^u_{t-1} \tag{85}$$

$$\frac{\lambda^u_t}{\lambda^u_t + \alpha_M \mu^u_t} w_t = \zeta^u_0 (L^u_t)^\zeta \tag{86}$$

$$\lambda^u_t + \alpha_M \mu^u_t = \frac{1}{C^u_t - \zeta^u_0 \frac{(L^u_t)^{1+\zeta}}{1+\zeta}} \tag{87}$$

$$\beta \mathbb{E}_t \xi_{t+1} \frac{\lambda^u_{t+1} + \mu^u_{t+1}}{\pi_{t+1}} = \lambda^u_t \left[1 + \phi_M (M_t - M)\right] \tag{88}$$

$$\lambda^u_t \left(1 + \varkappa^{DC} \frac{DC^u_t}{\widetilde{DC}^u}\right) = \beta \mathbb{E}_t \xi_{t+1} \lambda^u_{t+1} \frac{R^{DC}_t}{\pi_{t+1}} \tag{89}$$

$$\alpha_M C^u_t = \frac{M_{t-1}}{\pi_t} \tag{90}$$

Production.

$$Q_t = 1 + \frac{\kappa_I}{2} \left(\frac{I_t}{I} - 1\right)^2 - \frac{I_t}{I} \kappa_I \left(\frac{I_t}{I} - 1\right) \tag{91}$$

$$K_t = (1-\delta) K_{t-1} + I_t \tag{92}$$

$$Y_t = A_t K^\alpha_{t-1} L^{1-\alpha}_t \tag{93}$$

$$\frac{z^k_t K_{t-1}}{w_t L_t} = \frac{\alpha}{1-\alpha} \tag{94}$$

$$MC_t = \frac{1}{A_t} \left(\frac{z^k_t}{\alpha}\right)^\alpha \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{95}$$

$$\pi_t(\pi_t - 1) = \frac{\epsilon - 1}{\kappa} \left(\mathcal{M}_t MC_t + \tau - 1\right) + \mathbb{E}_t \left[\Lambda^h_{t,t+1} (\pi_{t+1} - 1) \pi_{t+1} \frac{Y_{t+1}}{Y_t}\right] \tag{96}$$

Banks.

$$\psi_t = \theta^b \phi_t \tag{97}$$

$$\phi_t = \frac{\upsilon_t}{\theta^b - \mu_t} \tag{98}$$

$$\mu_t = \mathbb{E}_t \left[\Omega_{t,t+1} \left\{ \frac{z_{t+1}^k + (1-\delta)Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}} \right\} \right] \tag{99}$$

$$\upsilon_t = \mathbb{E}_t \left[\Omega_{t,t+1} \frac{R_t}{\pi_{t+1}} \right] \tag{100}$$

$$\Omega_{t,t+1} = \Lambda_{t,t+1}^h (1 - \sigma_b + \sigma_b \psi_{t+1}) \tag{101}$$

Monetary and fiscal policy.

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_Y} \right]^{1-\rho_R} \exp(\varepsilon_t^R) \tag{102}$$

$$\frac{R_{t-1}^{DC}}{\Pi_t} DC_{t-1} + \frac{M_{t-1}}{\Pi_t} = \tau Y_t + \Gamma_h T_t^h + \Gamma_u T_t^u + DC_t + M_t \tag{103}$$

$$R_t^{DC} = R_t \tag{104}$$

Market clearing.

$$C_t = \Gamma_h C_t^h + \Gamma_u C_t^u \tag{105}$$

$$L_t = \Gamma_h L_t^h + \Gamma_u L_t^u \tag{106}$$

$$DC_t = \Gamma_h DC_t^h + \Gamma_u DC_t^u \tag{107}$$

$$\begin{aligned} Y_t = C_t + \left[1 + \Phi \left(\frac{I_t}{I} \right) \right] I_t + \frac{\kappa}{2} (\pi_t - 1)^2 Y_t \\ + \Gamma_h (\chi_t^h + \chi_t^{DC,h}) + \Gamma_u (\chi_t^M + \chi_t^{DC,u}) \end{aligned} \tag{108}$$

$$K_t = \Gamma_h (K_t^h + K_t^b) \tag{109}$$

$$\begin{aligned} N_t = \sigma_b \left[(z_t^k + (1-\delta)Q_t) K_{t-1}^b - \frac{R_{t-1}}{\pi_t} D_{t-1} \right] \\ + \gamma_b (z_t^k + (1-\delta)Q_t) \frac{K_{t-1}}{\Gamma_h} \end{aligned} \tag{110}$$

$$Q_t K_t^b = \phi_t N_t \tag{111}$$

$$Q_t K_t^b = D_t + N_t \tag{112}$$

Exogenous processes.

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A \tag{113}$$

$$\mathcal{M}_t = (1-\rho_M)\mathcal{M} + \rho_M \mathcal{M}_{t-1} + \varepsilon_t^M \tag{114}$$

$$\ln \xi_t = \rho_b \ln \xi_{t-1} + \varepsilon_t^b \tag{115}$$

A.2.7 Model Steady State

In the non-stochastic steady state, we have the following:

$$\begin{aligned}
Q &= 1, \\
\pi &= 1, \\
R &= \frac{1}{\beta}, \\
R^{DC} &= R.
\end{aligned}$$

We define the discounted spreads on equity as:

$$s = \beta[z^k + (1-\delta)] - 1, \tag{116}$$

which we consider to be endogenous.

From the BHH's FOC with respect to equity, (72), we have:

$$\begin{aligned}
1 &= \beta \left[\frac{z^k + (1-\delta)}{1 + \varkappa^h \Gamma_h \frac{K^h}{K}} \right] \\
1 + \varkappa^h \Gamma_h \frac{K^h}{K} &= \beta \left[z + (1-\delta) \right] \\
\Gamma_h \frac{K^h}{K} &= \frac{s}{\varkappa^h}.
\end{aligned} \tag{117}$$

Additionally, in steady state we have:

$$\Omega = \beta(1 - \sigma_b + \sigma_b \psi),$$

$$
\begin{aligned}
\upsilon &= \frac{\Omega}{\beta}, \\
\mu &= \Omega \left[z^k + (1-\delta) - \frac{1}{\beta} \right],
\end{aligned}
$$

and so, using (116) we can write:

$$
\frac{\mu}{\upsilon} = s.
$$

Next, define J as:

$$
J = \frac{n_{t+1}}{n_t} = \left[z^k + (1-\delta) \right] \frac{K^b}{N} - R\frac{D}{N},
$$

and use the following:

$$
\begin{aligned}
\frac{D}{N} &= \phi - 1, \\
\phi &= \frac{K^b}{N},
\end{aligned}
$$

to write J as:

$$
\begin{aligned}
J &= (z^k + (1-\delta) - R)\phi + R \\
&= \frac{1}{\beta} \left[s\phi + 1 \right].
\end{aligned}
$$

Then, from (40) we have:

$$
\begin{aligned}
N &= \sigma_b \left\{ \left[z^k + (1-\delta)\right] K^b - RD \right\} + \gamma_b \left[z^k + (1-\delta)\right] \frac{K}{\Gamma} \\
\frac{N}{N} &= \sigma_b \left\{ \left[z^k + (1-\delta)\right] \frac{K^b}{N} - R\frac{D}{N} \right\} + \frac{\gamma_b}{N} \left[z^k + (1-\delta)\right] \frac{K}{\Gamma} \\
\beta &= \sigma_b \beta J + \frac{\gamma_b}{N} \beta \left[z^k + (1-\delta)\right] \frac{K}{\Gamma} \\
&= \sigma_b \beta J + \frac{\gamma_b K^b}{N} \left(1 + \varkappa^h \Gamma \frac{K^h}{K}\right) \frac{K}{\Gamma K^b} \\
&= \sigma_b \beta J + \gamma_b (1+s) \phi \frac{1}{\frac{\Gamma K^b}{K}} \\
&= \sigma_b \beta J + \gamma_b (1+s) \phi \frac{1}{\frac{K - \Gamma K^h}{K}} \\
&= \sigma_b \left[s\phi + 1\right] + \gamma_b (1+s) \phi \frac{1}{1 - \frac{s}{\varkappa^h}} \\
\beta &= \sigma_b + \left[\sigma_b s + \gamma_b \frac{1+s}{1 - \frac{s}{\varkappa^h}}\right] \phi,
\end{aligned}
$$

or

$$
\phi = \frac{\beta - \sigma_b}{\sigma_b s + \gamma_b \frac{1+s}{1 - \frac{s}{\varkappa^h}}}
$$

Equation (24) in steady state gives us:

$$
\begin{aligned}
\psi &= \beta (1 - \sigma_b + \sigma_b \psi) J \\
&= \beta J - \beta \sigma_b J + \beta \sigma_b \psi J \\
&= \beta (1 - \sigma_b) J + \beta \sigma_b \psi J \\
&= \frac{\beta (1 - \sigma_b) J}{1 - \beta \sigma_b J} \\
&= \frac{(1 - \sigma_b) \left[s\phi + 1\right]}{1 - \sigma_b \left[s\phi + 1\right]} \\
&= \frac{(1 - \sigma_b) \left[s\phi + 1\right]}{1 - \sigma_b - \sigma_b s \phi},
\end{aligned}
$$

and from (79) we have

$$
\psi = \theta^b \phi.
$$

Combine the expressions for ϕ and ψ to get:

$$\frac{\theta^b(\beta-\sigma_b)}{\sigma_b s+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}} = \frac{(1-\sigma_b)\left[\frac{s(\beta-\sigma_b)}{\sigma_b s+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}}+1\right]}{1-\sigma_b-\sigma_b\left[\frac{s(\beta-\sigma_b)}{\sigma_b s+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}}\right]},$$

then rearrange:

$$\begin{aligned} H(s) = (1-\sigma_b)&\left[s\beta+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}\right]\left[s\sigma_b+\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}\right] \\ &-\theta^b(\beta-\sigma_b)\left[\sigma_b(1-\beta)s+(1-\sigma_b)\gamma_b\frac{1+s}{1-\frac{s}{\varkappa^h}}\right]. \end{aligned}$$

We can observe that as $\gamma_b \to 0$,

$$\begin{aligned} &H(s) = (1-\sigma_b)s^2\beta\sigma_b - \theta^b(\beta-\sigma_b)\left[\sigma_b(1-\beta)s\right] \\ \implies\ & s \to \theta^b\frac{(\beta-\sigma_b)(1-\beta)}{(1-\sigma_b)\beta}. \end{aligned}$$

Thus, there exists a unique steady state equilibrium with positive spread $s>0$ for a small enough γ_b.

Given s, we then yield:

$$z^k = \frac{1}{\beta}(1+s)-(1-\delta),$$

and from (11) in the steady state:

$$MC = \frac{1-\tau}{\mathcal{M}},$$

and with (9), (10), and (12) we get:

$$MC = \frac{z^k}{\alpha}\frac{K}{Y},$$

or

$$\frac{K}{Y} = MC\frac{\alpha}{z^k}.$$

From the FOCs of the BHH and UHH problem, we have:

$$w = \zeta_0^h (L^h)^\zeta,$$
$$w = \frac{\zeta_0^u (L^u)^\zeta (1 + \frac{\alpha_M}{\beta} - \alpha_M)}{\left[C^u - \zeta_0^u \frac{(L^u)^{1+\zeta}}{1+\zeta}\right]}.$$

But since we have that $\zeta_0^u = \frac{\zeta_0^h}{(1+\frac{\alpha_M}{\beta}-\alpha_M)}$, we can write:

$$w = \zeta_0^h L^\zeta.$$

We can then use our previous expression for w to express L as a function of z^k:

$$L = \left[\frac{1-\alpha}{\zeta_0^h}\left(\frac{z^k}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{1}{\zeta}}.$$

Since we know that

$$w = (1-\alpha)\frac{Y}{L},$$

we yield:

$$Y = \frac{\zeta_0^h}{\alpha}\left[\frac{1-\alpha}{\zeta_0^h}\left(\frac{z^k}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{1+\zeta}{\zeta}}.$$

Additionally, we have:

$$\frac{I}{K} = \delta,$$

and

$$\frac{1}{\beta} = \frac{\alpha\frac{Y}{K} + 1 - \delta}{1 + \varkappa^h \Gamma_h \frac{K^h}{K}}$$
$$\Leftrightarrow \frac{Y}{K} = \frac{\beta^{-1}(1+s) + \delta - 1}{\alpha},$$

from (117), and

$$\frac{I}{Y} = \frac{I/K}{Y/K} = \frac{\alpha\delta}{\beta^{-1}(1+s)+\delta-1}.$$

These of course imply:

$$K = \left[\frac{1-\alpha}{\zeta_0^h}\left(\frac{z^k}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{1+\zeta}{\zeta}} \frac{\zeta_0^h}{\beta^{-1}(1+s)+\delta-1}$$

With K and s in hand, we can then turn back to the BHH's FOC wrt to equity, (72), to find K^h:

$$K^h = \frac{s}{\varkappa^h}\frac{K}{\Gamma_h},$$

and also get K^b:

$$K^b = \frac{K}{\Gamma_h} - K^h.$$

This then gives us N as we already solved ϕ:

$$N = \frac{K^b}{\phi}.$$

Then D is also solved as a residual from (20):

$$D = K^b - N.$$

Given Y, I, and K, we can get C:

$$\frac{C}{Y} = 1 - \frac{I}{Y} - \frac{\varkappa^h}{2}(\Gamma_h K^h)^2\left(\frac{K}{Y}\right)^{-1}.$$

From the UHH's FOC with respect to M, we have:

$$\mu^u = \lambda^u\left(\frac{1}{\beta}-1\right),$$

and the FOC with respect to consumption gives us an expression for the marginal utility from consumption:

$$\left(C^u - \zeta_0^u \frac{(L^u)^{1+\zeta}}{1+\zeta}\right)^{-1} = \lambda^u \left(1 + \frac{\alpha_M}{\beta} - \alpha_M\right).$$

Thus, we can express λ^u as a function of the marginal utility of consumption:

$$\frac{1}{\lambda^u} = \left(1 + \frac{\alpha_M}{\beta} - \alpha_M\right)\left(C^u - \zeta_0^u \frac{(L^u)^{1+\zeta}}{1+\zeta}\right),$$

noting that because of the values of ζ_0^h and ζ_0^u, we have:

$$L^u = \left(\frac{w}{\zeta_0^h}\right)^{\frac{1}{\zeta}}.$$

Finally, much like aggregate digital currency holdings, the BHH will not hold any digital currency holdings in steady state due to the presence of management costs. This means that in steady state:

$$DC^h = \frac{\beta R^{DC} - 1}{\varkappa^{DC}} + \widetilde{DC}^h$$

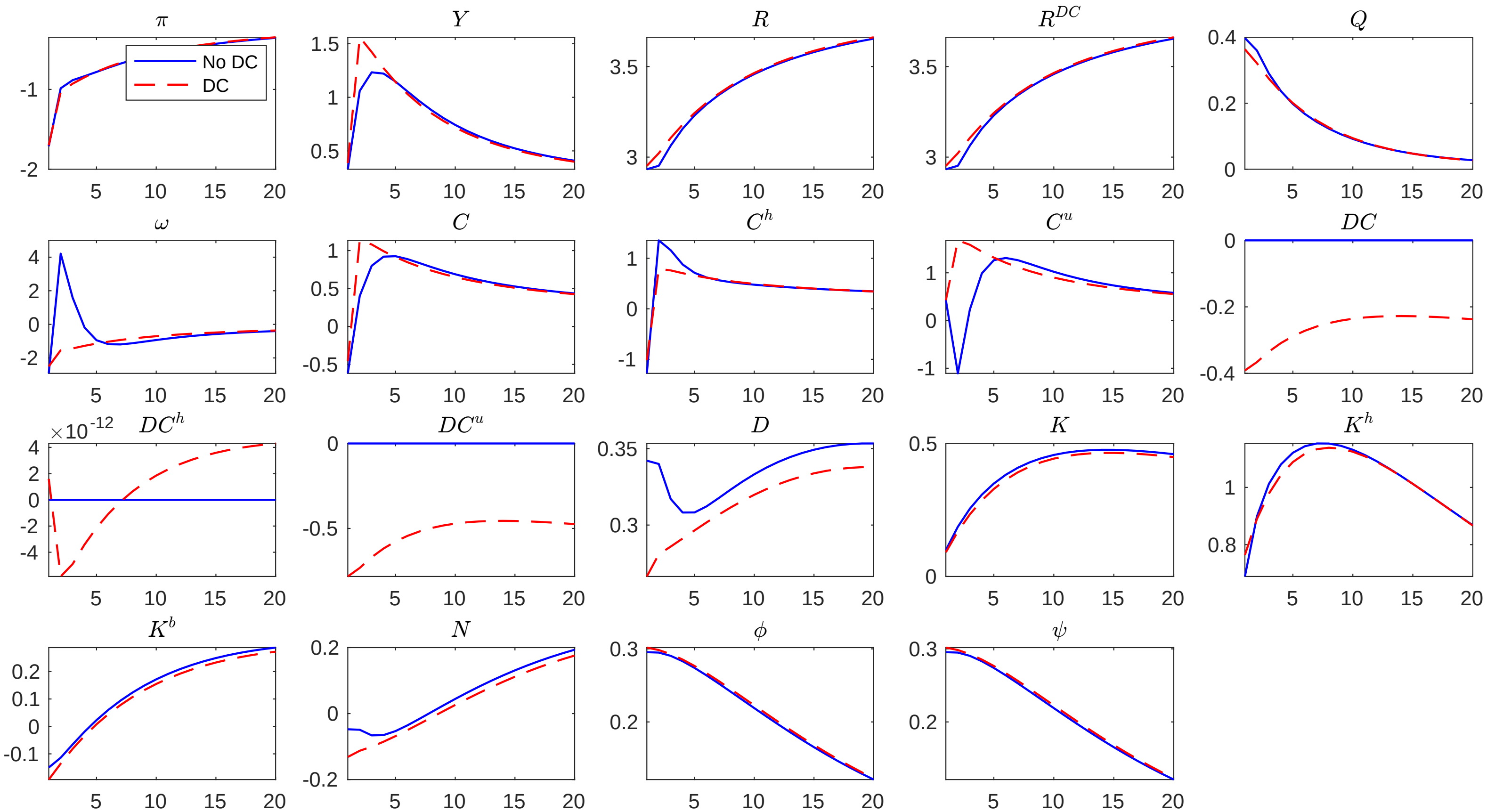
which, of course, implies:

$$DC^u = \frac{\beta R^{DC} - 1}{\varkappa^{DC}} + \widetilde{DC}^u.$$

A.2.8 Additional Impulse Responses to Shocks

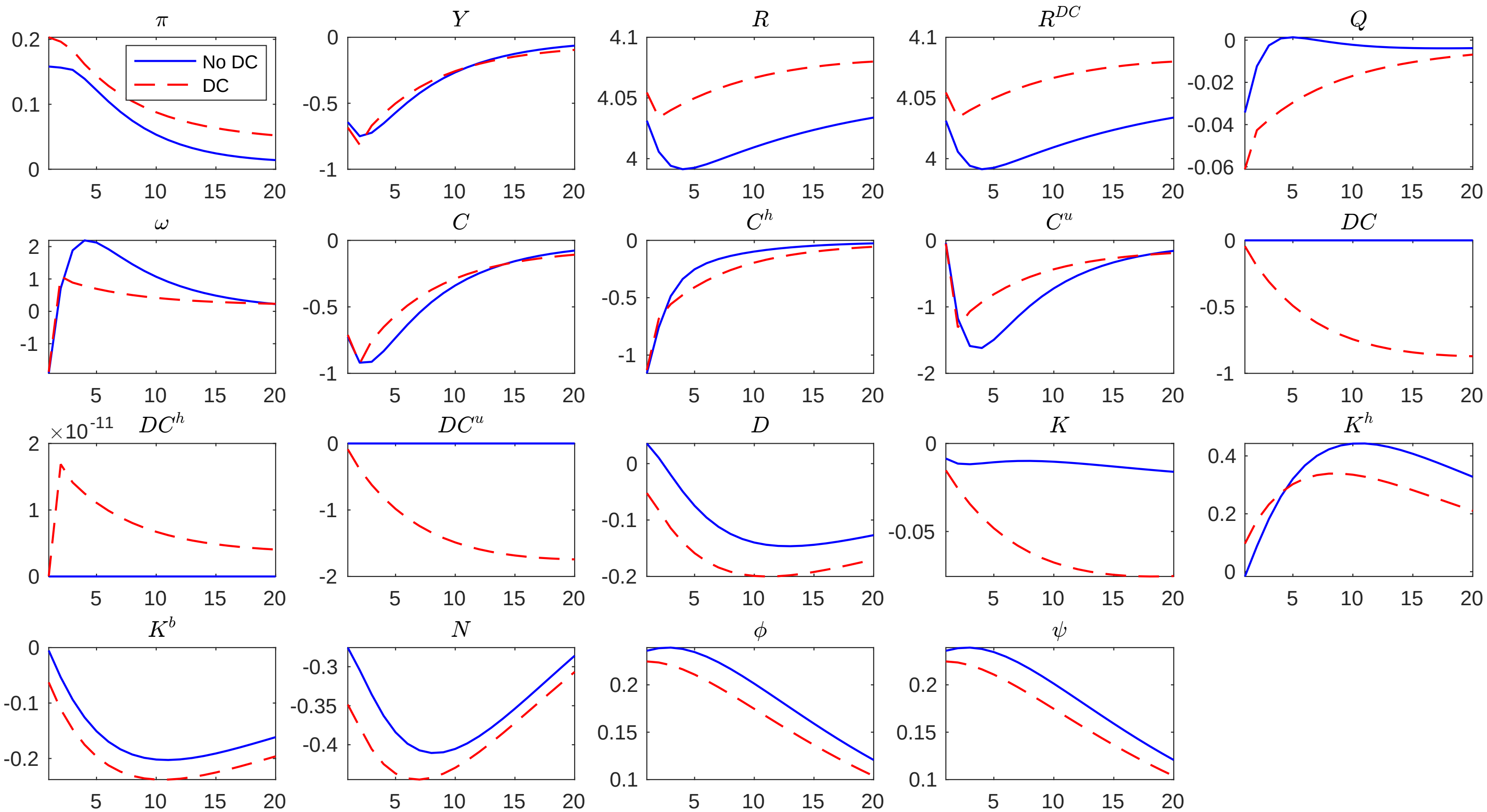
Figures 9, 10, and 11 present results in response to an annualized 1% orthogonal innovation to TFP, cost-push, and preference shocks, respectively. The figures compare IRFs for a no-CBDC economy and to an economy with CBDCs. Meanwhile, Figures 12, 13, and 14 plot IRFs in response to a TFP, cost-push, and preference shock, respectively, for two regimes: a Taylor rule with a constant spread on R^{DC} and a 2-instrument Ramsey optimal policy regime.

Figure 9: **IRFs to a 1% TFP shock**



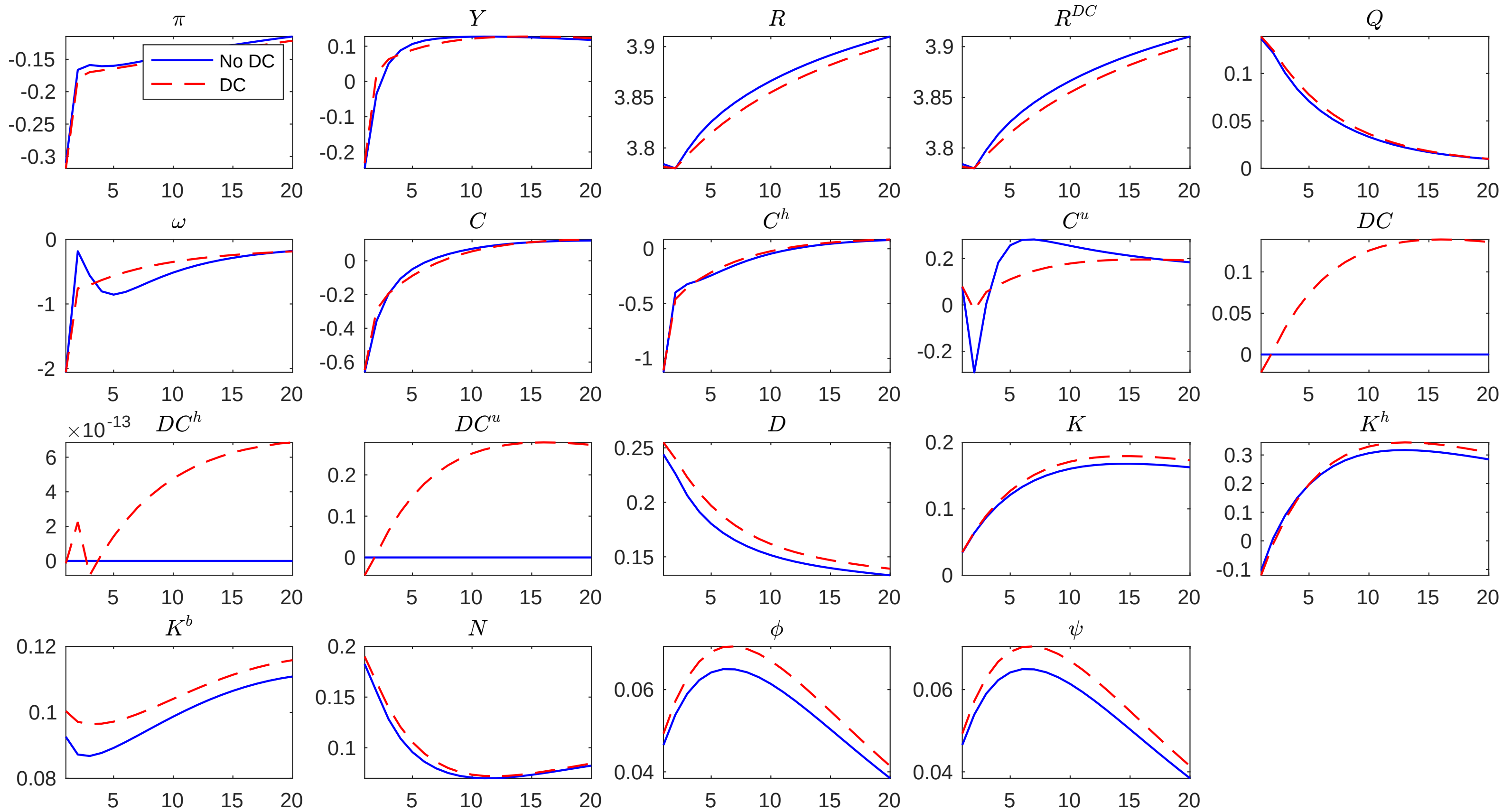
Note: Figure plots impulse responses of model variables with respect to a 1 % annualized innovation to TFP. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R), and Digital Currency Returns (R^{DC}) which are expressed as annualized net rates.

Figure 10: **IRFs to a 1% cost-push shock**



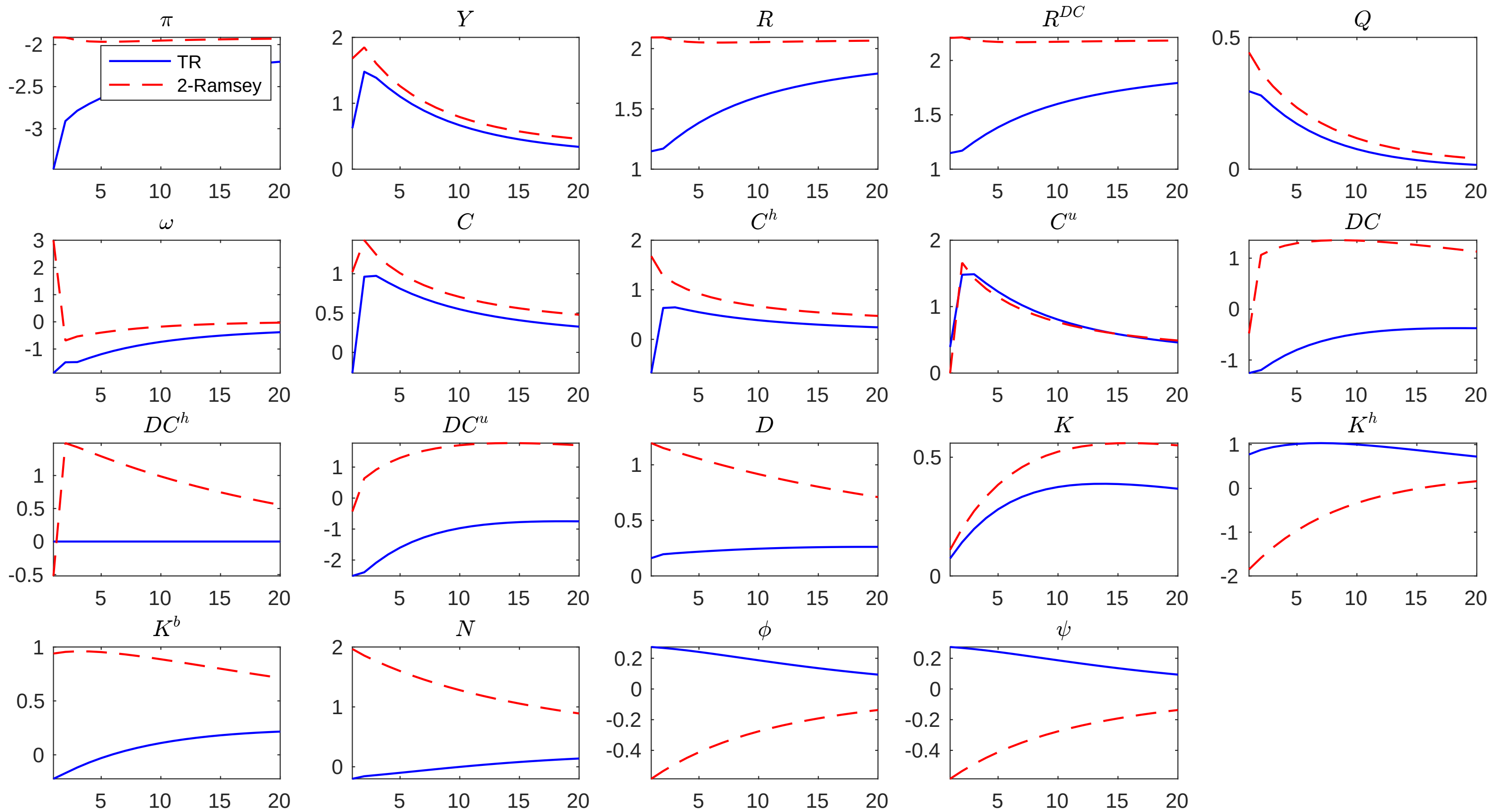
Note: Figure plots impulse responses of model variables with respect to a 1 % annualized innovation to markups. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R), and Digital Currency Returns (R^{DC}) which are expressed as annualized net rates.

Figure 11: **IRFs to a 1% annualized demand shock**



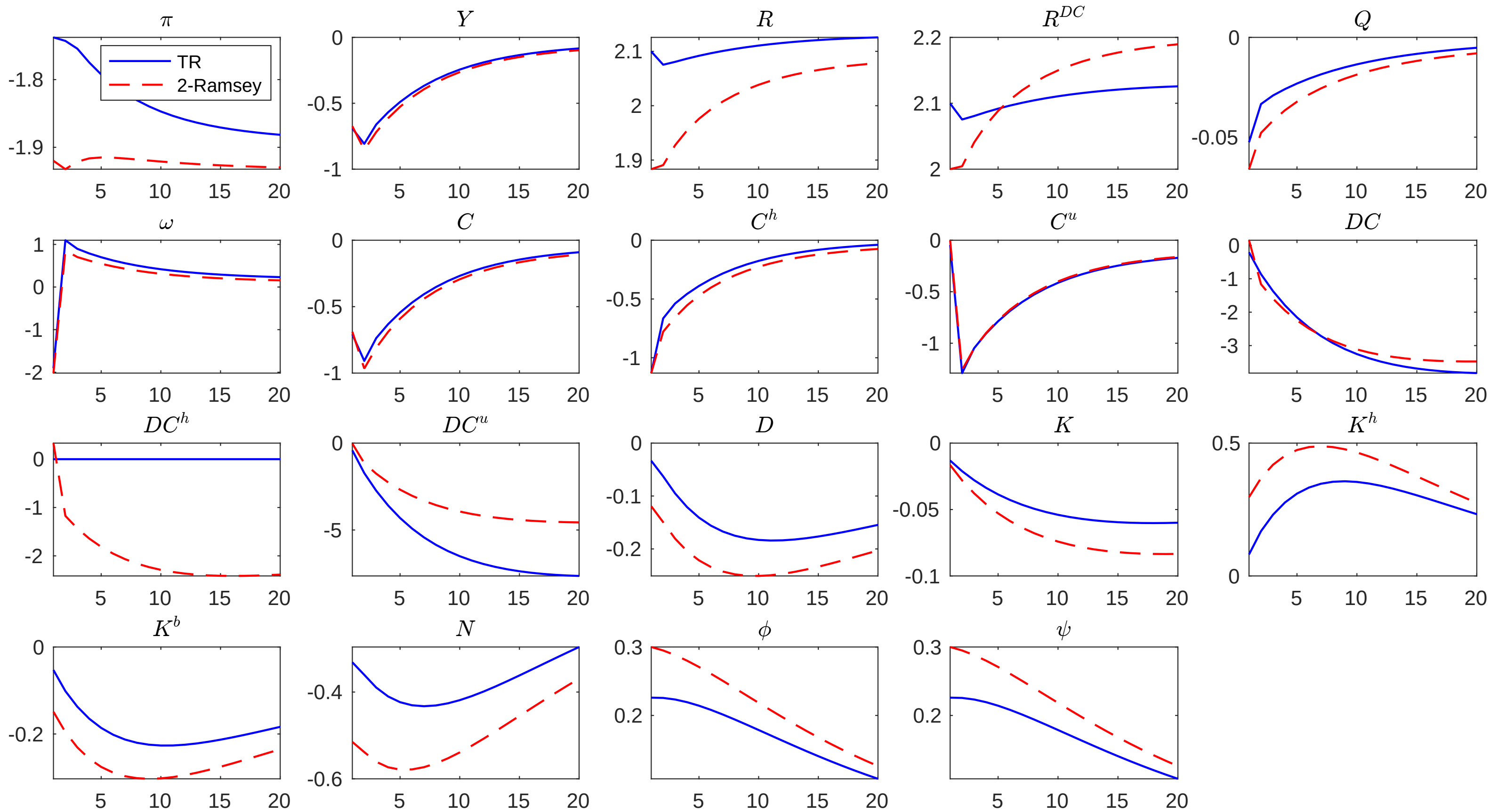
Note: Figure plots impulse responses of model variables with respect to a 1% annualized preference shock. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R) and Digital Currency Returns (R^{DC}) which are expressed as annualized net rates.

Figure 12: **IRFs to a 1% TFP shock**



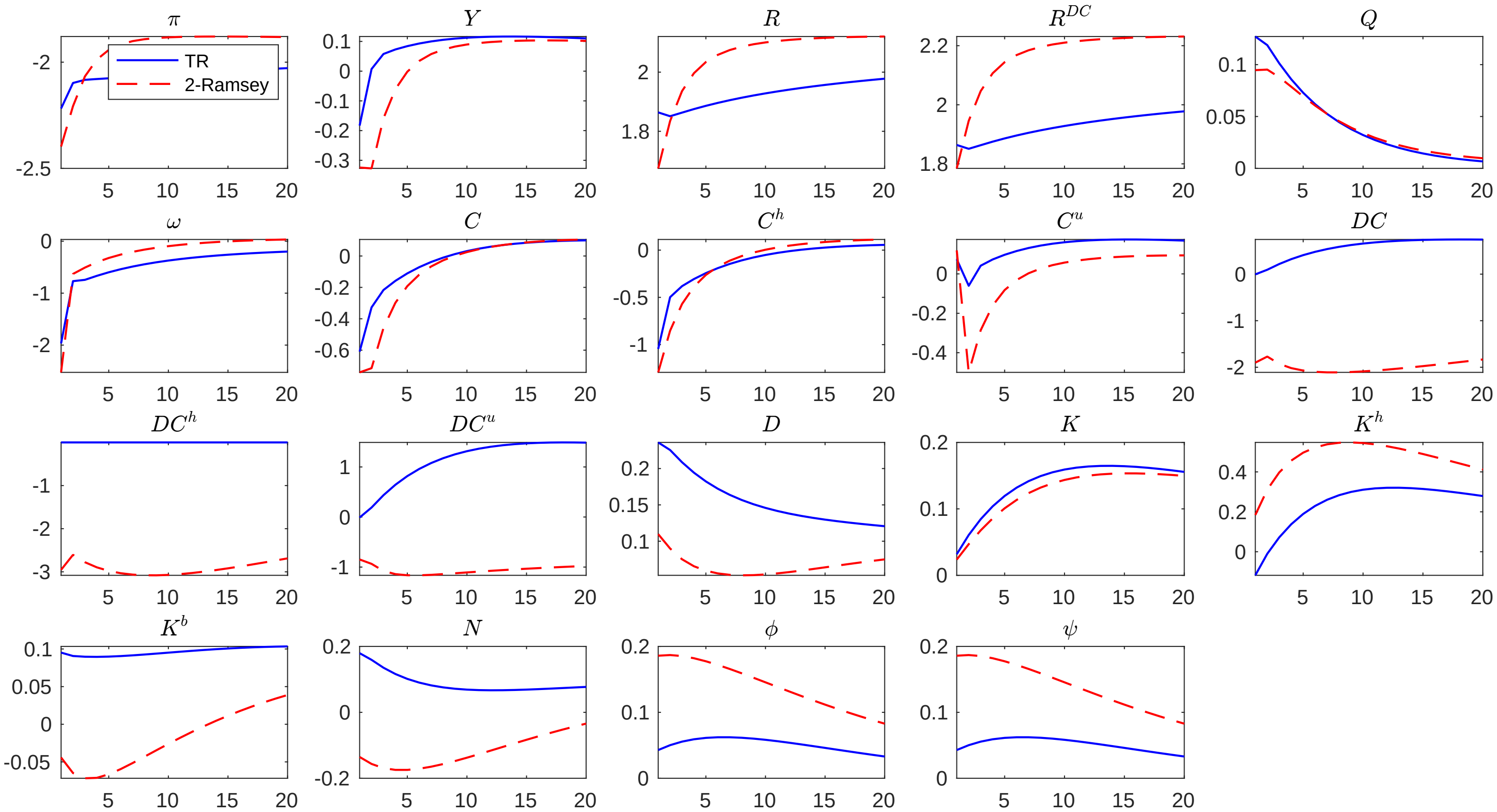
Note: Figure plots impulse responses of model variables with respect to a 1% annualized innovation to the markup. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R) and Digital Currency Returns (R^{DC}) which are expressed as annualized net rates.

Figure 13: **IRFs to a 1% annualized cost-push shock**



Note: Figure plots impulse responses of model variables with respect to a 1% annualized innovation to the markup. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R) and Digital Currency Returns (R^{DC}) which are expressed as annualized net rates.

Figure 14: **IRFs to a 1% annualized demand shock**



Note: Figure plots impulse responses of model variables with respect to a 1% annualized preference shock. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation (π), Nominal Interest Rates (R) and Digital Currency Returns (R^{DC}) which are expressed as annualized net rates.