Investment Analysis[1]

Lecture 5: Bonds and the Yield Curve

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[1] These lecture slides are based loosely on the set of lectures by Professors Eric Sims and Raymond da Silva Rosa.

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Introduction

- Bonds are broadly classified as "fixed income securities". This is as opposed to equity (stocks) which offer unknown payout streams. Furthermore, bonds (especially government/Treasury bonds) are seen as safe assets.
- Bonds entitle the holder to periodic cash flows.
- There are many different types of bonds, differing by things such as default risk, time to maturity, whether they come with coupon payments, and so on.
- The interest rate of the bond is commonly referred to as "yield to maturity" (YTM). Hence, I will switch back and forth between "interest rate" and "yield".
- In fact, perhaps the most important aspect of a bond is its yield, as that acts as a sort of barometer for broader market and economic conditions.

Present Value

- We have covered present discounted value (PDV): something received in the future is worth less to you than if you received the thing today.
- Discount rate: the rate at which you discount future cash flows from an asset.
- Suppose the discount rate is i and is constant.
- For a future cash flow (CF), how many dollars would be equivalent to you today would be:

$$
\begin{aligned}
PV_t &= \frac{CF_{t+n}}{(1+i)(1+i)(1+i)\times \ldots \times (1+i)} \\
&= \frac{CF_{t+n}}{(1+i)^n},
\end{aligned}
$$

where t is the time today, $t+n$ is the future (n periods away), and i is the discount rate.

Present Value and Asset Prices

- In the previous lecture on the CAPM, we showed a way to calculate the discount rate on a particular asset such as a stock. Additionally, recall from previous lectures that a small change in the discount rate (or perceived risk of an asset) had a profound effect on the asset's present value.
- The price of an asset is equal to the PDV of all of its future cash flows; the price of a bond is just the PDV of its cash flows.
- The yield is the discount rate you use to discount those future cash flows.
- The yield is also equal to the expected or required return: not necessarily equal to realised return if security is sold prior to maturity.

Bond Types

- Zero-coupon (simple) bond: you buy the bond for P which has a face value of F, and at maturity you earn X. Typically, $P < F$, and so the bond sells "at a discount".
- Coupon bond: You purchase the bond for P dollars and earn fixed coupon payments, C, each period (e.g. annually) for a specified period of time (e.g. 10 years), and at maturity the bond will pay off its face value.

Yield to Maturity

- The YTM is the (fixed) interest rate that equates the PDV of cash flows with the price of the bond in the present.
- This measures the return (expressed at an annualised rate) that would be earned on holding a bond if it is held until maturity.
- YTM may not correspond to the realised return if the bond is not held until maturity.
- The YTM is another way of calculating the price of a bond, taking future cash flows as given.

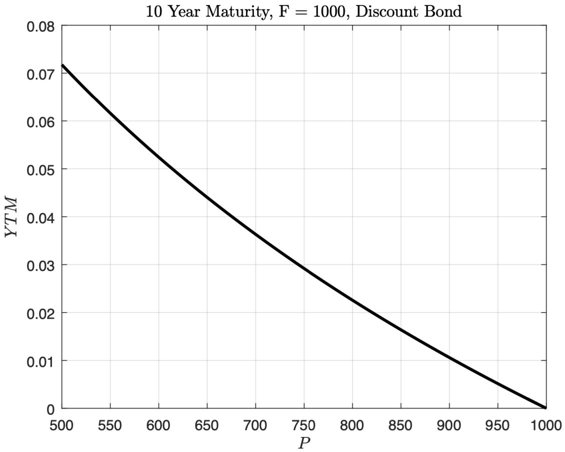
YTM on a Zero-Coupon Bond

- For a bond with a face value of F, maturity of n, and price of P, the YTM satisfies

$$P = \frac{F}{(1+i)^n}$$

$$1 + i = \left(\frac{F}{P}\right)^{\frac{1}{n}}.$$

Price and Yield on a Zero-Coupon Bond



Source: Eric Sims (2020)

YTM on a Coupon Bond

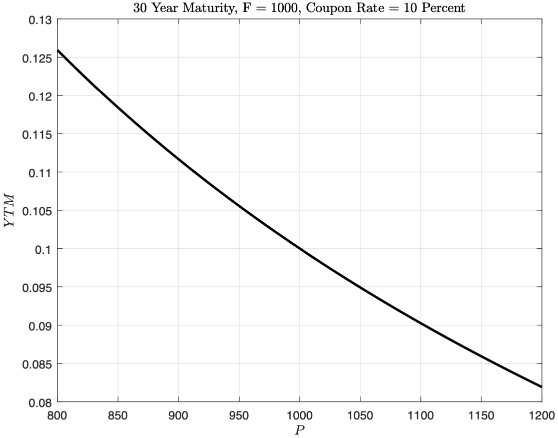
- This is a bit trickier (which is why we often assume bonds pay no coupons in models). Suppose a bond has a face value of \$100 and a maturity of three years.
- It pays coupon payments of \$10 in years $t+1$, $t+2$, and $t+3$, implying a coupon rate of 10 percent.
- The face value is paid out after period $t+3$.
- The period t price of the bond is \$100.
- Then the YTM solves:

$$100 = \frac{10}{1+i} + \frac{10}{(1+i)^2} + \frac{10}{(1+i)^3} + \frac{100}{(1+i)^3}$$
$$\implies i = 0.1.$$

- More generally, for an n period maturity coupon bond,

$$P = \sum_{j=1}^{n} \frac{C}{(1+i)^j} + \frac{F}{(1+i)^n}.$$

Price and Yield on a Coupon Bond



Source: Eric Sims (2020)

Coupon Rate vs Yield

- In the previous example, the coupon rate (10 percent) and the yield (10 percent) are the same.
- But, in general, this is not the case.
- This only holds if the coupon bond "sells at par" – meaning the price of the bond equals the face value. If the bond sells at a discount to face value, the yield is greater than coupon rate (and vice versa).
- i.e., coupon rate is not the same as the interest rate.

Prices and Yields

- Probably the most important takeaway from this lecture: The bond price and yield are negatively related, i.e., they move in opposite directions.
- For zero-coupon bonds, we would not expect price to be greater than face value – this would imply a negative yield.
- For a coupon bond, when the bond is priced at face value, the YTM equals the coupon rate. When the bond is priced less than face value, the YTM is greater than the coupon rate (and vice-versa).

Yields and Returns

- Returns and yields are, in general, not the same thing.
- Rate of return: cash flow plus new security price, divided by current price.
- Useful way to think about it (in terms of equities): dividend rate + capital gain, where capital gain is the change in the security's price.
- The return on a coupon bond held from t to $t+1$ is

$$R = \frac{C + P_{t+1} - P_t}{P_t}$$
$$\Leftrightarrow R = \underbrace{\frac{C}{P_t}}_{\text{Current yield}} + \underbrace{\frac{P_{t+1} - P_t}{P_t}}_{\text{Capital gain}}.$$

- Return will differ from current yield if bond prices fluctuates in unexpected ways.

Yields and Returns: Zero-Coupon Bond Example

- Suppose $F = \$1000$, maturity is 30 years, and a current YTM of 10 percent. Then,

$$P = \frac{1000}{1.1^{30}} = 57.31.$$

- If the interest rate stays the same after a year. Then the bond has a price of

$$P = \frac{1000}{1.1^{29}} = 63.04.$$

- Since there is no coupon payment, the one year holding period return is just the capital gain:

$$\begin{aligned} R &= \frac{P_{t+1} - P_t}{P_t} \\ &= \frac{63.04 - 57.31}{57.31} = 0.1. \end{aligned}$$

- If interest rates do not change, then the return and the YTM are the same thing.

Interest Rate Risk

- Continue with the same setup.
- But then suppose that interest rates go up to 15 percent in period $t+1$ and are expected to remain there.
- Then the price of the bond in period $t+1$ will be

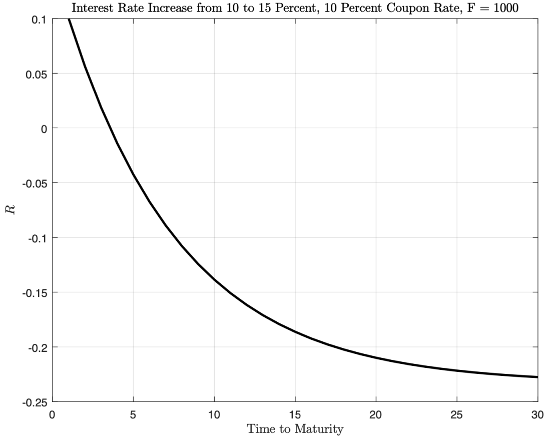
$$P = \frac{1000}{1.15^{29}} = 17.37.$$

- Your return is then

$$\begin{aligned} R &= \frac{P_{t+1} - P_t}{P_t} \\ &= \frac{17.37 - 57.31}{57.31} = -0.69. \end{aligned}$$

- On a zero-coupon bond, an increase in interest rates exposes you to capital loss. This is also true, in general, for a coupon bond.
- This is often referred to as interest rate risk (related to another concept: duration).

Return and Time to Maturity (Coupon Bond)



Source: Eric Sims (2020)

Returns and Yields

- Returns and initial YTM are equal if the holding period is the same as time to maturity (1 period). The capital gain is simply the face value (which is fixed) minus the initial price.
- Increase in interest rates results in returns being less than initial yield. The reverse is also true.
- Returns are more affected by interest rate changes the longer is the time to maturity.
- If you hold the bond until maturity, your return is locked in at initial YTM.
- Thinking of returns is relevant even if you do not sell the bond and realise the capital loss. There is an opportunity cost: If interest rates rise, had you not locked yourself in on a long maturity bond you could have purchased a bond in the future with a higher yield.
- Longer maturity bonds are therefore risker than short maturity bonds.

Determinants of Bond Prices

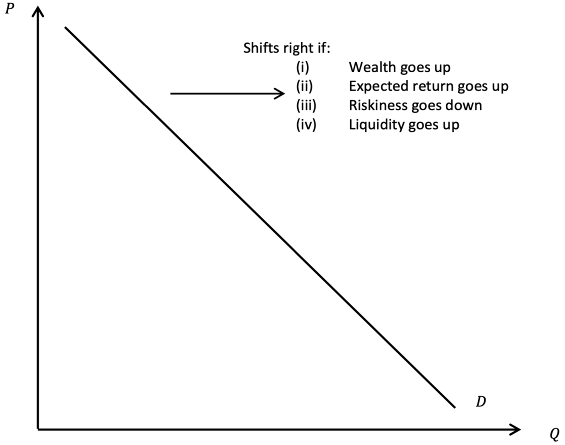
- What determines prices (and hence yields)?
- Two related approaches:
  - Conceptual: demand and supply (portfolio choice)
  - Micro-founded: explicit consumption-saving maximisation problem.
- For simplicity, assume all bonds are zero-coupon bonds.

Bond Prices: Portfolio Choice

- Based on the idea of demand and supply.
- Demand for bonds is based on the following:
  - Wealth/income
  - Expected returns
  - Risk
  - Liquidity
- Supply for bonds is based on:
  - Expected profitability
  - Expected inflation
  - Government spending

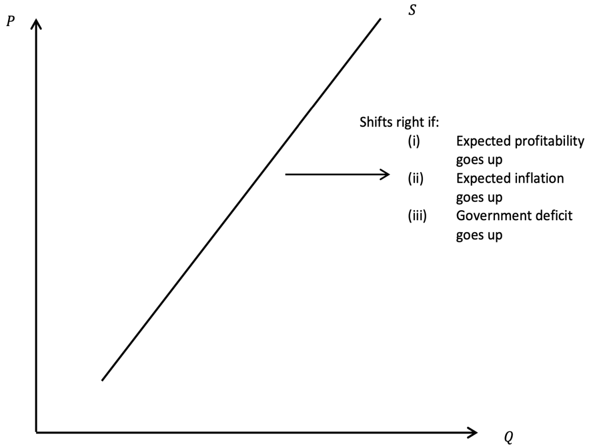
Bond Prices: Portfolio Choice

Figure: Bond Demand



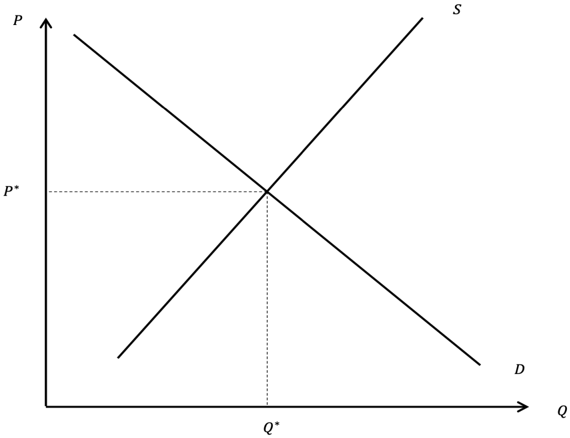
Bond Prices: Portfolio Choice

Figure: Bond Supply



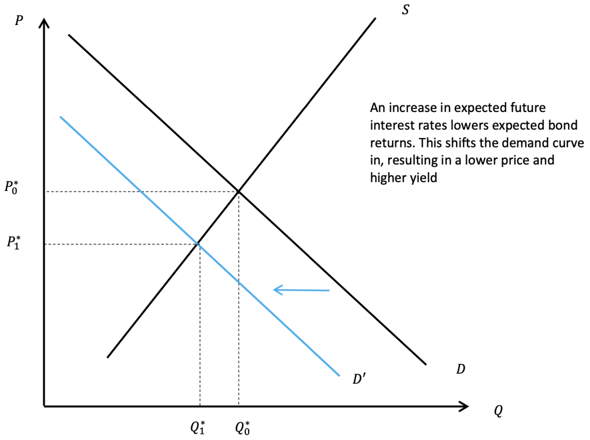
Bond Prices: Portfolio Choice

Figure: Bond Market Equilibrium



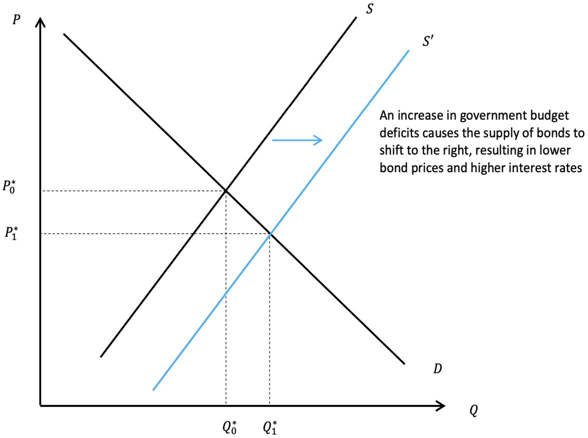
Bond Prices: Portfolio Choice

Figure: An Increase in Expected Future Interest Rates



Bond Prices: Portfolio Choice

Figure: An Increase in Government Budget Deficit



Bond Prices: Micro-Foundations

- The term "micro-foundations" basically means that there is some kind of economic model which rationalises prices and quantities. For example, here we are micro-founding the behaviour of bond prices – we're saying that fluctuations in bond prices are not due to random external factors.
- Consider the case of a household choosing consumption over two periods (simple Fisher like model).
- Simple set up: Household lives for two periods, t and $t+1$, and it consumes consumption, C, from income, Y. Bonds yield a return of i. Household chooses C_t and C_{t+1}.
- The household's perfect foresight allows it to know Y_t, Y_{t+1}, i, and price levels in the two periods, P_t and P_{t+1}. This also gives the annual rate of inflation:

$$\pi = \frac{P_{t+1} - P_t}{P_t}, \tag{1}$$

and

$$P_{t+1} = P_t(1 + \pi_t). \tag{2}$$

Bond Prices: Micro-Foundations

- So, the household must choose consumption in periods t and $t+1$, as well as how much to save in period t by investing in bonds (remember that since the household "dies" in period $t+1$, it will not save in $t+1$).
- Assuming no uncertainty, the utility that a household gains from consumption is:

$$U = u(C_t) + \beta u(C_{t+1}), \tag{3}$$

given budget constraints (in real terms):

$$C_t + B_t = Y_t, \tag{4}$$

$$C_{t+1} = Y_{t+1} + (1+r)B_t, \tag{5}$$

where $u(\cdot)$ is period utility and β is the household's discount factor – the more impatient the household is, the smaller is β.

Bond Prices: Micro-Foundations

- In the first period, the budget constraint in nominal or monetary terms is

$$\begin{aligned} P_t Y_t &= P_t C_t + P_t B_t \\ P_t B_t &= P_t Y_t - P_t C_t. \end{aligned}$$

- This money saved will, in the second period, be worth

$$P_t B_t (1+i) = (P_t Y_t - P_t C_t)(1+i),$$

and so second period consumption expenditure will be

$$\begin{aligned} P_{t+1} C_{t+1} &= P_t B_t (1+i) + P_{t+1} Y_{t+1} \\ \Leftrightarrow P_{t+1} C_{t+1} &= (P_t Y_t - P_t C_t)(1+i) + P_{t+1} Y_{t+1}. \end{aligned}$$

- But we know the rate of inflation, π, so we can convert all prices back to the first period:

$$P_t (1+\pi) C_{t+1} = (P_t Y_t - P_t C_t)(1+i) + P_t (1+\pi) Y_{t+1}. \tag{6}$$

Bond Prices: Micro-Foundations

- Then, divide the previous expression by P_t to obtain an expression in real terms:

$$\begin{aligned}(1+\pi)C_{t+1} &= (Y_t - C_t)(1+i) + (1+\pi)Y_{t+1} \\ C_{t+1} &= (Y_t - C_t)\frac{1+i}{1+\pi} + Y_{t+1} \\ &= (Y_t - C_t)(1+r) + Y_{t+1} \\ &= B_t(1+r) + Y_{t+1}, \end{aligned} \tag{7}$$

  which is what we had previously!

- Solving the household's optimisation problem yields the "consumption Euler equation":

$$\frac{1}{1+r} = \underbrace{\beta\frac{u'(C_{t+1})}{u'(C_t)}}_{M_{t,t+1}}, \tag{8}$$

  where the RHS is known as the stochastic discount factor (SDF), $M_{t,t+1}$.

Bond Prices: Micro-Foundations

- As an aside: the general asset pricing condition is

$$P_t^A = \mathbb{E}_t \left[\sum_{j=1}^{\infty} M_{t,t+j} D_{t+j}^A \right], \tag{9}$$

where A indexes an asset, D_{t+j}^A is the cash flow generated by the asset in period $t + j$, $M_{t,t+j}$ is the SDF, and the summation term allows the security to pay cash flows in multiple periods.

Bond Prices: Micro-Foundations

- Some more intuition on the real interest rate: Consider a household with assets worth \$100 this year.
- Prices this year are normalised to \$1 per unit, and next year the price will be $\$(1+\pi)$ per unit.
- The household can therefore consume 100 units of output this year, or save it by purchasing a \$100 bond.
- Next year, the bond will nominally be worth $\$100(1+i)$.
    - But how much real output will the household be able to buy with it next year?

Bond Prices: Micro-Foundations

- Prices will be $\$(1+\pi)$ next year, so the output purchasing power will be
$$100\frac{1+i}{1+\pi} \text{ units.}$$
- By purchasing a bond with nominal interest i, the household has raised its purchasing power of real output by
$$100\frac{1+i}{1+\pi} - 100 \text{ units.}$$
- This represents a real rate of return, or real interest rate, of
$$r = \frac{100\frac{1+i}{1+\pi} - 100}{100} = \frac{1+i}{1+\pi} - 1$$
$$\Leftrightarrow 1+r = \frac{1+i}{1+\pi}. \tag{10}$$
- This is the gross return on the household's saving – or the opportunity cost of its current consumption – the amount it can consume next year per unit of consumption denied in the present.

Bond Prices: Micro-Foundations

- Assuming that the household is able to smooth consumption, such that $C_t = C_{t+1}$, then we have
$$\frac{1}{1+r} = \beta.$$
- Then, using the asset pricing condition from the Euler equation, we see that
$$1 + r = \frac{1}{P_t^B},$$
which is an expression in real terms. We could simply assume no inflation and we would get:
$$1 + i = \frac{1}{P_t^B}.$$

Bond Prices

- Prefer to compare bonds in terms of yields instead of prices.
- Why? Comparing yields puts assets with different levels of cash flow on "equal footing".
  - Bonds with the same cash flow details may nevertheless have very different yields.
- Aside form details about cash flows, bonds differ in price due to: risk (default) and time to maturity.

Default Risk

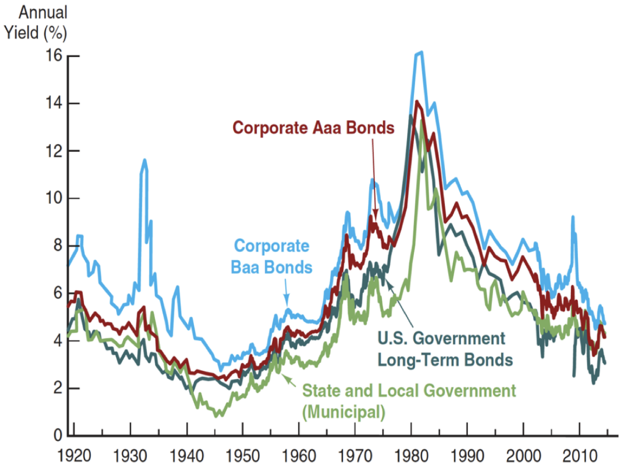
- The risk of default generates uncertainty – we know that people don't like uncertainty. It's safe to say that bonds which are more uncertain to be paid off are cheaper, and therefore feature a higher yield.
- Hence why we think of [advanced economy] government bonds as being risk-free. As such their prices are high and their yields are low.
- Corporate bonds (and bonds from less reputable governments) are considered to have some risk.
- Companies such as Moody's and Standard & Poor's rate these bonds: AAA is the most safe, followed by B's, and then C's (junk).
- Assuming the same time to maturity, the risk premium is defined as:

$$RP = r_{-f} - r_f,$$

where r_{-f} denotes the yield on a non-risk-free bond.

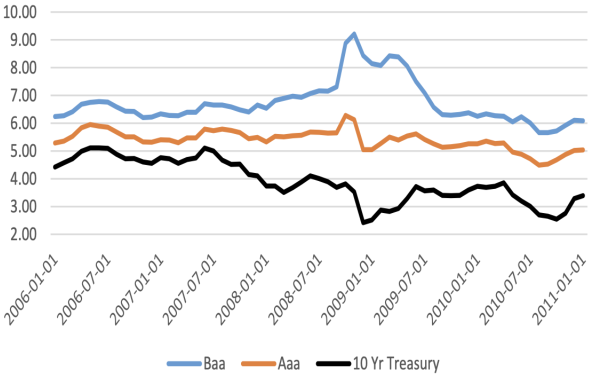
Bond Yields and Risk

Figure: Bond Yields



Yields and the GFC

Figure: Yields and the Global Financial Crisis



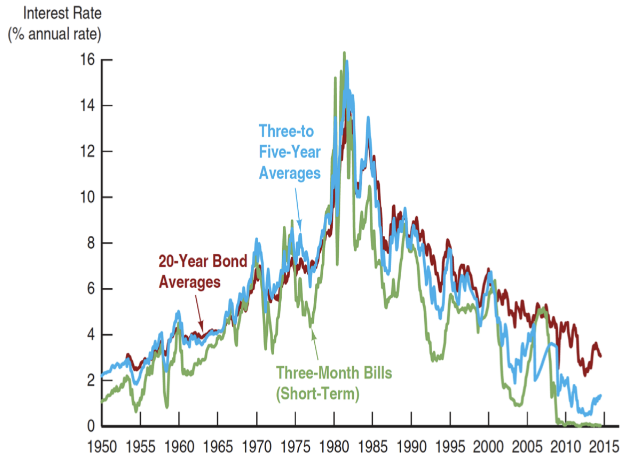
Source: Eric Sims (2020)

Term Structure of Interest Rates

- Conditional on having the same risk-profile – and for ease of discussion, let's assume we're only talking about Treasury Bills – bonds with different maturities will have different yields.
  - Why?
- Comparing yields on bonds and the time to maturity is known as a yield curve.

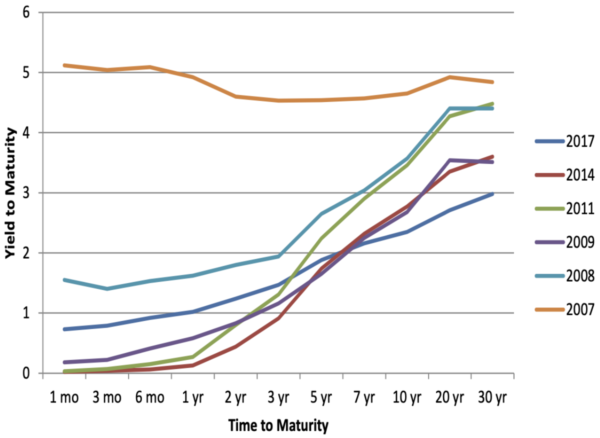
Term Structure of Interest Rates

Figure: US Government Bond Yields



Term Structure of Interest Rates

Figure: Yield Curve for 30-yr T-Bills



Source: Eric Sims (2020)

Term Structure of Interest Rates

- Yields on bonds with different maturities move together.
- Yield curves are concave (usually). They do not have this trait preceding a recession.

Term Structure of Interest Rates

- Economists have theories to explain the term structure of interest rates:
  - Expectations hypothesis: Bonds of different maturities are perfect substitutes.
  - Segmented markets: Bonds with different maturities are not substitutes.
  - Liquidity premium theory: Bonds with different maturities are imperfect substitutes.

Expectations Hypothesis

- Basic definition: the yield on a long term bond is the average of the expected yields on shorter maturity bonds.
- Example: Consider a 1 year bond and a 2 year bond.
  - Suppose that the yield on the 1 year bond is 4%, and suppose you EXPECT the yield on 1 year bonds 1 year later to be 6%.
  - Then the yield on the two year bond today should be 5%: $\frac{1}{2} \times (4+6) = 5$.
  - Why? You buy one 1 year bond today and one 1 year bond next year. Then your yield over two years should be: $1.04 \times 1.06 = 1.1024$, so roughly 10%.
  - If the 1 year bond and 2 year bond are perfect substitutes, then the yield on the 2 year bond today should give us:

$$(1+i)^2 = 1.1024 \implies i \approx 0.05.$$

Segmented Markets

- This idea is quite straightforward: there are many different types of investors with different investment horizons. Some investors prefer short term investments, some prefer long term investments.
- Investors who prefer short term bonds are very risk averse, and hence they're willing to pay more to purchase bonds. This causes short term bond yields to be relatively low.
- Investors who prefer long term bonds are less risk averse, and so bond prices are relatively cheap leading to higher yields.
- Hence, yield curves are generally upward sloping and concave.
- Simple explanation but not very convincing.

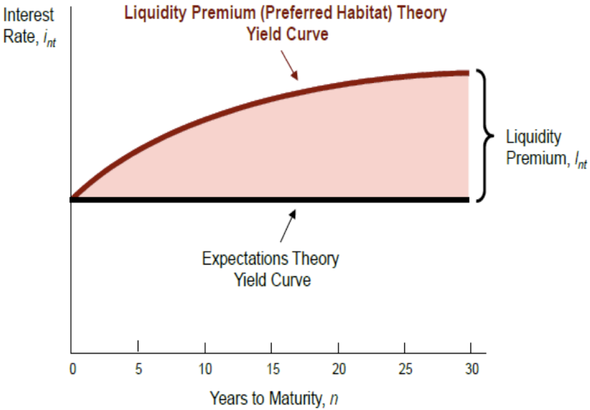
Liquidity Premium

- This is basically a mix of the expectations hypothesis mixed with some uncertainty.
  - Probably the most common way of understanding bond yields.
- Bonds with different maturities are not perfect substitutes, but rather imperfect substitutes.
- This is mostly because long term bonds are more risky than short term bonds.
  - With long bonds, you are exposed to potential interest rate risk when prices fluctuate.
- Investors demand some compensation for this risk:

$$i_{t,t+n} = \underbrace{\frac{i_{t,t+1} + i^e_{t+1,t+2} + \cdots + i^e_{t+n-1,t+n}}{n}}_{\text{Expectations hypothesis}} + \underbrace{l_{n,t}}_{\text{Liquidity premium}}$$

- The liquidity premium is increasing in bond maturity.

Liquidity Premium



Source: Eric Sims (2020)

Liquidity Premium

- Another intuitive way of thinking about the liquidity premium theory: opportunity cost.
- Suppose you excess savings which you wish to invest. But you want a mix of bonds and cash reserves – just in case something urgent comes up.
- A short term bond only requires you to lock away your funds for a shorter amount of time. If in case you need liquid assets, the short term bond gives you a lot of flexibility.
- Long term bonds lock away your funds for a significant amount of time. Yes, you can sell the bond off and bond markets are fairly liquid, but they're not as liquid as cash reserves. Thus, you as an investor require some compensation for the opportunity cost of investing in the long term bond as opposed to a short term bond or cash.

Conclusion

- Bonds are a very important financial instrument.
- They form the basis of many economic models, and they're integral to the CAPM.
- Their yields are seen as being risk-free, and after factoring for inflation, can be a proxy for real interest rates.
- Bond yields and prices are inversely related.
- Yield curves are usually upward sloped, concave shaped.
- Bonds are in high demand when market uncertainty rises.