Investment Analysis[1]

Lecture 4: Modern Portfolio Theory and CAPM

David Murakami[2]

11 October 2021

[1] These lecture slides are based loosely on the set of lectures by Professor Raymond da Silva Rosa.

[2] St Cross College, University of Oxford
Email: david.murakami@economics.ox.ac.uk

Introduction

- If one 2L bottle of Heinz tomato sauce costs $6, how much is one 500mL bottle worth?

MPT and CAPM

- Modern portfolio theory (MPT) shows how an investor who aims to maximise return and minimise risk should select investments (Markowitz, 1952).
- The capital asset pricing model (CAPM) provides the discount rate used to value individual investments, given that investors follow MPT (Linter and Sharpe, 1965).

MPT and CAPM

- MPT shows which combination of assets maximised expected return for a given level of risk, or put differently, minimise risk for a given expected return.
- MPT leads to development of model for valuing individual assets: the CAPM,

$$V = D_t + \frac{\mathbb{E}D_{t+1}}{1+r} + \frac{\mathbb{E}D_{t+2}}{(1+r)^{t+2}} + \cdots + \frac{\mathbb{E}D_{t+n}}{(1+r)^{t+n}}.$$

- Two challenges for any investment analysis:
  - Estimate expected cash flows in each period
  - Choose an appropriate discount rate, r.
- CAPM gives basis for estimating r.

A naive diversification strategy

- Babylonian Talmud (1500 years ago): "A man should always place his money, one third in land, a third into merchandise, and keep a third in hand."
- This is a naive diversification strategy, a "$1/n$ rule", where n is the number of assets available.
- How is MPT different?

A thought experiment

- Suppose you had to invest your retirement savings into six different funds. What proportion would you invest in each?

- Given a choice, most people would tend to use naive diversification.

Diversification

- To diversify efficiently:
  - Do not invest equally across available assets
  - Focus on adding assets whose returns you expect to move in the opposite direction to existing assets in your portfolio

Another thought experiment

- Assume three stocks, each with a standard deviation of 40% and expected return on 15%: Ford, Microsoft, and Starbucks.
  - if you buy all three stocks, what is the expected return to your portfolio?
  - Will the standard deviation of your portfolio be greater or less than 40%? Why?
  - Who would pay more for Ford stock: (a) an investor who already owns Microsoft and Starbucks stock, or (b) an investor who has just Ford stock in her portfolio? Why?

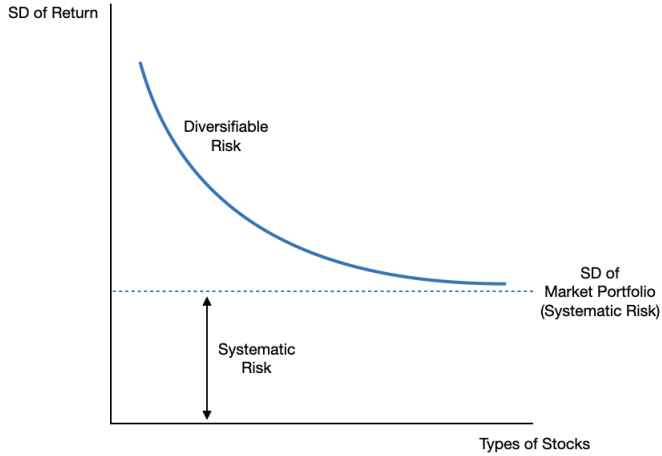
Another note

- Assume Apple, Ford, and Google have the same expected return of 15%.
- You already have Apple stock. Would you choose to buy Ford or Google stock?
- Ford for diversification, and because it shares less covariance with your Apple stock.

Modern portfolio theory

- Efficient diversification reduces risk but it is not, in practice, possible to diversify away all risk.
- To the extent that asset returns covary, that shared variance cannot be diversified away.
- How many different assets must a portfolio have before it eliminates all the risk it is possible to remove?
  - Prior research shows that picking roughly 25 randomly selected equities will remove almost all of the common component of variance.
  - You could of course do better by strategically selecting the stocks.

Portfolio standard deviation (risk)



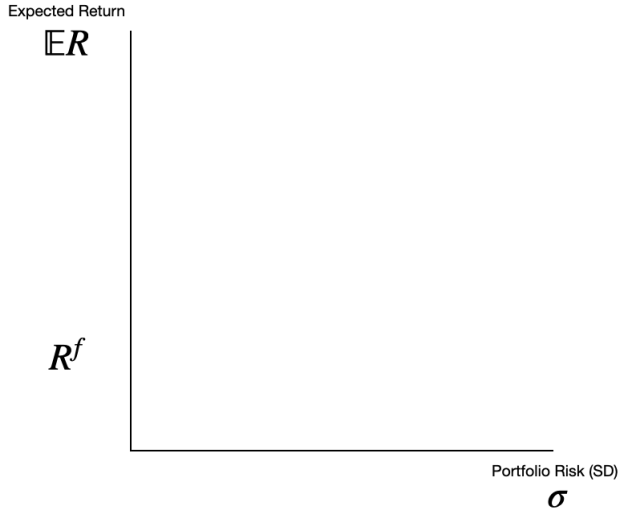
Issues with MPT

- Using historical returns to determine the extent to which the returns of different assets covary is often not a good guide to how much they will covary in the future.
- Given this practical problem, research has found that a naive diversification strategy is often very good
- In times of economic crisis, the covariance of assets' returns move towards unity: the returns of assets tend to plunge down together (but diversification still helps).

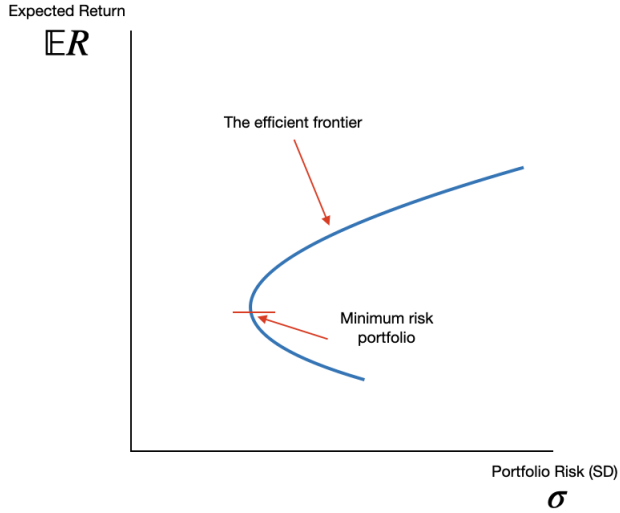
The CAPM

- An equilibrium model which assumes investors share the same information, and that there are no costs to buying and selling assets.
- CAPM provides answer to the question: How would investors price individual assets (i.e., what is the discount rate they would use to value expected cash flows) if they followed the principles of MPT when constructing their portfolios?
- The assumptions are, of course, strong. But a model need not be 100% realistic to be useful.

The efficient frontier



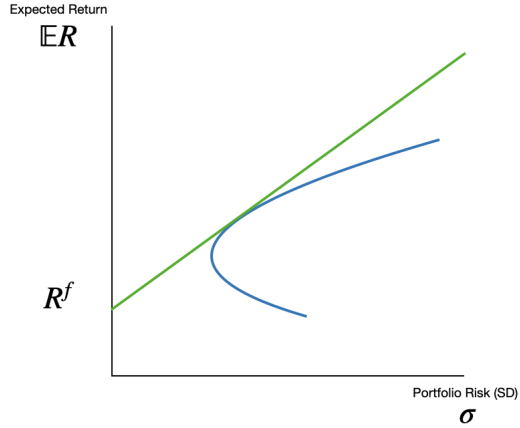
The efficient frontier



The Sharpe ratio

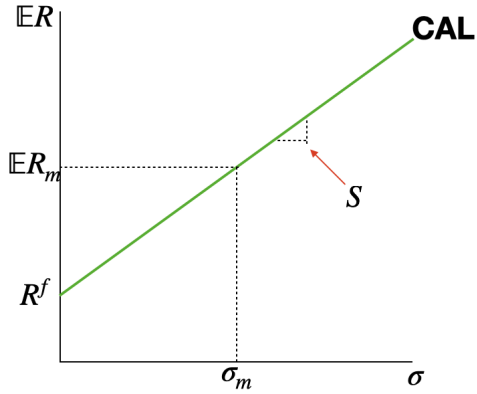
- Suppose you have two portfolio options, X and Y.
- The Sharpe ratio is given as
$$S = \frac{\mathbb{E}R}{\sigma}.$$
- Portfolio X has a Sharpe ratio of 1.6, while Y has a ratio of 2. Which would you prefer?

The efficient frontier with a risk-free asset



- Now, the straight line starting at R^f is the new efficient frontier. The point of tangency is referred to as the "market portfolio".

The capital allocation line (CAL)



The CAL and the Separation Theorem

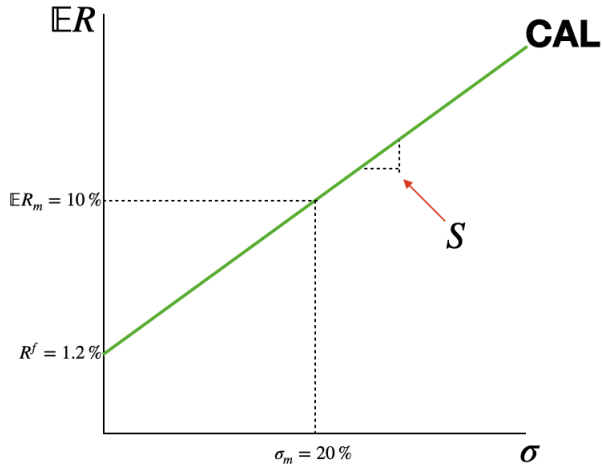
- The CAL leads all investors to invest in the market portfolio
- Individual investors should differ in position on the CAL depending on risk preferences
  - Risk averse investors will lend part of the portfolio at the risk-free rate and invest the remainder in the market portfolio
  - Risk seeking investors will borrow at the risk-free rate and invest in the market portfolio

The CAL and the Separation Theorem

- The CAL leads all investors to invest in the market portfolio in an efficient market. Stock selection becomes separate from the risk-taking decision.
- However, if you believe:
  - the market is inefficient, and
  - you can identify mispriced assets, and
  - that the mispricing will correct over your investment horizon, then

  you will choose not invest in the market portfolio; you will buy disproportionately more underpriced assets and disproportionately less overpriced assets.

Some examples of the CAL



Some examples of the CAL

- Scenario 1: Invest \$100,000 in market portfolio with an expected return of 10%
  - Risk-free rate is 1.2%
  - Risk premium is therefore 8.8%
  - If the standard deviation is 20%, then the Sharpe ratio is

$$S = \frac{8.8\%}{20\%} = 0.44.$$

- Scenario 2: Invest \$50,000 in market portfolio and \$50,000 in the risk-free asset
  - The expected return is: $\frac{1}{2} \times 1.2\% + \frac{1}{2} \times 10\% = 5.6\%$
  - Risk premium: $5.6 - 1.2 = 4.4\%$
  - Standard deviation (portfolio): $\frac{1}{2} \times 20\% = 10\%$
  - Sharpe ratio: $S = \frac{4.4\%}{10\%} = 0.44$

Some examples of the CAL

- Scenario 3: Invest \$100,000 and borrow an extra \$100,000 to invest in the market portfolio
  - Expected return: $-1.2\% + 2 \times 10\% = 18.8\%$
  - Risk premium: $18.8 - 1.2 = 17.6\%$
  - Standard deviation: $2 \times 20\% = 40\%$
  - Sharpe ratio: $S = \frac{17.6\%}{40\%} = 0.44$

Focusing on discount rate for individual assets

- Assume that the market is efficient and MPT holds. What is the discount rate, $\mathbb{E}R$, for each asset?

$$\mathbb{E}R = R_f + R_p$$

- All risky assets are expected to yield at least the risk-free rate plus a risk premium
- The size of the risk premium demanded for each asset depends on how closely its returns covary with the market portfolio. This is the key point of the CAPM.

An intuitive explanation

- Investor A owns an ice cream shop
  - \$100,000 profit during a warm year, \$0 otherwise. Average profit per year is \$50,000.
- Investor B owns a hot chocolate shop
  - \$100,000 profit during a cold year, \$0 otherwise. Average profit per year is \$50,000.
- Assume that half the year is warm and half is cold.
- Investors C and D jointly own an ice-cream and a hot chocolate shop.
- Which investors face more risk?

An intuitive explanation

- Now, assume that the risk-free rate is 5%
- How much would an investor pay for each shop?
- Why would Investors A and B be willing to buy each of the shops for up to $1 million?
- How much would Investors C&D be willing to pay for both shops?
- Who sets the price?

An intuitive explanation

- Owning both the shops delivers a risk-free return of \$100,000 per year – which is 5% of \$2 million dollars.
- Investors will value the shops accordingly: the shops' value will not reflect the individual business' risk but their zero-risk as a portfolio.
- Note: ice cream and hot chocolate shops have profits that move in opposite directions (negatively correlated)
- For investors, the relevant risk of a project is how correlated its returns are with the returns of the other assets in their portfolio.

Back to CAPM

- If the relevant risk of an asset is the extent to which it covaries with other assets in the investor's portfolio
- And all investors hold the market portfolio
- Then it follows that the covariance with the market portfolio is the relevant systematic risk of an asset.

Application of CAPM to pricing of a single security

- Let $\mathrm{Cov}(i, m)$ stand for covariance of security i with the market portfolio, m.
- If you had to choose between security X and Y with the same expected return but X had a higher covariance with the market portfolio, which would you choose? Why?
- If other investors thought the same way as you, what would happen to the price of X? What would happen to its expected return?
- What would be the outcome in equilibrium?

Standardised measure of covariance

- You should learn this in your econometrics/stats undergrad courses, but here's a quick overview:
- $\mathrm{Cov}(i, m)$ is the absolute measure of the covariance of security i and market portfolio m.
  - The value will depend on the extent i moves with m, but also on the absolute variance of i and the absolute variance of m
  - So we can divide $\mathrm{Cov}(i, m)$ by $\mathrm{Var}(m)$ to get a "standardised covariance", which is often referred to as β (beta) in the finance literature.
- This leads to the famous CAPM equation

$$\mathbb{E}R_i = R_f + \beta_i(R_m - R_f).$$

- So, β_i captures the activity of i and how it responds to swings in the entire market.

Interpreting beta

- Assets with returns that change exactly with the market (in percentage terms) have $\beta = 1$
- Average beta of all assets is equal to 1, since the market portfolio is equal to all assets
- Assets with $\beta > 1$ have returns that move in the same direction as the market but proportionally more sensitive
- Assets with $0 < \beta < 1$ have returns that move in the same direction as the market but proportionally less sensitive
- Assets with $\beta = 0$ do not covary with the market at all
- Assets with $\beta < 0$ have returns that move in the opposite direction to the market

Key point of CAPM

- For investors, the relevant risk of a project is how correlated its returns are with their portfolio's returns
- Well diversified investors hold the market portfolio. The relevant risk of a project is therefore the correlation of the project's returns with the market portfolio's returns
- The CAPM model is known as a "single factor" model because the model states that variation in risk, and therefore in the expected return, across investments is due to one thing only: covariance of the investment return with the market portfolio. Nothing else matters.

Example of CAPM

- Assume that $R_f = 5\%$ and to invest in a well diversified but risky portfolio investors require an additional 7% expected return, i.e., 7% is the risk premium:
$$\mathbb{E}R_m = 5 + 7 = 12\%$$
- If a particular asset moves exactly in line with the market ($\beta = 1$), investors would also require 12%.
- But what if an asset moves twice as much as the market portfolio ($\beta = 2$)?

$$\begin{aligned}\mathbb{E}R_i &= R_f + \beta(\mathbb{E}R_m - R_f) \\ &= 5 + 2(12 - 5) \\ &= 19\%\end{aligned}$$

- But, where did we get the 7% risk premium from?

Biggest critique of CAPM (and finance)

- Where on earth did we get 7% from?
- There is no way to unambiguously understand the risk premium
- The risk premium is the cornerstone of finance and the CAPM, but there is no way to observe it!

Finance = ketchup economics

- Because we cannot observe the market risk premium, it can range and vary wildly depending on how you calculate it.
- Example: in 2011 finance professors (in the US) estimated the market risk premium at roughly 5.7%. Market analysts had it varying between 5% and 5.6%.
- These fluctuations in the estimates of the market risk premium can lead to large changes in the expected return/valuation of assets:

$$\frac{\$100}{0.05} = \$2,000$$
$$\frac{\$100}{0.06} = \$1,666.67$$

Conclusion

- MPT gives us an environment (a set of assumptions) to construct an equilibrium model, the CAPM
- "All models are unrealistic, but some are useful"
- A big weakness of finance (and indeed economics and most other social sciences) is that one of the key variables is unobservable