PhD Macroeconomics: The New Keynesian Model

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24 April 2024

Introduction

- In the RBC model, everything is in real terms. There is no role for money (i.e., money is neutral).
- But there is a lot of empirical evidence to suggest that monetary policy influences short-run economic fluctuations.
- Need to merge classic Keynesian ideas of price stickiness with the RBC DSGE framework.
- New Keynesian theory attempted to provide “microfoundations” and to formalise Keynesian concepts.
- The synthesis of New Keynesian economics and the RBC theory resulted in the “new neoclassical synthesis” (Goodfriend and King, 1997) or the “neo-Wicksellian” approach (Woodford, 2003).
- In these slides I will do my best to stick to the notation of Galí (2015).

A Baseline New Keynesian Model

Households I

- The representative household derives utility from consumption C_t of final goods, labour N_t, and real money balances M_t:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t, \frac{M_t}{P_t}\right),$$

  where $\beta \in (0,1)$ is the household discount factor and the utility function is "well behaved" and satisfies the usual Inada condition.

- The period utility function is given by:

$$u\left(C_t, N_t, \frac{M_t}{P_t}\right) = \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \varphi_0 \frac{N_t^{1+\varphi}}{1+\varphi} + \zeta_M \ln\left(\frac{M_t}{P_t}\right)\right] Z_t,$$

  where σ is the Arrow-Pratt coefficient of relative risk aversion and φ is the inverse-Frisch elasticity of labour supply.

Households II

- Z_t follows a stationary AR(1) process:

$$z_t \equiv \ln Z_t = \rho_z z_{t-1} + \varepsilon_t^z.$$

- The HH nominal flow budget constraint is:

$$P_t C_t + B_t + M_t + P_t T_t \leq W_t N_t + M_{t-1} + D_t + R_{t-1} B_{t-1},$$

  where money is the numeraire, P_t: price of goods in terms of money, B_{t-1}: stock of nominal bonds a household enters the period with, and $R_{t-1} = 1 + i_{t-1}$: gross nominal interest rate.

- Households enter period t with nominal money balances of M_{t-1}, earn a nominal wage of W_t, earn nominal profits D_t remitted to them by firms, and T_t is a lump sum tax paid to the government.

Households III

- Using our timing trick from when we solved the RBC model, the Lagrangian for the household is:

$$
\begin{aligned}
\mathcal{L} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \varphi_0 \frac{N_{t+s}^{1+\varphi}}{1+\varphi} + \zeta_M \ln\left(\frac{M_{t+s}}{P_{t+s}}\right) \right] Z_{t+s} \\
+ \lambda_t \left(W_t N_t + D_t - P_t T_t + R_{t-1} B_{t-1} - P_t C_t - B_t - M_t + M_{t-1} \right) \\
+ \beta \mathbb{E}_t \left[W_{t+1} N_{t+1} + D_{t+1} - P_{t+1} T_{t+1} + R_t B_t - P_{t+1} C_{t+1} - B_{t+1} - M_{t+1} + M_t \right].
\end{aligned}
$$

Households IV

- The FOCs – which look very familiar – are:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &= C_t^{-\sigma} Z_t - \lambda_t P_t = 0, && (1)\\
\frac{\partial \mathcal{L}}{\partial N_t} &= -\varphi_0 N_t^{\varphi} Z_t + \lambda_t W_t = 0, && (2)\\
\frac{\partial \mathcal{L}}{\partial B_t} &= -\lambda_t + \beta \mathbb{E}_t \lambda_{t+1} R_t = 0, && (3)\\
\frac{\partial \mathcal{L}}{\partial M_t} &= \zeta_M \frac{1}{M_t} Z_t - \lambda_t + \beta \mathbb{E}_t \lambda_{t+1} = 0. && (4)
\end{aligned}
$$

Households V

- From (3) we know that $\lambda_t = \beta \mathbb{E}_t \lambda_{t+1} R_t$, so we can write the FOCs as:

$$C_t^{-\sigma} Z_t = P_t \beta \mathbb{E}_t \lambda_{t+1} R_t, \tag{5}$$

$$\varphi_0 N_t^{\varphi} Z_t = \lambda_t W_t, \tag{6}$$

$$\zeta_M \frac{1}{M_t} Z_t = \lambda_t - \beta \mathbb{E}_t \lambda_{t+1}. \tag{7}$$

- Then, from (5), like we do in the RBC models, we have:

$$\frac{C_t^{-\sigma}}{P_t} Z_t = \beta \mathbb{E}_t \lambda_{t+1} R_t = \lambda_t,$$

so we can roll one period ahead to get:

$$\frac{C_{t+1}^{-\sigma}}{P_{t+1}} Z_{t+1} = \lambda_{t+1},$$

Households VI

and combining (5) and (6) we can get rid of λ_t from our FOCs:

$$\varphi_0 N_t^{\varphi} = \frac{C_t^{-\sigma}}{P_t} W_t, \tag{8}$$

$$C_t^{-\sigma} = \frac{P_t}{Z_t} \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{P_{t+1}} Z_{t+1} R_t = \beta \mathbb{E}_t C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} \frac{Z_{t+1}}{Z_t} R_t, \tag{9}$$

and our third FOC (after some derivation) is:

$$\zeta_M \left(\frac{M_t}{P_t} \right)^{-1} = \frac{i_t C_t^{-\sigma}}{R_t} Z_t. \tag{10}$$

Firms and production I

- Split production into two sectors.
- There is a representative perfectly competitive final good firm, and monopolistically competitive intermediate goods firms.
- The final output good is a CES aggregate, utilising the Dixit-Stiglitz aggregator, of a continuum of intermediate goods:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 0,$$

  so final good firms maximise their profits by selecting how much of each intermediate good to purchase, and so their problem is:

$$\max_{Y_t(j)} \left\{ P_t Y_t - \int_0^1 P_t Y_t(j) dj \right\}.$$

Firms and production II

- The FOC for a typical intermediate good j is:

$$Y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\epsilon} Y_t. \tag{11}$$

- From Blanchard and Kiyotaki (1987), we can derive a price index for the aggregate economy:

$$P_t Y_t \equiv \int_0^1 P_t(j) Y_t(j) dj.$$

- Then, plugging in the demand for good j from (11) we have:

$$\implies P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}. \tag{12}$$

Intermediate firms I

- A typical intermediate firm produces output according a constant returns to scale technology in labour, with a common productivity shock, A_t:

$$Y_t(j) = A_t N_t(j)^{1-\alpha}. \tag{13}$$

- Intermediate firms pay a common wage.
- They are not freely able to adjust price so as to maximise profit each period, but will always act to minimise cost.

Intermediate firms II

- The cost minimisation problem is to minimise total cost subject to the constraint producing enough to meet demand:

$$\min_{N_t(j)} \; W_t N_t(j),$$

subject to

$$A_t N_t(j)^{1-\alpha} \geq Y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\epsilon} Y_t.$$

Intermediate firms III

- The Lagrangian for an intermediate firm j's problem is:

$$\mathcal{L} = W_t N_t(j) - \Psi_t(j) \left(A_t N_t(j)^{1-\alpha} - \left[\frac{P_t(j)}{P_t} \right]^{-\epsilon} Y_t \right),$$

where $\Psi_t(j)$ is the Lagrangian multiplier for firm j. The FOC is:

$$\frac{\partial \mathcal{L}}{\partial N_t(j)} = W_t - (1-\alpha)\Psi_t(j) A_t N_t(j)^{-\alpha} = 0,$$

which then implies:

$$\Psi_t(j) = \frac{W_t}{(1-\alpha)A_t} N_t(j)^{\alpha} = \frac{W_t}{(1-\alpha)A_t} \left[\frac{Y_t(j)}{A_t} \right]^{\frac{\alpha}{1-\alpha}}. \tag{14}$$

Intermediate firms IV

- Notice that when $\alpha = 0$ neither W_t nor A_t are firm j specific, so in fact we can write $\Psi_t(j)$ as simply Ψ_t.
- Now, what is the economic interpretation of $\Psi_t(j)$? It is an intermediate firm's nominal marginal cost – how much costs change if you are forced to produce an extra unit of output.
- For the case where $\alpha \neq 0$, define the economy-wide average marginal cost as

$$\Psi_t = \frac{W}{(1-\alpha)A_t}\left(\frac{Y_t}{A_t}\right)^{\frac{\alpha}{1-\alpha}}. \tag{15}$$

- Then we can write:

$$\Psi_t(j) = \Psi_t\left[\frac{Y_t}{Y_t(j)}\right]^{\frac{-\alpha}{1-\alpha}} = \Psi_t\left[\frac{P_t(j)}{P_t}\right]^{\frac{\epsilon\alpha}{\alpha-1}}, \tag{16}$$

which shows that firms with high prices have low marginal costs.

Intermediate firms V

- Now, formulate the intermediate firm's real flow profit as:

$$\frac{D_t(j)}{P_t} = \frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t}{P_t} N_t(j),$$

  and substitute in the nominal wage from (14):

$$\begin{aligned} \frac{D_t(j)}{P_t} &= \frac{P_t(j)}{P_t} Y_t(j) - (1-\alpha) \frac{\Psi_t(j) A_t}{P_t} N_t(j)^{1-\alpha} \\ &= \frac{P_t(j)}{P_t} Y_t(j) - MC_t Y_t(j), \end{aligned} \tag{17}$$

  where $MC_t(j) = \frac{\Psi_t(j)}{P_t}$ is the real marginal cost for an intermediate firm.
- Now, buckle up because this is where the fun begins...

Monopolistic competition with Calvo pricing I

- Intermediate firms are not able to freely adjust prices each period.
- Each period they are unable to adjust prices wp θ. Conversely, they can adjust prices in any given period wp $1-\theta$.
    - They get a visit from the "Calvo fairy" wp $1-\theta$.
- So, the probably a firm is stuck with today's price s periods ahead is θ^s.
- Firms will discount s periods into the future by:

$$\Lambda_{t,t+s}\theta^s,$$

where

$$\Lambda_{t,t+s} = \beta^s \frac{u_{C,t+s}}{u_{C,t}},$$

is the household stochastic discount factor (SDF).

Monopolistic competition with Calvo pricing II

- The dynamic problem of an updating firm can be written as

$$\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\theta^s \left(\frac{P_t(j)}{P_{t+s}} \underbrace{\left[\frac{P_t(j)}{P_{t+s}}\right]^{-\epsilon} Y_{t+s}}_{Y_{t+s}(j)} - MC_{t+s}(j) \underbrace{\left[\frac{P_t(j)}{P_{t+s}}\right]^{-\epsilon} Y_{t+s}}_{Y_{t+s}(j)} \right), \tag{18}$$

  where we assume that output will equal demand.
- Expanding the terms, we get

$$\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \left(\Lambda_{t,t+s}\theta^s P_t(j)^{1-\epsilon} P_{t+s}^{\epsilon-1} Y_{t+s} - \Lambda_{t,t+s}\theta^s MC_{t+s}(j)(j) P_t(j)^{-\epsilon} P_{t+s}^{\epsilon} Y_{t+s}\right),$$

Monopolistic competition with Calvo pricing III

and so the FOC is:

$$0 = (1-\epsilon)P_t(j)^{-\epsilon}\mathbb{E}_t\sum_{s=0}^{\infty}\Lambda_{t,t+s}\theta^s P_{t+s}^{\epsilon-1}Y_{t+s} + \epsilon P_t(j)^{-\epsilon-1}\mathbb{E}_t\sum_{s=0}^{\infty}\Lambda_{t,t+s}\theta^s MC_{t+s}(j)P_{t+s}^{\epsilon}Y_{t+s}.$$

- Move the first summation to the LHS and do some algebra to write:

$$P_t(j) = \frac{\epsilon}{\epsilon-1}\frac{\mathbb{E}_t\sum_{s=0}^{\infty}\beta^s u_{C,t+s}\theta^s MC_{t+s}(j)P_{t+s}^{\epsilon}Y_{t+s}}{\mathbb{E}_t\sum_{s=0}^{\infty}\beta^s u_{C,t+s}\theta^s P_{t+s}^{\epsilon-1}Y_{t+s}}.$$

Monopolistic competition with Calvo pricing IV

- Use (16) (adjust appropriately to get MC_t) to write the above expression as:

$$P_t(j) = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_{C,t+s} \theta^s MC_t \left[\frac{P_t(j)}{P_{t+s}}\right]^{\frac{\epsilon\alpha}{\alpha-1}} P_{t+s}^{\epsilon} Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_{C,t+s} \theta^s P_{t+s}^{\epsilon-1} Y_{t+s}}$$

$$P_t(j)^{1+b} = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_{C,t+s} \theta^s MC_t P_{t+s}^{\epsilon + \frac{\epsilon\alpha}{1-\alpha}} Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_{C,t+s} \theta^s P_{t+s}^{\epsilon-1} Y_{t+s}},$$

  where $b = \frac{\epsilon\alpha}{1-\alpha}$.
- Note that none of the variables on the RHS depend on j.

Monopolistic competition with Calvo pricing V

- This means that any firm able to update their price will update to a common optimal price, say, $P_t(j) = P_t^*$. Write this compactly as:

$$(P_t^*)^{1+b} = \mathcal{M}\frac{X_{1,t}}{X_{2,t}}, \tag{19}$$

where $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$ is the optimal markup charged by monopolistically competitive firms, and our auxiliary variables $X_{1,t}$ and $X_{2,t}$ are:

$$X_{1,t} = u_{C,t}MC_tP_t^{\epsilon+b}Y_t + \theta\beta\mathbb{E}_tX_{1,t+1}, \tag{20}$$

$$X_{2,t} = u_{C,t}P_t^{\epsilon-1}Y_t + \theta\beta\mathbb{E}_tX_{2,t+1}. \tag{21}$$

- Notice that the second terms of our auxiliary variables are equal to 0 when $\theta = 0$, i.e., if all firms are able to change their prices freely, then prices are flexible which means that $MC_tP_t = \Psi_t$ and $\Psi_t = \mathcal{M}^{-1}$. This is important to note down.

Equilibrium and aggregation I

- To close the model, we need to specify an exogenous process for our technology shocks A_t, some kind of monetary policy rule to determine M_t, and a fiscal rule to determine T_t.
- As before, let the aggregate productivity term follow an AR(1) process such as:

$$a_t \equiv \ln A_t = \rho_a a_{t-1} + \varepsilon_t^a. \tag{22}$$

- Suppose that the nominal money supply also follows an AR(1) process in the growth rate:

$$\Delta m_t \equiv \Delta \ln M_t = (1 - \rho_m)\pi + \rho_m \Delta m_{t-1} + \varepsilon_t^m, \tag{23}$$

  where $\Delta m_t \equiv m_t - m_{t-1}$.
- Mean growth rate of money is equal to the steady state net inflation rate π, as we want real money balances to be stationary.

Equilibrium and aggregation II

- For both the law of motion of technology and nominal money, I assume that they contain white noise shock terms such that $\varepsilon_t^a \overset{\text{IID}}{\sim} \mathcal{N}(0, \sigma_a^2)$ and $\varepsilon_t^m \overset{\text{IID}}{\sim} \mathcal{N}(0, \sigma_m^2)$.
- The government prints money, so it earns seignorage. Right now, we assume that the government does not consume, and that it does not take part in bond markets. The nominal government budget constraint is:

$$0 \leq P_t T_t + M_t - M_{t-1}.$$

- In words, the change in the nominal money supply, $M_t - M_{t-1}$ is nominal revenue for the government. Since it does no spending, at equality lump sum taxes must satisfy:

$$T_t = -\frac{M_t - M_{t-1}}{P_t}.$$

Equilibrium and aggregation III

- So if money growth is positive, e.g. $M_t > M_{t-1}$, then lump sum taxes will be negative – the government will be rebating its seignorage revenue to the households via lump sum transfers.
- In equilibrium, bond-holding is always zero in all periods: $B_t = 0$. Using this, plus the relationship between the lump sum tax and money growth derived above, the household budget constrain can be written in real terms:

$$
\begin{aligned}
P_t C_t + B_t + M_t + P_t T_t &\leq W_t N_t + M_{t-1} + D_t + R_{t-1} B_{t-1} \\
\Leftrightarrow P_t C_t + M_t - M_{t-1} + P_t \left(-\frac{M_t - M_{t-1}}{P_t} \right) &\leq W_t N_t + D_t \\
\Leftrightarrow C_t &= \frac{W_t N_t}{P_t} + \frac{D_t}{P_t}.
\end{aligned}
$$

Equilibrium and aggregation IV

- Real dividends received by the household are just the sum of real profits from intermediate goods firms (since the final good firm is competitive and earns no economic profit):

$$\begin{aligned}\frac{D_t}{P_t} &= \int_0^1 \left[\frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t}{P_t} N_t(j)\right] dj \\ &= \int_0^1 \frac{P_t(j)}{P_t} Y_t(j) dj - \frac{W_t}{P_t} N_t\end{aligned}$$

Equilibrium and aggregation V

- So, the household budget constraint becomes:

$$
\begin{aligned}
C_t &= \frac{W_t N_t}{P_t} + \int_0^1 \frac{P_t(j)}{P_t} Y_t(j) dj - \frac{W_t N_t}{P_t} \\
\implies C_t &= \int_0^1 \frac{P_t(j)}{P_t} Y_t(j) dj,
\end{aligned}
$$

and since:

$$
Y_t(j) = \left[\frac{P_t(j)}{P_t} \right]^{-\epsilon} Y_t,
$$

Equilibrium and aggregation VI

we have:

$$
\begin{aligned}
C_t &= \int_0^1 \frac{P_t(j)}{P_t}\left[\frac{P_t(j)}{P_t}\right]^{-\epsilon} Y_t dj \\
&= \int_0^1 P_t(j)^{1-\epsilon} P_t^{\epsilon-1} Y_t dj \\
&= P_t^{\epsilon-1} Y_t \int_0^1 P_t(j)^{1-\epsilon} dj,
\end{aligned}
$$

but $\int_0^1 P_t(j)^{1-\epsilon} dj = P_t^{1-\epsilon}$ from (12), so the P_t terms drop out and we have the market clearing condition:

$$C_t = Y_t \tag{24}$$

Equilibrium and aggregation VII

- Now, we need to solve for Y_t. But we first need to get aggregate labour demand by firms, N_t: (11) and (13) give:

$$N_t(j) = \left[\frac{Y_t(j)}{A_t}\right]^{\frac{1}{1-\alpha}}.$$

Aggregate this across firms to then get

$$N_t \equiv \int_0^1 N_t(j)dj = \int_0^1 \left[\frac{Y_t(j)}{A_t}\right]^{\frac{1}{1-\alpha}} dj,$$

then substitute in for $Y_t(j)$ using the demand for intermediate goods:

$$N_t = \int_0^1 \left\{ \frac{\left[\frac{P_t(j)}{P_t}\right]^{-\epsilon} Y_t}{A_t} \right\}^{\frac{1}{1-\alpha}} dj. \tag{25}$$

Equilibrium and aggregation VIII

- Now, do a bit of algebra and solve for Y_t:

$$Y_t = \frac{A_t N_t^{1-\alpha}}{\underbrace{\left\{ \int_0^1 \left[\frac{P_t(j)}{P_t} \right]^{-\frac{\epsilon}{1-\alpha}} dj \right\}^{1-\alpha}}_{V_t^P}}$$

$$= \frac{A_t N_t^{1-\alpha}}{V_t^P}. \tag{26}$$

- The new variable we have defined, V_t^P, is a measure of “price dispersion”.
- If there were no pricing frictions, all firms would charge the same price, and $V_t^P = 1$.
- Since $V_t^P \geq 1$, price dispersion leads to lower output. This is the gist for why price stability is a good thing.

Full set of equilibrium conditions I

$$
\begin{aligned}
C_t^{-\sigma} &= \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma} R_t P_t Z_{t+1}}{P_{t+1} Z_t}, \\
\varphi_0 N_t^{\varphi} &= C_t^{-\sigma} \frac{W_t}{P_t}, \\
\frac{M_t}{P_t} &= \zeta_M \frac{R_t}{i_t Z_t} C_t^{\sigma}, \\
MC_t &= \frac{W_t / P_t}{(1-\alpha) A_t (Y_t / A_t)^{\frac{-\alpha}{1-\alpha}}}, \\
C_t &= Y_t, \\
Y_t &= \frac{A_t N_t^{1-\alpha}}{V_t^P},
\end{aligned}
$$

Full set of equilibrium conditions II

$$
\begin{aligned}
V_t^P &= \left\{ \int_0^1 \left[\frac{P_t(j)}{P_t} \right]^{-\frac{\epsilon}{1-\alpha}} dj \right\}^{1-\alpha}, \\
P_t^{1-\epsilon} &= \int_0^1 P_t(j)^{1-\epsilon} dj, \\
(P_t^*)^{1+\frac{\epsilon\alpha}{1-\alpha}} &= \mathcal{M} \frac{X_{1,t}}{X_{2,t}}, \\
X_{1,t} &= C_t^{-\sigma} Z_t MC_t P_t^{\epsilon+\frac{\epsilon\alpha}{1-\alpha}} Y_t + \theta\beta\mathbb{E}_t X_{1,t+1}, \\
X_{2,t} &= C_t^{-\sigma} Z_t P_t^{\epsilon-1} Y_t + \theta\beta\mathbb{E}_t X_{2,t+2}, \\
\ln Z_t &= \rho_z \ln Z_{t-1} + \varepsilon_t^z, \\
\ln A_t &= \rho_a \ln A_{t-1} + \varepsilon_t^a,
\end{aligned}
$$

Full set of equilibrium conditions III

$$\Delta m_t = (1 - \rho_m)\pi + \rho_m \Delta m_{t-1} + \varepsilon_t^m,$$
$$\Delta m_t = m_t - m_{t-1}.$$

- This is 15 equations in 15 aggregate variables. But there are three issues with the way we have written up this system of equations:
  - We have heterogeneity (j shows up);
  - The price level shows up and it isn't stationary;
  - Nominal money growth shows up and it isn't stationary.
- So we will rewrite these conditions using Calvo pricing to get rid of the j terms, using inflation instead of price levels.
- We also need to ensure that trending variables are detrended.

Rewriting the equilibrium conditions I

- Begin by rewriting gross inflation as $\Pi_t = 1 + \pi_t = \frac{P_t}{P_{t-1}}$. The consumption Euler equation can be re-written as:

$$C_t^{-\sigma} = \beta \mathbb{E}_t C_{t+1}^{-\sigma} R_t \Pi_{t+1} \frac{Z_{t+1}}{Z_t}.$$

- The demand for money equation is already written in terms of real money balances, M_t/P_t, which is stationary so it's fine:

$$\frac{M_t}{P_t} = \zeta_M \frac{R_t}{i_t Z_t} C_t^{\sigma}.$$

Now we need to get rid of the j terms in the price level and price dispersion expansions. The expression for the price level is:

$$P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} dj.$$

Rewriting the equilibrium conditions II

- Now, recall that a fraction $1-\theta$ of these firms will update their price to the same optimal price, P_t^*.
- The other fraction θ will charge the price they charged in the previous period.
- Since it doesn't matter how we "order" these firms along the unit interval, this means we can break up the integral on the RHS above as:

$$P_t^{1-\epsilon} = \int_0^{1-\theta} (P_t^*)^{1-\epsilon}\, dj + \int_{1-\theta}^{1} P_{t-1}(j)^{1-\epsilon} dj$$
$$\Leftrightarrow P_t^{1-\epsilon} = (1-\theta)\,(P_t^*)^{1-\epsilon} + \int_{1-\theta}^{1} P_{t-1}(j)^{1-\epsilon} dj.$$

Rewriting the equilibrium conditions III

- Now, watch the Calvo magic:

$$\int_{1-\theta}^{1} P_t(j)^{1-\epsilon} dj = \theta \int_{0}^{1} P_{t-1}(j)^{1-\epsilon} dj = \theta P_{t-1}^{1-\epsilon},$$

  and therefore we have

$$P_t^{1-\epsilon} = (1-\theta)\left(P_t^*\right)^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon}.$$

- Tada! We've gotten rid of the heterogeneity. The Calvo assumption allows us to integrate out the heterogeneity.

Rewriting the equilibrium conditions IV

- Now, we want to write things in terms of inflation, so divide both sides by $P_{t-1}^{1-\epsilon}$, and define $\Pi_t^* = 1 + \pi_t^* = \frac{P_t^*}{P_{t-1}}$ as "optimal price inflation":

$$\begin{aligned} \frac{P_t^{1-\epsilon}}{P_{t-1}^{1-\epsilon}} &= (1-\theta)\frac{\left(P_t^*\right)^{1-\epsilon}}{P_{t-1}^{1-\epsilon}} + \theta\frac{P_{t-1}^{1-\epsilon}}{P_{t-1}^{1-\epsilon}} \\ \Leftrightarrow \Pi_t^{1-\epsilon} &= (1-\theta)\left(\Pi_t^*\right)^{1-\epsilon} + \theta. \end{aligned} \tag{27}$$

Rewriting the equilibrium conditions V

- Now, look at the price dispersion term. Notice we can use the same Calvo trick we used above here:

$$
\begin{aligned}
(V_t^P)^{\frac{1}{1-\alpha}} &= \int_0^1 \left[\frac{P_t(j)}{P_t}\right]^{-\frac{\epsilon}{1-\alpha}} dj = \int_0^{1-\theta} \left[\frac{P_t(j)}{P_t}\right]^{-\frac{\epsilon}{1-\alpha}} dj + \int_{1-\theta}^1 \left[\frac{P_{t-1}(j)}{P_t}\right]^{-\frac{\epsilon}{1-\alpha}} dj \\
&= \int_0^{1-\theta} \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} dj + \int_{1-\theta}^1 \left[\frac{P_{t-1}(j)}{P_t}\right]^{-\frac{\epsilon}{1-\alpha}} dj \\
&= \int_0^{1-\theta} \left(\frac{P_t^*}{P_{t-1}}\right)^{-\frac{\epsilon}{1-\alpha}} \left(\frac{P_{t-1}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} dj + \int_{1-\theta}^1 \left[\frac{P_{t-1}(j)}{P_{t-1}}\right]^{-\frac{\epsilon}{1-\alpha}} \left(\frac{P_{t-1}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} dj \\
&= (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{-\frac{\epsilon}{1-\alpha}} \left(\frac{P_t}{P_{t-1}}\right)^{\frac{\epsilon}{1-\alpha}} + \left(\frac{P_t}{P_{t-1}}\right)^{\frac{\epsilon}{1-\alpha}} \int_{1-\theta}^1 \left[\frac{P_{t-1}(j)}{P_{t-1}}\right]^{-\frac{\epsilon}{1-\alpha}} dj \\
&= (1-\theta)(\Pi_t^*)^{-\frac{\epsilon}{1-\alpha}} \Pi_t^{\frac{\epsilon}{1-\alpha}} + \Pi_t^{\frac{\epsilon}{1-\alpha}} \int_{1-\theta}^1 \left[\frac{P_{t-1}(j)}{P_{t-1}}\right]^{-\frac{\epsilon}{1-\alpha}} dj,
\end{aligned}
$$

Rewriting the equilibrium conditions VI

and use the Calvo trick, and the definition of price dispersion, on the last term of the RHS to get:

$$(V_t^P)^{\frac{1}{1-\alpha}} = (1-\theta)(\Pi_t^*)^{-\frac{\epsilon}{1-\alpha}}\Pi_t^{\frac{\epsilon}{1-\alpha}} + \theta\Pi_t^{\frac{\epsilon}{1-\alpha}}(V_{t-1}^P)^{\frac{1}{1-\alpha}}. \tag{28}$$

- Now, adjust the reset price expression. First, divide the following auxiliary variables by P_t raised to the relevant powers:

$$\frac{X_{1,t}}{P_t^{\epsilon+b}} = \frac{C_t^{-\sigma}Z_tMC_tP_t^{\epsilon+b}Y_t}{P_t^{\epsilon+b}} + \theta\beta\frac{\mathbb{E}_tX_{1,t+1}}{P_t^{\epsilon+b}},$$

$$\frac{X_{2,t}}{P_t^{\epsilon-1}} = \frac{C_t^{-\sigma}Z_tP_t^{\epsilon-1}Y_t}{P_t^{\epsilon-1}} + \theta\beta\frac{\mathbb{E}_tX_{2,t+1}}{P_t^{\epsilon-1}}.$$

Rewriting the equilibrium conditions VII

- Multiplying and dividing the second terms on the RHS by P_{t+1} to the right powers yields:

$$
\begin{aligned}
\frac{X_{1,t}}{P_t^{\epsilon+b}} &= C_t^{-\sigma} Z_t MC_t Y_t + \theta\beta\mathbb{E}_t \frac{X_{1,t+1}}{P_{t+1}^{\epsilon+b}} \left(\frac{P_{t+1}}{P_t}\right)^{\epsilon+b} \\
&= C_t^{-\sigma} Z_t MC_t Y_t + \theta\beta\mathbb{E}_t \Pi_{t+1}^{\epsilon+b} \frac{X_{1,t+1}}{P_{t+1}^{\epsilon+b}}, \\
\frac{X_{2,t}}{P_t^{\epsilon-1}} &= C_t^{-\sigma} Z_t Y_t + \theta\beta\mathbb{E}_t \frac{X_{2,t+1}}{P_{t+1}^{\epsilon-1}} \left(\frac{P_{t+1}}{P_t}\right)^{\epsilon-1} \\
&= C_t^{-\sigma} Z_t Y_t + \theta\beta\mathbb{E}_t \Pi_{t+1}^{\epsilon-1} \frac{X_{2,t+1}}{P_{t+1}^{\epsilon-1}}.
\end{aligned}
$$

Rewriting the equilibrium conditions VIII

- So the reset price expression can now be written as:

$$(P_t^*)^{1+b} = \mathcal{M}\frac{X_{1,t}/P_t^{\epsilon+b}}{X_{2,t}/P_t^{\epsilon-1}}P_t^{1+b},$$

and by dividing both sides by P_{t-1}^{1+b} we can write this in terms of inflation:

$$(\Pi_t^*)^{1+b} = \mathcal{M}\frac{X_{1,t}/P_t^{\epsilon+b}}{X_{2,t}/P_t^{\epsilon-1}}\Pi_t^{1+b}. \tag{29}$$

Rewriting the equilibrium conditions IX

- The process for real money balances can be converted into real terms quite easily:

$$\Delta m_t = m_t - m_{t-1},$$

and then do some add and subtraction:

$$\begin{aligned}\Delta m_t &= m_t - m_{t-1} + p_t - p_t + p_{t-1} - p_{t-1} \\ &= m_t - p_t - (m_{t-1} - p_{t-1}) + p_t - p_{t-1} \\ &= \Delta(m_t - p_t) + \pi_t.\end{aligned}$$

- If you really want to, you could make some new auxiliary variables to easily track the real variables from their nominal counterparts.
  - For example, you could define, say, $M_t^r = M_t/P_t$ or $\tilde{X}_{1,t} = X_{1,t}/P_t^{\epsilon+b}$.

Rewriting the equilibrium conditions X

- So we can write the process for money growth in terms of real balance growth as:

$$\Delta(m_t - p_t) + \pi_t = (1 - \rho_m)\pi + \rho_m \Delta(m_{t-1} - p_{t-1}) + \rho_m \pi_{t-1} + \varepsilon_t^m. \tag{30}$$

Rewriting the equilibrium conditions XI

- The full set of rewritten equilibrium conditions is:

$$
\begin{aligned}
C_t^{-\sigma} &= \beta \mathbb{E}_t C_{t+1}^{-\sigma} \Pi_{t+1} R_t \frac{Z_{t+1}}{Z_t}, \\
\varphi_0 N_t^{\varphi} &= C_t^{-\sigma} \frac{W_t}{P_t}, \\
\frac{M_t}{P_t} &= \zeta_M \frac{R_t}{i_t Z_t} C_t^{\sigma}, \\
MC_t &= \frac{W_t / P_t}{(1-\alpha) A_t (Y_t / A_t)^{\frac{-\alpha}{1-\alpha}}}, \\
C_t &= Y_t, \\
Y_t &= \frac{A_t N_t^{1-\alpha}}{V_t^P},
\end{aligned}
$$

Rewriting the equilibrium conditions XII

$$
\begin{aligned}
(V_t^P)^{\frac{1}{1-\alpha}} &= (1-\theta)(\Pi_t^*)^{-\frac{\epsilon}{1-\alpha}}\Pi_t^{\frac{\epsilon}{1-\alpha}} + \theta\Pi_t^{\frac{\epsilon}{1-\alpha}}(V_{t-1}^P)^{\frac{1}{1-\alpha}},\\
\Pi_t^{1-\epsilon} &= (1-\theta)(\Pi_t^*)^{1-\epsilon} + \theta,\\
(\Pi_t^*)^{1+b} &= \mathcal{M}\frac{X_{1,t}/P_t^{\epsilon+b}}{X_{2,t}/P_t^{\epsilon-1}}\Pi_t^{1+b},\\
\frac{X_{1,t}}{P_t^{\epsilon+b}} &= C_t^{-\sigma}MC_tY_t + \theta\beta\mathbb{E}_t\Pi_{t+1}^{\epsilon+b}\frac{X_{1,t+1}}{P_{t+1}^{\epsilon+b}},\\
\frac{X_{2,t}}{P_t^{\epsilon-1}} &= C_t^{-\sigma}Y_t + \theta\beta\mathbb{E}_t\Pi_{t+1}^{\epsilon-1}\frac{X_{2,t+1}}{P_{t+1}^{\epsilon-1}},
\end{aligned}
$$

Rewriting the equilibrium conditions XIII

$$
\begin{aligned}
z_t &= \rho_z z_{t-1} + \varepsilon_t^z, \\
a_t &= \rho_a a_{t-1} + \varepsilon_t^a, \\
\Delta m_t &= \Delta(m_t - p_t) + \pi_t, \\
\Delta(m_t - p_t) + \pi_t &= (1 - \rho_m)\pi + \rho_m \Delta(m_{t-1} - p_{t-1}) + \rho_m \pi_{t-1} + \varepsilon_t^m.
\end{aligned}
$$

- These equations are all in closed-form, so we could put it into `Dynare` and solve it – we just need the steady state!

The steady state I

- We now solve for the non-stochastic steady state of the model.
- We have $A = 1$, and since output and consumption are always equal, it must be that $Y = C$.
- Steady state inflation is equal to the exogenous target, π, which we will assume to be 0.
    * In other words, in what follows, we will assume that there is no trend inflation in the steady state. The case of trend inflation see Ascari (2004) and Ascari and Ropele (2009).

The steady state II

- Next, from the consumption Euler equation, we have:

$$
\begin{aligned}
C^{-\sigma} &= \beta C^{-\sigma} \Pi R \frac{Z}{Z} \\
\implies R &= \frac{1+\pi}{\beta} \\
\Leftrightarrow 1+i &= \frac{1+\pi}{\beta} \\
\implies i &= \rho + \pi, \qquad (31)
\end{aligned}
$$

where

$$
\beta = \frac{1}{1+\rho}.
$$

- (31) is the familiar Fisher equation, and ρ in the expression for β is the discount rate (whereas β is the discount factor), and is also referred to as the net real interest rate.

The steady state III

- From the price evolution equation, we can derive the steady state expression for reset price inflation:

$$
\begin{aligned}
\Pi^{1-\epsilon} &= (1-\theta)(\Pi^*)^{1-\epsilon} + \theta \\
\frac{\Pi^{1-\epsilon} - \theta}{1-\theta} &= (\Pi^*)^{1-\epsilon} \\
\implies \Pi^* &= \left(\frac{\Pi^{1-\epsilon} - \theta}{1-\theta}\right)^{\frac{1}{1-\epsilon}} .
\end{aligned} \tag{32}
$$

- If $\Pi = 1$, then $\Pi^* = \Pi$, since the RHS of the above expression is equal to 1. If $\Pi > 1 \implies \Pi^* > \Pi$, and if $\Pi < 1 \implies \Pi^* < \Pi$.

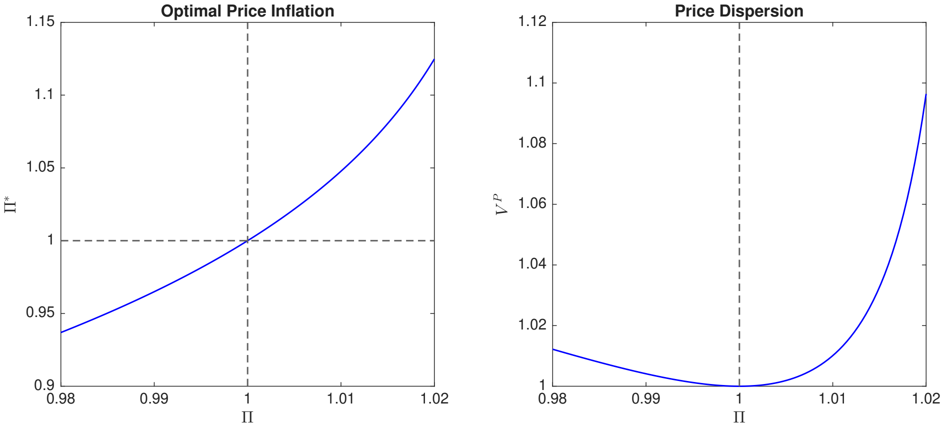
The steady state IV

- With this in hand, we can solve for steady state price dispersion:

$$\begin{aligned}(V^P)^{\frac{1}{1-\alpha}} &= (1-\theta)(\Pi^*)^{-\frac{\epsilon}{1-\alpha}}\Pi^{\frac{\epsilon}{1-\alpha}} + \theta\Pi^{\frac{\epsilon}{1-\alpha}}(V^P)^{\frac{1}{1-\alpha}} \\ \left(1-\theta\Pi^{\frac{\epsilon}{1-\alpha}}\right)(V^P)^{\frac{1}{1-\alpha}} &= \frac{(1-\theta)(\Pi)^{\frac{\epsilon}{1-\alpha}}}{(\Pi^*)^{\frac{\epsilon}{1-\alpha}}}.\end{aligned} \tag{33}$$

- If $\Pi = 1$, then $V^P = 1$. If $\Pi \neq 1$, then $V^P > 1$.

The steady state V



NOTE: $\epsilon = 10$, $\theta = 0.75$, $\alpha = 0$.

The steady state VI

- Now, we can solve for the steady state ratio of $(X_1/P^{\epsilon+b})/(X_2/P^{\epsilon-1})$:

$$\frac{X_1/P^{\epsilon+b}}{X_2/P^{\epsilon-1}} = \mathcal{M}^{-1}\left(\frac{\Pi^*}{\Pi}\right)^{1+b}.$$

- Then take the equation for the auxiliary variable $X_{1,t}/P_t^{\epsilon+b}$, and do some rearranging:

$$\begin{aligned}\frac{X_1}{P^{\epsilon+b}}(1-\theta\beta\Pi^{\epsilon+b}) &= MC\frac{Y}{C}\\ \frac{X_1}{P^{\epsilon+b}} &= MC\frac{Y}{C}(1-\theta\beta\Pi^{\epsilon+b})^{-1},\end{aligned}$$

and do the same for $X_{2,t}/P_t^{\epsilon-1}$:

$$\frac{X_2}{P^{\epsilon-1}} = \frac{Y}{C}(1-\theta\beta\Pi^{\epsilon-1})^{-1}.$$

The steady state VII

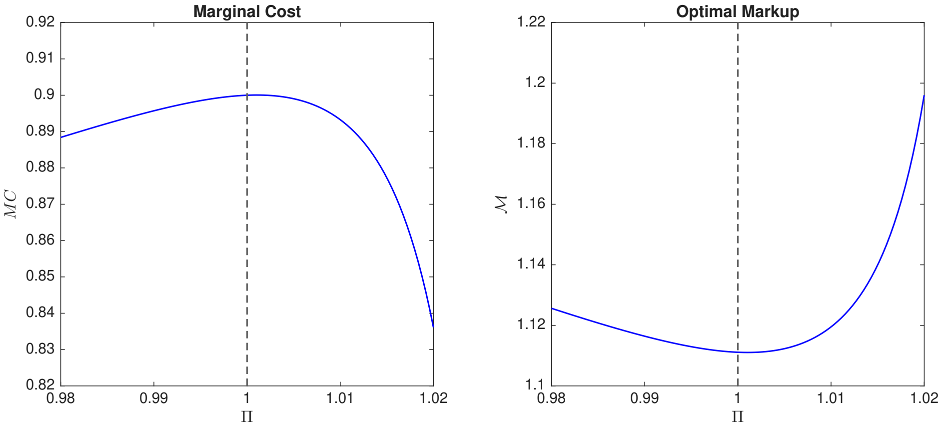
- So then we can divide the two:

$$\frac{X_1/P^{\epsilon+b}}{X_2/P^{\epsilon-1}} = MC\frac{1-\theta\beta\Pi^{\epsilon-1}}{1-\theta\beta\Pi^{\epsilon+b}}$$

$$MC = \mathcal{M}^{-1}\left(\frac{\Pi^*}{\Pi}\right)^{1+b}\frac{(1-\theta\beta\Pi^{\epsilon+b})}{(1-\theta\beta\Pi^{\epsilon-1})}.$$

- In words, the steady state real marginal cost is inverse to the price markup. If $\Pi = 1$, then $MC = \mathcal{M}^{-1} = \frac{\epsilon-1}{\epsilon}$. In other words, if steady state net inflation is zero, then the steady state markup will be what it would be if prices were fully flexible (also corresponding to $\theta = 0$).
- If $\Pi \neq 1$, then $MC < \frac{\epsilon-1}{\epsilon}$, which means that the steady state markup will be higher than it would be if net inflation were zero.

The steady state VIII



NOTE: $\epsilon = 10$, $\theta = 0.75$, $\alpha = 0$.

The steady state IX

- With the steady steady state marginal cost in hand, we now look at the labour supply condition. We have that

$$MC = \frac{W}{P} Y^{\frac{\alpha}{1-\alpha}},$$

since $A = 1$.

- The lower is the marginal cost, the bigger is the wedge between the wage and the marginal product of labour (i.e., the more distorted the economy is).
- Then we have:

$$\begin{aligned} \varphi_0 N^{\varphi} &= \frac{W}{C^{\sigma} P} \\ \Leftrightarrow \varphi_0 N^{\varphi} &= \frac{W}{Y^{\sigma} P}, \end{aligned}$$

The steady state X

and since $Y = \frac{AN^{1-\alpha}}{V^P}$, we have:

$$\begin{aligned}
\varphi_0 N^{\varphi} &= \frac{W}{P}\left(\frac{AN^{1-\alpha}}{V^P}\right)^{-\sigma} \\
&= \frac{W}{P}\left(V^P\right)^{\sigma} N^{-\sigma(1-\alpha)} \\
&= \frac{MC}{Y^{\frac{\alpha}{1-\alpha}}}\left(V^P\right)^{\sigma} N^{-\sigma(1-\alpha)} \\
&= MC(V^P)^{\sigma} N^{-\sigma(1-\alpha)}\left[\frac{N^{1-\alpha}}{V^P}\right]^{-\frac{\alpha}{1-\alpha}} \\
\varphi_0 N^{\sigma(1-\alpha)+\alpha+\varphi} &= MC(V^P)^{\frac{\sigma(1-\alpha)+\alpha}{1-\alpha}} \\
\therefore N &= \left[\frac{MC}{\varphi_0}(V^P)^{\frac{\sigma(1-\alpha)+\alpha}{1-\alpha}}\right]^{\frac{1}{\sigma(1-\alpha)+\alpha+\varphi}}.
\end{aligned}$$

The steady state XI

- Finally, since we have Y, steady state M/P is easy:

$$\frac{M}{P} = \frac{\zeta_M R}{i} Y^{\sigma}.$$

The flexible price equilibrium I

- We now consider the hypothetical equilibrium case where all prices are flexible (i.e., when $\theta = 0$).
- Sometimes referred to as the “natural allocation”.
- But even with flex prices, output is lower than an “RBC economy” since there is monopolistic competition.
- Let superscript n denote the hypothetical flex price allocation of a variable.
- When $\theta = 0$, firms will always price their goods optimally to P_t^*. Then, from the price dispersion equation we have:

$$V_t^{P,n} = \left(\frac{\Pi^*}{\Pi}\right)^{-\epsilon} = 1,$$

The flexible price equilibrium II

and combining this result with the (19) we have

$$\frac{P_t^*}{P_t} = \mathcal{M} MC_t(j).$$

- But when prices are flexible, the equilibrium is symmetric and we have: $P_t^* = P_t$, $MC_t(j) = \mathcal{M}^{-1}$, $Y_t(j) = Y_t$, and $N_t(j) = N_t$, $\forall j$.
- In words, if prices are flexible, all firms charge the same price, and price dispersion is at its lower bound of 1 and marginal costs are constant.
- Then, from the average marginal cost equation (15), we have

$$\frac{W_t^n}{P_t} = \mathcal{M}^{-1}(1-\alpha)A_t(N_t^n)^{-\alpha}.$$

The flexible price equilibrium III

- Use this with the labour supply condition to get:

$$\varphi_0 \frac{(N_t^n)^{\varphi}}{(C_t^n)^{-\sigma}} = \frac{W_t^n}{P_t} = \mathcal{M}^{-1}(1-\alpha)A_t(N_t^n)^{-\alpha}.$$

- The aggregate resource constraint will give us:

$$C_t^n = Y_t^n = A_t(N_t^n)^{1-\alpha}.$$

- These equations will allow us to price the flexible price – or “natural” – level of output This implies that the flexible price output is:

$$Y_t^n = \left(\frac{1-\alpha}{\varphi_0 \mathcal{M}}\right)^{\frac{1-\alpha}{\sigma(1-\alpha)+\alpha+\varphi}} A_t^{\frac{1+\varphi}{\sigma(1-\alpha)+\alpha+\varphi}}. \tag{34}$$

The flexible price equilibrium IV

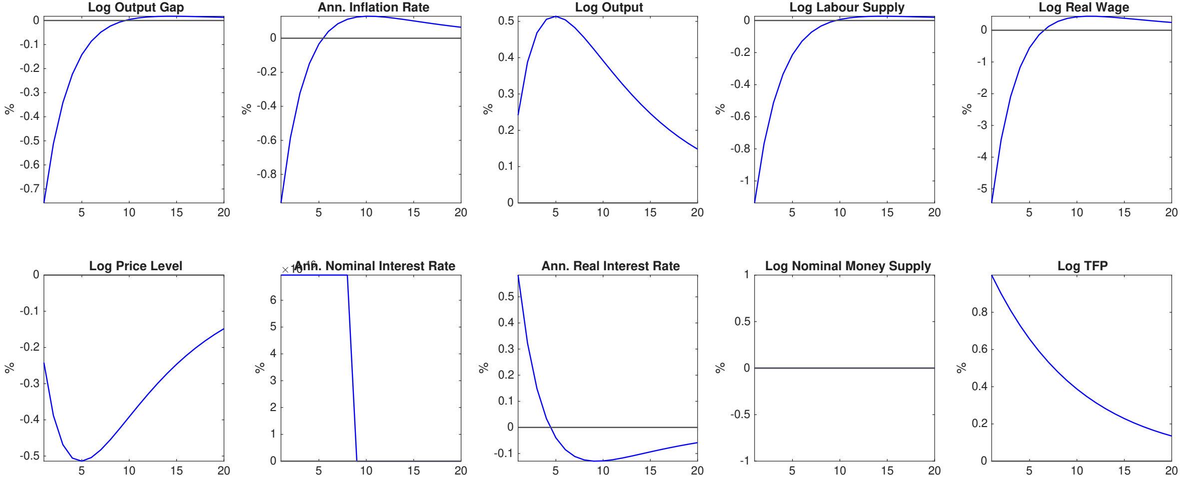
- Note that if $\sigma = 1$, then N_t^n is a constant and not a function of A_t.
- In other words, if prices are flexible and $\sigma = 1$ (meaning we have log utility), labour hours would not react to technology shocks A_t. What is driving this is that, if $\sigma = 1$, then preferences are consistent with King, Plosser, and Rebelo (1988) preferences in which the income and substitution effects of changes in A_t exactly offset.
- Also note that in the flex price equilibrium, nominal shocks have no real effects. This makes sense as we no longer have any nominal rigidities or stickiness.

Quantitative Performance

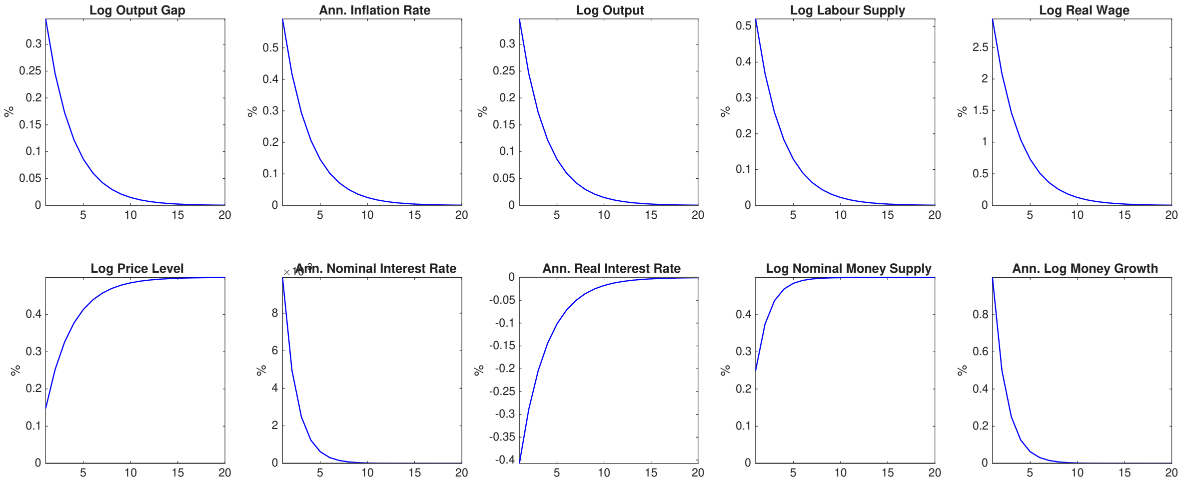
Quantitative analysis

- We can solve the model quantitatively in Dynare using a first order approximation about the steady state.
- I calibrate the model using the parameter values as in Galí (2015).
- Also, full credit to Dynare extraordinaire Johannes Pfeifer (who is also a great macroeconomist!) for doing the original replication – I've plugged his fantastic work already, but check out his `GitHub` for a collection of replication files he has done.

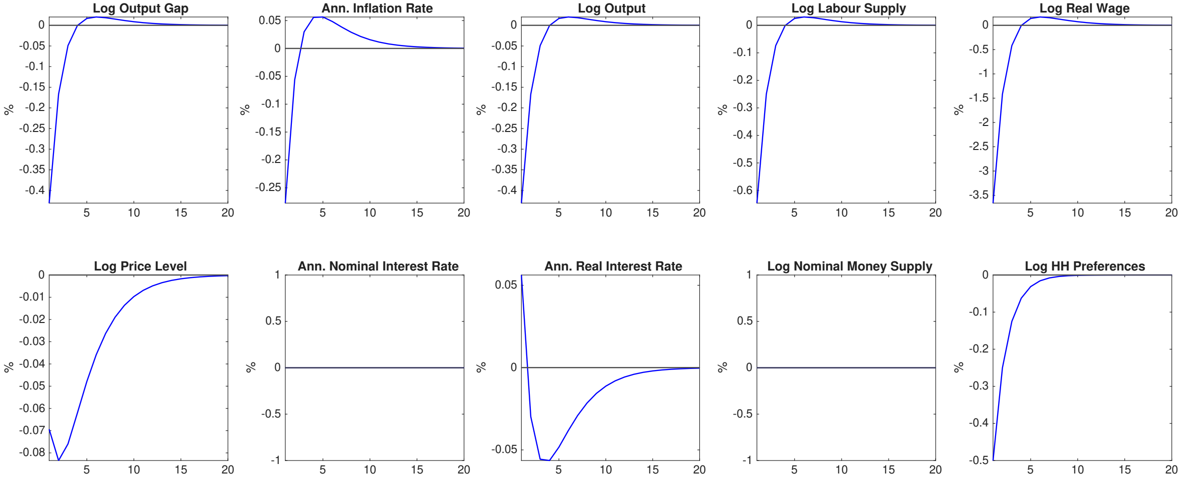
IRFs to a TFP shock



IRFs to a monetary shock (money supply rule)



IRFs to a preference shock



Conclusion

- Next class: log linearising, the Taylor rule, and more on the Blanchard-Kahn conditions.

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