MATH 124, PROBLEM SET 4

Due: 12:00PM, October 10, 2014. ***Late submissions will not be graded.***
Note: Collaboration is permitted and encouraged. We only ask that you write up your solutions independently, and list your collaborators on your problem sets. Hard copies typed in $\TeX$ are preferred. Please separate your submissions as indicated below.

Notation: Davenport uses a non-standard square-bracket notation (that we also adopt in this class) for the numerators and denominators of the convergents. This is separate from the standard square-bracket notation for continued fractions, where $[3;\overline{1,2}]$ would denote:

$$3+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cdots}}}}$$

For Hirsh Jain

1.) Using the manner of the Euclidean algorithm to show your work, express these as continued fractions: $\frac{155}{131}$, $\frac{782}{99}$.

Let F_n denote the *n-th Fibonacci number*, where $F_0 = 0$, $F_1 = 1$, and $F_{n+2} = F_{n+1} + F_n$.

2.) Find and prove the general form of the continued fraction of $\frac{F_{n+1}}{F_n}$ for $n > 0$.

3.) Find and prove the general form of the continued fraction of $\frac{F_{n+1}^2}{F_n^2}$ for $n > 0$.

For Julian Salazar

A *generalized continued fraction* does not require the numerators to be 1.

4.) Derive closed expressions for both

$$1+\cfrac{2}{3+\cfrac{2}{1+\cfrac{2}{3+\cdots}}} \qquad \text{and} \qquad 1+\cfrac{x-1}{2+\cfrac{x-1}{2+\cfrac{x-1}{2+\cdots}}}$$

Given a continued fraction $\alpha = [a_0; a_1, a_2, \ldots]$, we call $[a_0; a_1, a_2, \ldots, a_k]$ the *k-th convergent* of α.

5.) Prove that the subsequence of odd-index convergents is decreasing.

6.) Find the general solution to $91x - 55y = 1$ using the convergents of $\frac{91}{55}$ (see Davenport). Show your work.