Department of Electronic and Telecommunication Engineering

University of Moratuwa

EN 2040 – Random Signals and Processes

Simulation Assignment

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This is submitted as a partial fulfillment for the module

EN 2040: Random Signals and Processes

Department of Electronic and Telecommunication

University of Moratuwa

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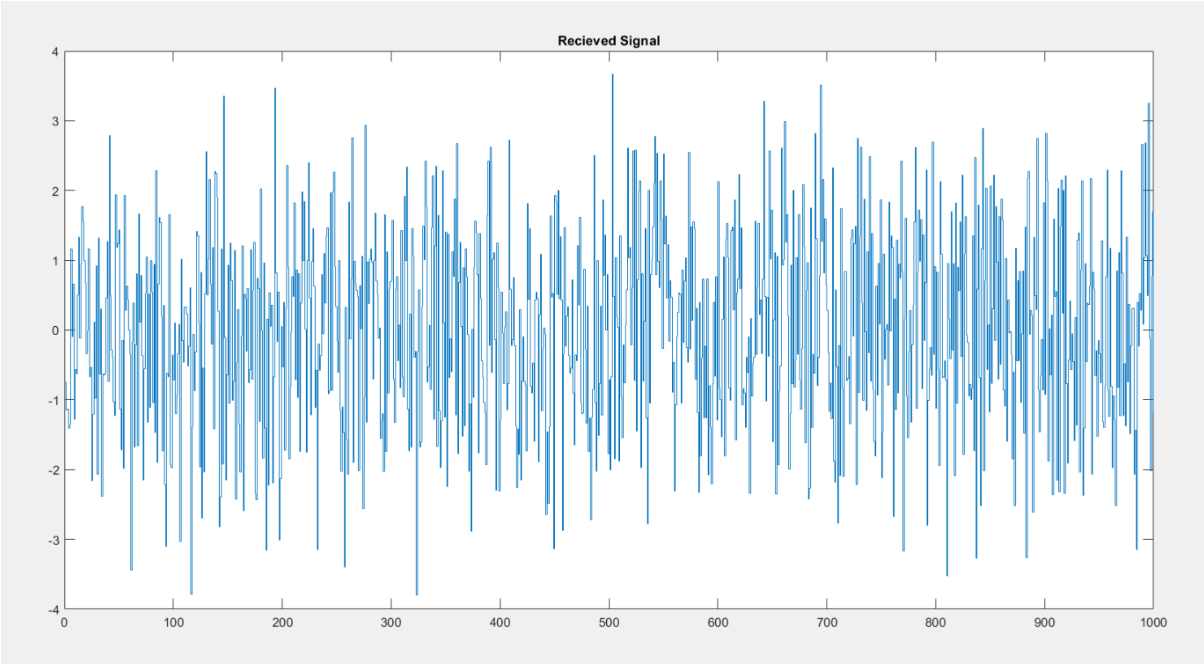
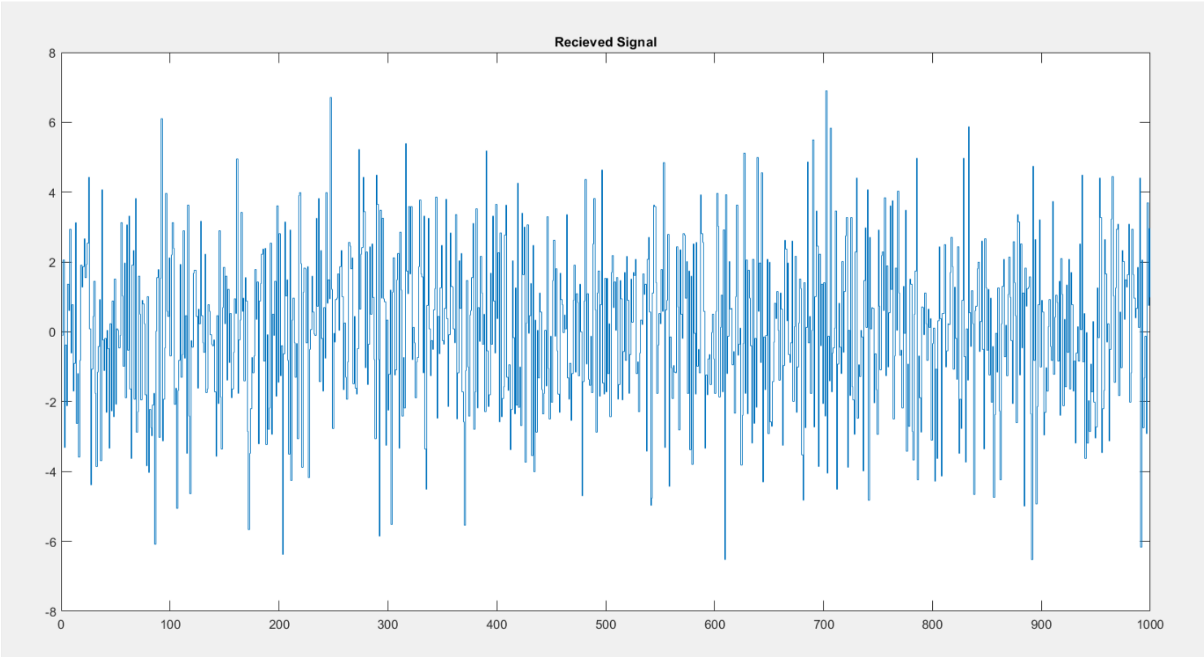
Question 3

$\sigma^2 = 1$

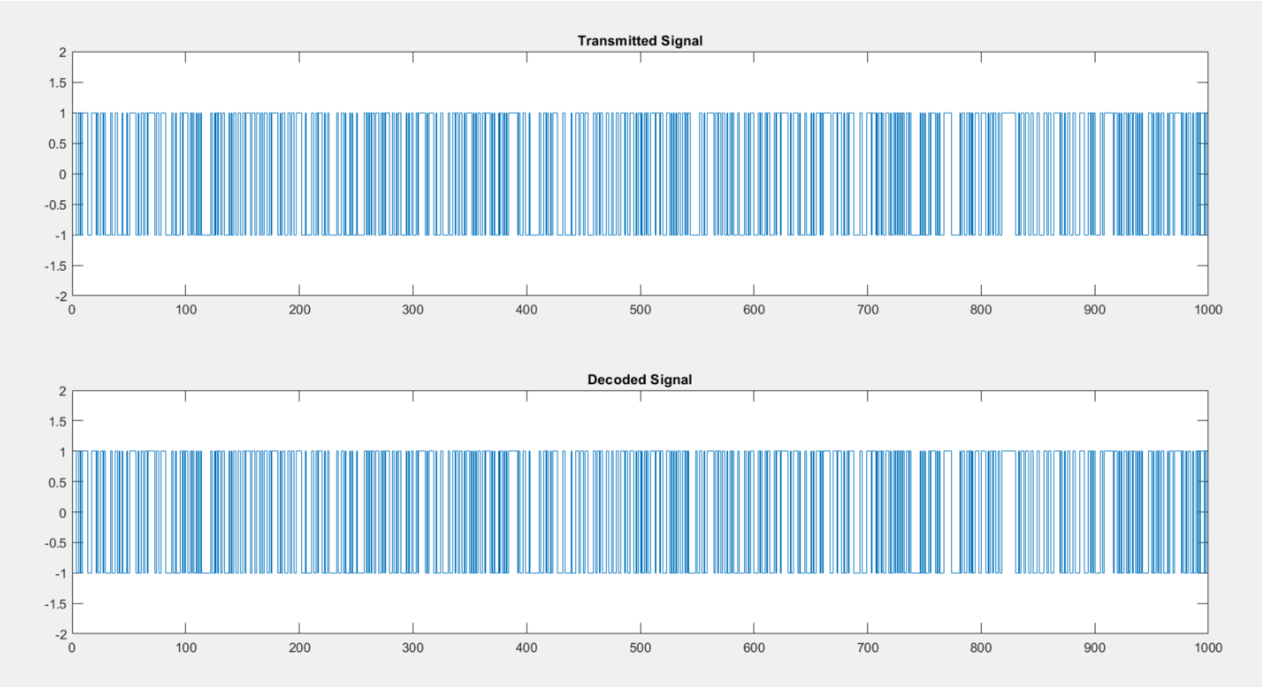


$\sigma^2 = 4$

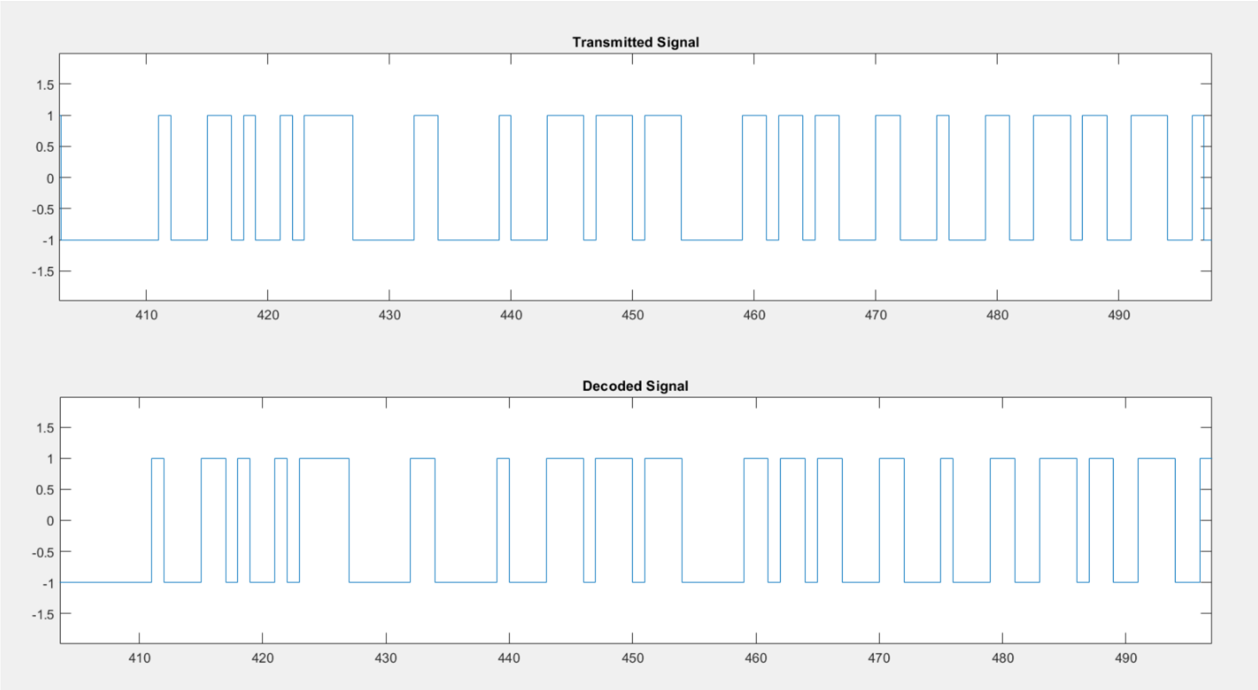


- When the variance of the noise increases the amplitudes of the received signal increases.
- The code for this section is shown in appendix I.

Question 4



- Zoomed in for comparison

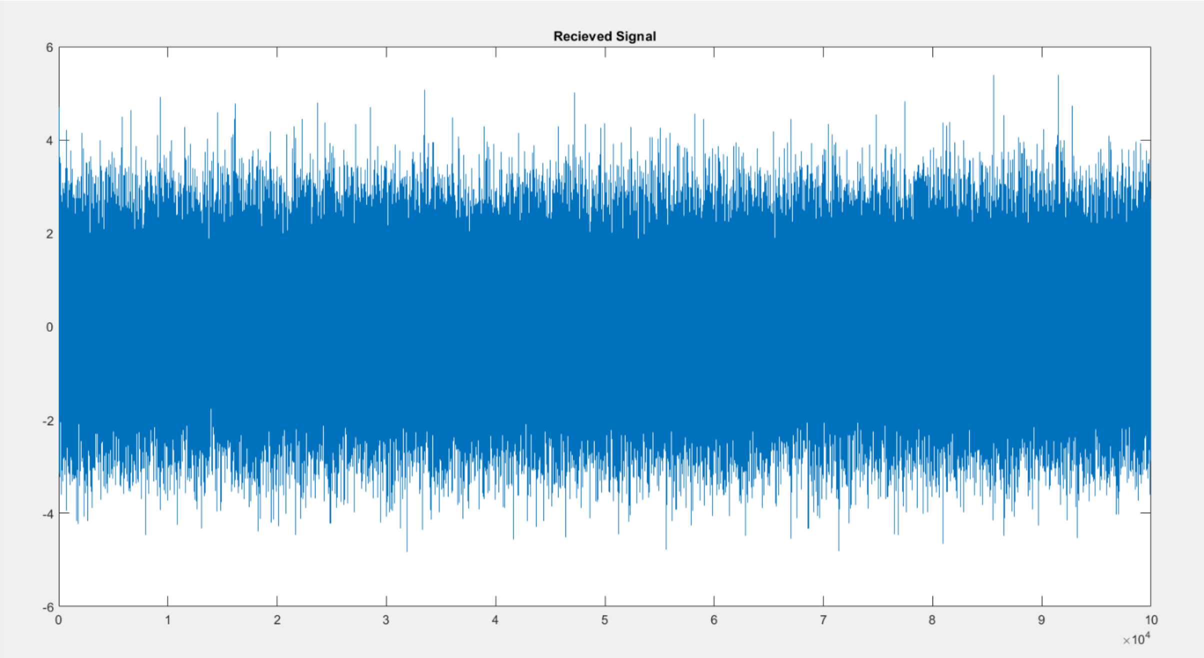


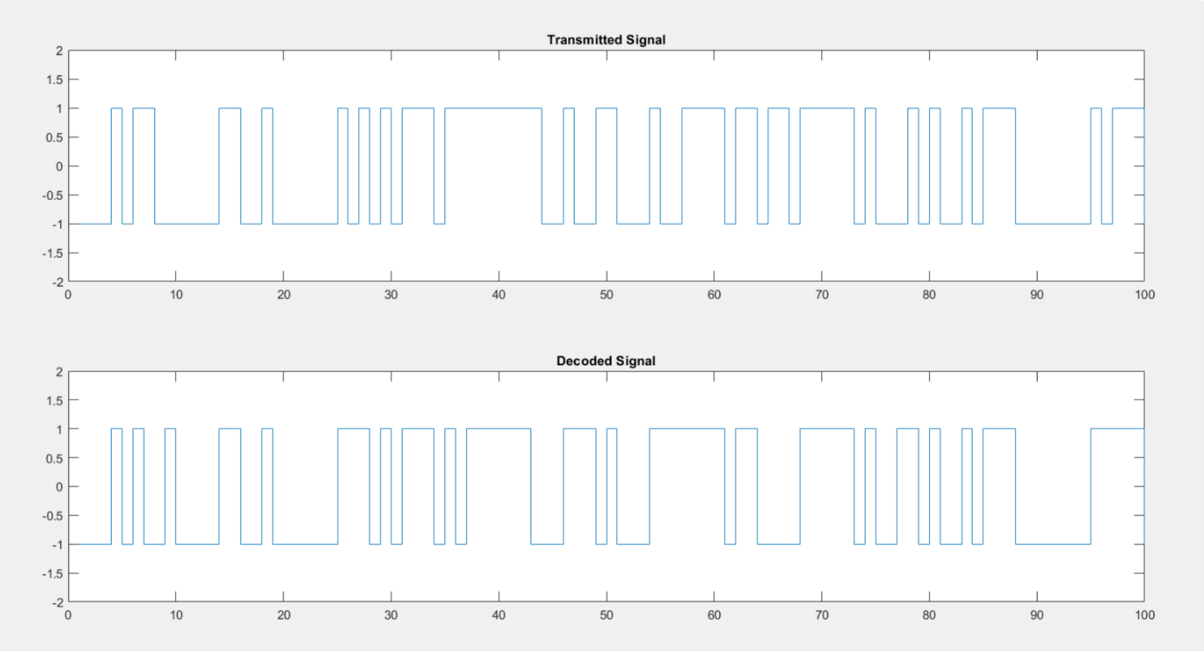
- Most of the bits are identified correctly. (same as the transmitted bit) But there are errors in decoding some of the bits.

- Code for this part is shown in appendix II.

Question 5

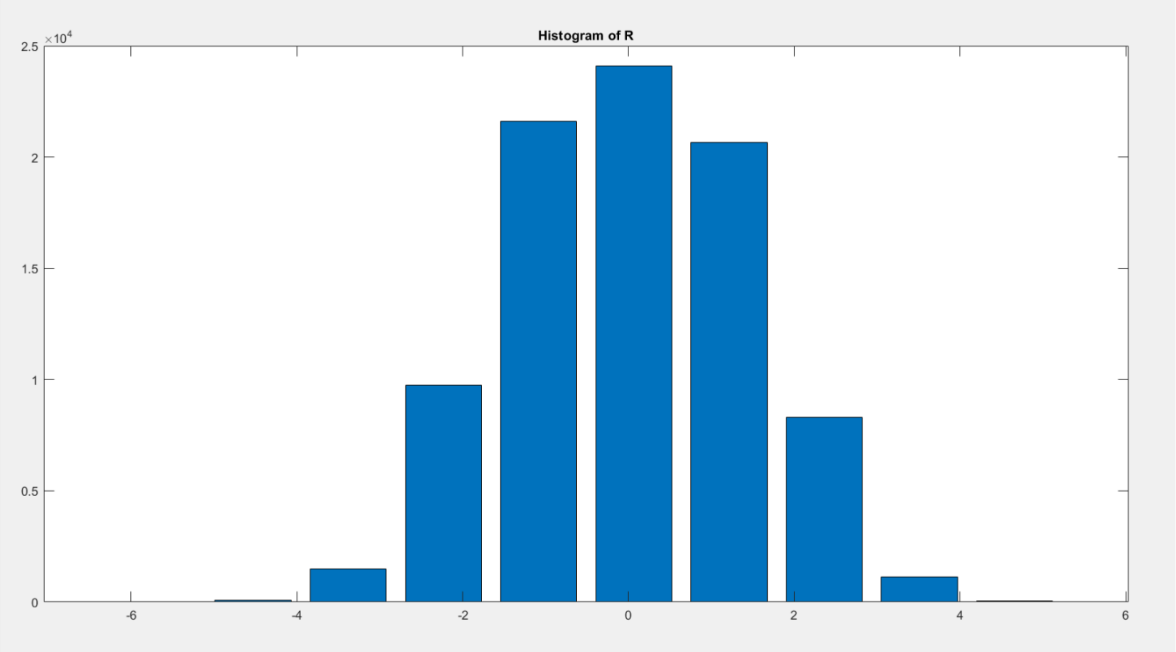
L = 100000



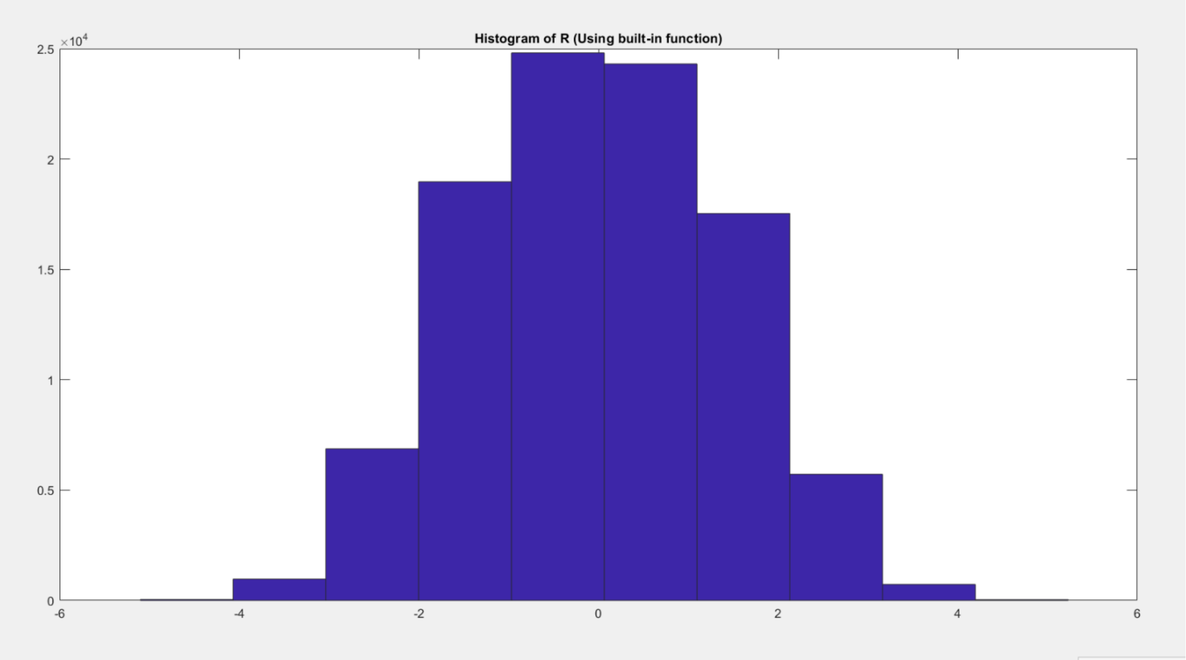


- Similar to the case of L = 1000, most of the bits are decoded without an error, but there are some bit errors.
- Code for this is given in appendix I and II.

- Histogram plotted **without** using the MATLAB built-in function hist( ) . (Number of bins = 10)



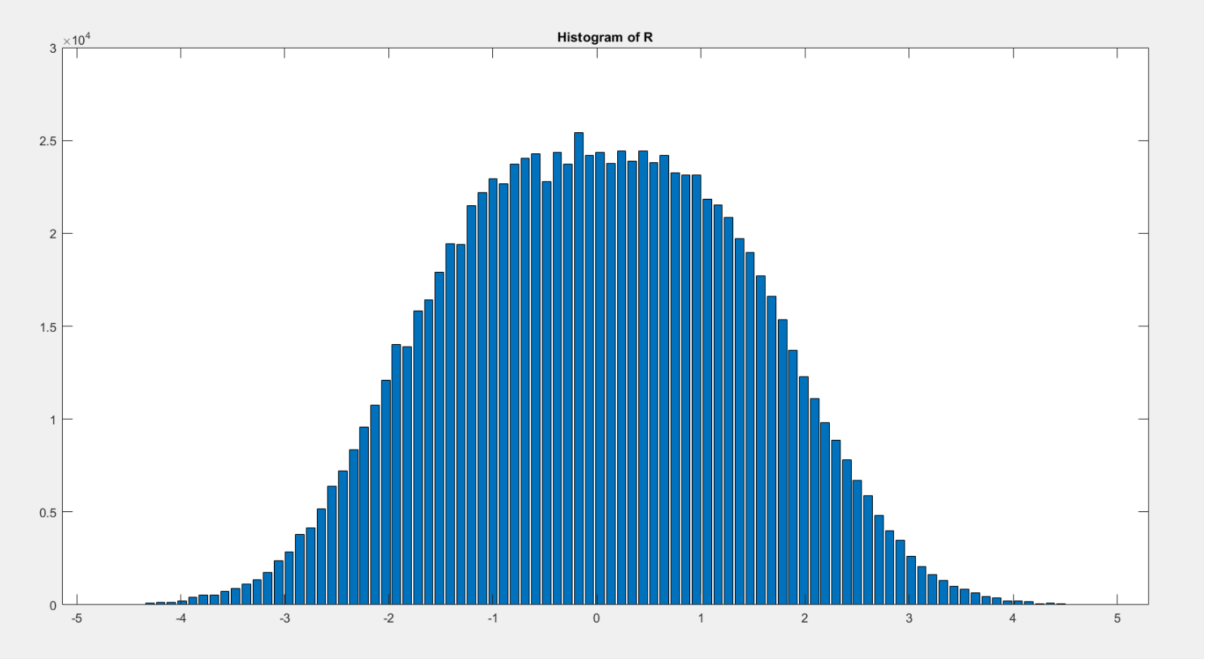
- Histogram plotted using the built-in function hist().



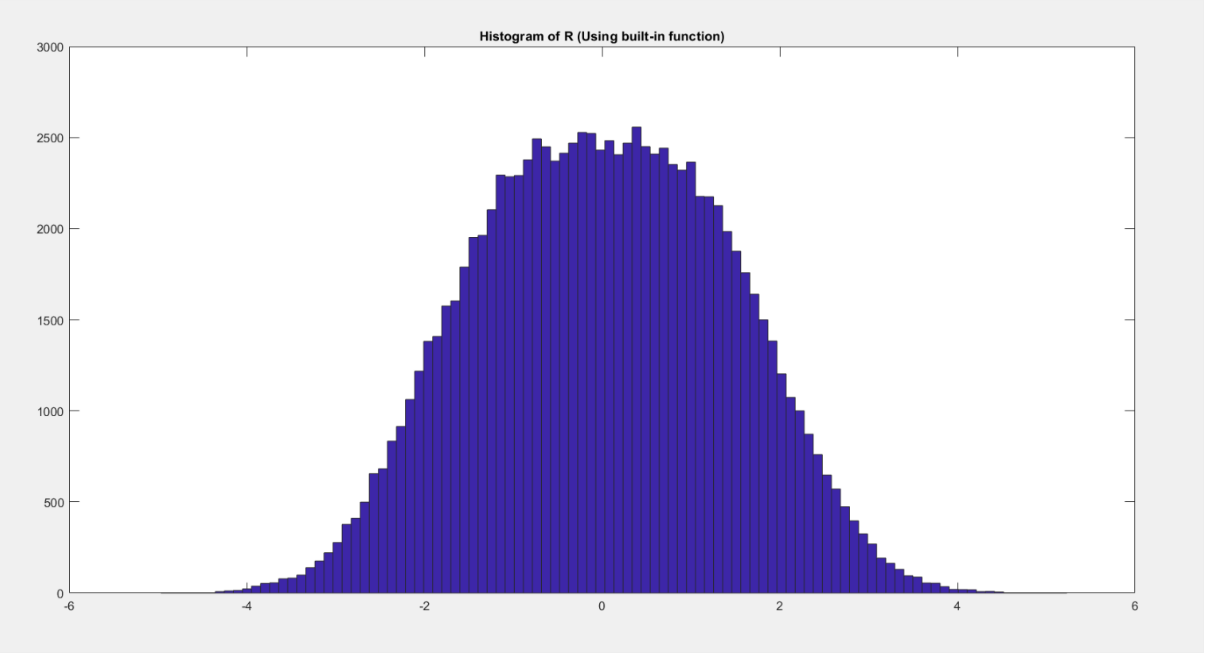
- Two histograms are very much similar. There is a slight difference in the frequency values for each bin. This could be due to the difference in the range for each bin.
- Code for this part is given in appendix III.

Question 5 (a)

- Histogram plotted **without** using the MATLAB built-in function hist( ).(Number of bins = 100)



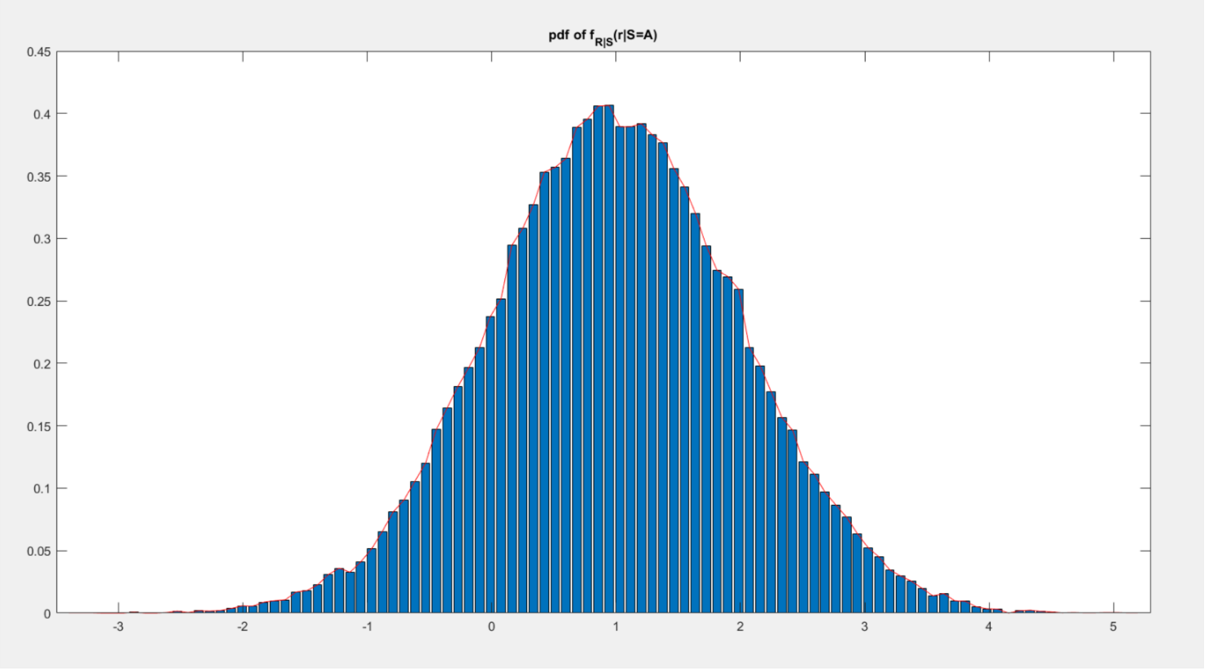
- Histogram plotted using the built-in function hist( ).



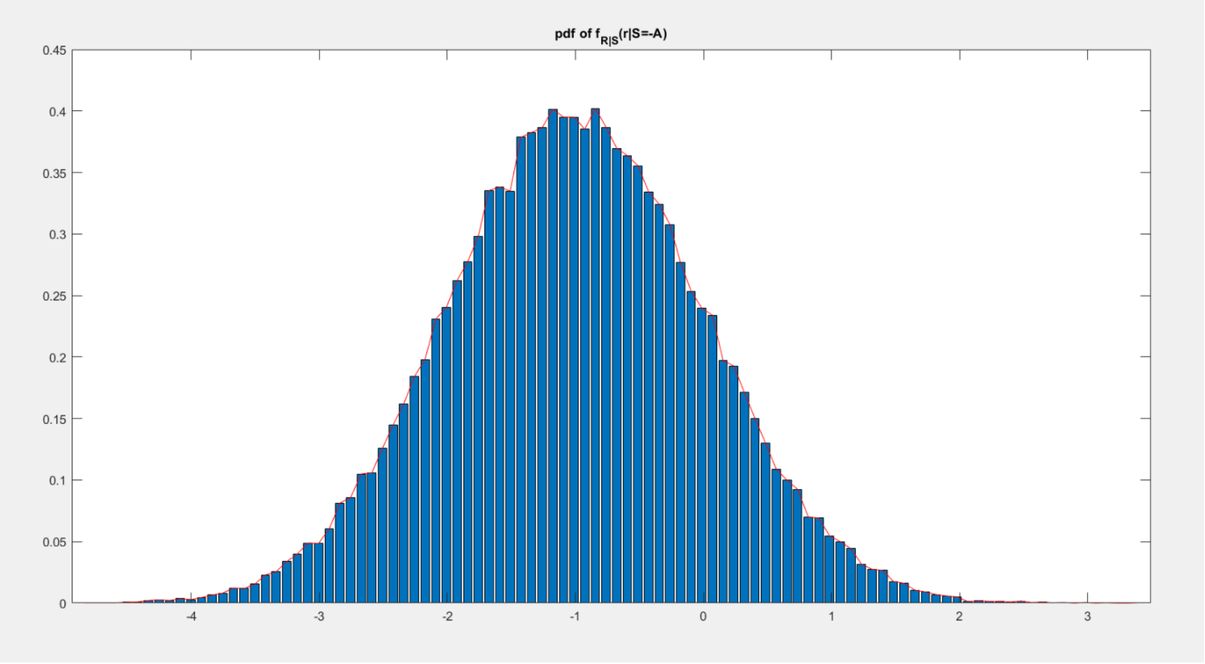
- When the number of bins is increased to 100, the two histograms look much similar in shape. But the values in the y axis has a significant variation. This is because the built in function gives a normalized frequency value.
- Appendix III shows the code for this part.

Question 5 (b)

$f_{R|S}(r|S = A)$, A = 1

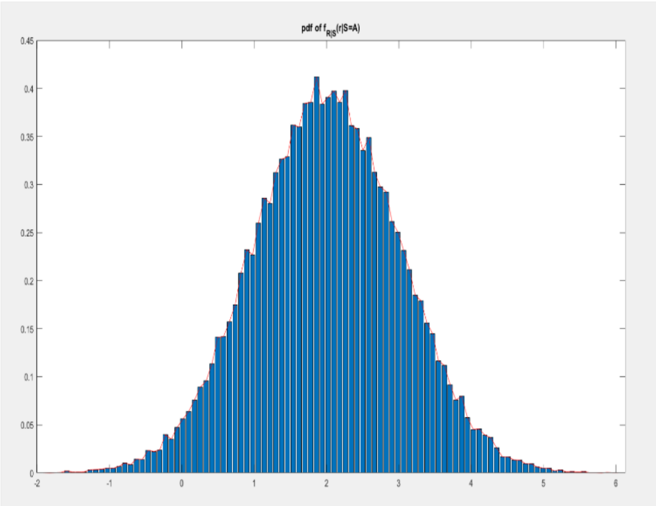


$f_{R|S}(r|S = -A)$, A = 1

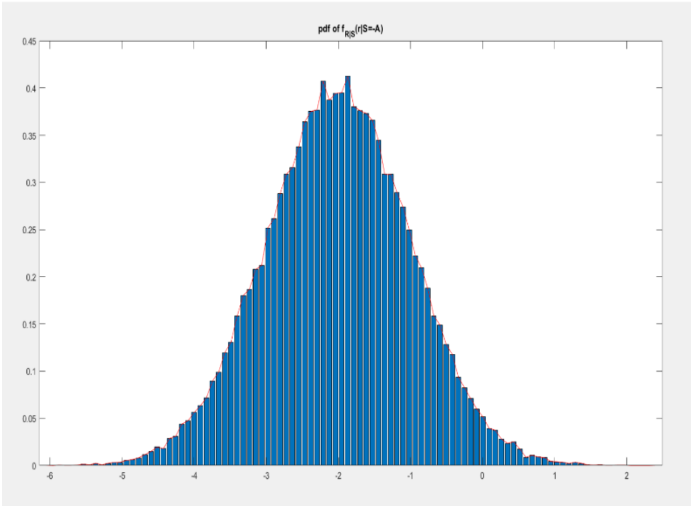


- The code for this part is given in appendix IV.

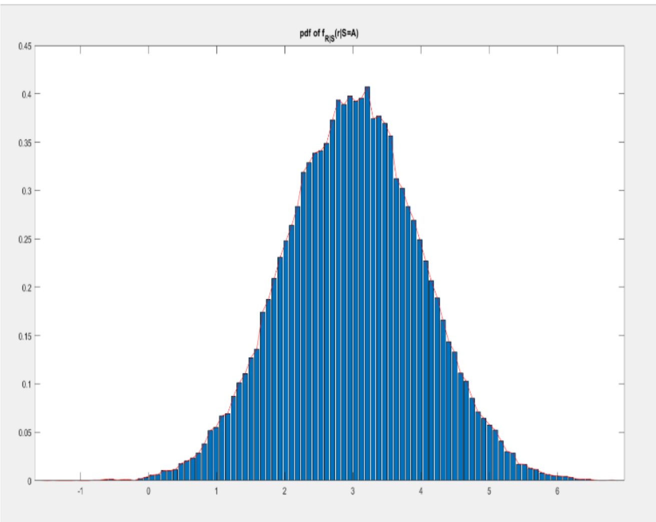
$f_{R|S}(r|S = A)$, A = 2



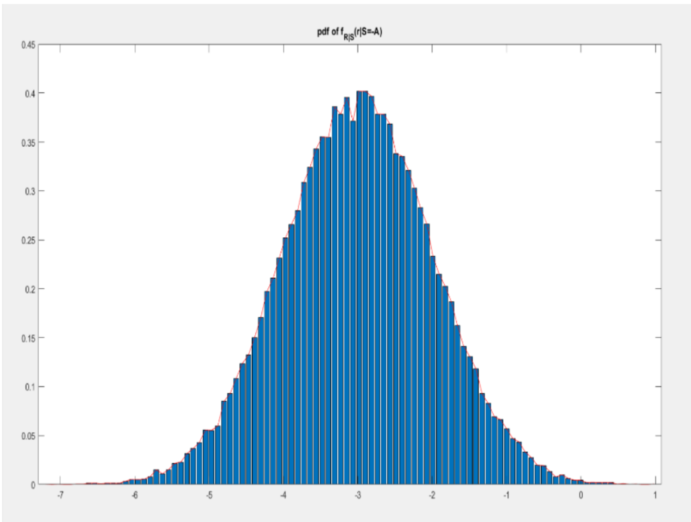
$f_{R|S}(r|S = -A)$, A = 2



$f_{R|S}(r|S = A)$, A = 3



$f_{R|S}(r|S = -A)$, A = 3



- When the value of A increases the range of the x-axis also increases. The mode and mean values also shift away from zero.

Question 5 (c)

- Expected value is calculated by $E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx$. In this case since we have the pdf as a discrete set of points we can calculate the expected values as follows.

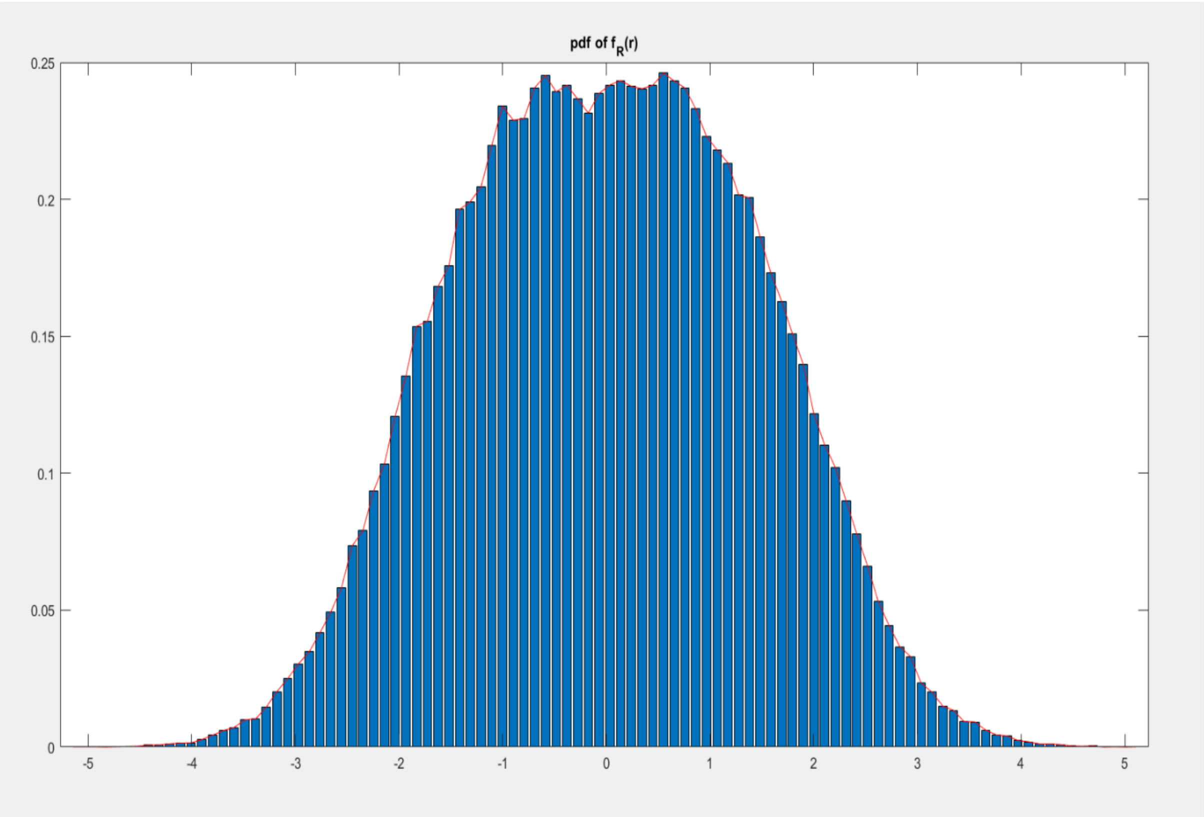
$$E[X] = \sum_{i=1}^{N} x_i . f_{x_i}(x_i) . \Delta x_i$$

- Expected values calculated for different values of A. The code is given in appendix V.

A	$E[R\|S = A]$	$E[R\|S = -A]$	E[R]
1	0.9974	-1.0031	-0.0019
2	2.0049	-2.0023	0.0021
3	3.0028	-3.0037	-0.0004

Question 5 (d)

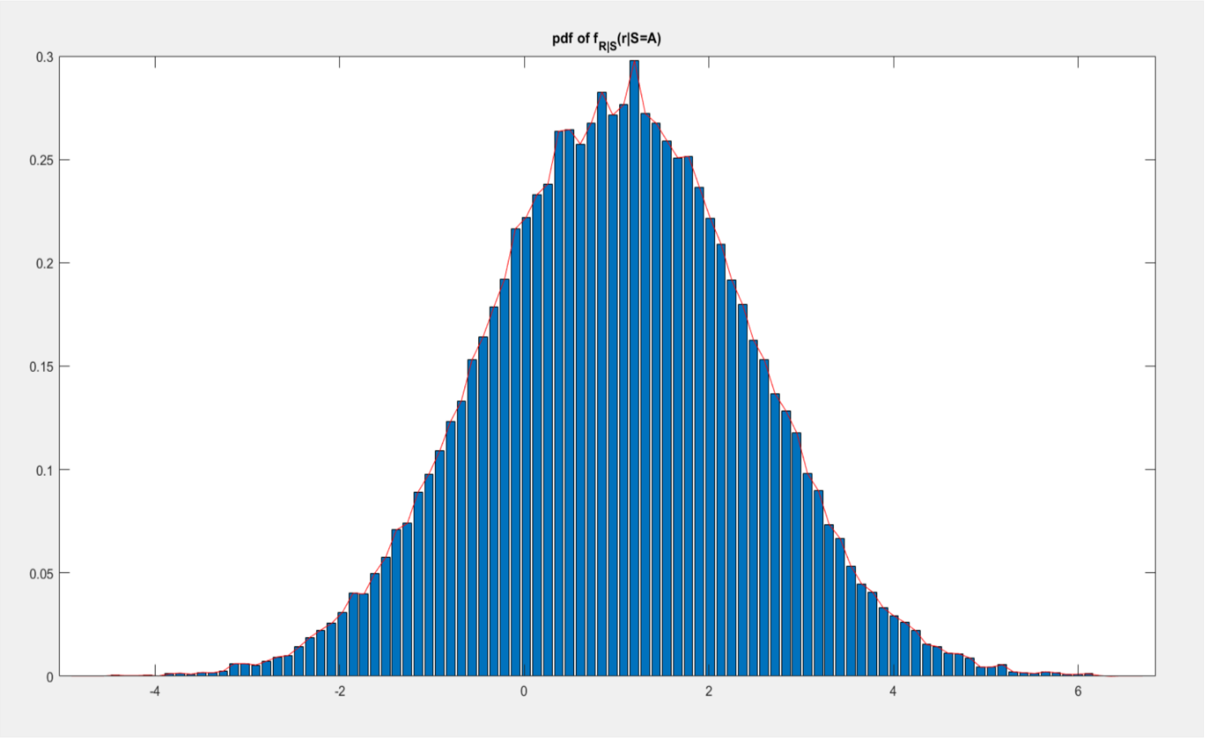
$f_R(r)$



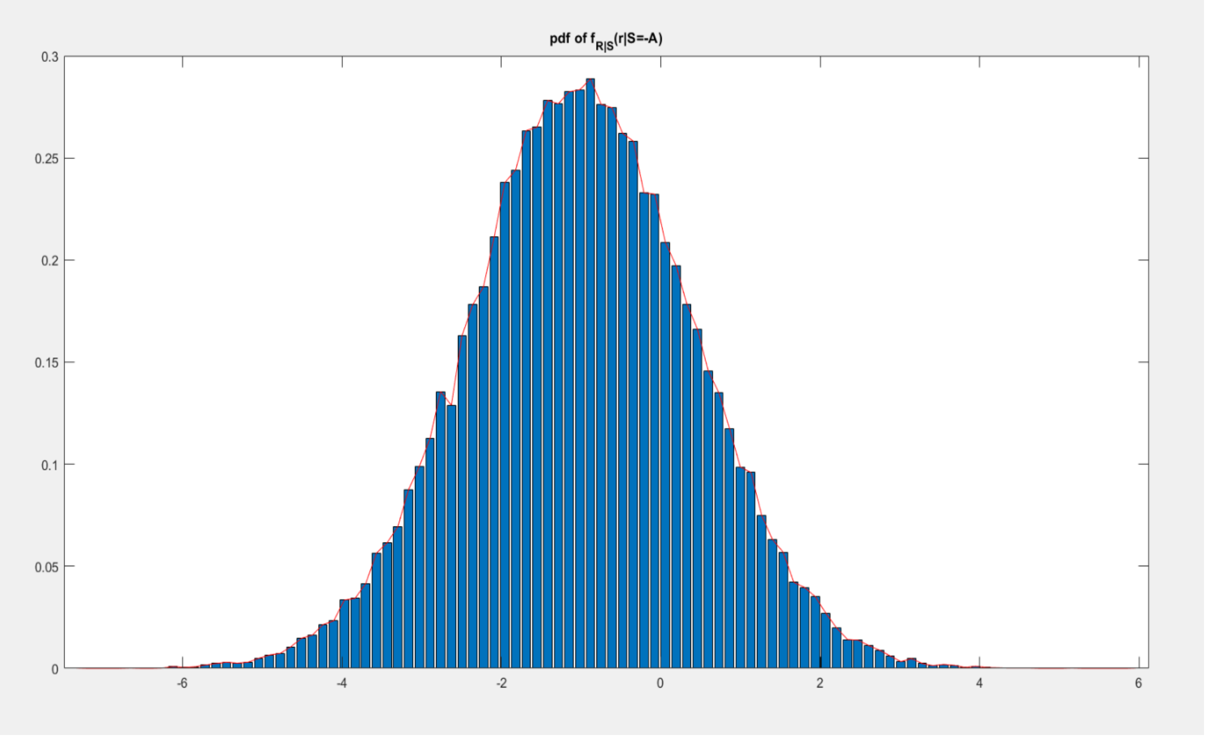
- Appendix V contains the code for this part.

Question 6

$f_{R|S}(r|S = A)$, A = 1



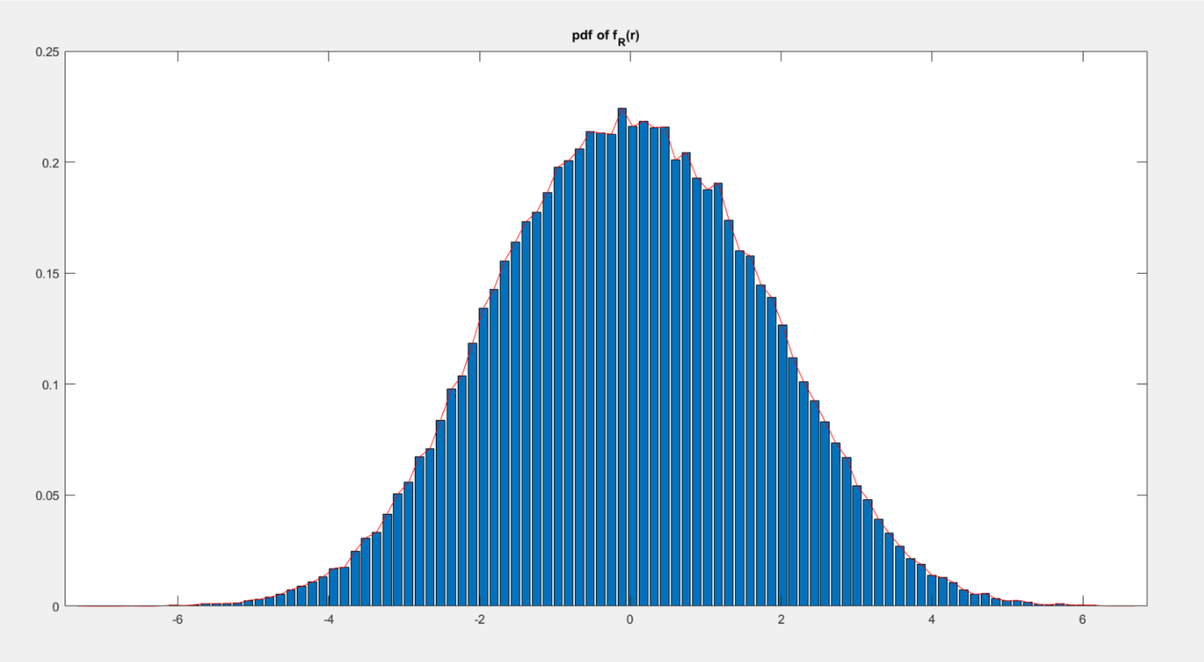
$f_{R|S}(r|S = -A)$, A = 1



A	$E[R\|S = A]$	$E[R\|S = -A]$	$E[R]$
1	1.0136	-1.0050	0.0044
2	2.0000	-2.0050	-0.0026
3	3.0015	-2.9958	0.0027

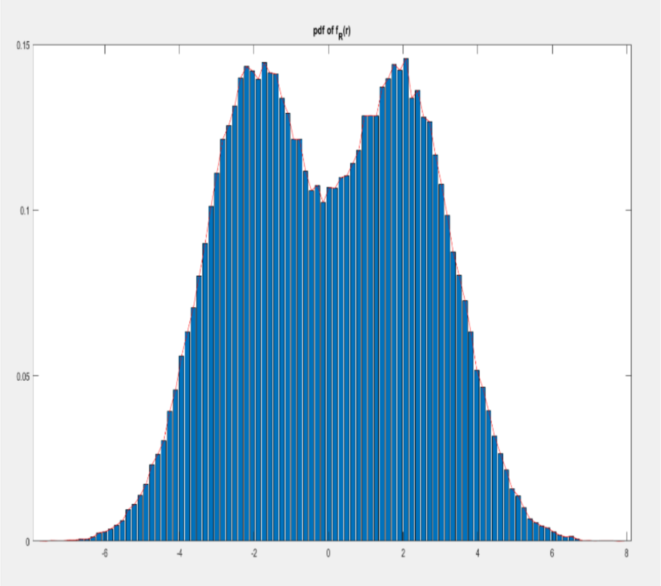
- When interference is added, the range in the x-axis of the pdfs increase. This is due to the increasing of the amplitude of the received signal. Interference generating code is given in appendix VI.

$f_R(r)$

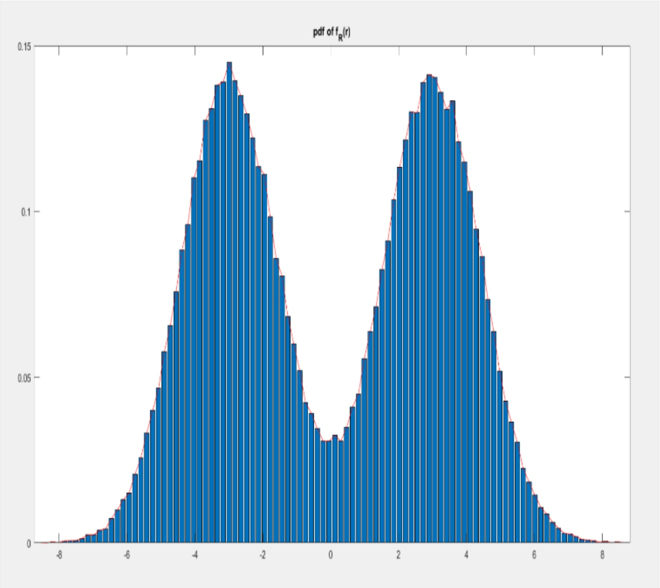


- When the value of A is changed, there is a significant difference in the pdf of $f_R(r)$.

A = 2



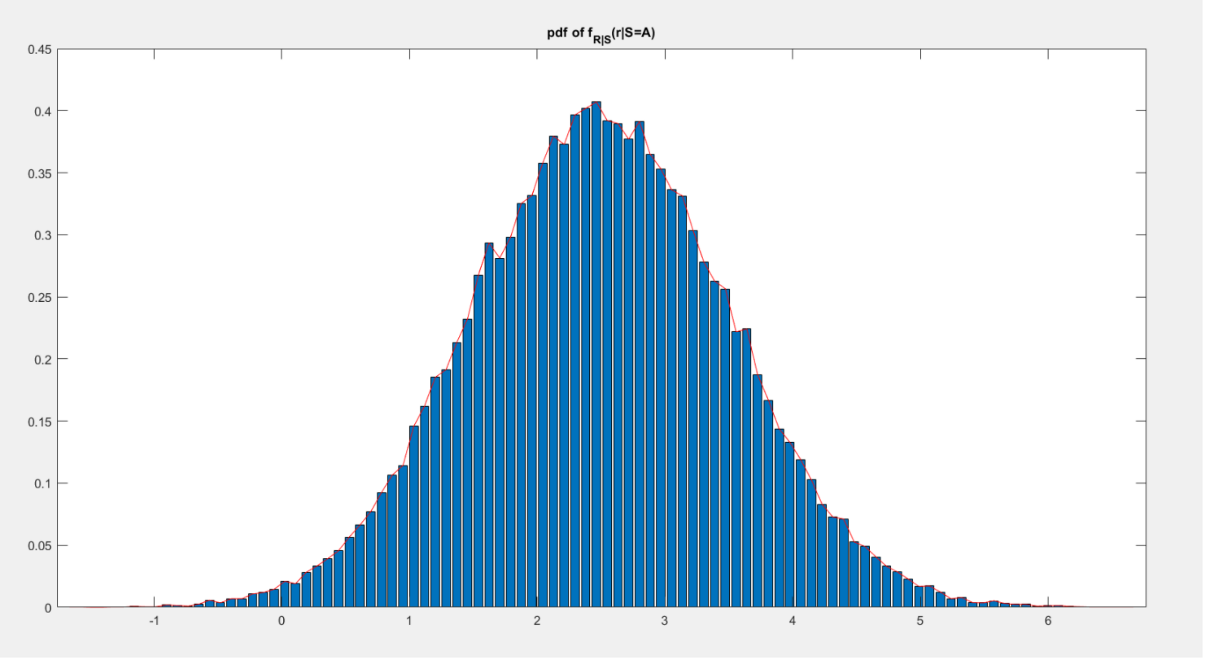
A = 3



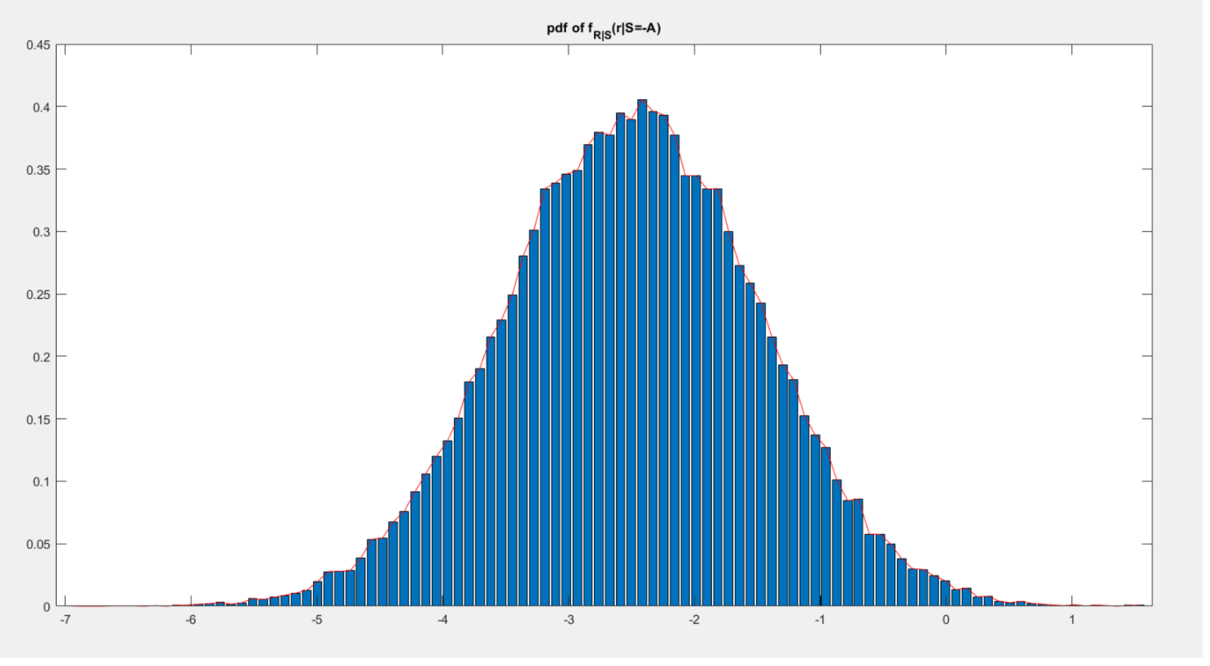
Question 7

- Amplifying factor, $\alpha = 2.5$;

  $f_{R|S}(r|S = A)$, A = 1



  $f_{R|S}(r|S = -A)$, A = 1

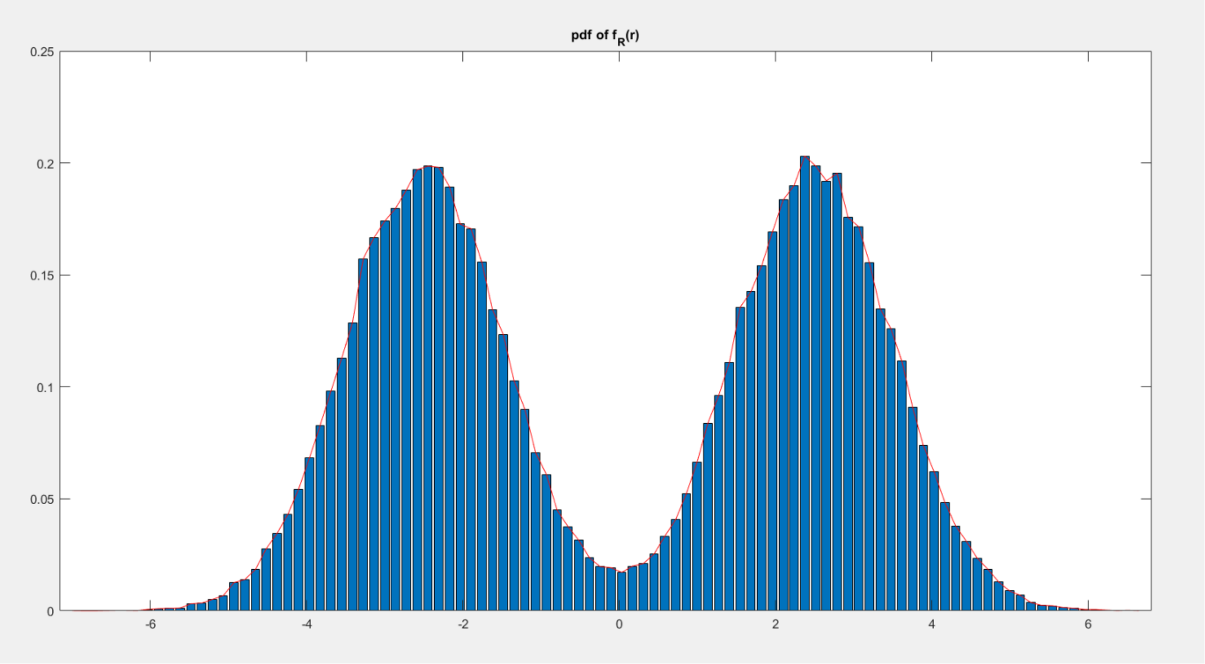


- The code for this question is given in appendix VII.

A	$E[R|S=A]$	$E[R|S=-A]$	$E[R]$
1	2.4951	-2.5001	-0.0026
2	5.0017	-4.9968	0.0023
3	7.4953	-7.4947	0.0009

- When the value of A increases the pdfs shift further away from zero.

$f_R(r)$



Appendix I

```
L = 1000; %change this to 100000 for question 5
%generating a equiprobable binary sequence
D = zeros(1,L);             %generating a sequence of thousand zeros
p = randperm(L,L/2);        %choosing 500 numbers randomly between 1 and 1000 without
replacement
D(p) = ones(1,L/2);            %replacing the zeros in D with ones in the randomly
chosen places

A = 1;
%generating a sequence of pulses
S = zeros(1,L);                  %generating a sequence of thousand zeros
for i = 1:L
    if D(i) == 0                 %assigning -A if D = 0
        S(i) = -1*A;
    else
        S(i) = A;                %assigning A if D = 1
    end
end

%generating AWGN with mean = 0 and variance = 1
m = 0;
sigma = 1;
N = m + sigma*randn(1,L);

%generating the received signal and plotting it
R = S + N;
figure;
stairs([1:L],R);
title("Received Signal");
```

Appendix II

```
%generating Y sequence
tau = 0;
Y = zeros(1,L);
for j = 1:L
    if R(j) > tau
        Y(j) = A;
    else
        Y(j) = -1*A;
    end
end

%plotting transmitted signal and Y sequence and comparing
figure;
subplot(2,1,1);
stairs([1:L],S);
title("Transmitted Signal");
xlim([0 L]);            %xlim([0 L/1000]) is taken when L = 100000
ylim([-1*A-1 A+1]);
subplot(2,1,2);
stairs([1:L],Y);
title("Decoded Signal");
xlim([0 L]);            %xlim([0 L/1000]) is taken when L = 100000
ylim([-1*A-1 A+1]);
```

Appendix III

```
%generating the bins sequence
bin_n = 10;       %change this to 100 for question 5(a)
R_max = max(R);
R_min = min(R);
width = (R_max-R_min)/(bin_n-1);
bins = [R_min-width/2:width:R_max];

%counting y values for each bin
yvalues = zeros(1,bin_n);
for k = 1:L
    for a = 1:bin_n
        if (R(k) >= bins(a)-width/2) && (R(k) < bins(a)+width/2)
            yvalues(a) = yvalues(a) + 1;
        end
    end
end
new = yvalues/width;
%plotting the histogram
figure;
bar(bins,new);
title("Histogram of R");

%using the buit in function hist()
figure;
hist(R,bin_n);
title("Histogram of R (Using built-in function)");
```

Appendix IV

```
%plotting the pdf of f_R|S(r|S=A)
rifs_one = [];    %creating a list containing R values when S = A
ind = 1;
for b = 1:L
    if S(b) == A
        rifs_one(ind) = R(b);
        ind = ind + 1;
    end
end

bin_1 = 100;
R_max1 = max(rifs_one);
R_min1 = min(rifs_one);
width_1 = (R_max1-R_min1)/(bin_1-1);      %setting bin width
bins_1 = [R_min1-width_1/2:width_1:R_max1]; %creating the bins list

[yval1,xval1] = hist(rifs_one,bins_1);    %plotting the histogram
yval1 = yval1/((ind-1)*width_1);
figure;
bar(xval1,yval1);
hold on;
plot(xval1,yval1,'r');                   %plotting the pdf
title("pdf of f_{R|S}(r|S=A)");
```

```
%plotting the pdf of f_R|S(r|S=-A)
rifs_zero = [];   %creating a list containing R values when S = -A
ind1 = 1;
for c = 1:L
    if S(c) == -1*A
        rifs_zero(ind1) = R(c);
        ind1 = ind1 + 1;
    end
end

bin_2 = 100;
R_max2 = max(rifs_zero);
R_min2 = min(rifs_zero);
width_2 = (R_max2-R_min2)/(bin_2-1);      %setting bin width
bins_2 = [R_min2-width_2/2:width_2:R_max2];      %creating the bins list

[yval2,xval2] = hist(rifs_zero,bins_2);   %plotting the histogram
yval2 = yval2/((ind1-1)*width_2);
figure;
bar(xval2,yval2);
hold on;
plot(xval2,yval2,'r');               %plotting the pdf
title("pdf of f_{R|S}(r|S=-A)");
```

Appendix V

```
%calculating E[R|S=A]
ER_SA = 0;
for i1 = 1:bin_1
    ER_SA = ER_SA + (xval1(i1)*yval1(i1)*width_1);
end
ER_SA

%calculating E[R|S=-A]
ER_SMA = 0;
for i2 = 1:bin_2
    ER_SMA = ER_SMA + (xval2(i2)*yval2(i2)*width_2);
end
ER_SMA

%calculating E[R]
[yval,xval] = hist(R,bins);
yval = yval/(L*width);

E_R = 0;
for i3 = 1:bin_n
    E_R = E_R + (xval(i3)*yval(i3)*width);
end
E_R

%plotting the pdf of f_R(r)
figure;
bar(xval,yval);
hold on;
plot(xval,yval,'r');
title("pdf of f_R(r)");
```

Appendix VI

```
%generating intterference
m_i = 0;
sigma_i = 1;
I = m_i + sigma_i*randn(1,L);

%generating the recieved signal
R = S + N + I;
```

Plotting the PDFs and calculating probabilities are as same as appendix IV and V.

Appendix VII

```
%amplifying factor
alpha = 2.5;

%generating the recieved signal and plotting it
R = alpha*S + N;
```

Plotting the PDFs and calculating probabilities are as same as appendix IV and V.