

**Department of Electronic & Telecommunication Engineering**

**University of Moratuwa**

**EN 2073 – Analog and Digital Communications**



## **Assignment - 01**

Name : Rathnayaka R.G.H.V.

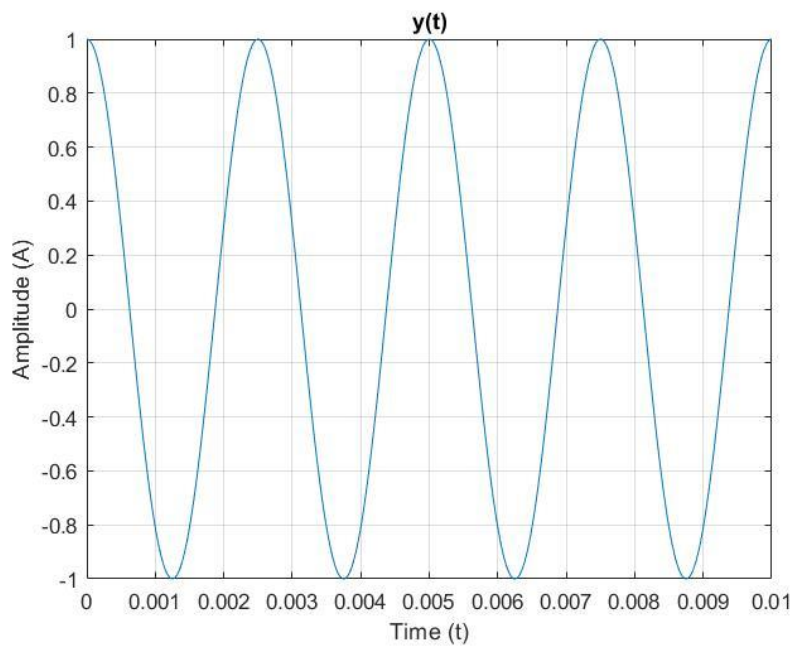
Index No : 180529E

**April 2, 2021**

This Report is submitted as a partial fulfillment of the requirement  
for the module EN2073 – Analog and Digital Communications

### 1. Plot of $y(t)$

The plot of the given  $y(t) = A * \cos(2 * \pi * f * t)$  signal for 10ms where  $f = 400$  Hz and  $A = 1$  (a.u) is given below in *Figure (01)*.



*Figure (01) – Plot of  $y(t)$*

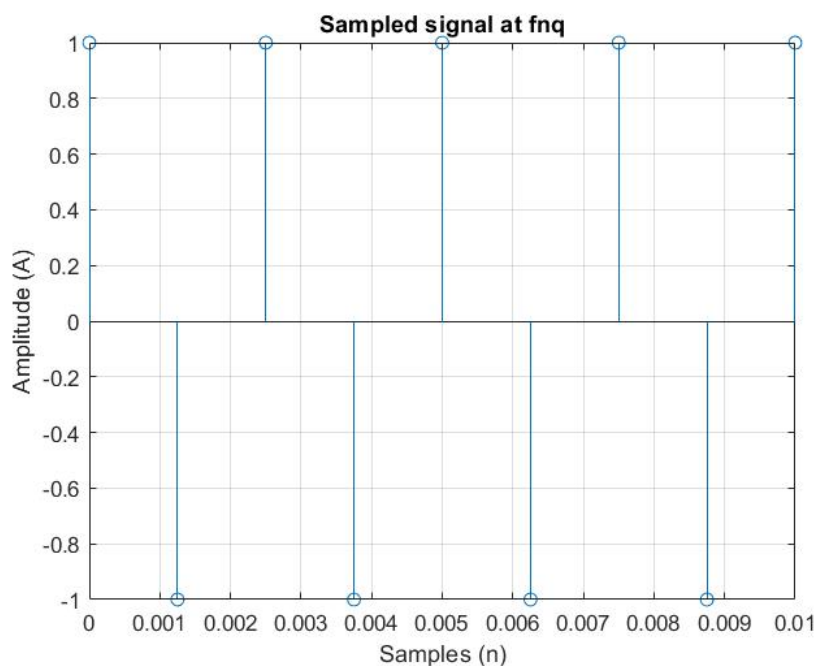
### 2. Nyquist sampling frequency ( $f_{nq}$ )

The Nyquist sampling frequency for the for the given signal can be easily calculated as it is twice of the frequency of the signal.

Therefore, Nyquist sampling frequency ( $f_{nq}$ ) =  $2 \times \text{frequency} = 2 \times 400 \text{ Hz} = \mathbf{800 \text{ Hz}}$

### 3. Ideal sampling the signal at Nyquist sampling rate ( $f_{nq}$ )

The plot after ideal sampling at the Nyquist sampling rate is given below in *Figure (02)*.



*Figure (2) – Sampled signal at  $f_{nq}$*

#### 4. Sampling the signal at 2 times Nyquist sampling rate & at one-half of Nyquist sampling rate

The plots for the sampled signals at Nyquist sampling rate, twice of the Nyquist sampling rate and one-half of the Nyquist sampling rate are depicted from the below Figure (03).

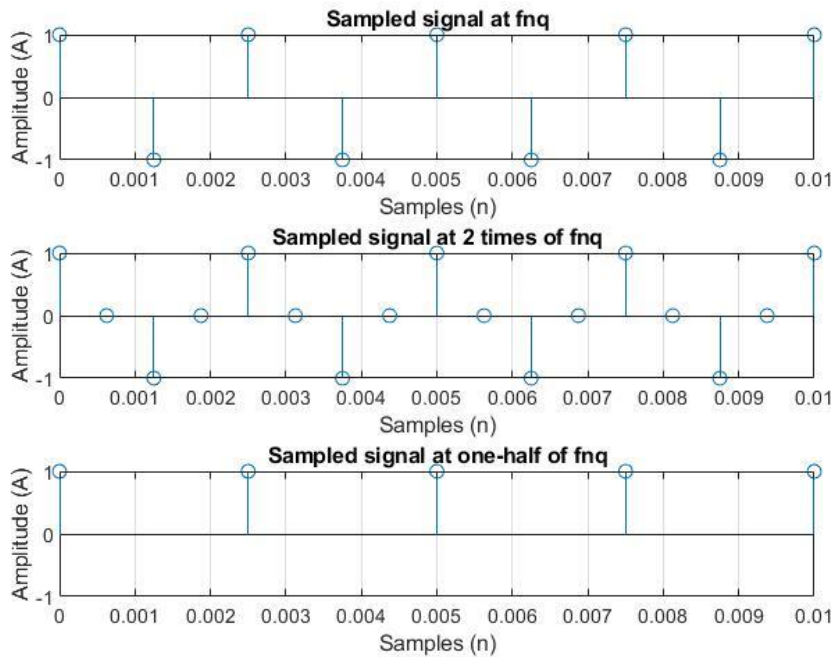


Figure (03) – Sampled signal at  $F_{nq}$ ,  $2F_{nq}$  &  $F_{nq}/2$

- The above three plots show the ideally sampled signal at the Nyquist rate, sampled signal at twice of Nyquist rate and the sampled at one-half of Nyquist rate.
- The ideally sampled signal, which was sampled at the Nyquist rate, shows only the minimum and maximum points. So, we can predict the original signal to some extent.
- The sampled signal at twice of the Nyquist rate shows mid values in addition to the minimum and the Maximum points represented by the ideally sampled signal. Although the mid values are zero, it will be advantageous in discovering the original signal from the sampled signal. This gives a high accuracy than the earlier situation.
- It is unable to discover the original signal from the sampled signal at one-half of the Nyquist rate as there are only the maximum points in the sampled signal.
- So, as a whole,
  - if the **frequency of the sampled signal ( $f_s$ ) = Nyquist sampling rate ( $f_{nq}$ )**, it is able to recover the original signal.
  - if the **frequency of the sampled signal ( $f_s$ ) > Nyquist sampling rate ( $f_{nq}$ )**, it is very easy to recover the original signal with a high accuracy.
  - if the **frequency of the sampled signal ( $f_s$ ) < Nyquist sampling rate ( $f_{nq}$ )**, it is unable to recover the original signal.

#### 5. The minimum number of bits ( $n_b$ ) required per a sample and number of minimum quantization levels ( $L$ ) required

$$\text{Signal Power} = \frac{A^2}{2} \quad Q \text{ Noise Power} = \frac{\Delta A^2}{12} \quad \text{since } \Delta A = \frac{2A}{(L-1)}, \quad Q \text{ Noise Power} = \frac{A^2}{3(L-1)^2}$$

$$SNqR = 10 \log \left( \frac{\text{Signal Power}}{Q \text{ Noise Power}} \right)$$

$$SNqR = 10 \log \left( \frac{\frac{A^2}{2}}{\frac{A^2}{3(L-1)^2}} \right) = 10 \log \left( \frac{3(L-1)^2}{2} \right)$$

To exist a  $SNqR$  ratio greater than 25dB,  
 $SNqR > 25\text{dB}$

$$10 \log \left( \frac{3(L-1)^2}{2} \right) > 25$$

$$\log \left( \frac{3(L-1)^2}{2} \right) > 2.5$$

$$\left( \frac{3(L-1)^2}{2} \right) > 10^{2.5}$$

$$L - 1 > 14.5196$$

$$L > 15.5196$$

$L$  is an integer.

Therefore, **the minimum number of quantization levels ( $L$ ) required is 16.**

As  $2^{nb} \geq L$ ,

$$nb \geq 4$$

**The minimum number of bits ( $nb$ ) required is 4.**

#### 6. MATLAB function to take a sampled value, number of quantization levels ( $L$ ) and range (maximum amplitude) and to output the quantized value

The MATLAB function which can output the quantized value by taking sample value, number of quantization levels and amplitude range is given below under *Figure (04)*.

**Defined function for calculating quantized Value**

```
function quantized_value = quantize(y,L,A)
    delta = 2*A/(L-1); % Gap between two quantization levels
    if y == A && (mod(y,delta) == 0)
        quantized_value = y - (mod(y,delta)) - delta/2;
    else
        quantized_value = y - (mod(y,delta)) + delta/2;
    end
end
```

*Figure (04) – Function for calculating quantized value.*

#### 7. Sampling the signal at 8 times Nyquist sampling frequency ( $8f_{nq}$ ) and quantization

The plots for the sampled signal at 8 times of Nyquist sampling frequency and the Quantized signal with 16 quantization levels is given below in the *Figure (05)*.

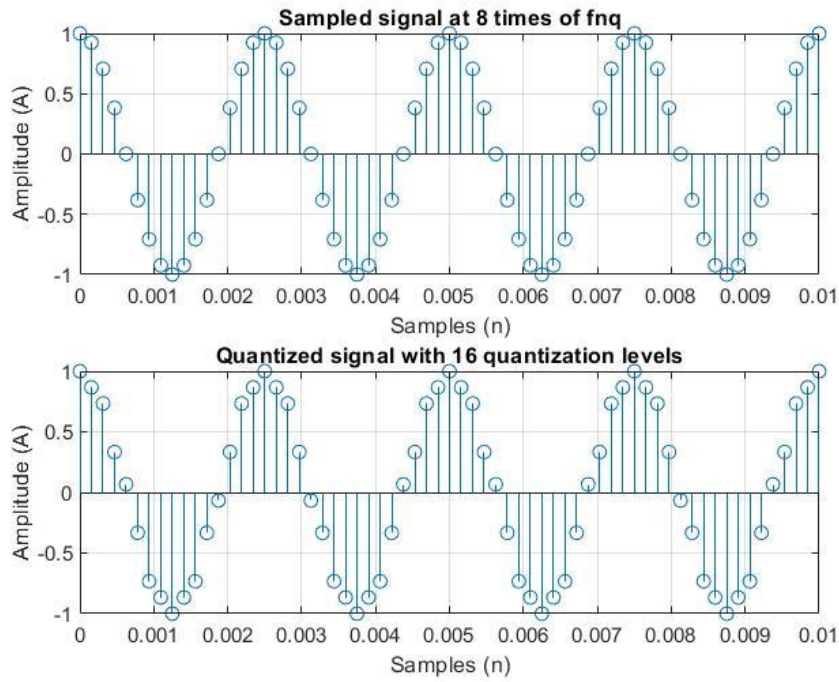


Figure (05) – Sampled signal at 8 times of Nyquist sampling frequency and Quantized signal with 16 quantization levels

## 8. Quantizing the above sampled signal for $L*2$ and $L/2$ quantization levels

The plots for the quantized signal with 16 quantization levels, 32 quantization levels and 8 quantization levels are given below under *Figure (06)*, *Figure (07)* and *Figure (08)*.

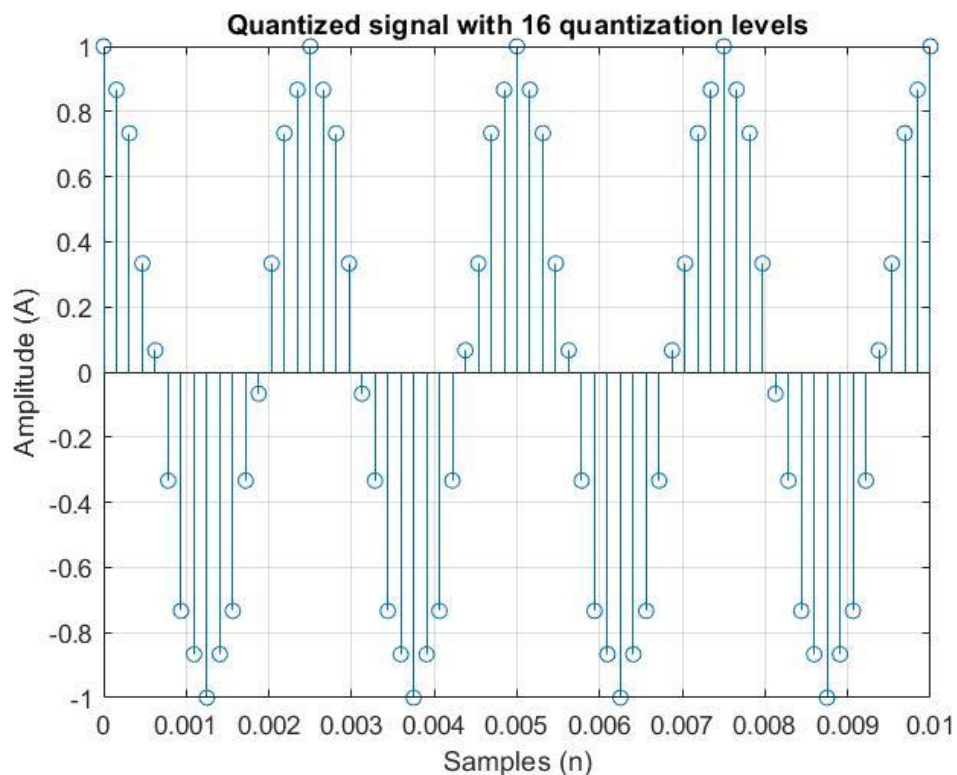


Figure (06) - quantized signal with 16 quantization levels

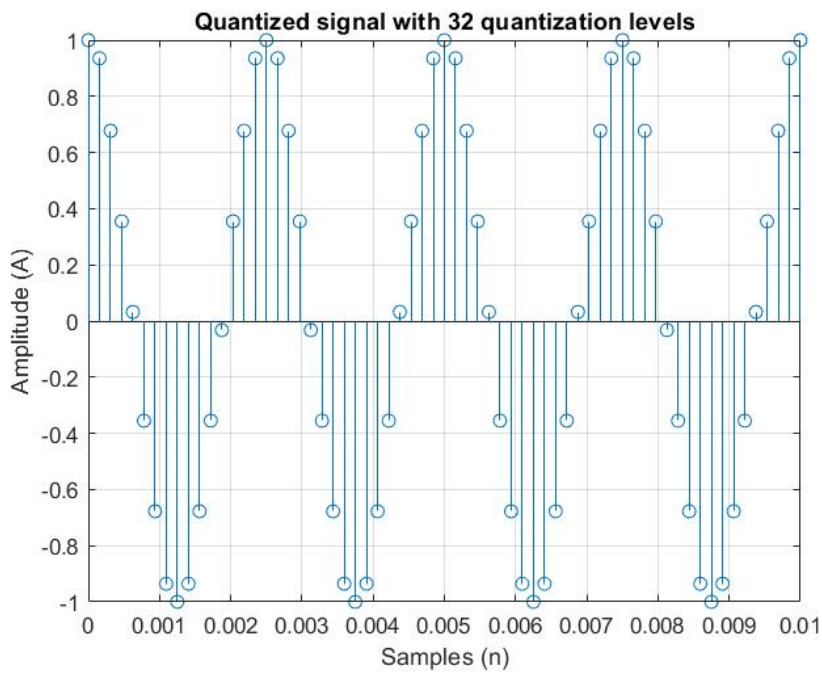


Figure (07) - quantized signal with 32 quantization levels

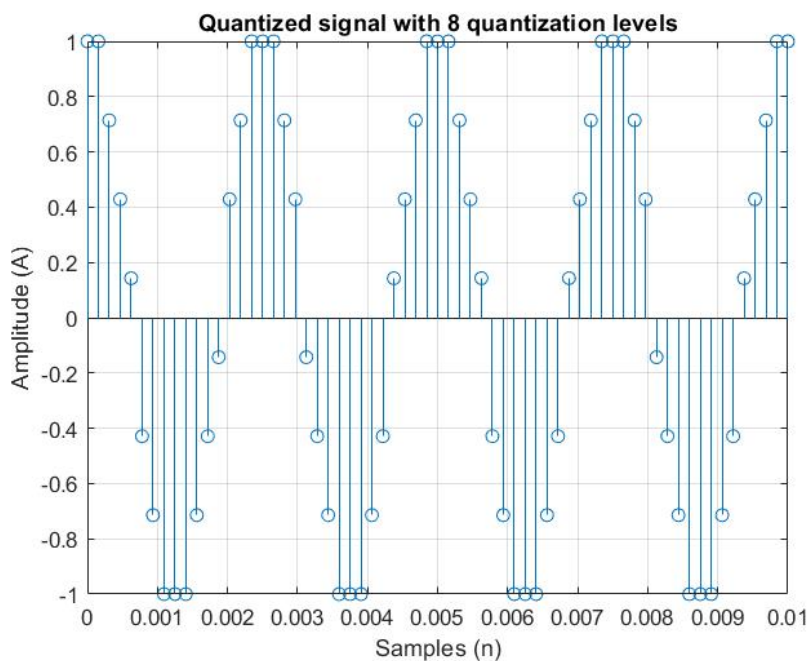


Figure (08) - quantized signal with 8 quantization levels

### Observations :

- ❖ With the decrease of the number of quantization levels accuracy of the signal get reduced. Number of bits needed in representation get reduces. As a result, complexity may reduce.
- ❖ With the increase of the number of quantization levels accuracy of the signal get increased. Number of bits needed in representation get increases. As a result, complexity may increase.
- ❖ It is better to select the optimum number of quantization levels which provide a high accuracy and a good  $SN_qR$ .

# EN2073 - Analog and Digital Communications

## Assignment 1

Name : Rathnayaka R.G.H.V.

Index Number : 180529E

### 1. Plot of $y(t)$

```
f = 400; % frequency (Hz)
A = 1; % Amplitude (a.u)
t_lim= 0.01; % Time limit (s)
t = linspace(0,t_lim,1000);
y_t = A*cos(2*pi*f*t); % Required function
figure;
plot(t,y_t); % Plot for y(t)
xlabel('Time (t)');
ylabel('Amplitude (A)');
title('y(t)');
grid on;
```

### 2. Nyquist sampling frequency (fnq)

```
f_nq = 2*f; % Nyquist sampling frequency
fprintf('Nyquist sampling frequency = %d Hz', f_nq);
```

Nyquist sampling frequency = 800 Hz

### 3. Ideal sampling the signal at Nyquist sampling rate (fnq)

```
f = 400; % frequency (Hz)
A = 1; % Amplitude (a.u)
t_lim= 0.01; % Time limit (s)
f_nq = 2*f; % Nyquist frequency
[n1,y_n1] = sampling(f_nq,t_lim,A,f); % Ideal sampling at Nyquist sampling rate
figure;
stem(n1,y_n1);
xlabel('Samples (n)');
ylabel('Amplitude (A)');
title('Sampled signal at fnq');
grid on;
```



## 4. Sampling the signal at 2 times Nyquist sampling rate & at one-half of Nyquist sampling rate

```
f_nq = 2*f; % Nyquist frequency
[n1,y_n1] = sampling(f_nq,t_lim,A,f); % Ideal sampling at Nyquist sampling rate
[n2,y_n2] = sampling(2*f_nq,t_lim,A,f); % Sampling at 2 times Nyquist sampling rate
[n3,y_n3] = sampling(0.5*f_nq,t_lim,A,f); % Sampling at one-half of Nyquist sampling rate
titles = ['Sampled signal at twice of fnq','Sampled signal at one-half of fnq'];
figure;
subplot(3,1,1);
stem(n1,y_n1); % Plotting the sampled signal at Nyquist sampling frequency
xlabel('Samples (n)');
ylabel('Amplitude (A)');
ylim([-A,+A]);
title('Sampled signal at fnq');
grid on;
subplot(3,1,2);
stem(n2,y_n2); % Plotting the sampled signal at 2 times Nyquist sampling frequency
xlabel('Samples (n)');
ylabel('Amplitude (A)');
ylim([-A,+A]);
title('Sampled signal at 2 times of fnq');
grid on;
subplot(3,1,3);
stem(n3,y_n3); % Plotting the sampled signal at one-half of Nyquist sampling frequency
xlabel('Samples (n)');
ylabel('Amplitude (A)');
ylim([-A,+A]);
title('Sampled signal at one-half of fnq');
grid on;
```

### Observations :

- The above three plots show the ideally sampled signal at the Nyquist rate, sampled signal at twice of Nyquist rate and the the sampled at one-half of Nyquist rate.
- The ideally sampled signal which was sampled at the Nyquist rate, shows only the minimum and maximum points. So, we can predict the original signal to some extent.
- The sampled signal at twice of the Nyquist rate shows mid values in addition to the minimum and the maximum points represented by the ideally sampled signal. Although the mid values are zero, it will be beneficial in discovering the original signal from the sampled signal. This give a high accuracy than the earlier situation.
- It is unable to discover the origiinal signal from the sampled signal at one-half of the Nyquist rate as there are only the maximum points in the sampled signal.
- **So, as a whole if the frequency of the sampled signal ( $f_s$ ) = Nyquist sampling rate ( $f_{nq}$ ), it is able to recover the original signal, if the frequency of the sampled signal ( $f_s$ ) > Nyquist**



sampling rate ( $f_{\text{ny}}$ ), it is very easy to recover the original signal with a high accuracy and if the frequency of the sampled signal ( $f_s$ ) < Nyquist sampling rate ( $f_{\text{ny}}$ ), it is unable to recover the original signal.

## 5. The minimum number of bits (nb) required per a sample and number of minimum quantization levels (L) required.

$$\text{Signal Power} = \frac{A^2}{2}$$

$$Q \text{ Noise Power} = \frac{\Delta A^2}{12}$$

$$\text{since } \Delta A = \frac{2A}{(L-1)}, \quad Q \text{ Noise Power} = \frac{A^2}{3(L-1)^2}$$

$$SN_{qR} = 10 \log \left( \frac{\text{Signal Power}}{Q \text{ Noise Power}} \right)$$

$$SN_{qR} = 10 \log \left( \frac{\frac{A^2}{2}}{\frac{A^2}{3(L-1)^2}} \right) = 10 \log \left( \frac{3(L-1)^2}{2} \right)$$

In order to exist a  $SN_{qR}$  ratio greater than 25dB,

$$SN_{qR} > 25\text{dB}$$

$$10 \log \left( \frac{3(L-1)^2}{2} \right) > 25$$

$$\log \left( \frac{3(L-1)^2}{2} \right) > 2.5$$

$$\left( \frac{3(L-1)^2}{2} \right) > 10^{2.5}$$

$$L - 1 > 14.5196$$

$$L > 15.5196$$

L is an interger.

Therefore, **the minimum number of quantization levels (L) required is 16.**

$$\text{As } 2^{\text{nb}} \geq L,$$

$$\text{nb} \geq 4$$

The minimum number of bits (nb) required is 4.

## 6. matlab function to take a sampled value, number of quantization levels (L) and range (maximum amplitude) and to output the quantized value

```
%function quantized_value = quantizing(y,L,A)
    %delta = 2*A/(L-1); % Gap between two quantization levels
    %if y == A & (mod(y,delta) == 0)
        %quantized_value = y - (mod(y,delta)) - delta/2;
    %else
        %quantized_value = y - (mod(y,delta)) + delta/2;
    %end
%end
```

Function has made commented to keep the continuity of the code. It is added to function at the end. This is the answer for question 6.

## 7. Sampling the signal at 8 times Nyquist sampling frequency (8f<sub>nq</sub>) and quantization

```
L = 16; % Number of Quantization levels
[n4,y_n4] = sampling(8*f_nq,t_lim,A,f); % Sampled signal at 8 times of Nyquist sampling frequency
num = size(n4,2); % Number of samples in the sampled signal
y_q1 = quantizing(num,y_n4,L,A); % Quantized signal with L number of quantization levels
figure;
subplot(2,1,1);
stem(n4,y_n4); % Plotting the sampled signal at 8 times of Nyquist sampling frequency
xlabel('Samples (n)');
ylabel('Amplitude (A)');
ylim([-A,+A]);
title('Sampled signal at 8 times of fnq');
grid on;
subplot(2,1,2);
stem(n4,y_q1); % Plotting the quantized signal with L number of quantization levels
xlabel('Samples (n)');
ylabel('Amplitude (A)');
ylim([-A,+A]);
title('Quantized signal with 16 quantization levels');
grid on;
```

## 8. Quantizing the above sampled signal for $L*2$ and $L/2$ quantization levels

```
y_q2 = quantizing(num,y_n4,2*L,A); % Quantized signal with 2L number of quantization levels
y_q3 = quantizing(num,y_n4,round(0.5*L),A); % Quantized signal with L/2 number of quantization levels
figure;
stem(n4,y_q1); % Plotting the quantized signal with L number of quantization levels
xlabel('Samples (n)');
ylabel('Amplitude (A)');
ylim([-A,+A]);
title('Quantized signal with 16 quantization levels');
grid on;
```

```
figure;
stem(n4,y_q2); % Plotting the quantized signal with 2L number of quantization levels
xlabel('Samples (n)');
ylabel('Amplitude (A)');
ylim([-A,+A]);
title('Quantized signal with 32 quantization levels');
grid on;
```

```
figure;
stem(n4,y_q3); % Plotting the quantized signal with L/2 number of quantization levels
xlabel('Samples (n)');
ylabel('Amplitude (A)');
ylim([-A,+A]);
title('Quantized signal with 8 quantization levels');
grid on;
```

### Defined function for calculating sampled signal

```
function [n,y_s] = sampling(f_s,t_lim,A,f)
    T_s = 1/f_s; % Calculating the period
    n = 0:T_s:t_lim; % Calculating the number of periods
    y_s = A*cos(2*pi*f*n); % Calculating sampled function
end
```

### Defined function for calculating quantized Value

```
function quantized_value = quantize(y,L,A)
    delta = 2*A/(L-1); % Gap between two quantization levels
    if y == A && (mod(y,delta) == 0)
```

```
        quantized_value = y - (mod(y,delta)) - delta/2;
    else
        quantized_value = y - (mod(y,delta)) + delta/2;
    end
end
```

### Defined function for calculating quantized signal

```
function y_q = quantizing(num,y,L,A)
    y_q = zeros(1,num); % Zero array is created in order to store the outputs
    i = 1;
    for k = y
        y_q(1,i) = quantize(k,L,A); % Calculating quantized values
        i = i + 1;
    end
end
```