EN 2570 - Digital Signal Processing

Assignment 01

Index No : 180529E

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Q1

a) A signal $x[n]$ is periodic if $x[n+N] = x[n]$ $\forall n$ where $n \in \mathbb{Z}^+$

if $x[n] = A\cos[\omega_0 n]$

then $x[n+N] = A\cos[\omega_0 n + \omega_0 N]$

In order to be $x[n+N] = x[n]$

$$\omega_0 N = 2\pi k \; ; \; k \in \mathbb{Z}$$

$\omega_0 = \pi/8$,

Since $N = \frac{2\pi k}{\omega_0}$

$$N = \frac{2\pi k}{\pi/8} = 16k \; ; \; k \in \mathbb{Z}$$

$$N = 16 \quad (\text{for } k = 1)$$

$\therefore$ $x[n]$ is periodic with period 16

b) $\omega_0 = 3\pi/4$

Since $N = \frac{2\pi k}{\omega_0}$ (From a)

$$N = \frac{2\pi k}{3\pi/4} = \frac{8k}{3}$$

$$N = 8 \quad (\text{for } k = 3)$$

$\therefore$ $x[n]$ is periodic with period 8

c) $\omega_0 = 1/2$

Since $N = \frac{2\pi k}{\omega_0}$ (From a)

$$N = \frac{2\pi k}{1/2} = 4\pi k$$

There is no $k \in \mathbb{Z}$ such that $N \in \mathbb{Z}^+$

$\therefore$ $x[n]$ is not periodic.

d) $(\omega_0)_1 = \pi/8$

Since $N_1 = \frac{2\pi k_1}{\omega_0}$

$$N_1 = \frac{2\pi k_1}{\pi/8} = 16k_1$$

$(\omega_0)_2 = 3\pi/4$

Since $N_1 = \frac{2\pi k_2}{\omega_0}$

$$N_1 = \frac{2\pi k_2}{3\pi/4} = \frac{8k_2}{3}$$

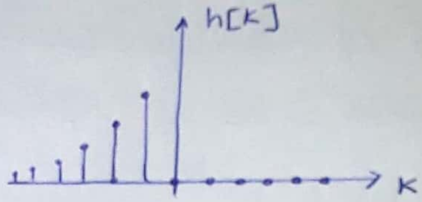
When $k_1 = 1$ & $k_2 = 6$

$N_1 = N_2 = 16$

$\therefore$ $x[n]$ is periodic with period 16

$\boxed{Q2}$

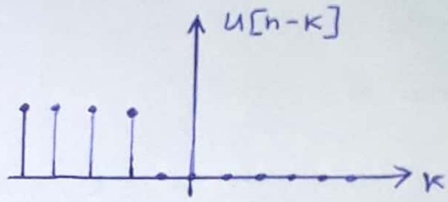
a) $h[n] = a^n u[-n-1]$ $\quad x[n] = u[n]$



$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, u[n-k]$$

$$= \sum_{k=-\infty}^{\infty} a^k u[-k-1]\, u[n-k]$$



for $n \leq -1$

$$y[n] = \sum_{k=-\infty}^{n} a^k = \sum_{k=-n}^{\infty} a^{-k}$$

$$= \frac{a^n}{1-a^{-1}} \quad (\because a^{-1} < 1)$$

$$= \frac{a^{n+1}}{a-1} \;//$$

for $n > -1$

$$y[n] = \sum_{k=-\infty}^{-1} a^k = \sum_{k=1}^{\infty} a^{-k}$$

$$= \frac{a^{-1}}{1-a^{-1}} \quad (\because a^{-1} < 1)$$

$$= \frac{1}{a-1} \;//$$

$$\therefore \; y[n] = \begin{cases} \dfrac{a^{n+1}}{a-1} & ; \; n \leq -1 \\[2ex] \dfrac{1}{a-1} & ; \; n > -1 \end{cases} \;//$$

b) i. $h[n] = 3^n u[-n-1]$ $\quad x[n] = u[n-3]$

$\boxed{a = 3}$

Considering time-invariant property

$$y_1[n] = h[n] * u[n-3] = y[n-3]$$

$$y_1[n] = \begin{cases} \dfrac{3^{n-2}}{2} & ; \; n \leq 2 \\[2ex] \dfrac{1}{2} & ; \; n > 2 \end{cases} \;//$$

ii. $h[n] = 2^n u[-n-3]$ $\quad x[n] = u[n] - u[n-1]$

$x[n] = \delta[n]$

$y_2[n] = h[n] * \delta[n]$

$y_2[n] = h[n]$

$y_2[n] = 2^n u[-n-3]$ //

$\boxed{Q3}$ $h[n] = a^n u[n] + b^n u[-n-1]$ $\quad a, b \in \mathbb{C}$

In order to have a stable system,

$$\sum_{n=-\alpha}^{\alpha} |h[n]| < \alpha$$

Let take $\sum_{n=-\alpha}^{\alpha} |h[n]| = B_n$

$$B_n = \sum_{n=-\alpha}^{\alpha} |a^n u[n] + b^n u[-n-1]$$

$$B_n = \sum_{n=-\alpha}^{-1} |b^n| + \sum_{n=0}^{\alpha} |a^n|$$

AS $b = |b| e^{j\theta}$

$b^n = |b|^n e^{jn\theta}$

$|b^n| = |b|^n$

So, $|a^n| = |a|^n$

$$\therefore B_n = \underbrace{\sum_{n=-\alpha}^{-1} |b|^n}_{G_1} + \underbrace{\sum_{n=0}^{\alpha} |a|^n}_{G_2}$$

So both G_1 & G_2 are geometric series. G_1 & G_2 gives finite values if

$|b| > 1$ and $|a| < 1$ //

$\boxed{Q4}$ $x_1(t) = 2\cos(40\pi t)$ $\quad x_2(t) = 2\cos(70\pi t)$

a) $\Omega_s = 100\pi \text{ rads}^{-1}$

$$\omega = \Omega T = \frac{2\pi \Omega}{\Omega_s}$$

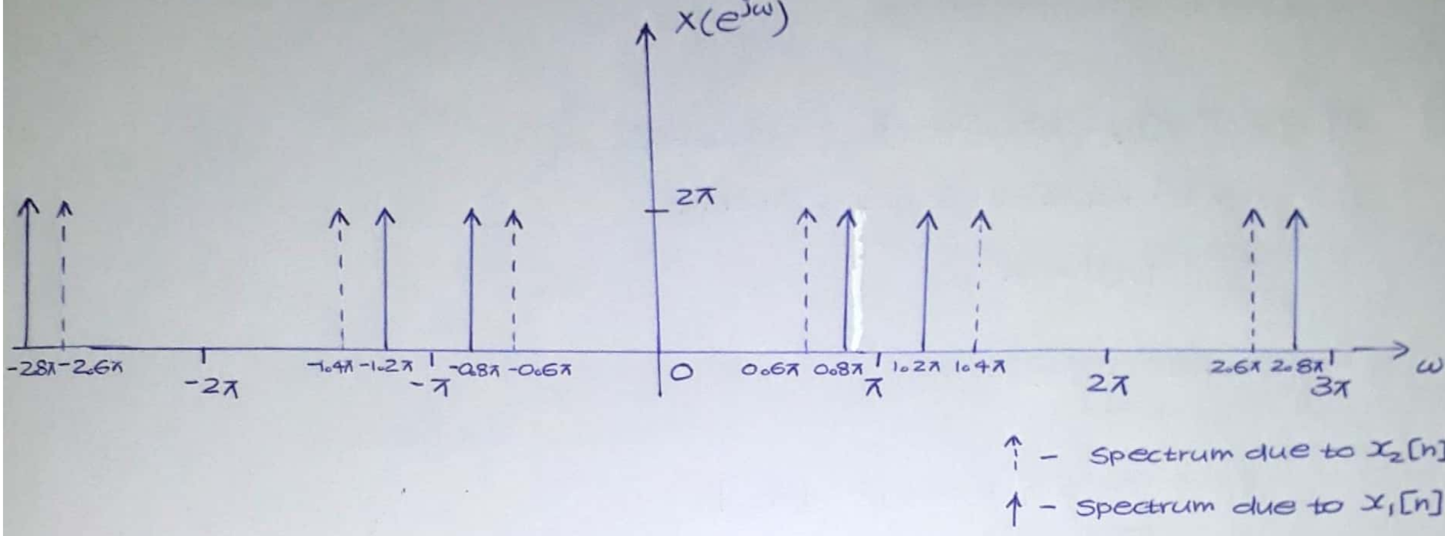
$x_1(t) \rightarrow \omega_1 = \dfrac{2\pi(40\pi)}{100\pi}$ $\quad x_2(t) \rightarrow \omega_2 = \dfrac{2\pi(70\pi)}{100\pi}$

$\omega_1 = 0.8\pi$ rad/sample $\quad \omega_2 = 1.4\pi$ rad/sample

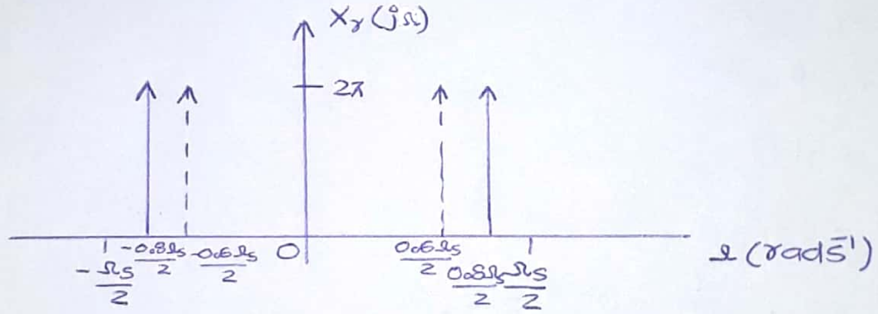
b) $x[n] = x_1[n] + x_2[n]$

$= 2\cos(0.8\pi n) + 2\cos(1.4\pi n)$

Since $\cos(\omega_0 n) \Leftrightarrow \sum_{k=-\infty}^{\infty} [\pi\,\delta(\omega-\omega_0+2\pi k) + \pi\,\delta(\omega+\omega_0+2\pi k)]$



c) When we are using an ideal reconstruction filter, the spectrum of the reconstructed signal is,



$x_r(t)$ should have two cosines.

$$\Omega_1 = \frac{0.8\,\Omega_s}{2} = \frac{0.8 \times (100\pi)}{2} = 40\pi \text{ rads}^{-1}$$

$$\Omega_2 = \frac{0.6\,\Omega_s}{2} = \frac{0.6 \times (100\pi)}{2} = 30\pi \text{ rads}^{-1}$$