FIR Filter Design

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Abstract

This report deals with the design and analysis of a FIR (Finite extended Impulse Response) band pass filter in direct closed form method. For simplicity, windowing method is used to generate the impulse response. Furthermore, for an optimum high quality solution, Kaiser Window is used since it is more flexible and it contains more adjustable parameters than other windowing methods. In this project, the Kaiser Window is generated manually using the MATLAB software without using in built functions for the proper understanding of the theory.

Keywords: FIR filter, Kaiser Window, Fourier Transform

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1 Intoduction

Filters play a major role in designing signal processing systems. Between analog filters and digital filters, digital filters are superior to the analog filters in many aspects.

When designing, two types of digital filters can be observed based on their impulse response.

- FIR filters Finite extended Impulse Response
- IIR filters Infinite extended Impulse Response

1.1 FIR filters

System function of the FIR filter is given by,

$$H(z) = \sum_{n=0}^{N-1} h[n]Z^{-n}$$

1.1.1 Kaiser Window

There are two methods to design FIR digital filters,

- Windowing method
- Weighted Chebyshev method

Detailed descriptions of the two methods are available in [1]. Amongst these two methods, the windowing method is simpler than optimizing method.

Windowing method is used in conjunction with the Fourier Transform. In this method, the infinite length of the Fourier Transformed transfer function is truncated using a window function. Text [1] includes several different window functions. Among all the window functions, Kaiser Window contains adjustable main lobe width and a ripple ratio, resulting high quality filter transfer function which fulfills all the specifications.

2 Basic Theory

2.1 Specifications

Parameter	Value
Maximum passband ripple, $\widetilde{A}_p$	0.12 dB
Minimum stopband attenuation, $\widetilde{A}_a$	54dB
Lower passband edge, ω_{p1}	1000rad/sec
Upper passband edge, ω_{p2}	1300rad/sec
Lower stopband edge, ω_{a1}	900rad/sec
Upper stopband edge, ω_{a2}	1450rad/sec
Sampling frequency, ω_s	3400rad/sec

Table 1: Design Specifications

2.2 Computations

2.2.1 Compute the Fourier Transform of the ideal band pass filter

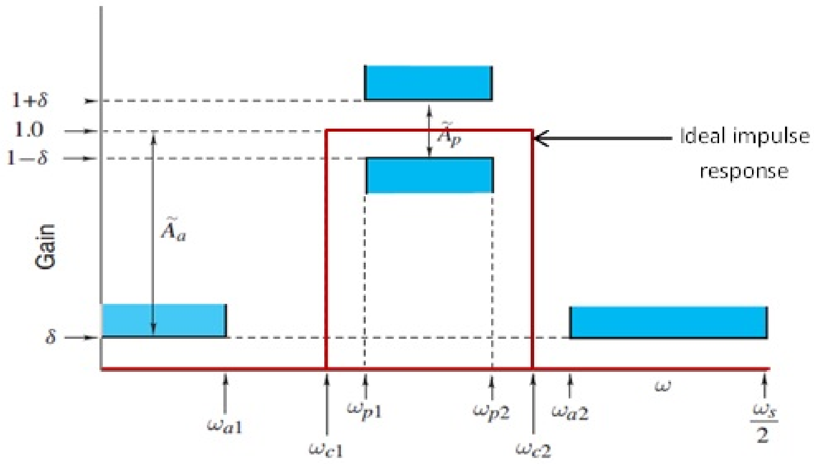


Figure 1: Impulse response of a band pass filter

$$H(e^{j\omega t}) = \begin{cases} 1, & \text{for } -\omega_{c2} \geq \omega \geq -\omega_{c1} \\ 1, & \text{for } \omega_{c1} \geq \omega \geq \omega_{c2} \\ 0, & \text{otherwise} \end{cases}$$

$$h(nT) = \frac{1}{\omega_s} \int_{-\omega_s}^{\omega_s} H(e^{j\omega t})dx$$

$$h(nT) = \begin{cases} \frac{1}{\omega_s}(\omega_{c2} - \omega_{c1}), & \text{for } n = 0 \\ \frac{1}{n\pi}(sin(\omega_{c2}nT) - sin(\omega_{c1}nT)), & \text{otherwise} \end{cases}$$

2.2.2 Compute the Kaiser Window

1 Find the pass band ripple and stop band ripple.

$$\widetilde{\delta}_p = \frac{10^{0.05\widetilde{A}_p}-1}{10^{0.05\widetilde{A}_p}+1}$$

$$\widetilde{\delta}_a = 10^{-0.05\widetilde{A}_a}$$

$$\delta = min(\widetilde{\delta}_p, \widetilde{\delta}_a)$$

$$\delta = 0.002dB$$

2 Calculate the actual stop band attenuation with δ

$$A_a = -20log\delta$$

$$A_a = 54dB$$

3 Choose the α value
Since $A_a = 54dB$

$$\alpha = 0.1102(A_a - 8.7)$$

$$\alpha = 4.9921$$

4 Choose D
Since $A_a > 21$

$$D = \frac{A_a - 7.95}{14.36}$$

$$D = 3.2068$$

5 Find transition band width and cut off frequencies
Transition bandwidth,

$$B_t = min\{(\omega_{p1} - \omega_{a1}), (\omega_{a2} - \omega_{p2})\}$$

$$B_t = 100rad/s$$

$$\omega_{c1} = \omega_{p1} - \frac{B_t}{2}$$

$$\omega_{c2} = \omega_{p2} + \frac{B_t}{2}$$

$$\omega_{c1} = 950rad/s$$

$$\omega_{c1} = 1350rad/s$$

6 Find the order of the filter

$$N \geq \frac{\omega_s D}{B_t} + 1$$

$$N \geq 111$$

7 Calculate the Kaiser window function

$$w_K(nT) = \begin{cases} \frac{I_0(\beta)}{I_0(\alpha)}, & \text{for } |n| \geq \frac{N-1}{2} \\ 0, & \text{otherwise} \end{cases}$$

Where $\beta = \alpha\sqrt{1 - (\frac{2n}{N-1})^2}$
$I_0(x)$ is the Bessel function of first kind

8 Compute the impulse response of the filter

$$h_w(nT) = w_K(nT)h(nT)$$

3 Results

3.1 Impulse Response

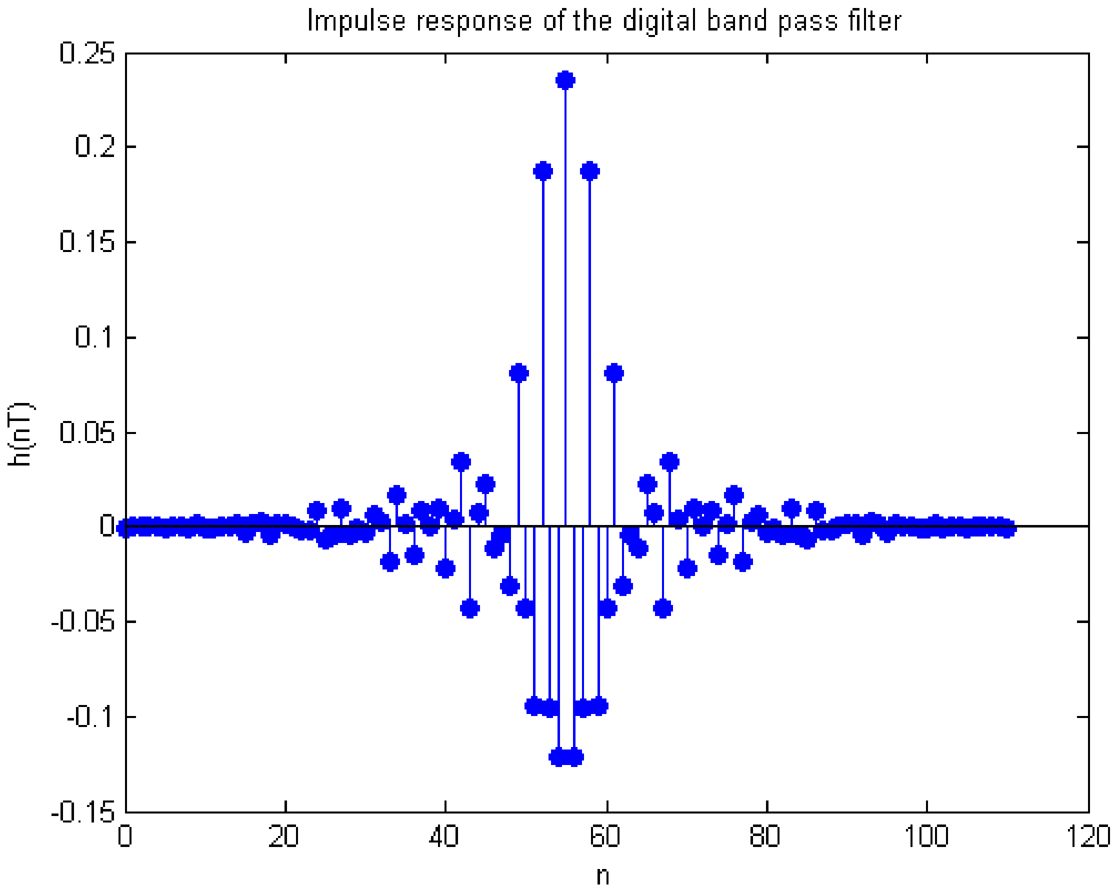


Figure 2: Impulse response in time domain

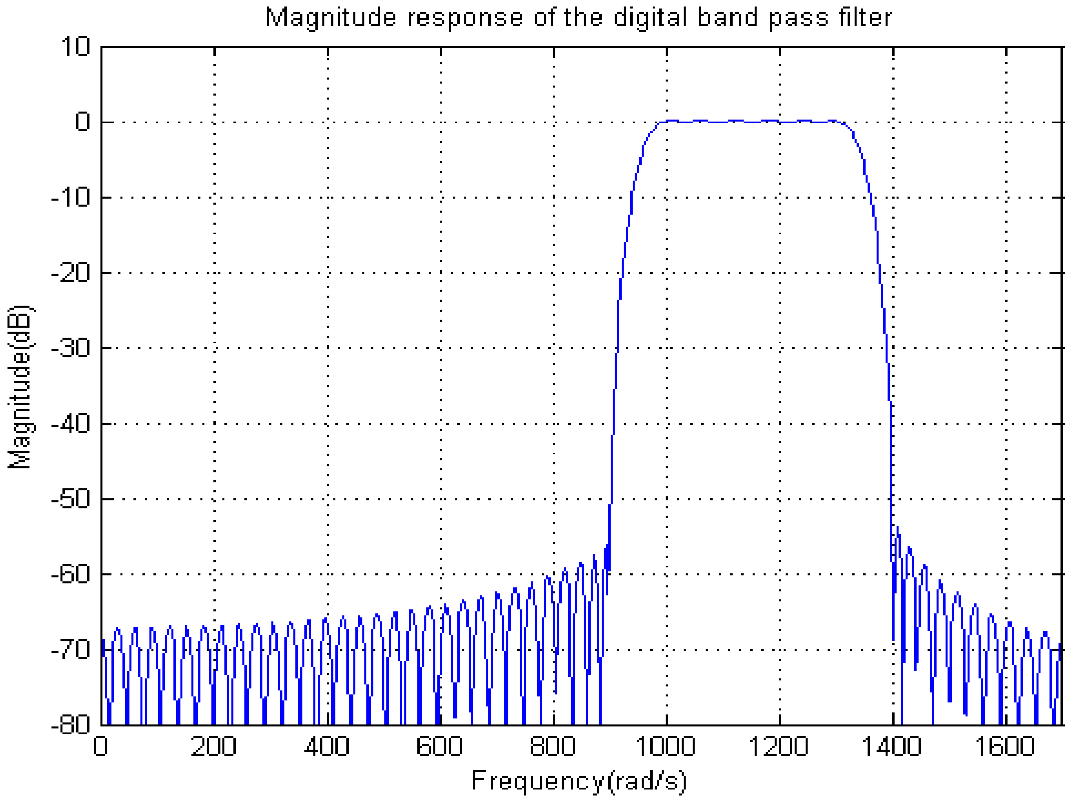


Figure 3: Frequency spectrum of the filter

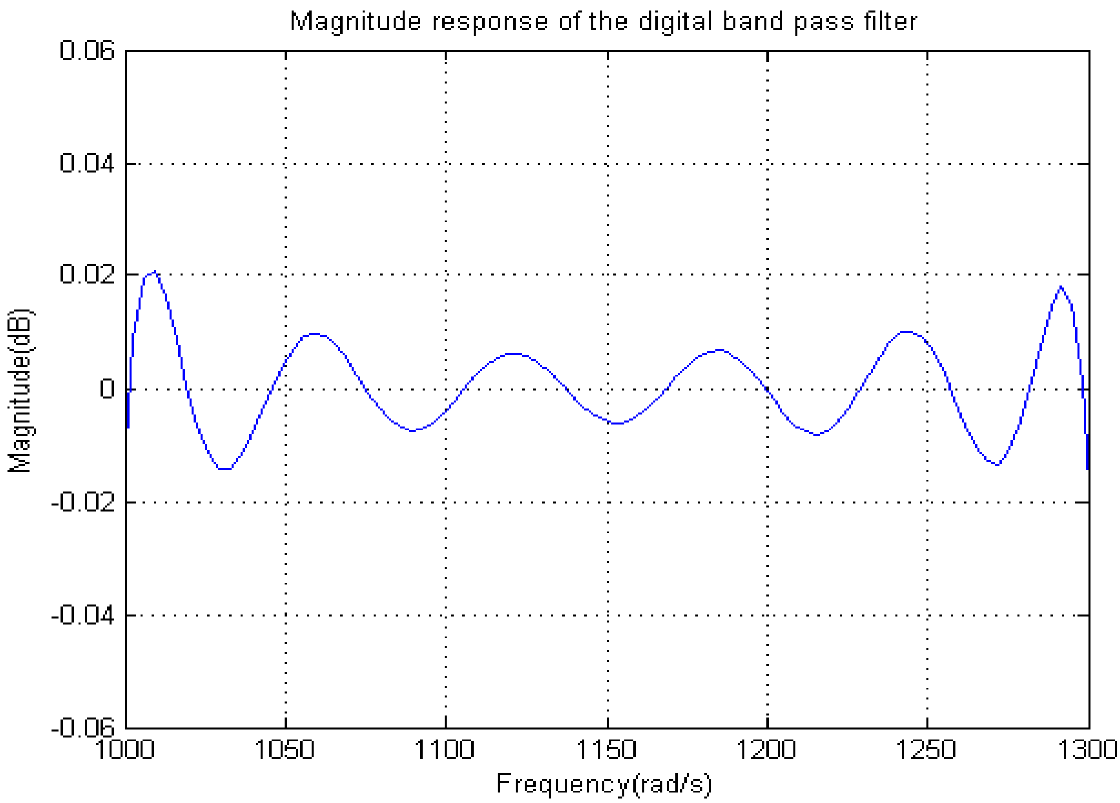


Figure 4: Magnitude of the pass band

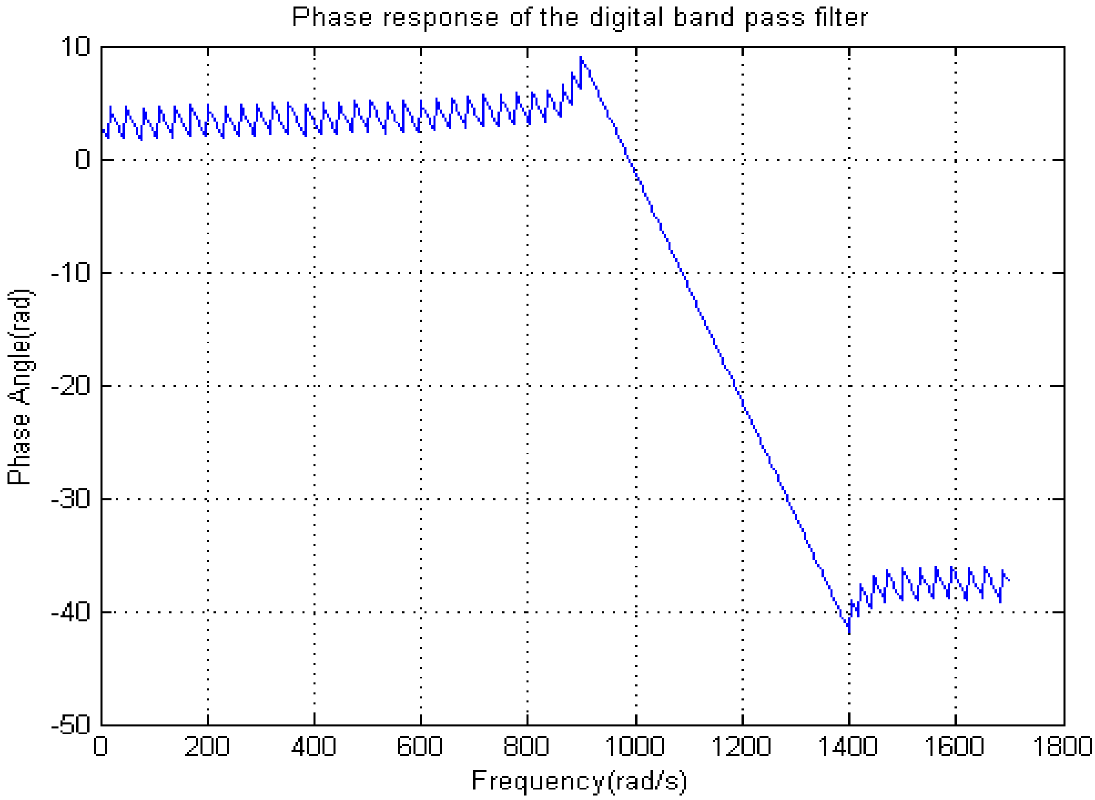


Figure 5: Phase response of the filter

Compute the step response of the filter

$$h_U(nT) = h_w(nT) * U(nT)$$

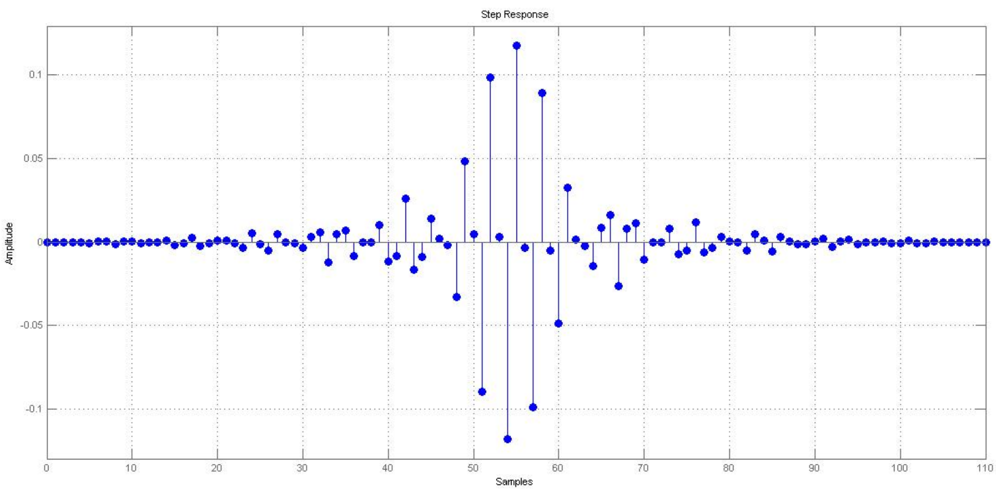


Figure 6: Step response

3.2 Implementation of the filter

Input to the filter

$$x(nT) = sin(\omega_1 nT) + sin(\omega_2 nT) + sin(\omega_3 nT)$$

$$\text{Where } \omega_1 = \frac{\omega_{a1}}{2}$$

$$\omega_1 = 450rad/s$$

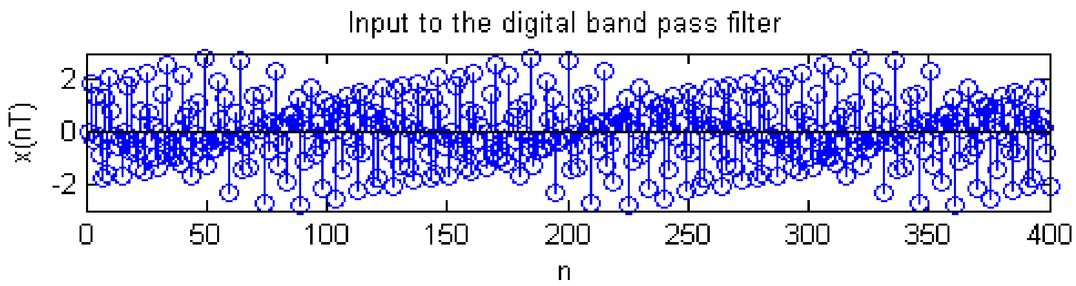
$$\omega_2 = \frac{\omega_{p1} + \omega p2}{2}$$

$$\omega_2 = 1150rad/s$$

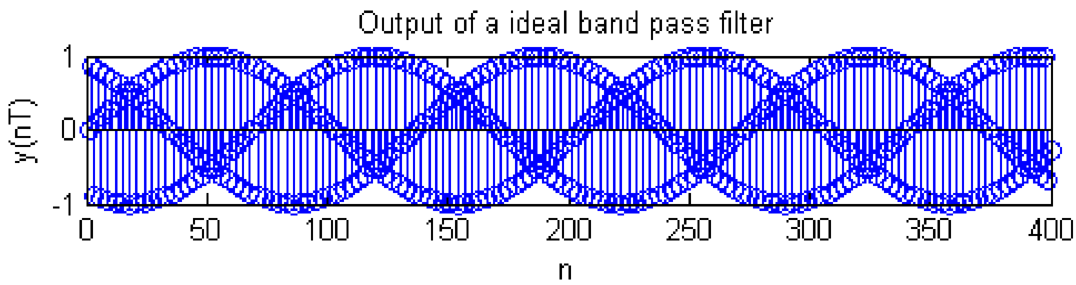
$$\omega_3 = \frac{\omega_{a2} + \omega_s}{2}$$

$$\omega_3 = 1575rad/s$$

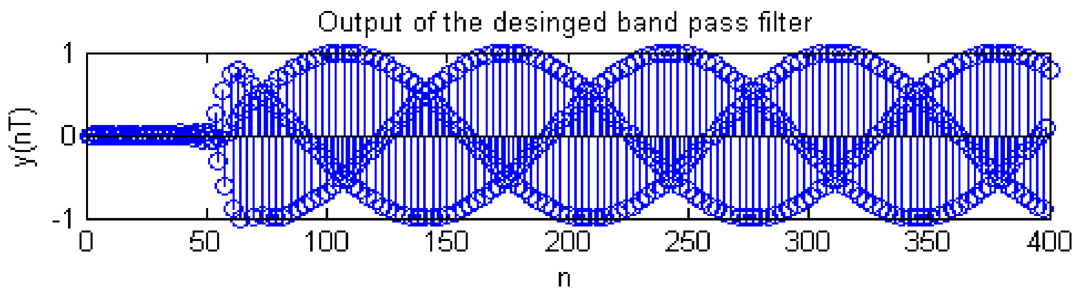
Designed filter should be able to filter out all the signals in the stop band



(a) Input signal in time domain



(b) Output from the ideal filter



(c) Output of the designed filter

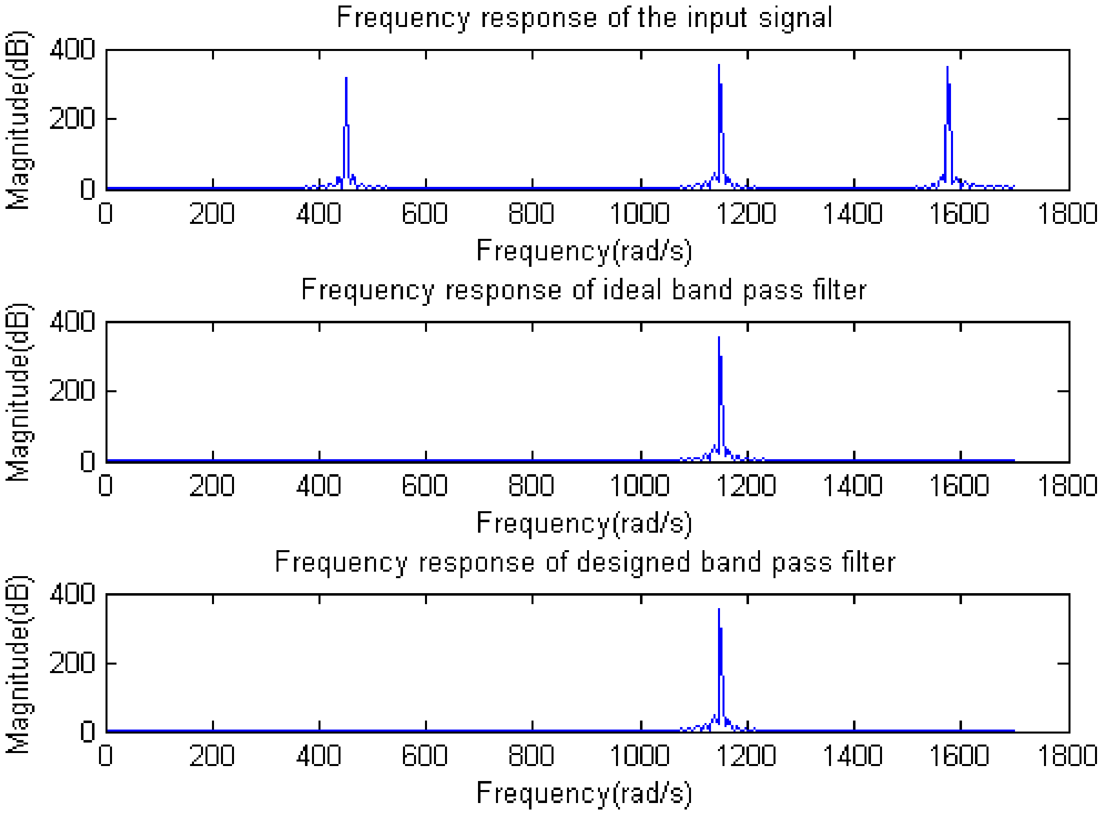


Figure 8: Frequency responses of the above signals

4 Analysis of results

4.1 Impulse response

- Order of the band pass filter is 110.
- As shown in Figure2, Impulse response is symmetric about n=0.
- Figure4 shows a reduction of the ripples in the pass band from 0.12dB to 0.02dB due to the choice of the δ. This ripple ratio overly satisfies the specification requirements.
- Stop band ripples are not uniform as in Dolph-Chebyshev window method.
- Ripples closer to the pass band have 54dB attenuation.
- As illustrated in Figure5, Phase response is linear in the pass band.
- Relationship between phase response and magnitude response is clearly shown in the Figure9 below.

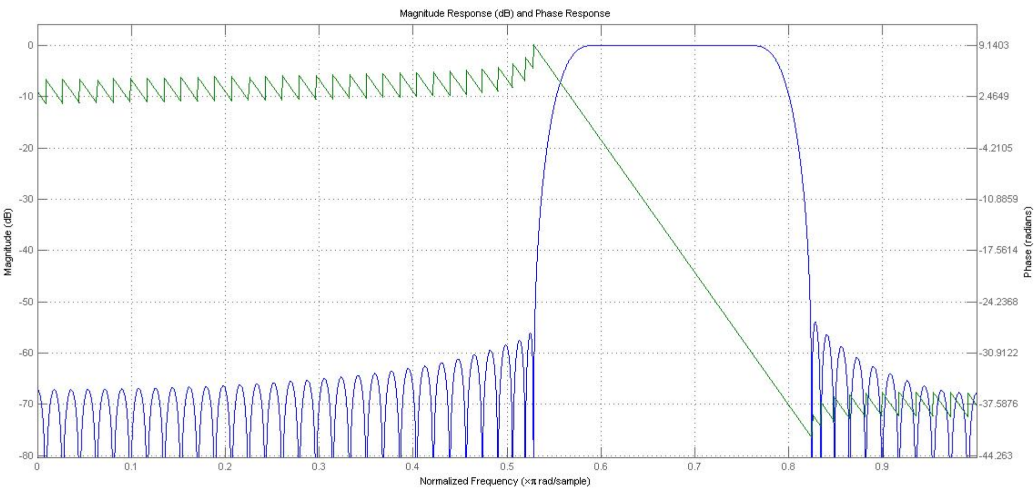


Figure 9: Phase and Magnitude responses

- This linear phase response is result of the constant phase delay and the group delay as shown in Figure10 and Figure11

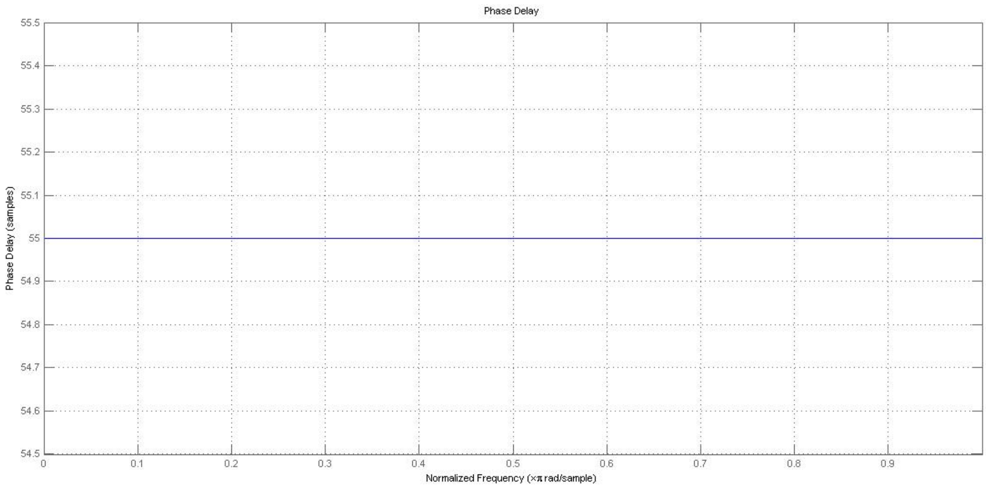


Figure 10: Phase delay

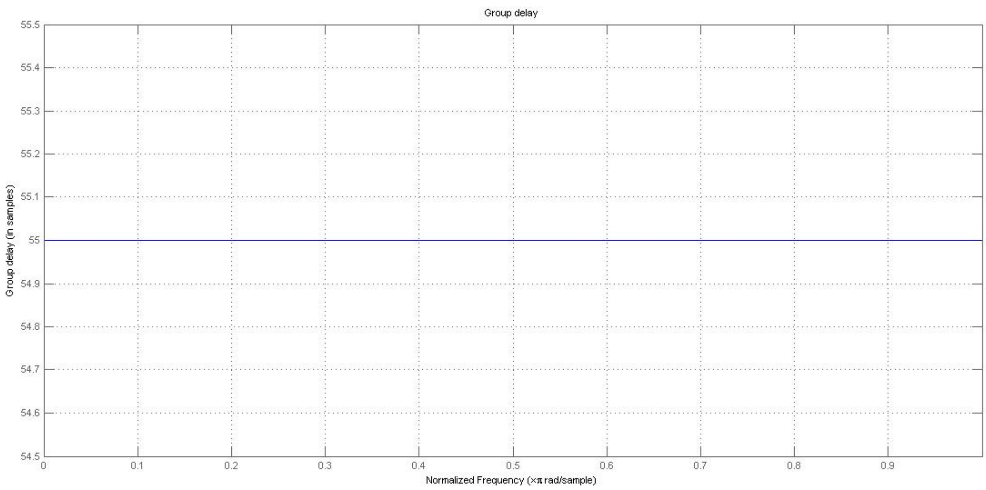


Figure 11: Group delay

- For the symmetrisity of the impulse response, poles and zeros has restrictions.
- Figure12 below shows the pole/zero plot of the designed filter which satisfies all the restrictions for a symmetric impulse response and a stable system.
  Which results,
  $$N(Z^-1) = \pm N(Z)$$
  Where
  $$N(Z) = \sum_{k=0}^{\frac{(N-1)}{2}} 2h[(\frac{N-1}{2} - k)T](Z^k \pm Z^-k)$$
- 110 (ie.(N-1)) poles located at the origin ensuring stability of the system.

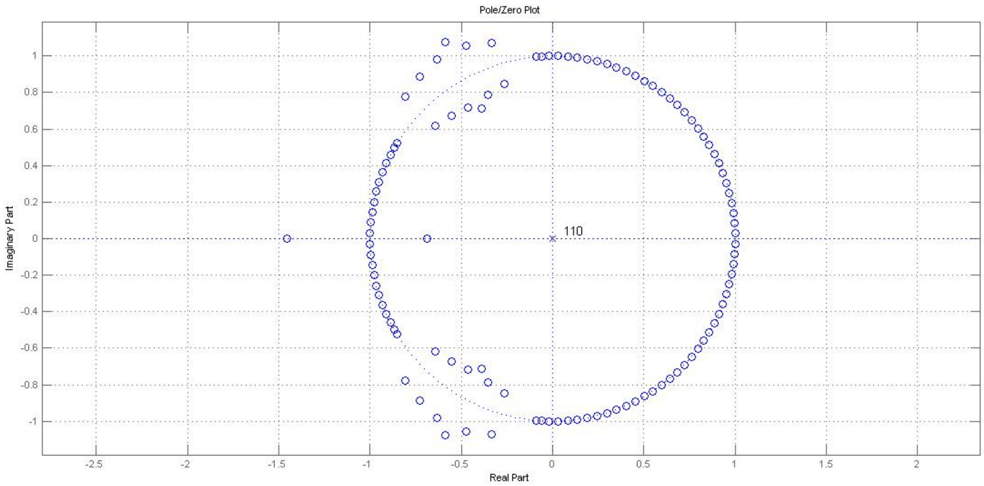


Figure 12: Pole/Zero plot

4.2 Implementation of the filter

- When the $x(nT)$, shown in the Figure7a gives to the filter, $sin(\omega_1 nT)$ and $sin(\omega_3 nT)$ suppress by the filter.
- Figure7c is very similar to the ideal output shown in Figure7b.
- Initial response is deviated due to the transient response of the designed filter.
- For the frequency response, same results are observed and are shown in the Figure8.

5 Realization of the filter

When the filter is generated as a digital filter network, the required number of adders and multipliers are shown in Figure13. As the order of the filter increases, the number of adders and multipliers that are needed for realization increases. Also computations for a sample is at a much higher rate.

```
Discrete-Time FIR Filter (real)
-------------------------------
Filter Structure    : Direct-Form FIR
Filter Length       : 111
Stable              : Yes
Linear Phase        : Yes (Type 1)

Implementation Cost
Number of Multipliers             : 107
Number of Adders                  : 106
Number of States                  : 110
Multiplications per Input Sample  : 107
Additions per Input Sample        : 106
```

Figure 13: Filter realization information

6 Study of arithmetic errors

Effect of the quantization errors occurring due to truncating and round off is computed in order to identify whether the noise is violate the specifications. Noise power for pass band and stop band is shown in Figure14 below.

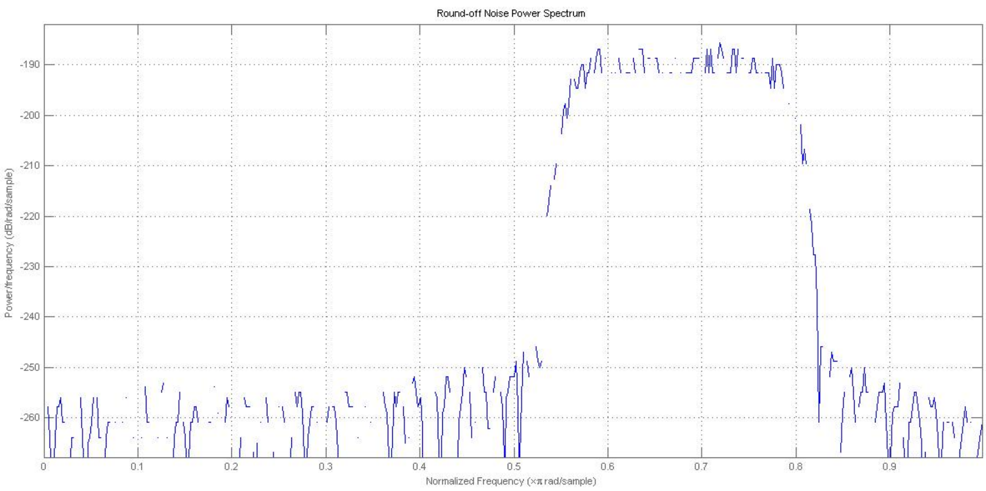


Figure 14: Noise power spectrum

Since magnitude of the noise power is below -190dB the effect of the quantization errors on the performance of the filter can be neglected.

7 Conclusion

This report contains the manual design and implementation of the digital FIR band pass filter using the Kaiser Window method. Since parameter values such as main lobe width, transition bandwidth and ripple ratio can be adjusted in the Kaiser window, it provides a high quality filter which satisfies all the specification requirements and meets all the aspects of the FIR filters. Even though designing a FIR filter is much simpler, realization requires more adders and multipliers which results in an expensive design, as observed in Figure13. Furthermore, for software implementation, it requires higher rate of computations per sample.

Due to the simplicity and flexibility as in the Kaiser Window, windowing method is a quality way to design an FIR filter.

8 Reference

1. A.Antoniou, *Digital Signal Processing : Signals, Systems and Filters*, McGraw-Hill, 2006.