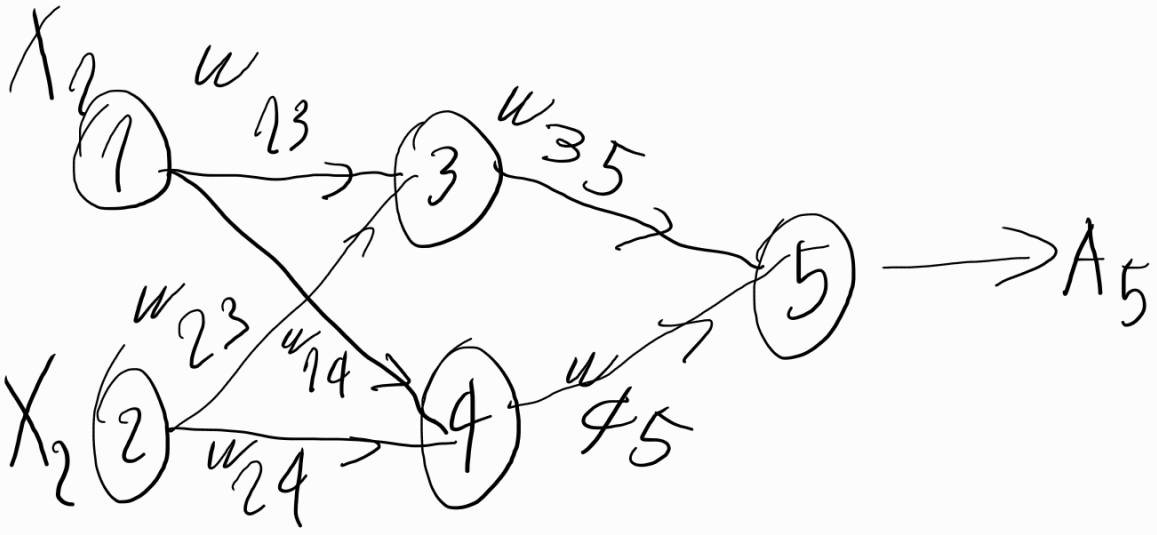
Implementing Backpropagation by Hands And-Logic Gate



$$Loss = \frac{1}{n}\sum(y-\hat{y})^2$$

$$Loss = \frac{\sum(y-\hat{y})^2}{4} = \frac{\sum(y-A_5)^2}{4}$$

x_1	x_2	y_i
1	1	1
1	0	0
0	1	0
0	0	0

Input Hidden

$Z_3 = w_{13} \cdot x_1 + w_{23} \cdot x_2 + b_3$

$A_3 = ReLu(Z_3)$

y : observed (Reais)

$Z_4 = W_{24} \cdot X_2 + W_{14} \cdot X_1 + b_4$

$A_4 = ReLu(Z_4)$

$Z_5 = A_3 \cdot W_{35} + A_4 \cdot W_{45} + b_5$

$A_5 = sig(Z_5)$

$\hat{y}$: Predicted (Modelo)

Calcular todas
las derivadas

Cálculo $\frac{\partial Loss}{\partial W_{13}}$

$$\frac{\partial Loss}{\partial W_{13}} = \frac{\partial Loss}{\partial A_5} \cdot \frac{\partial A_5}{\partial Z_5} \cdot \frac{\partial Z_5}{\partial A_3} \cdot \frac{\partial A_3}{\partial Z_3} \cdot \frac{\partial Z_3}{\partial W_{13}} = W_{13} \cdot grad$$

$$\frac{\partial Loss}{\partial A_5} = \frac{\partial \left(\frac{\sum (y - A_5)^2}{4} \right)}{\partial A_5} = \frac{-2 \cdot \sum (y - A_5)}{4}$$

$$\frac{\partial A_5}{\partial Z_5} = \frac{\partial sig(Z_5)}{\partial Z_5} = sig(X)\left[1 - sig(X)\right]$$

$$\frac{\partial Z_5}{\partial A_3} = W_{35}$$

$$\frac{\partial A_3}{\partial Z_3} = \frac{\partial [ReLu(Z_3)]}{\partial Z_3}$$

$$\frac{\partial Z_3}{\partial W_{13}} = X_1$$

$[sig(x)]' = sig(x)\left[1 - sig(x)\right]$

$sig(x) = \frac{1}{1+e^{-x}}$

$sig'(x) = \frac{+e^{-x}}{\left[1+e^{-x}\right]^2}$

$sig(x) = \frac{1}{1+e^{-x}}$

$sig(y) = \frac{1}{1+e^{y}}$; $y = -x$

$\frac{\partial sig(x)}{\partial x} = \frac{\partial (1+e^{y})}{\partial y} \cdot \frac{\partial y}{\partial x}$

$e^{y} \cdot 1 = -e^{-x}$

$$\frac{\partial Loss}{\partial W_{13}} = -\frac{(y - A_5)}{2} \cdot sig(X)\left[1 - sig(X)\right] \cdot W_{35} \left[ReLu(Z_3)\right]' \cdot X_1$$

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$Z_3 = W_{13} \cdot X_1 + W_{23} \cdot X_2 + b_3$

$A_3 = ReLu(Z_3)$

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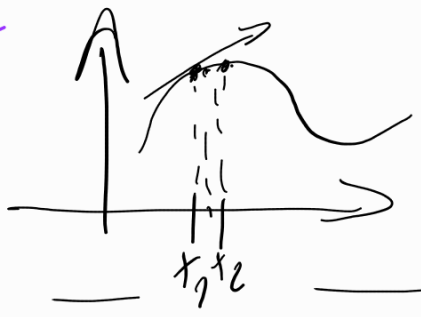
$A_5 = sig(Z_5)$

$$\frac{\partial Loss}{\partial W_{23}} = \frac{\partial Loss}{\partial A_5} \cdot \frac{\partial A_5}{\partial Z_5} \cdot \frac{\partial Z_5}{\partial A_3} \cdot \frac{\partial A_3}{\partial Z_3} \cdot \frac{\partial Z_3}{\partial W_{23}}$$

$$\frac{\partial Loss}{\partial W_{23}} = -\frac{(y - A_5)}{2} \cdot sig(x)[1 - sig(x)] \cdot W_{35} \cdot ReLu'(z_3) \cdot X_2 \quad 2$$

$$\frac{\partial Loss}{\partial b_3} = \frac{\partial Loss}{\partial A_5} \cdot \frac{\partial A_5}{\partial z_5} \cdot \frac{\partial z_5}{\partial A_3} \cdot \frac{\partial A_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial b_3}$$

$$\frac{\partial Loss}{\partial b_3} = -\frac{(y - A_5)}{2} \cdot sig(x)[1 - sig(x)] \cdot W_{35} \cdot ReLu'(z_3) \cdot 1 \quad 3$$



$$\frac{\partial Loss}{\partial W_{24}} = \frac{\partial Loss}{\partial A_5} \cdot \frac{\partial A_5}{\partial z_5} \cdot \frac{\partial z_5}{\partial A_4} \cdot \frac{\partial A_4}{\partial z_4} \cdot \frac{\partial z_4}{\partial W_{24}}$$

$$\frac{\partial Loss}{\partial W_{24}} = -\frac{(y - A_5)}{2} \cdot sig(x)[1 - sig(x)] \cdot W_{45} \cdot ReLu'(z_4) \cdot X_2 \quad 4$$

$$\frac{\partial Loss}{\partial W_{14}} = \frac{\partial Loss}{\partial A_5} \cdot \frac{\partial A_5}{\partial z_5} \cdot \frac{\partial z_5}{\partial A_4} \cdot \frac{\partial A_4}{\partial z_4} \cdot \frac{\partial z_4}{\partial W_{14}}$$

$$\frac{\partial Loss}{\partial W_{14}} = \frac{-(y - A_5)}{2} \cdot sig(x)[1 - sig(x)] \cdot W_{45} \cdot ReLu'(Z_4) \cdot X_1$$ 5

$$\left(\frac{\partial Loss}{\partial b_4} = \frac{\partial Loss}{\partial A_3} \cdot \frac{\partial A_5}{\partial Z_5} \cdot \frac{\partial Z_5}{\partial A_4} \cdot \frac{\partial A_4}{\partial Z_4} \cdot \frac{\partial Z_4}{\partial b_4} \right.$$

$$\frac{\partial Loss}{\partial b_4} = \frac{-(y - A_5)}{2} \cdot sig(x)[1 - sig(x)] \cdot W_{45} \cdot ReLu'(Z_4) \cdot 1$$ (6)

$$\frac{\partial Loss}{\partial W_{33}} = \frac{\partial Loss}{\partial A_5} \cdot \frac{\partial A_5}{\partial Z_5} \cdot \frac{\partial Z_5}{\partial W_{33}}$$

$$\frac{\partial Loss}{\partial W_{33}} = \frac{-(y - A_5)}{2} \cdot sig(x)[1 - sig(x)] \cdot \overbrace{ReLu(Z_3)}^{A_3}$$ (7)

$$\frac{\partial Loss}{\partial W_{45}} = \frac{\partial Loss}{\partial A_5} \cdot \frac{\partial A_5}{\partial Z_5} \cdot \frac{\partial Z_5}{\partial W_{45}} \qquad A_4 = ReLu(Z_4)$$

$$\frac{\partial Loss}{\partial W_{45}} = \frac{-(y - A_5)}{2} \cdot sig(x)[1 - sig(x)] \cdot A_4$$ (8)

$$\frac{\partial Loss}{\partial b_3} = \frac{\partial Loss}{\partial A_3} \cdot \frac{\partial A_5}{\partial Z_5} \cdot \frac{\partial Z_5}{\partial b_3}$$

$$\frac{\partial Loss}{\partial b_5} = -\frac{(y - A_5)}{2} \cdot sig(x)[1 - sig(x)] \cdot 1$$

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