Determinants (Part 2) # L1.5

- Upper triangular matrix and lower ~~to~~ triangular matrix. For such matrices, the determinant is the product of diagonal elements. 

- The transpose of $A_{m\times n}$ is the $(\overset{n\times m}{\cancel{m\times n}})$ matrix with (i,j)th entry of A_{ji} $\quad (A^T)_{ij} = A_{ji}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \qquad \det(A) = \det(A^T)$$

- If A is an $n\times n$ sq. with $n \le 4$. Then the minor of the entry is the i^{th} row & j^{th} column is the determinant of the submatrix formed by deleting the ith rows & jth column.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad \boxed{M_{11} = \det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}}$$

The $(i,j)^{th}$ cofactor $\boxed{C_{ij} = (-1)^{(i+j)} M_{ij}}$

- For $A_{3\times 3}$

$$\det(A) = a_{11} \times M_{11} - a_{12} \times M_{12} + a_{13} \times M_{13}$$

$$= a_{11} \times C_{11} + a_{12} \times C_{12} + a_{13} \times C_{13}$$

Must try to figureout during division.

- $\det(A) = \sum_{j=1}^{4} (-1)^{1+j} a_{1j} M_{1j} = \sum_{j=1}^{4} a_{1j} C_{1j}$

- $\det(A) = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} M_{1j} = \sum_{j=1}^{n} a_{1j} C_{1j}$