Determinants (Part-1) #L1.4

- Every sq. matrix A has an associated no., called its determinant.

  $\Rightarrow$ det (A) or |A|

  - Solving system of linear eqⁿ
  - Finding inverse of matrix
  - calculus and more

- If $A = [a]$, a 1x1 matrix, then $\boxed{\det(A) = a}$

  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\boxed{\det(A) = ad - bc}$

ex, $A = \begin{bmatrix} 2 & 3 \\ 6 & 10 \end{bmatrix}$

- $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$\det(A) = a_{11} \times \det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12} \times \det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13} \det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

Solve during Revision $\rightarrow$ $A = \begin{bmatrix} 2 & 4 & 1 \\ 3 & 8 & 7 \\ 5 & 6 & 9 \end{bmatrix}$

(Ans:) = 70

- det of I $\quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\det(I_2) = 1 - 0 = 1$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \det(I_3) = 1$$

- $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ $\quad AB = \begin{bmatrix} \overset{①}{ae + bg} & \overset{②}{af + bh} \\ \underset{③}{ce + dg} & \underset{④}{cf + dh} \end{bmatrix}$

$$\det(AB) = [(1)(4)] - [(2)(3)]$$

$$\boxed{\det(AB) = \det(A)\det(B)}$$ → do the proof during rivision

- Inverse of matrix

$(AA^{-1}) = I = A^{-1}A$ $\quad \boxed{\det(A^{-1}) = \frac{1}{\det(A)}}$

- $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad \bar{A} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$ } By switching 2 rows

$$\boxed{\det(\bar{A}) = -\det(A)}$$

$\boxed{\bar{A} = \begin{bmatrix} b & a \\ d & c \end{bmatrix} \quad \boxed{\det(\bar{A}) = -\det(A)}}$

by switching 2 columns.

- Adding multiples of a row to another row

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad \bar{A} = \begin{bmatrix} a + tc & b + tc \\ c & d \end{bmatrix}$

$$\boxed{\det(\bar{A}) = \det(A)}$$

- Scalar multiplication of a row by a constaint

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad \bar{A} = \begin{bmatrix} ta & tb \\ c & d \end{bmatrix}$

$$\boxed{\det(\bar{A}) = t\det(A)}$$