CSC236 Week 01: Introduction and Basic Induction

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September 9 – 15, 2021

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1	Why reason about computing	
	• You're not just hackers anymore Sometimes you need to analyze code before it runs. Sometimes it should never be run!	
	• Can you test everything? Infinitely many inputs: integers, strings, lists.	
	• Careful, you might get to like it(?!*)	

2 How to reason about computing

• It's messy...

interesting problems fight back.

You need to draft, re-draft, and re-re-draft.

You need to follow blind alleys until you find a solution.

You can also find a solution that isn't wrong, but could be better.

• It's art...

Strive for correctness, clarity, surprise, humor, pathos, and others.

3 How to do well in this course

- read the syllabus as a two-way promise
- question, answer, record, synthesize try annotating blank slides.
- collaborate with respect
 You need computer science friends who are respectful and constructively critical.

4 Assume that you already know

- Chapter 0 material from *Introduction to Theory of Computation*.
- CSC110/111 material, especially proofs and big-O.

5 By December you'll know

- understand and use several flavours of induction. some of these flavours will taste new
- Formal languages, regular languages, regular expressions Sets of strings
- complexity and correctness of programs both recursive and iterative

6 domino fates foretold

$$[P(0) \land (\forall n \in \mathbb{N}, P(n) \implies P(n+1))] \implies \forall n \in \mathbb{N}, P(n)$$

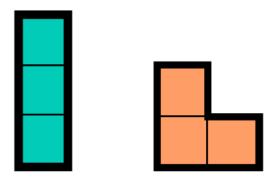
If the initial case works, and each case that works implies its successor works, then all cases work

7 Simple induction outline

- inductive step: introduce n and inductive hypothesis H(n)
 - derive conclusion C(n): show that C(n) follows from H(n), indicating **where** you use H(n) and why that is valid.
- Verify base case(s): verify that the claim is true for any cases not covered in the inductie step
- In simple induction C(n) is just H(n+1)

8 Trominoes

See https://en.wikipedia.org/wiki/Tromino



Can an $n \times n$ square grid, with one subsquare removed, be tiled (covered without overlapping) by "chair" trominoes?

- 1×1 : Yes.
- 2×2 : Yes.
- 3×3 : No. The remaining number of squares is not divisible by 3.
- 4×4 : Yes.

Proof: $\forall n \in \mathbb{N}$, define the predicate P(n) as a $2^n \times 2^n$ square grid, with one subsquare removed, can be tiled (covered without overlapping) by "chair" trominoes.

• Induction on n

Let n be an arbitrary, fixed, natural number. (Let $n \in \mathbb{N}$).

Assume P(n), that is a $2^n \times 2^n$ grid, with one square removed can be tiled with "chairs."

I will prove P(n+1), that is a $2^{n+1} \times 2^{n+1}$ grid, with one square removed can be tiled by chairs.

Let G be a $2^{n+1} \times 2^{n+1}$ grid with one square removed. Notice that G can be decomposed into four $2^n \times 2^n$ disjoint quadrant grids. We may assyme, WLOG (without loss of generality)

that the missing square is in the upper-right quadrant, since otherwise just rotate it there, and rotate back when done. By P(n) I can tile the upper-right quadrant, minus the missing square. By P(n) 3 more times, I can tile the remaining 3 quadrants, omitting for a moment the 3 tiles nearest the centre of G, with chairs. The briefly omitted squares form a chair! So I complete the tiling by adding one more chair. Thus P(n+1).

• Base Case

A $2^0 \times 2^0$ grid, with one square removed, is just empty space! This can be tiled with 0 charis. So P(0) is true.

And thus $\forall n \in \mathbb{N}, P(n)$.

9
$$3^n > n^3$$
?

9.1 Scratch Work

Check for a few values of n:

$$3^{0} = 1 \ge 0 = 0^{3}$$

$$3^{1} = 3 \ge 1 = 1^{3}$$

$$3^{2} = 9 \ge 8 = 2^{3}$$

$$3^{3} = 27 \ge 27 = 3^{3}$$

$$3^{4} = 81 \ge 64 = 4^{3}$$

$$3^{-1} = \frac{1}{3} \ge -1 = -1^{3}$$

$$3^{2.5} < 2.5^{3}$$
 \checkmark

9.2 Simple Induction

Proof: $\forall n \in \mathbb{N}$, define the predicate P(n) as $3^n \ge n^3$.

• Induction on n

Let $n \in \mathbb{N}$. Assume $H(n) : 3^n \ge n^3$. I will prove H(n+1) follows, that is $3^{n+1} \ge (n+1)^3$.

$$3^{n+1}$$
= $3 \cdot 3^n$

$$\geq 3 \cdot n^3$$
= $n^3 + n^3 + n^3$

$$\geq n^3 + 3n^2 + 9n$$
 (since $n \geq 3$)
$$\geq n^3 + 3n^2 + 3n + 6n$$
= $n^3 + 3n^2 + 3n + 1$ (since $6n \geq 1$)
= $(n+1)^3$

And thus we have shown that $\forall n \in \mathbb{N} \text{ s.t. } n \geq 3, H(n) \implies H(n+1).$

• Base Case

$$3^{3} \ge 3^{3}$$
 so $P(3)$ holds.
 $3^{2} \ge 2^{3}$ so $P(2)$ holds.
 $3^{1} \ge 1^{3}$ so $P(1)$ holds.
 $3^{0} \ge 0^{3}$ so $P(0)$ holds.

And thus, we have shown $\forall n \in \mathbb{N}, 3^n \geq n^3$, as needed.