CSC236 Week 06: Automata and Languages

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1 $L = \{x \in \{0,1\}^* \,|x$ begins and ends with a different bit$\}$

The language $L'((0\,(0+1)^*\,1) \cup (1\,(1+0)^*\,0))$ should be the same language as the one listed above.

If we want to prove this, we can show that $\forall x \in L, x \in L' \wedge \forall x \in L', x \in L$. This is the same as showing that $L \subseteq L' \wedge L' \subseteq L$, which is equivalent to $L = L'$, what we are trying to show.

Proof: First we show that $L' \subseteq L$: Let $x \in L'$. Then either $x = 1y0$ where $y \in \{0,1\}^*$ or $x = 0w1$, where $w \in \{1,0\}^*$. Without loss of generality, assume $x = 1y0$, otherwise just replace 1 with 0, 0 with 1, and y with w.

Then $1 \in L(1), 0 \in L(0),$ and $y \in L(0+1)^*$, since it is the concatenation of 0 or more strings from $L(0+1)$. So $x \in L(1)L(0+1)^*L(0)$, so it begins with 1 and ends with 0, which are different, so $x \in L$.

Next we show that $L \subseteq L$: left as an exercise to the reader

So since we have shown that $L \subseteq L' \wedge L' \subseteq L$, $L = L'$. ■

2 RE identities

Some of these follow from set properties, others require some proof

- Commutativity of union: $R + S \equiv S + R$
- Associativity of union: $(R + S) + T \equiv R + (S + T)$
- Associativity of concatenation: $(RS)T \equiv R(ST)$

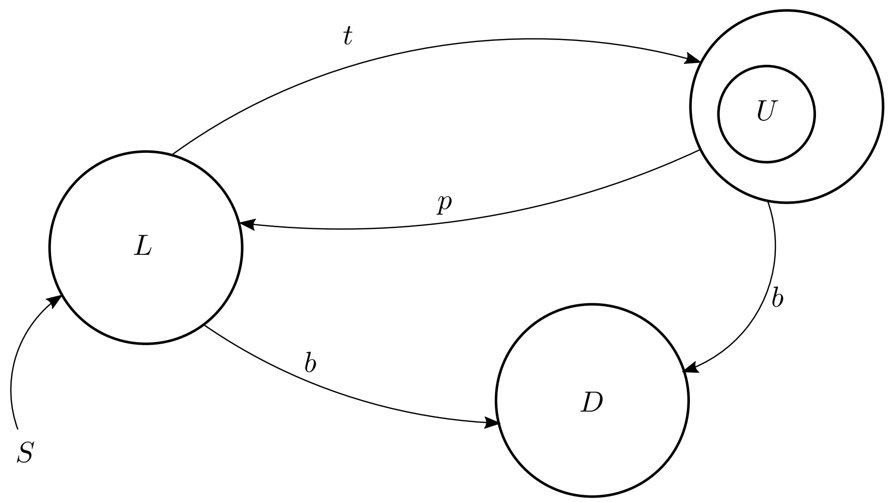
- Left distributivity: $R(S+T) \equiv RS + RT$
- Right distributivity: $(S+T)R \equiv SR + ST$
- Identity for union: $R + \emptyset \equiv R$
- Identity for concatenation: $R\varepsilon \equiv R \equiv \varepsilon R$
- Annihilator for concatenation: $\emptyset R \equiv \emptyset \equiv R\emptyset$
- Idempotence of Kleene star: $(R^*)^* \equiv R^*$

3 Turnstile finite-state machine

Let out alphabet be $\{t, p, b\}$, where p denotes push, t denotes tap or token, and b denotes bicycle.

We will study three different states: $Q = \{U, L, D\}$. U denotes unlocked, L denotes locked, D stands for dead (or jammed or deactivated).

Σ^* is all strings over $\{t, p, b\} = \{t, p, b\}^*$



Is *tptppt* accepted? We start at L, go to U with the first t, go back to L with p and so on. When we have a p but we are at L, then nothing happens as pushing on a locked turnstile will do nothing. Similarly, when we have a t when we are at a U, then nothing will happen as using a tap/token on an unlocked turnstile will also do nothing. Following these rules, we arrive at the following:

$$L \xrightarrow{t} U \xrightarrow{p} L \xrightarrow{t} U \xrightarrow{p} L \xrightarrow{p} L \xrightarrow{t} U$$

And thus the combination is accepted.