

CSC236 Week 01: Introduction and Basic Induction

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September 9 – 15, 2021

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1 Why reason about computing

- You're not just hackers anymore
Sometimes you need to analyze code before it runs. Sometimes it should never be run!
- Can you test everything?
Infinitely many inputs: integers, strings, lists.
- Careful, you might get to like it. . . (!*)

2 How to reason about computing

- It's messy...
interesting problems fight back.
You need to draft, re-draft, and re-re-draft.
You need to follow blind alleys until you find a solution.
You can also find a solution that isn't wrong, but could be better.
- It's art...
Strive for correctness, clarity, surprise, humor, pathos, and others.

3 How to do well in this course

- read the syllabus as a two-way promise
- question, answer, record, synthesize
try annotating blank slides.
- collaborate with respect
You need computer science friends who are respectful and constructively critical.

4 Assume that you already know

- Chapter 0 material from *Introduction to Theory of Computation*.
- CSC110/111 material, especially proofs and big- \mathcal{O} .

5 By December you'll know

- understand and use several flavours of induction.
some of these flavours will taste new
- Formal languages, regular languages, regular expressions
Sets of strings
- complexity and correctness of programs — both recursive and iterative

6 domino fates foretold

$$[P(0) \wedge (\forall n \in \mathbb{N}, P(n) \implies P(n+1))] \implies \forall n \in \mathbb{N}, P(n)$$

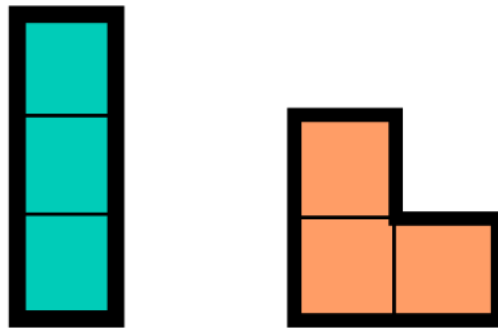
If the initial case works, and each case that works implies its successor works, then all cases work

7 Simple induction outline

- inductive step: introduce n and inductive hypothesis $H(n)$
 - derive conclusion $C(n)$: show that $C(n)$ follows from $H(n)$, indicating **where** you use $H(n)$ and why that is valid.
- Verify base case(s): verify that the claim is true for any cases not covered in the inductive step
- In simple induction $C(n)$ is just $H(n + 1)$

8 Trominoes

See <https://en.wikipedia.org/wiki/Tromino>



Can an $n \times n$ square grid, with one subsquare removed, be tiled (covered without overlapping) by “chair” trominoes?

- 1×1 : Yes.
- 2×2 : Yes.
- 3×3 : No. The remaining number of squares is not divisible by 3.
- 4×4 : Yes.

$P(n)$: a $2^n \times 2^n$ square grid, with one subsquare removed, can be tiled (covered without overlapping) by “chair” trominoes.

Pf:

i. Induction on n

Let n be an arbitrary, fixed, natural number. (Let $n \in \mathbb{N}$).

Assume $P(n)$, that is a $2^n \times 2^n$ grid, with one square removed can be tiled with "chairs."

I will prove $P(n + 1)$, that is a $2^{n+1} \times 2^{n+1}$ grid, with one square removed can be tiled by chairs.

Let G be a $2^{n+1} \times 2^{n+1}$ grid with one square removed. Notice that G can be decomposed into four $2^n \times 2^n$ disjoint quadrant grids. We may assume, WLOG (without loss of generality) that the missing square is in the upper-right quadrant, since otherwise just rotate it there, and rotate back when done. By $P(n)$ I can tile the upper-right quadrant, minus the missing square. By $P(n)$ 3 more times, I can tile the remaining 3 quadrants, omitting for a moment the 3 tiles nearest the centre of G , with chairs. The briefly omitted squares form a chair! So I complete the tiling by adding one more chair. Thus $P(n+1)$.

ii. Base Case

A $2^0 \times 2^0$ grid, with one square removed, is just empty space! This can be tiled with 0 chairs. So $P(0)$ is true.

9 $3^n \geq n^3$?

9.1 Scratch Work

scratch work: check for a few values of n :

$$3^0 = 1 \geq 0 = 0^3 \checkmark$$

$$3^1 = 3 \geq 1 = 1^3 \checkmark$$

$$3^2 = 9 \geq 8 = 2^3 \checkmark$$

$$3^3 = 27 \geq 27 = 3^3 \checkmark$$

$$3^4 = 81 \geq 64 = 4^3 \checkmark$$

$$3^{-1} = \frac{1}{3} \geq -1 = -1^3 \checkmark$$

$$3^{2.5} = 2.5^3 = 4^3 \times$$

9.2 Simple Induction

i. Induction on n

Let $n \in \mathbb{N}$. Assume $H(n) : 3^n \geq n^3$. I will prove $H(n+1)$ follows, that is $3^{n+1} \geq (n+1)^3$.

$$\begin{aligned} & 3^{n+1} \\ &= 3 \cdot 3^n \\ &\geq 3 \cdot n^3 \\ &= n^3 + n^3 + n^3 \\ &\geq n^3 + 3n^2 + 9n && \text{(since } n \geq 3\text{)} \\ &\geq n^3 + 3n^2 + 3n + 6n \\ &= n^3 + 3n^2 + 3n + 1 && \text{(since } 6n \geq 1\text{)} \\ &= (n+1)^3 \end{aligned}$$

And thus we have shown that, starting at $n = 3$, $H(n) \implies H(n + 1)$.

ii. Base Case

$3^3 \geq 3^3$ so $P(3)$ holds.

$3^2 \geq 2^3$ so $P(2)$ holds.

$3^1 \geq 1^3$ so $P(1)$ holds.

$3^0 \geq 0^3$ so $P(0)$ holds.

And thus, we have shown $\forall n \in \mathbb{N}, 3^n \geq n^3$, as needed.