CSC236 Week 08: Machines, Expressions: Equivalence

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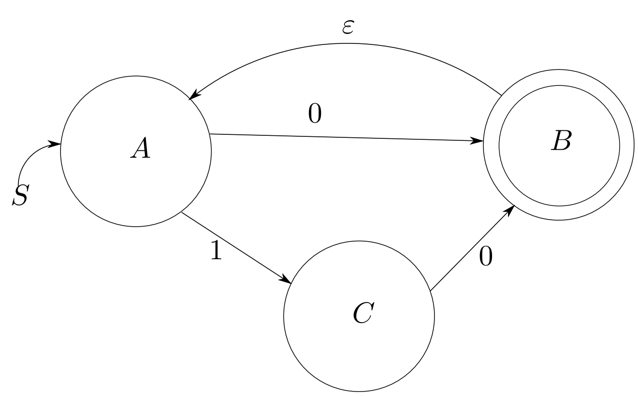
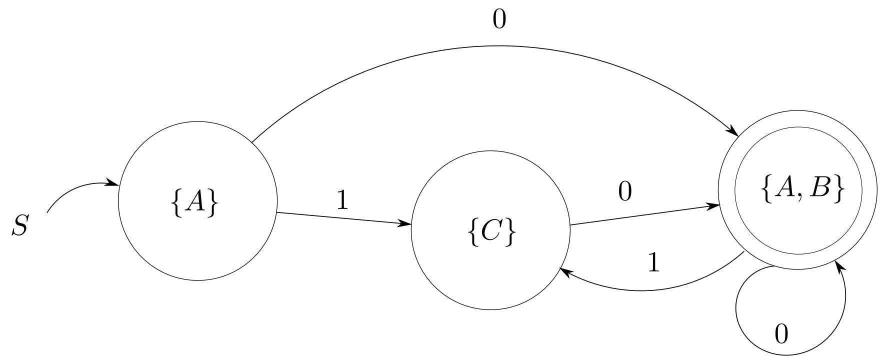
1 NFSA that accepts $L((0 + 10)(0 + 10)^*)$

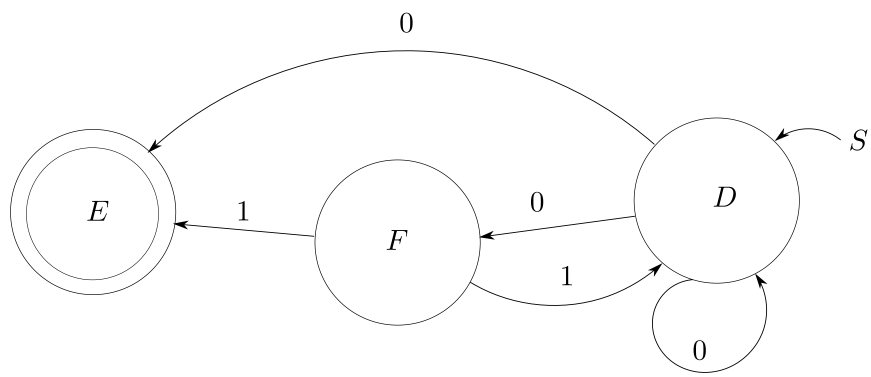


The ε transition makes it non deterministic. $A \xrightarrow{0} A \cup B$ and $C \xrightarrow{0} A \cup B$.

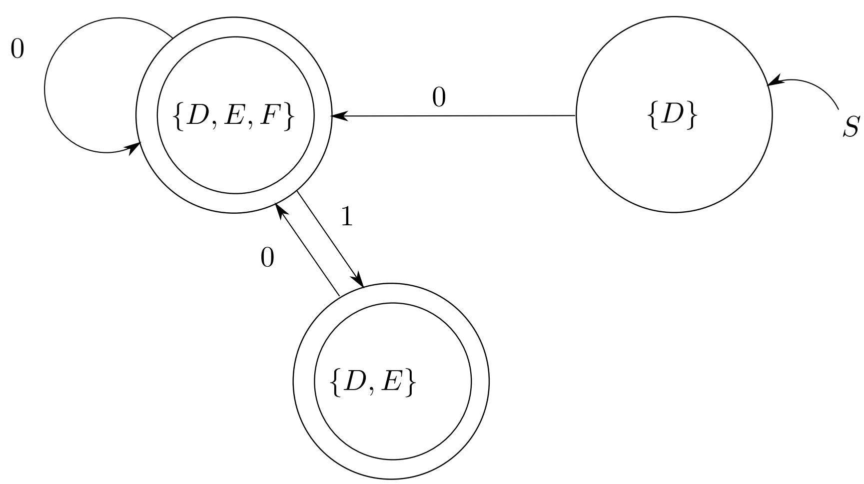
The corresponding DFSA is as follows:



2 NFSA that accepts $\text{Rev}(L((0+10)(0+10)^*))$



The corresponding DFSA is as follows:



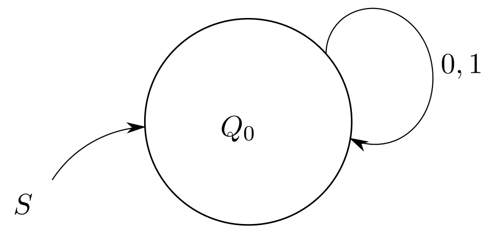
3 FSAs and regexes are equivalent.

$L = L(M)$ for some DFSA $M \iff L = L(M')$ for some NFSA $M' \iff L = L(R)$ for some regular expression R.

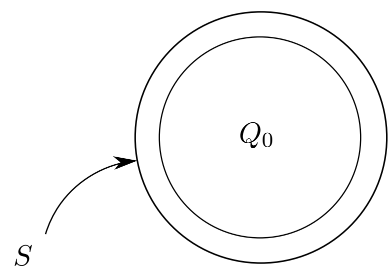
3.1 Step 1.0: convert $L(R)$ to $L(M')$.

Start with $\emptyset, \varepsilon, a \in \Sigma$.

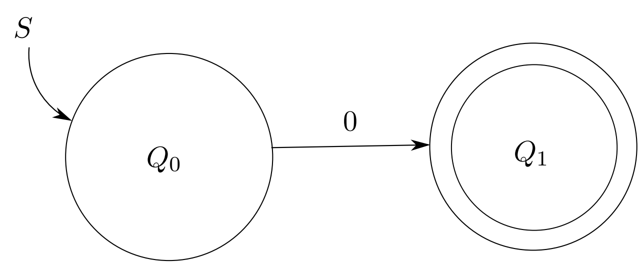
- Base case: Let $s\ in \{\emptyset, \varepsilon, a\}$ for some $a \in \Sigma$.
  $L(\emptyset) = L(M)$, where M is:



  $L(\varepsilon) = L(M)$, where M is:

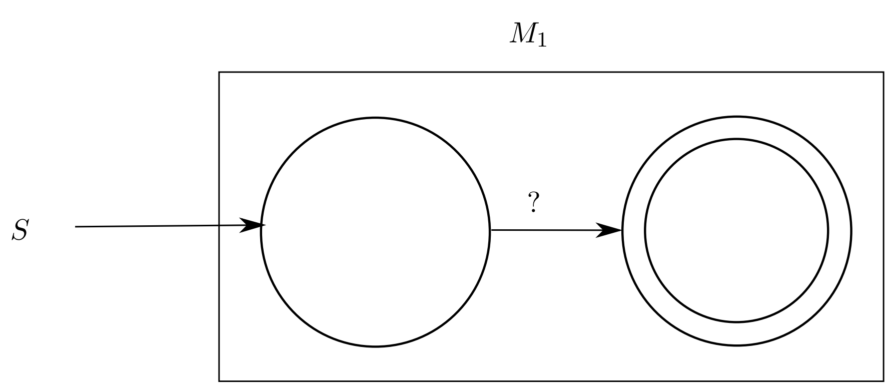


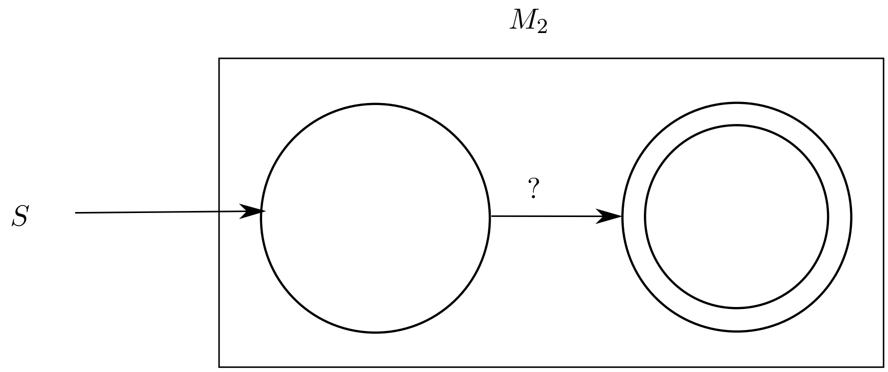
  $L(0) = L(M)$, where M is:



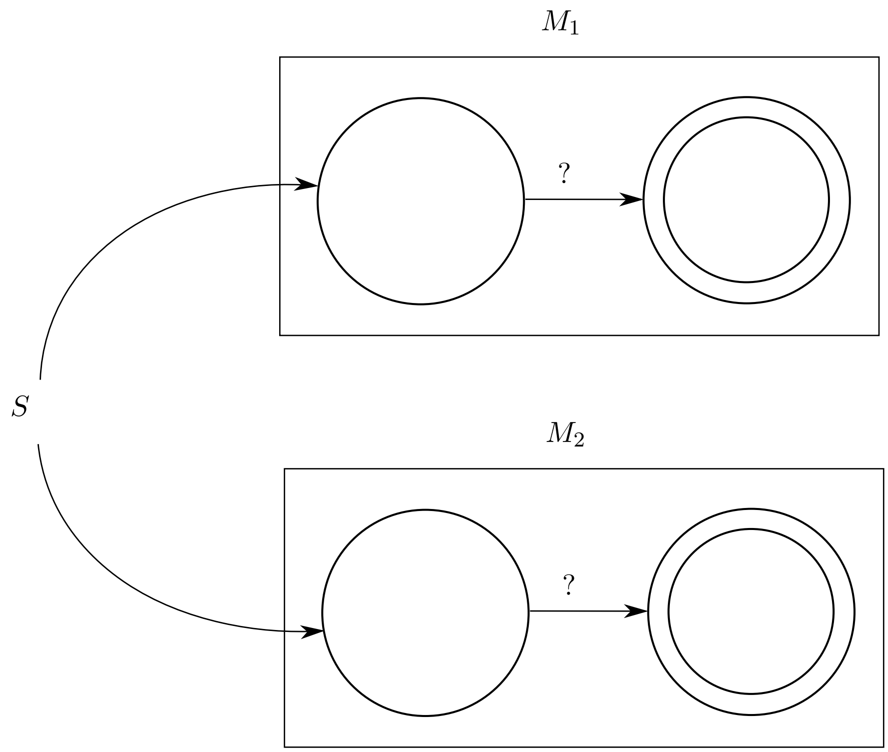
3.2 Step 1.5: Convert $L(R)$ to $L(M')$.

Suppose r_1 and r_2 denote languages accepted by M_1 and M_2 respectively.

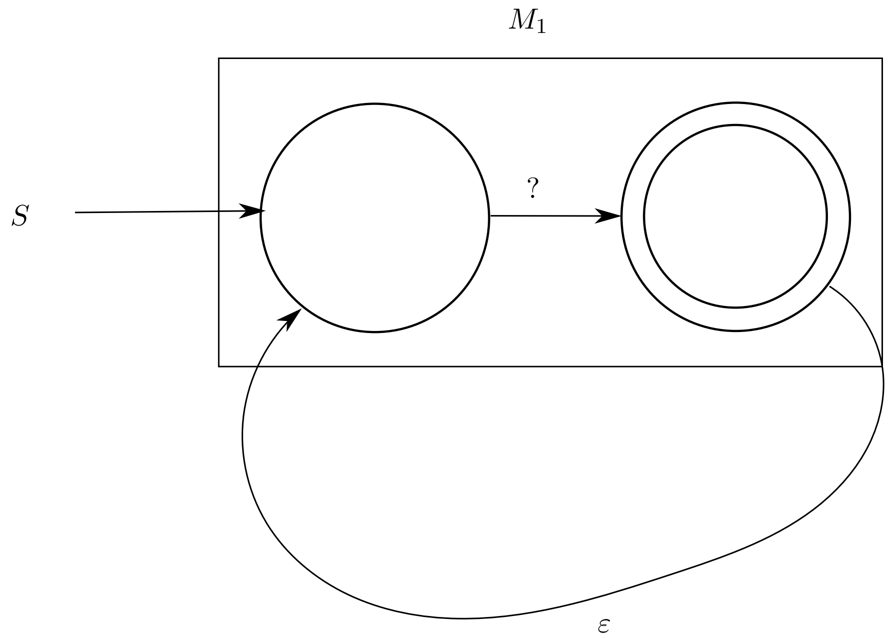




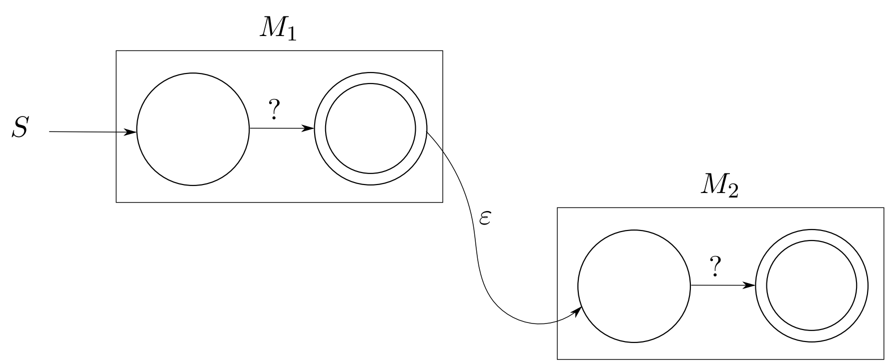
$L(r_1 + r_2) = L(r_1) \cup L(r_2)$. We could build the corresponding machine using product construction on M_1 and M_2 or we could use a NFSA.



$L(r_1^* = L(r_1)^*$. We want to transform M_1 into a machine that accepts the Kleene star of the language accpeted by M_1.



$L(r_1r_2) = L(r_1)L(r_2)$. How do we combine M_1 and M_2 to accept this concatenated language? We concatenate the two machines!



Not that all three techniques use non-determinism but we can use subset construction to create an equivalent deterministic machine so these answers are no less valid.

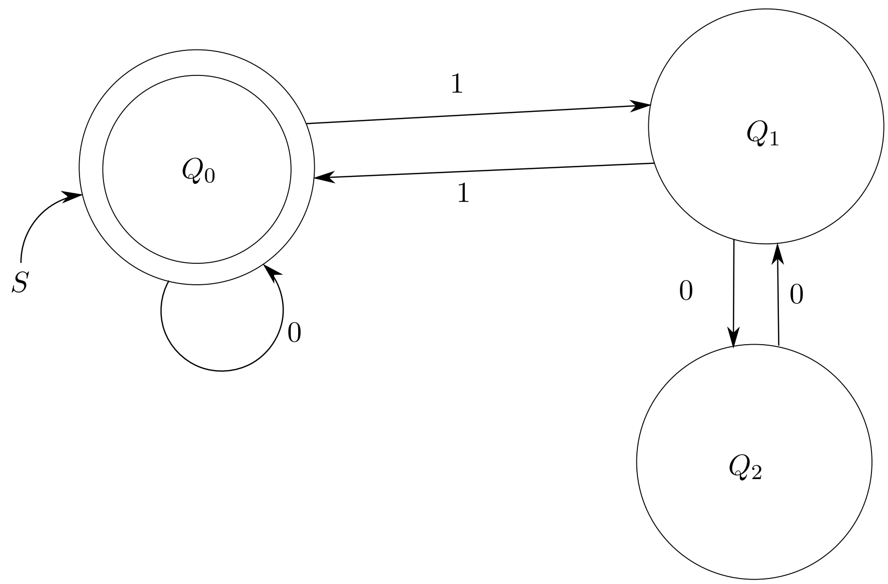
4 State elimination recipe for state q

1. $s_1 \dots s_n$ are states with transition to q, with labels $S_1 \dots S_n$.
2. $t_1 \dots t_n$ are states with transition from q, with labels $T_1 \dots T_n$.
3. Q is any self-loop state on q.
4. Eliminate q, and add (union) transition label $S_iQ^*T_j$ from s_i to t_j.

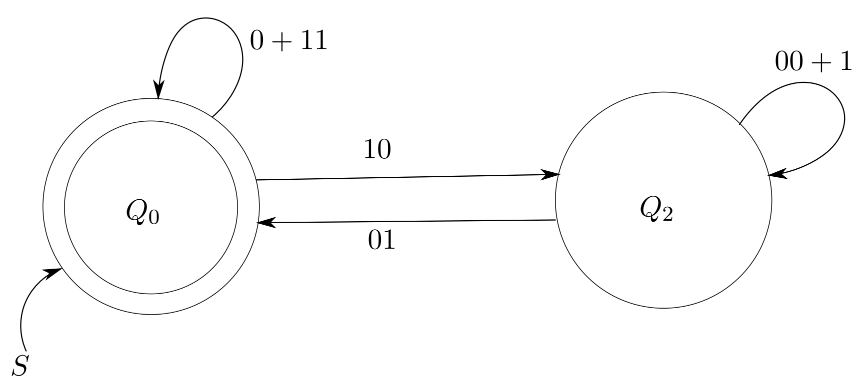
5 FSAs and regexes are equivalent:

$L = L(M)$ for some DFSA $M \iff L = L(M')$ for some NFSA $M' \iff L = L(R)$ for some regular expression R.

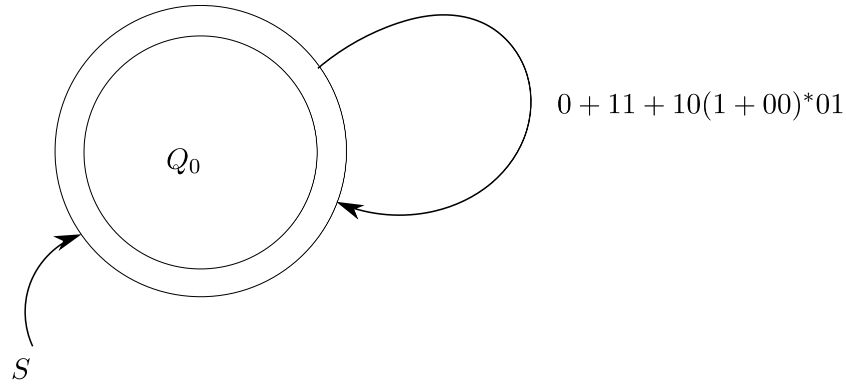
5.1 Step 3: convert $L(M)$ to $L(R)$ and eliminate states.



Then we eliminate Q_1:



Then we eliminate Q_2!



So our regular expression is $(0 + 11 + 10(1 + 00)^*01)^*$.

6 Regular languages closure

Regular languages are those that can be