CSC236 Week 09: Languages: The Last Words

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1 Regula languages closure

regular langes are those that cna be denoted by a regula expression or accept by an FSA. In addition:

- L regular $\implies \overline{L}$ regular.
- L regular $\implies \text{Rev}(L)$ regular.

If L has a finite number of strnigs, then L is regular.

2 Pumping Lemma

If $L \subseteq \Sigma^*$ is a regular language, then there is some $n_L \in \mathbb{N}$ (n_L depends on L such that n_L is the number of states in some FSA that accepts L) such that if $x \in L$ and $|x| \geq n_L$ then:

• $\exists u, v, w \in \Sigma^*, x = uvw$	x is a sandwich
• $\lvert v \rvert > 0$	middle of sandwich is non-empty
• $\lvert uv \rvert \leq n_L$	first two slices no longer than n_L
• $\forall k \in \mathbb{N}, uv^k w \in L$	

Idea: if machine $M(L)$ has $|Q| = n_L, x \in L \wedge |x| \geq n_L$, denote $q_i = \delta^*(q_0, x[:\, i])$, so x "visits" $q_0, q_1, \ldots, q_{n_L - 1}$ with the first n_L prefixes of x (including ε)... so there is at least one state that x "visits" twice (pigeonhold principle, and x has $n_L + 1$ prefixes).

3 Consequences of regularity

How about $L = \{1^n 0^n : n \in \mathbb{N}\}$?

Proof: Assume, for the sake of contradiction, that L is regular. Then, there must be a machine M_L that accepts L. So M_L has $|Q| = m > 0$ states. Consider the string $1^m 0^m$. By the pumping lemma, $x = uvw$, where $|uv| \leq m$ and $|v| > 0$, and $\forall k \in \mathbb{N}, uv^k w \in L$. But, then $uvvw \in L$, so $m + |v|$ 1s followed by just m 0s. *This is a contradiction.* Elements of L must have the same number of 1s as zeros, but $m + |v| > m$. ■

4 Another approach... Myhill-Nerode

Consider how many different states $1^k \in \text{Prefix}(L)$ and end up in ... for various k

Scratch work: Could 1, 11, 111 each take the machine to the same state?

Proof: Assume, for the sake of contradiction, that L (previous section) is regular. Then some machine M that accepts L has some number of states $|Q| = m$. Consider the prefixes $1^0, 1^1, \ldots, 1^m$. Since there are $m+1$ such prefixes, at least two drive M to the same state, so there are $0 \leq h < i \leq m$ such that 1^h and 1^i drive M to the same state but then $1^h 0^h$ drive the machine to an accepting state. But so does $1^i 0^h$! But $1^i 0^h$ (since $i \neq h$) sohuld not be accepted. *This is a contradiction.* By assuming that L was regular, we had to conclude there was a mchine that accepted L, which lead to a contradiction. So that assumption is false and L is not regular ■