# CSC236 Week 06: Automata and Languages

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October 14 – October 20, 2021

## Contents


**1 $L = \{x \in \{0,1\}^* \, | x$ begins and ends with a different bit$\}$ 1**

**2 RE identities 2**

**3 Turnstile finite-state machine 2**

**4 Float machine 3**

**5 States needed to classify a string 3**

**6 Integer multiples of 3 3**

**7 Build an automaton with formalities 4**


## 1 $L = \{x \in \{0,1\}^* \, | x$ begins and ends with a different bit$\}$

The language $L'((0\,(0+1)^*\,1) \cup (1\,(1+0)^*\,0))$ should be the same language as the one listed above.

If we want to prove this, we can show that $\forall x \in L, x \in L' \wedge \forall x \in L', x \in L$. This is the same as showing that $L \subseteq L' \wedge L' \subseteq L$, which is equivalent to $L = L'$, what we are trying to show.

*Proof:* First we show that $L' \subseteq L$: Let $x \in L'$. Then either $x = 1y0$ where $y \in \{0,1\}^*$ or $x = 0w1$, where $w \in \{1,0\}^*$. Without loss of generality, assume $x = 1y0$, otherwise just replace 1 with 0, 0 with 1, and $y$ with $w$.

Then $1 \in L(1), 0 \in L(0),$ and $y \in L(0+1)^*$, since it is the concatenation of 0 or more strings from $L(0+1)$. So $x \in L(1)L(0+1)^*L(0)$, so it begins with 1 and ends with 0, which are different, so $x \in L$.

Next we show that $L \subseteq L$: this is left as an exercise to the reader

So since we have shown that $L \subseteq L' \wedge L' \subseteq L$, $L = L'$. ■

## 2 RE identities

Some of these follow from set properties, others require some proof

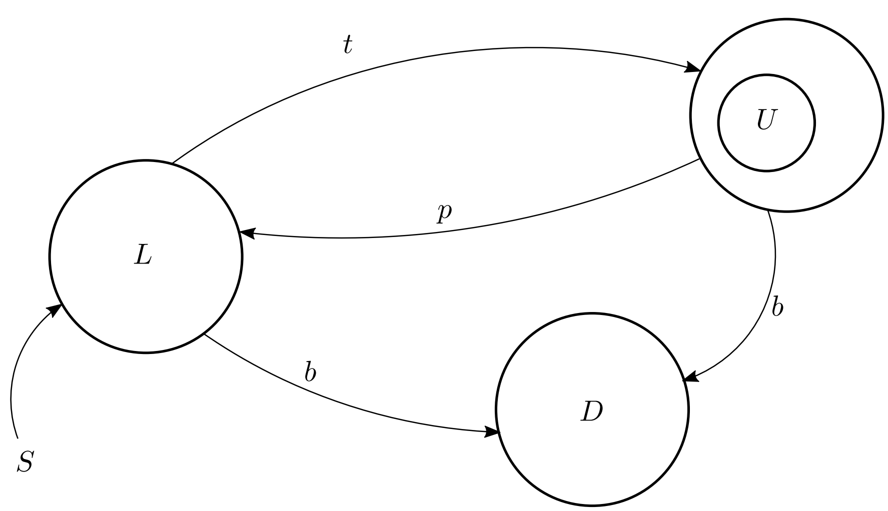
- Commutativity of union: $R + S \equiv S + R$
- Associativity of union: $(R + S) + T \equiv R + (S + T)$
- Associativity of concatenation: $(RS)T \equiv R(ST)$
- Left distributivity: $R(S + T) \equiv RS + RT$
- Right distributivity: $(S + T)R \equiv SR + ST$
- Identity for union: $R + \emptyset \equiv R$
- Identity for concatenation: $R\varepsilon \equiv R \equiv \varepsilon R$
- Annihilator for concatenation: $\emptyset R \equiv \emptyset \equiv R\emptyset$
- Idempotence of Kleene star: $(R^*)^* \equiv R^*$

## 3 Turnstile finite-state machine

Let out alphabet be $\{t, p, b\}$, where $p$ denotes push, $t$ denotes tap or token, and $b$ denotes bicycle.

We will study three different states: $Q = \{U, L, D\}$. $U$ denotes unlocked, $L$ denotes locked, $D$ stands for dead (or jammed or deactivated).

$\Sigma^*$ is all strings over $\{t, p, b\} = \{t, p, b\}^*$




Is *tptppt* accepted? We start at $L$, go to $U$ with the first $t$, go back to $L$ with $p$ and so on. When we have a $p$ but we are at $L$, then nothing happens as pushing on a locked turnstile will do nothing. Similarly, when we have a $t$ when we are at a $U$, then nothing will happen as using a tap/token on an unlocked turnstile will also do nothing. Following these rules, we arrive at the following:

$$L \xrightarrow{t} U \xrightarrow{p} L \xrightarrow{t} U \xrightarrow{p} L \xrightarrow{p} L \xrightarrow{t} U$$

And thus the combination is accepted.

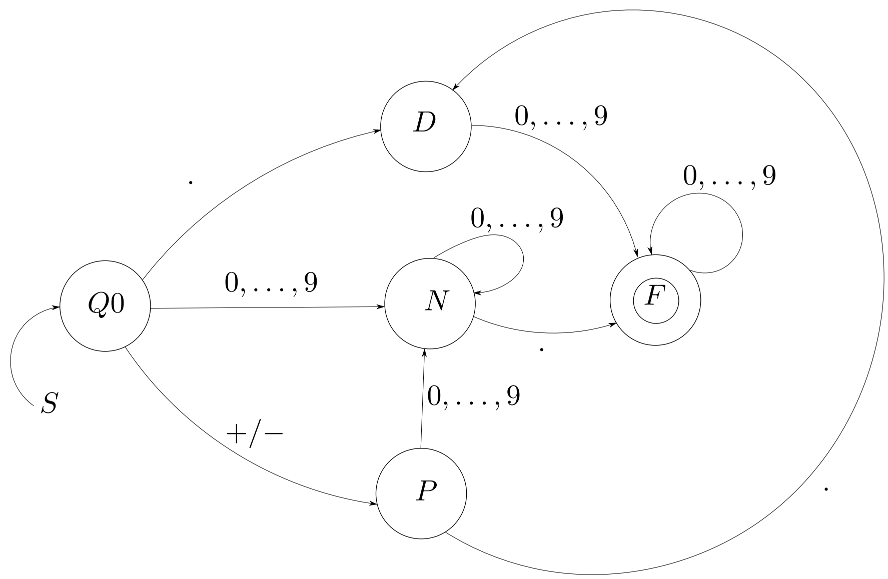
## 4 Float machine

Which strings are floats in Python?

$\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ., -, +\}$

Examples of accepted floats: $\{1.25, -0.3, .3, 125.\}$

Examples of rejected floats: $125, 1..2, 1.2.5, -., 1.25-$



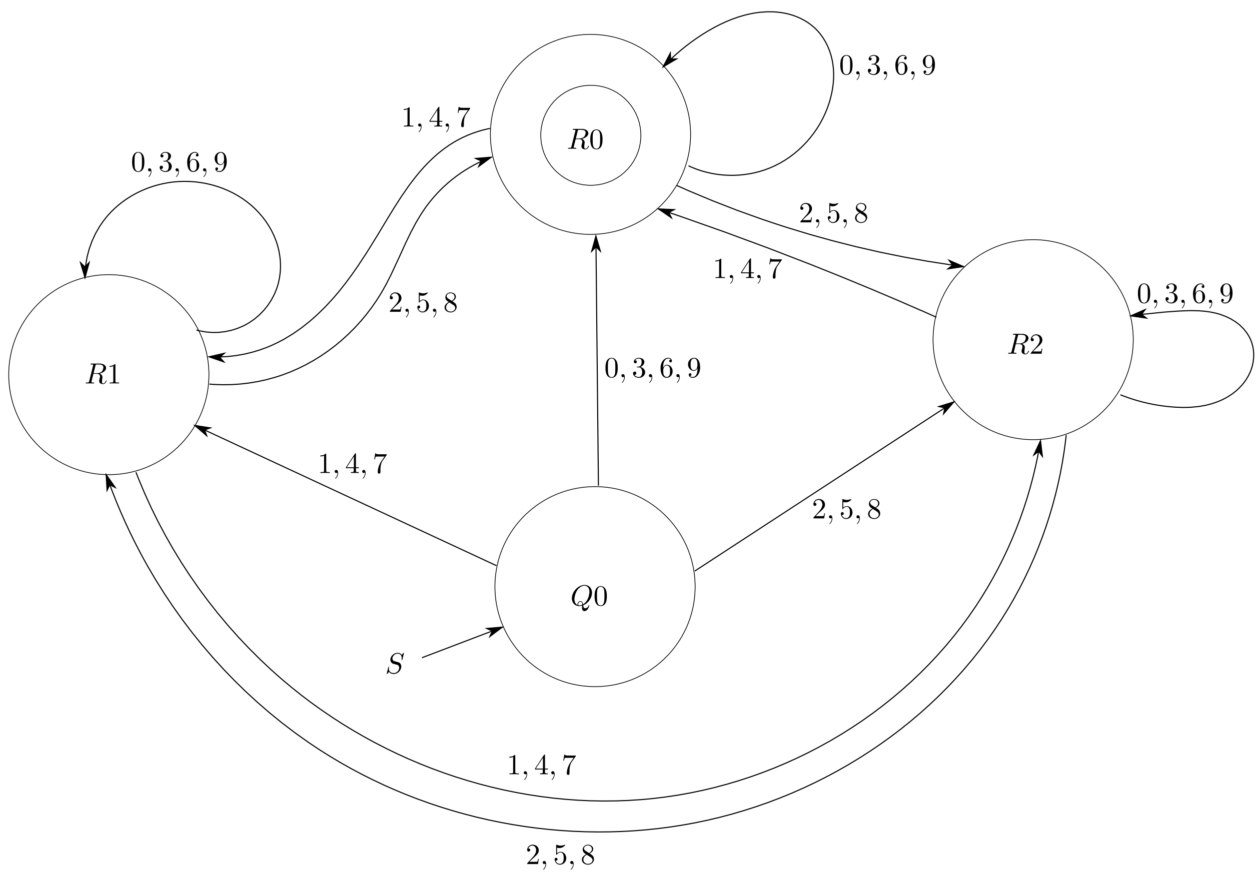

## 5 States needed to classify a string

What state is a stingy vending machine in, based on coins? Accepts only nickels, dimes, and quarters, no change given, and everything costs 30 cents.

| $\delta$ | 0 | 5 | 10 | 15 | 20 | 25 | $\geq 30$ |
|---|---|---|---|---|---|---|---|
| $n$ | 5 | 10 | 15 | 20 | 25 | $\geq 30$ | $\geq 30$ |
| $d$ | 10 | 15 | 20 | 25 | $\geq 30$ | $\geq 30$ | $\geq 30$ |
| $q$ | 25 | $\geq 30$ | $\geq 30$ | $\geq 30$ | $\geq 30$ | $\geq 30$ | $\geq 30$ |

## 6 Integer multiples of 3

If an integer is divided by 3, it falls into one of three groups. Remainder 0, remainder 1, and remainder 2. Concatenating a number onto the end of another number can have interesting effects depicted in the diagram below.




## 7 Build an automaton with formalities

The idea motivating this is to be able to describe the complicated systems above without drawing out messy and time consuming diagrams.

Quintuple: $(Q, \Sigma, q_0, F, \delta)$

$Q$ is a set of states, $\Sigma$ is finite, non-empty alphabet, $q_0$ is the start state, $F$ is the set of accepting states, and $\delta : Q \times \Sigma \mapsto Q$ is a transition function.

We can extend $\delta : Q \times \Sigma \mapsto Q$ to a transition function that tells us what state a string $s$ takes the automaton to:

$$\delta^* : Q \times \Sigma^* \mapsto Q \qquad \delta^*(q, s) = \begin{cases} q & \text{if } s = \varepsilon \\ \delta(\delta^*(q, s'), a) & \text{if } s' \in \Sigma^*, a \in \Sigma, s = s'a \end{cases}$$

String $s$ is accepted if and only if $\delta^*(q, s) \in F$, and it is rejected otherwise.

$\delta^*$ and $\delta$ are not in fact the same thing. $\delta$ takes a single valid character $\in \Sigma$, whereas $\delta^*$ takes any valid string of characters $\in \Sigma^*$. This is why $\delta : Q \times \Sigma \mapsto Q$ and $\delta^* : Q \times \Sigma^* \mapsto Q$.