

# CSC111 Lecture 5: Induction, Recursion, and Nested Lists

Hisbaan Noorani

January 25, 2020

## Contents

### 1 Additional exercises

1

**Note:** today's lecture will cover a lot of new material and we likely won't have time for exercises. Instead, we have included two additional exercises for you to complete as homework after class.

### 1 Additional exercises

1. We saw in class that the greatest common divisor function can be defined recursively over the natural numbers as follows:

$$\gcd(a, b) = \begin{cases} a, & \text{if } b = 0 \\ \gcd(b, a \% b), & \text{if } b > 0 \end{cases}$$

- (a) Modify this definition so that it is valid for all integers (including negative numbers).  
What mathematical properties of  $\gcd$  do you need to hold in order for your definition to be valid?
  - (b) Implement your new recursive definition in Python. It should no longer need the preconditions  $a \geq 0$  and  $b \geq 0$
2. In today's lecture, we learned about a formal recursive definition of a *nested list of integers*, and used this to define a function that calculates the sum of a nested list. In this exercise, you'll practice this technique with a new function: computing the number of integers in a nested list.

Let `num_ints` be a function that takes a nested list and returns the number of integers in that list.

- (a) What is `num_ints(7)`? `num_ints(111)`? `num_ints(x)`, for an arbitrary  $x \in \mathbb{Z}$ ?
- (b) Suppose we have three nested lists  $a_0, a_1, a_2$ , and we know the following:

- `num_ints(a0) = 10`
- `num_ints(a1) = 1`
- `num_ints(a2) = 3`

What is `num_ints([a0, a1, a2])`?

- (c) Let  $k \in \mathbb{N}$ , and let  $x = [a_0, a_1, \dots, a_{k-1}]$  be a nested list (where each of the  $a_i$  is also a nested list).  
Write a formula relating `num_ints(x)` to the values `num_ints(a0)`, `num_ints(a1)`,  $\dots$ , `num_ints(ak-1)`.
- (d) Write a recursive definition for the `num_ints` function:

$$\text{num\_ints}(x) = \begin{cases} \dots \\ \dots \end{cases}$$

How does this recursive definition compare with the one for `sum_nested` from lecture?

- (e) Finally, implement a recursive Python function `num_ints` that takes a nested list of integers and returns the number of integers in that list.  
How does this definition compare with the one for `sum_nested` from lecture?