CSC236 Lecture 02: Basic Induction

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Contents

1 $3^n \geq n^3$?

1.1 Scratch Work

scratch work: check for a few values of n:

$3^0 = 1 \geq 0 = 0^3$ ✓

$3^1 = 1 \geq 1 = 1^3$ ✓

$3^2 = 9 \geq 8 = 2^3$ ✓

$3^3 = 27 \geq 27 = 3^3$ ✓

$3^4 = 81 \geq 64 = 4^3$ ✓

$3^{-1} = \frac{1}{3} \geq -1 = -1^3$ ✓

$3^{2.5} =< 2.5^3 = 4^3$ $\times$

1.2 Simple Induction

$i.$ Induction on n

Let $n \in \mathbb{N}$. Assume $H(n)\ :\ 3^n \geq n^3$. I will prove $H(n+1)$ follows, that is $3^{n+1} \geq (n+1)^3$.

$$
\begin{aligned}
&3^{n+1} \\
&= 3 \cdot 3^n \\
&\geq 3 \cdot n^3 \\
&= n^3 + n^3 + n^3 \\
&\geq n^3 + 3n^2 + 9n && \text{(since } n \geq 3\text{)} \\
&\geq n^3 + 3n^2 + 3n + 6n \\
&= n^3 + 3n^2 + 3n + 1 && \text{(since } 6n \geq 1\text{)} \\
&= \ (n+1)^3
\end{aligned}
$$

And thus we have shown that, starting at $n = 3$, $H(n) \implies H(n+1)$.

ii. Base Case

$3^3 \geq 3^3$ so $P(3)$ holds.
$3^2 \geq 2^3$ so $P(2)$ holds.
$3^1 \geq 1^3$ so $P(1)$ holds.
$3^0 \geq 0^3$ so $P(0)$ holds.

And thus, we have shown $\forall n \in \mathbb{N}, 3^n \geq n^3$, as needed.