CSC236 Week 07: Autoamata and Languages

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October 21 – October 27, 2021

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1 Example — an odd machine

A machine that accepts strings over $\{0, 1\}$ with an odd number of 0s.

We will formally prove that this description can be represented by:

$$\delta^*(E, s) = \begin{cases} E & \text{only if } s \text{ has even number of 0s} \\ O & \text{only if } s \text{ has odd number of 0s} \end{cases}$$

Define Σ^* as the smallest set of strings over Σ such that:

- $\varepsilon \in \Sigma^*$
- $s \in \Sigma^* \implies s0, s0 \in \Sigma^*$

Define $P(s)$ as $\delta^*(E, s)$ correctly defines the machine.

Proof: We will show that $\forall s \in \Sigma^*, P(s)$.

- Base Case: The following implications hold vacuously:

  $$\delta^*(E, \varepsilon) = E \implies \varepsilon \text{ has an even number of 0s}$$

  $$\delta^*(E, \varepsilon) = O \implies \varepsilon \text{ has an odd number of 0s}$$

  And thus for all members of the basis, $P(s)$.

- Inductive step: Let $s \in \Sigma^*$ and assume $P(s)$. We want to show that $P(s0)$ and $P(s1)$ hold.

$P(s0)$: If $\delta^*(E, s) = E$

$$\begin{aligned} \delta^*(E, s0) &= \delta(\delta^*(E, s), 0) \\ &= \delta(E, 0) && \text{(by the current case)} \\ &= O \end{aligned}$$

So $s0$ has an odd number of 0s and $P(s0)$ follows in this case.
If $\delta^*(E, s) = O$

$$\begin{aligned} \delta^*(E, s0) &= \delta(\delta^*(E, s), 0) \\ &= \delta(O, 0) && \text{(by the current case)} \\ &= E \end{aligned}$$

So $s0$ has an even number of 0s and $P(s0)$ follows in this case.
So $P(s0)$ holds.

$P(s1)$: If $\delta^*(E, s) = E$

$$\begin{aligned} \delta^*(E, s1) &= \delta(\delta^*(E, s), 1) \\ &= \delta(E, 1) && \text{(by the current case)} \\ &= E \end{aligned}$$

So $s1$ has an even number of 0s and $P(s1)$ follows in this case.
If $\delta^*(E, s) = O$

$$\begin{aligned} \delta^*(E, s1) &= \delta(\delta^*(E, s), 1) \\ &= \delta(O, 1) && \text{(by the current case)} \\ &= O \end{aligned}$$

So $s1$ has an odd number of 0s and $P(s1$ follows in this case.
So $P(s1)$ holds.

Since $P(s0)$ and $P(s1)$ both hold, we have shown that $s \in \Sigma^* \wedge P(s) \implies P(s0) \wedge P(s1)$.

So $\forall s \in \Sigma^*, P(s)$, as needed. ■

2 More odd/even: intersection

L is the langauge of binary strings with an odd number of 0s, and at least one 0. We will devise a machine for L using product construction.