CENTRAL WASHINGTON UNIVERSITY, DEPARTMENT OF COMPUTER SCIENCE

CS471 Project5
Using NEH Heuristic to Solve Flow Shop Scheduling Problem

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1 INTRODUCTION

This project is to check how NEH heuristic works on the Flow Shop Scheduling problems. There are 120 work-time matrixes for the input, and each matrix will be processed with NEH in three situations: Flow Shop Scheduling(FSS), Flow Shop Scheduling with Blocking(FSSB), Flow Shop Scheduling with No Wait(FSSNW). Each situation with each function has a Cmax(makespan/total flow time), and a sequence of the ordering. Those can be found in the result table. The results for FSS is also compared to the optimal value found ever.

NEH heuristic is an algorithm published in 1983 by Nawaz, Enscore and Ham. It is the best heuristic solving FSS problems in the past 20 years. To use NEH, first, sort the working time matrix by each array according to their sums. Then, permutate first two arrays to see which makespan/total flow time is less and fix the order. Then insert another array, also compare their makespan time and fix the order of the arrays. After inserting all the arrays, the problem is solved.

2 EXPERIMENTAL DATAS

For each size of the result Matrix (12), 3 gantt chart is plotted for 3 objective functions– FSS, FSSB, and FSSNW. The matrixes are randomly picked.
The table for the experimental result is listed. On the table, there are datas of optimal value,

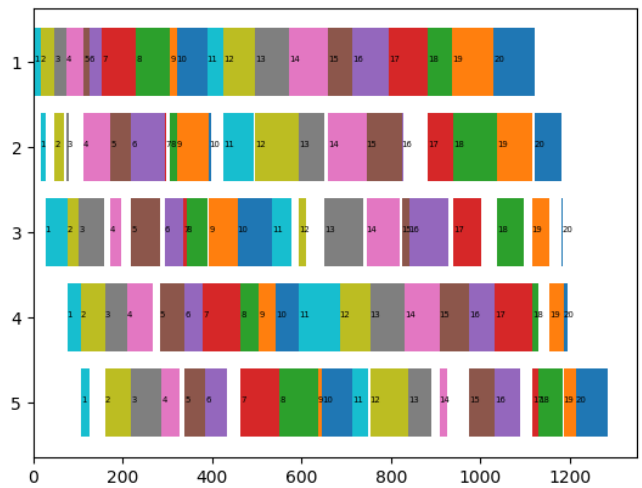

Figure 2.1: Matrix 1, FSS

time, function calls,sequence for each situation of 120 matrixs. Also, the optimal value of FSS is compared with the actual optimal value, the percentage of the difference is recorded.

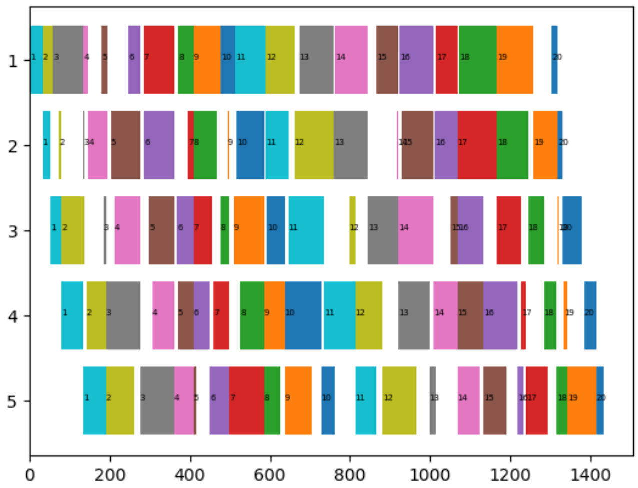

Figure 2.2: Matrix 1, FSSB

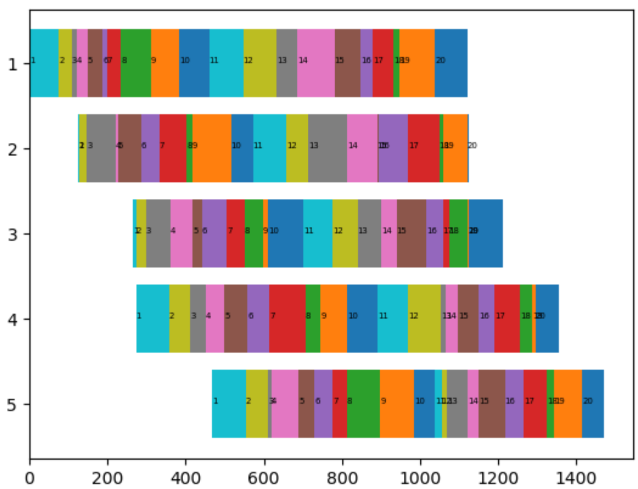

Figure 2.3: Matrix 1, FSSNW

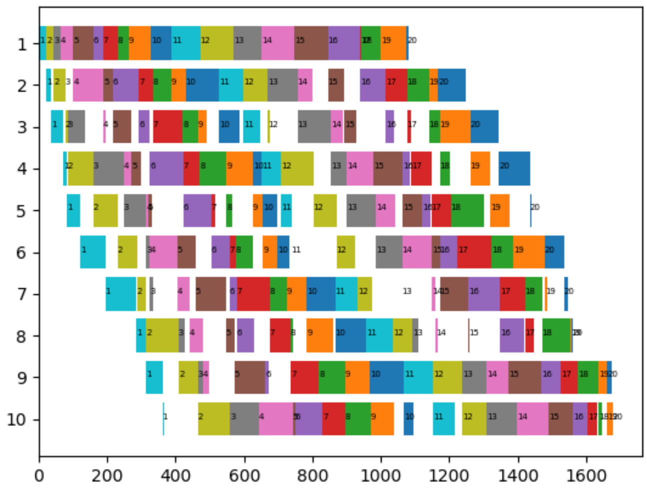

Figure 2.4: Matrix 11, FSS

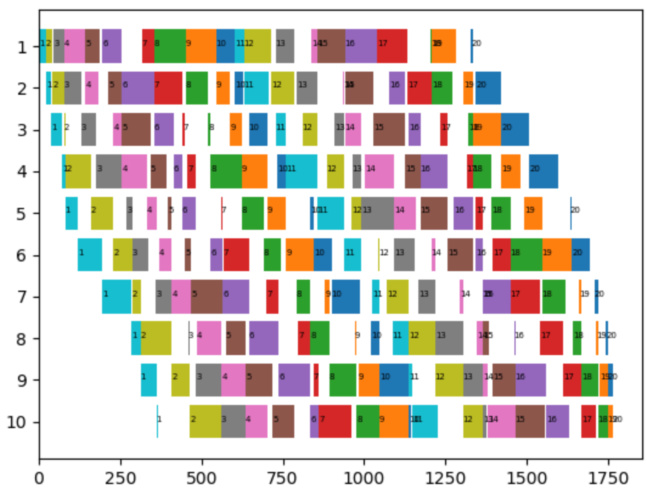

Figure 2.5: Matrix 11, FSSB

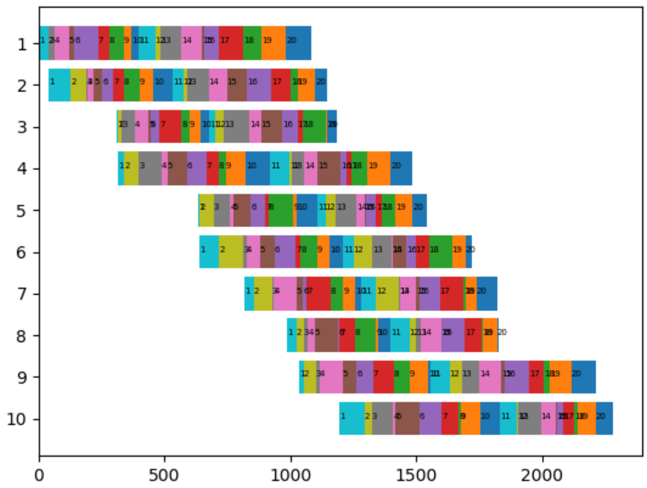

Figure 2.6: Matrix 11, FSSBW

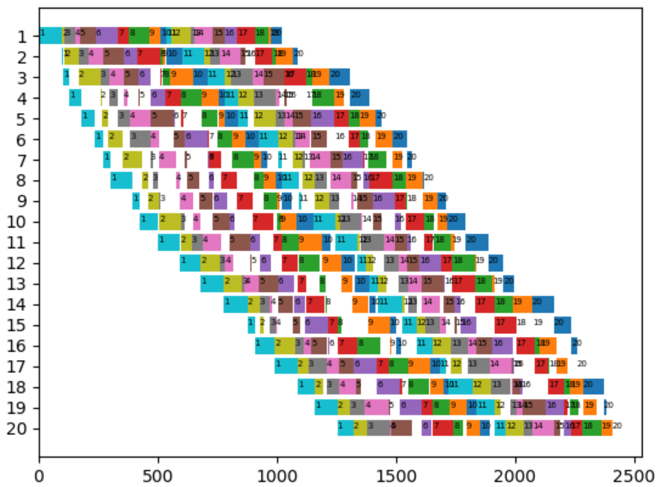

Figure 2.7: Matrix 23, FSS

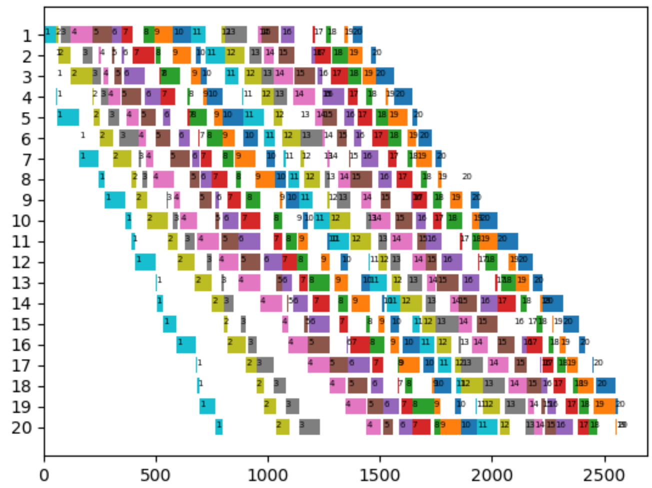

Figure 2.8: Matrix 24, FSSB

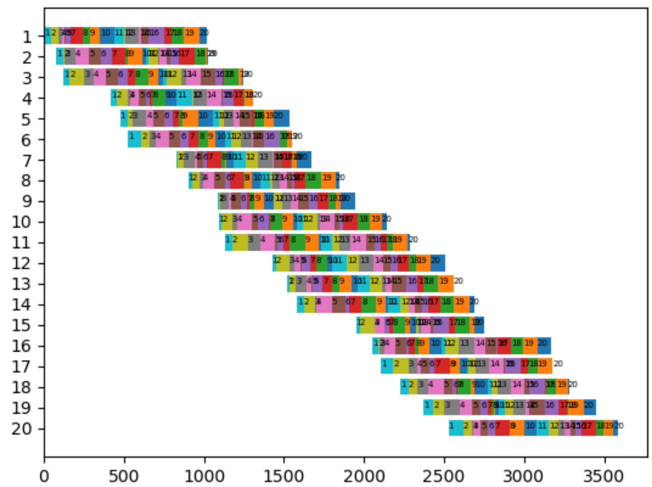

Figure 2.9: Matrix 23, FSSNW

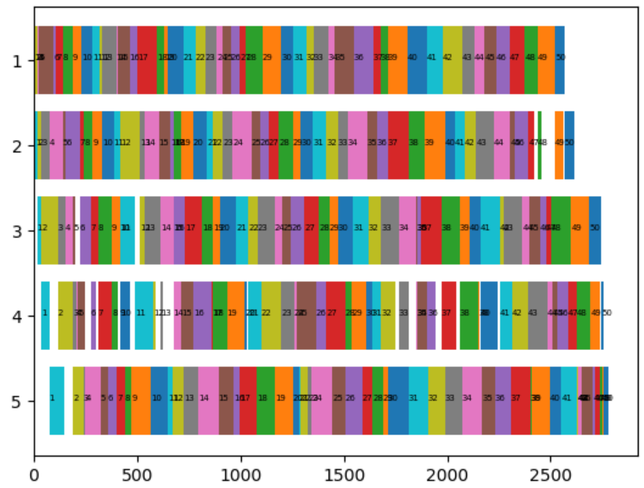

Figure 2.10: Matrix 34, FSS

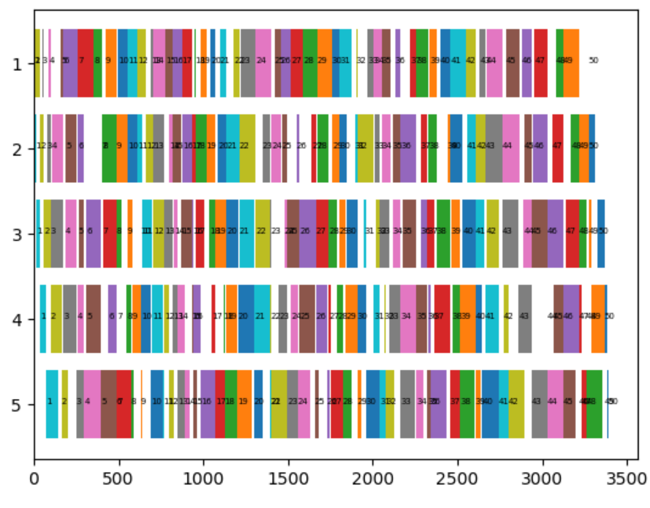

Figure 2.11: Matrix 34, FSSB

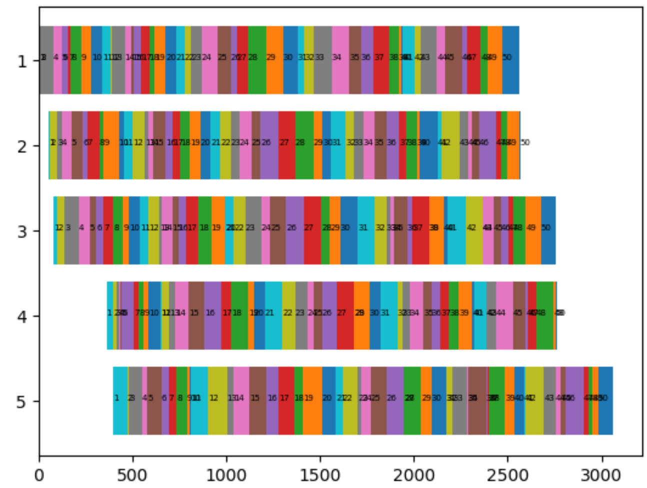

Figure 2.12: Matrix 34, FSSNW

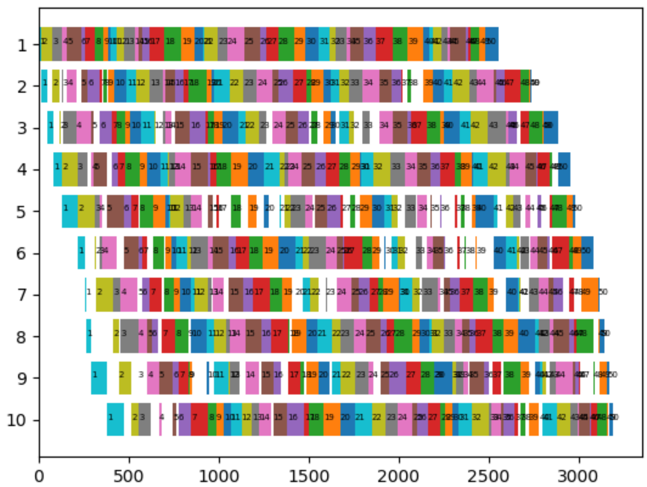

Figure 2.13: Matrix 48, FSS

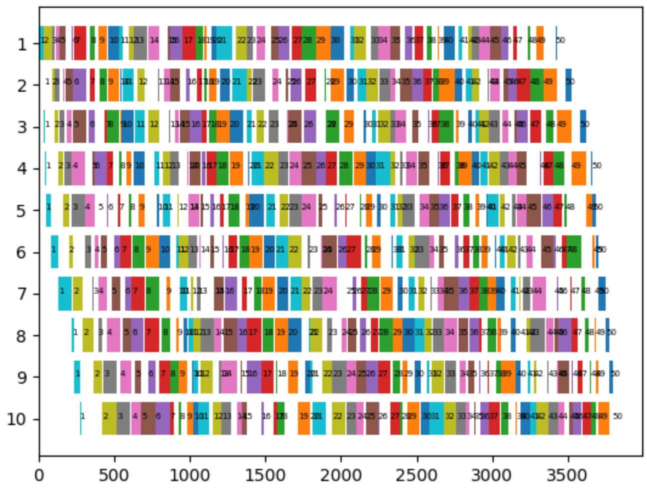

Figure 2.14: Matrix 48, FSSB

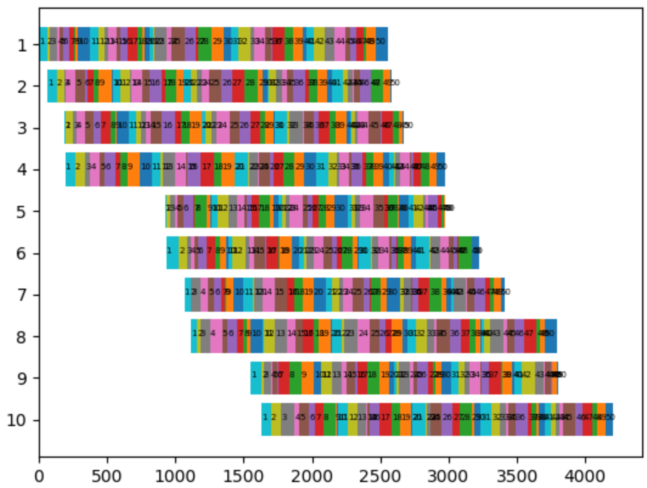

Figure 2.15: Matrix 49, FSSNW

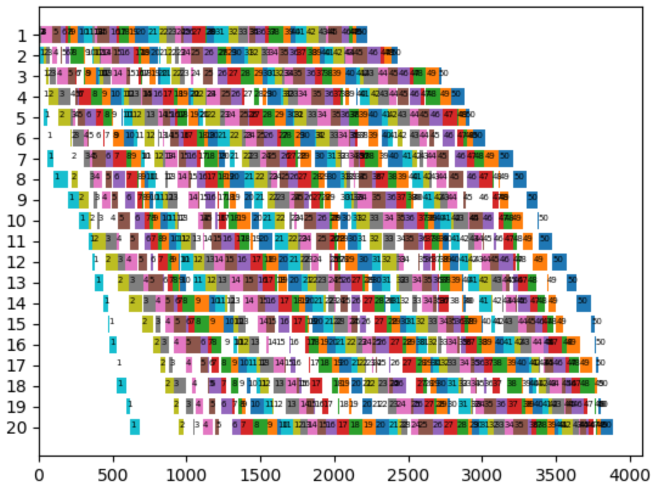

Figure 2.16: Matrix 53, FSS

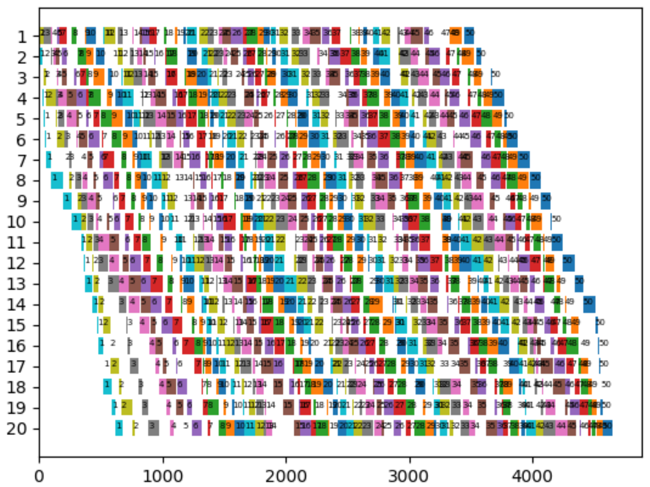

Figure 2.17: Matrix 53, FSSB

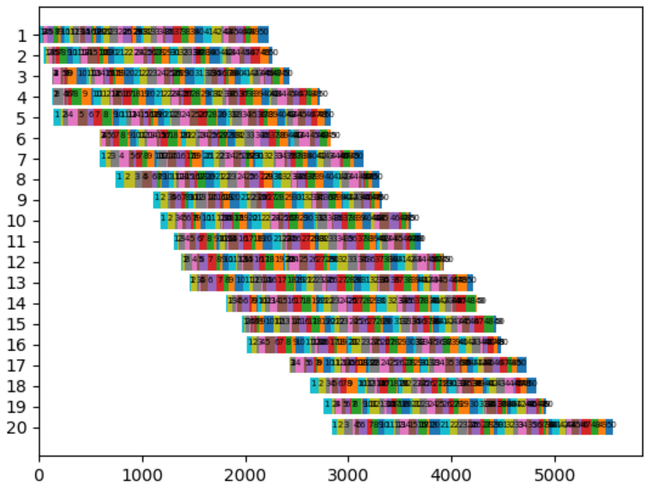

Figure 2.18: Matrix 53, FSSNW

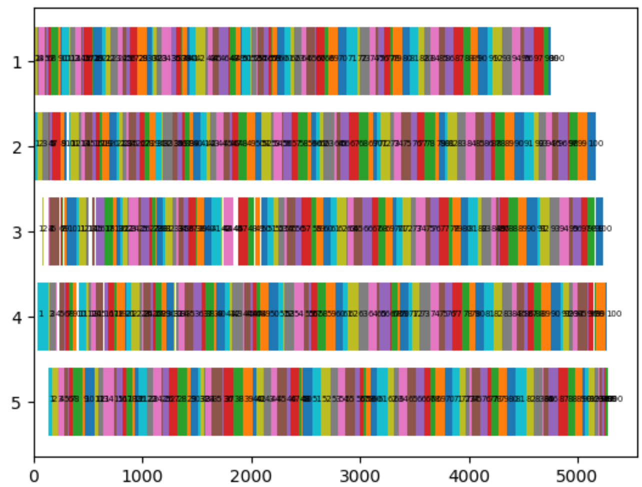

Figure 2.19: Matix 62, FSS

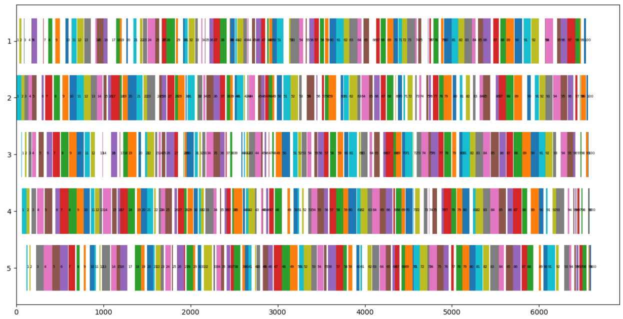

Figure 2.20: Matix 62, FSSB

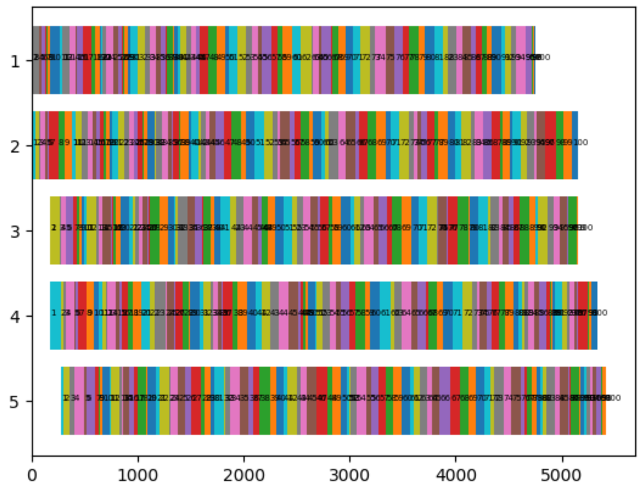

Figure 2.21: Matix 62, FSSNW

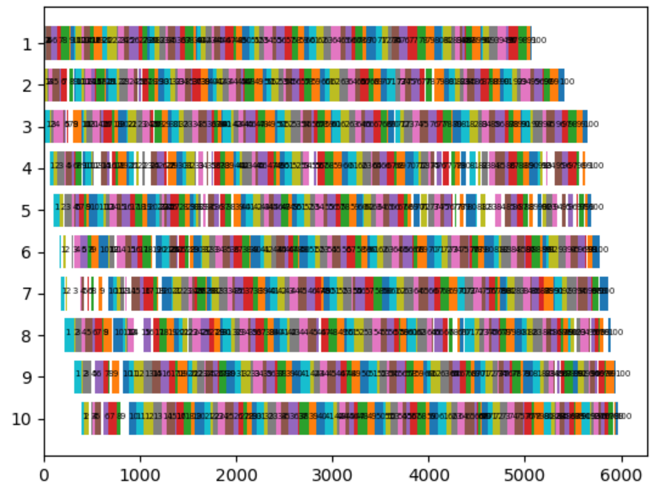

Figure 2.22: Matrix 74, FSS

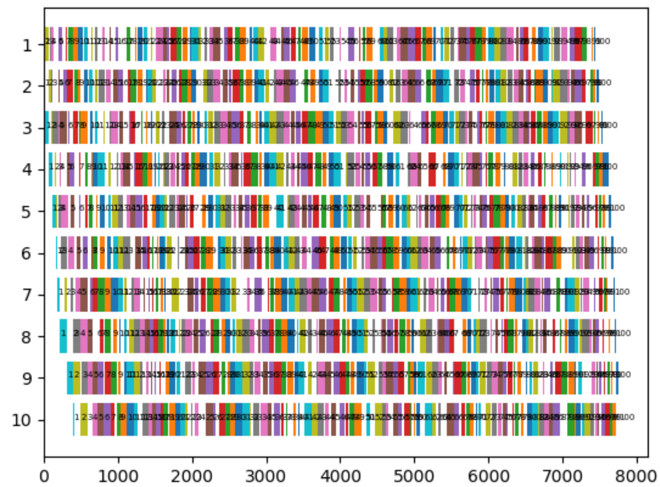

Figure 2.23: Matrix 74, FSSB

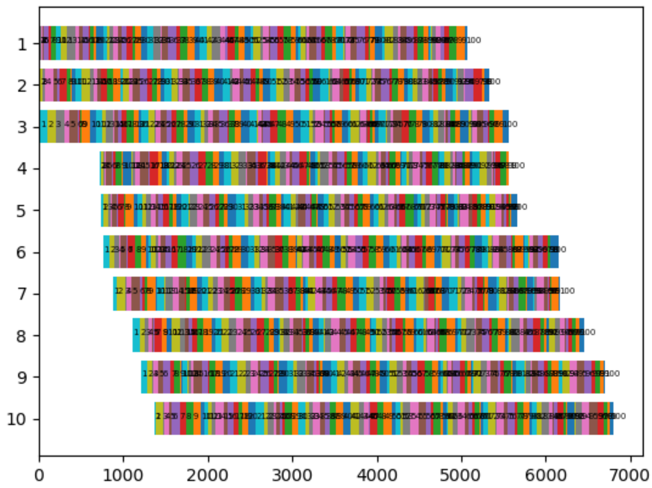

Figure 2.24: Matrix 74, FSSNW

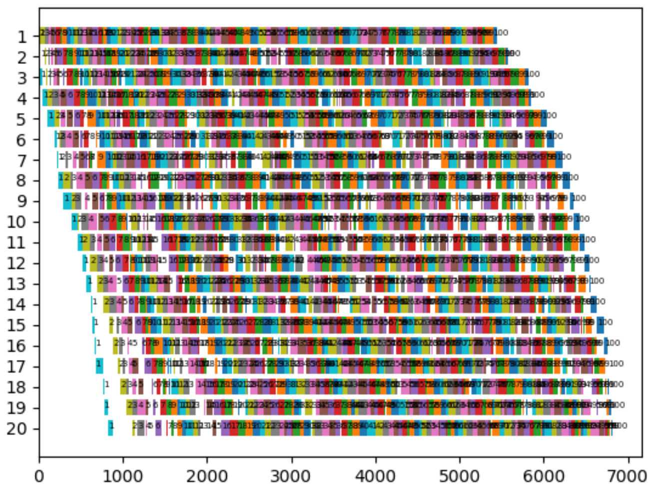

Figure 2.25: Matrix 88, FSS

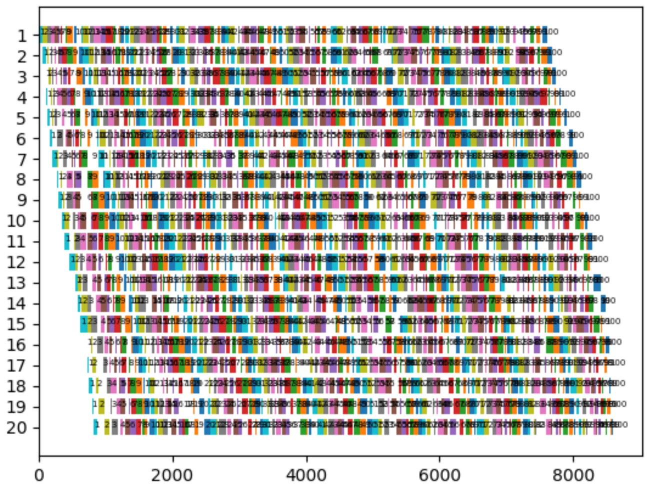

Figure 2.26: Matrix 88, FSSB

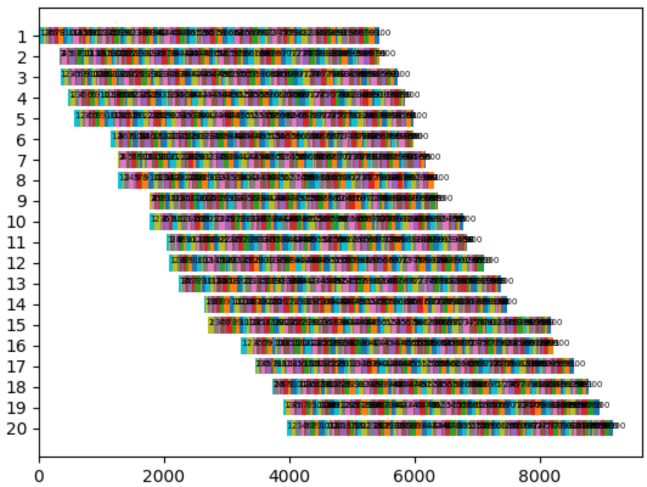

Figure 2.27: Matrix 88, FSSNW

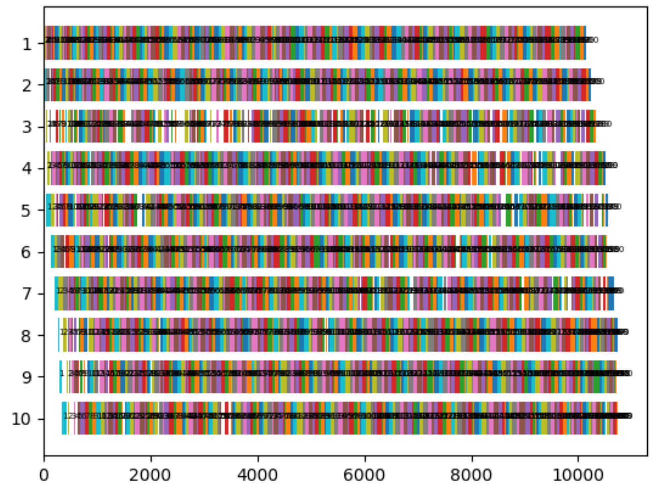

Figure 2.28: Matrix 92, FSS

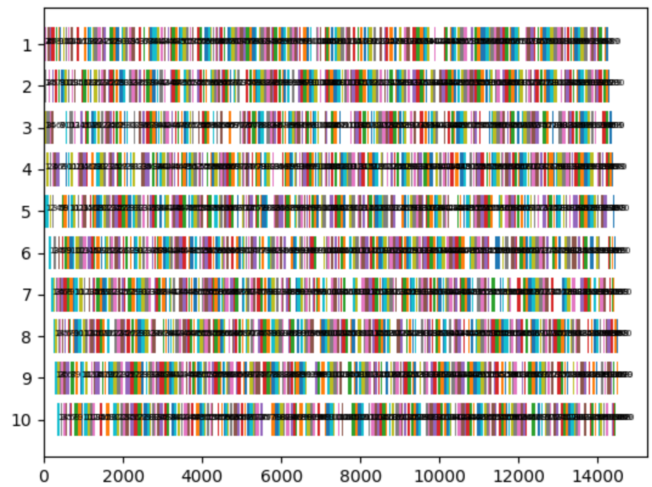

Figure 2.29: Matrix 92, FSSB

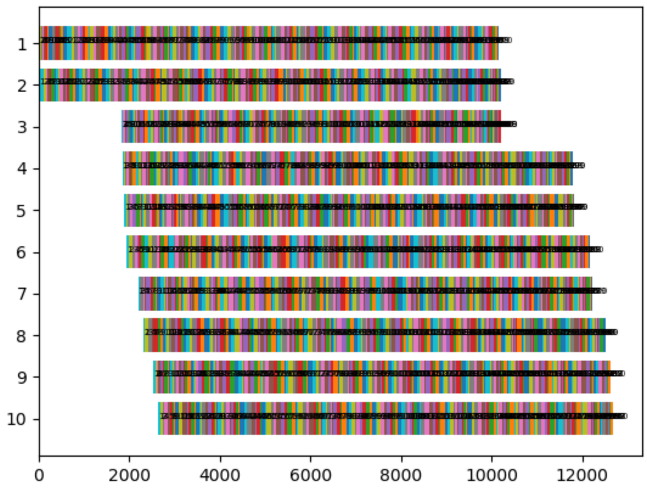

Figure 2.30: Matrix 92, FSSNW

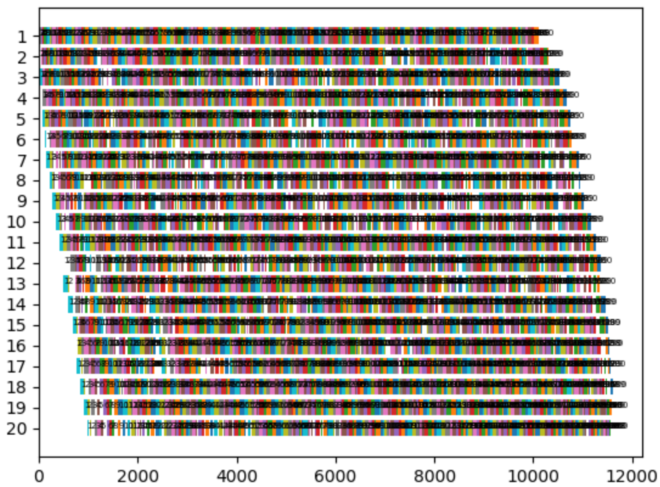

Figure 2.31: Matrix 101, FSS

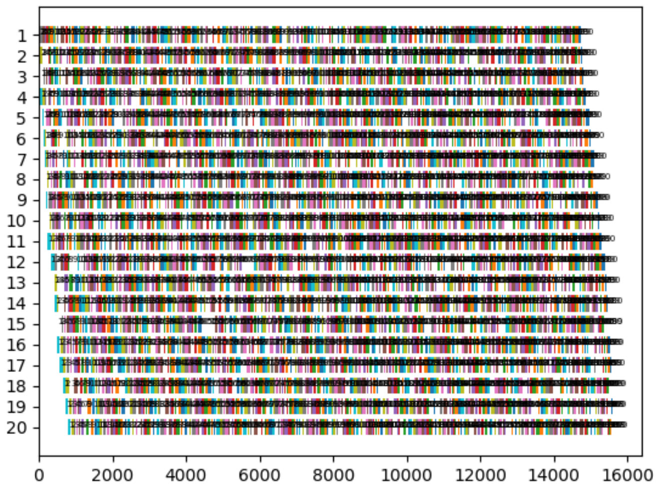

Figure 2.32: Matrix 101, FSSB

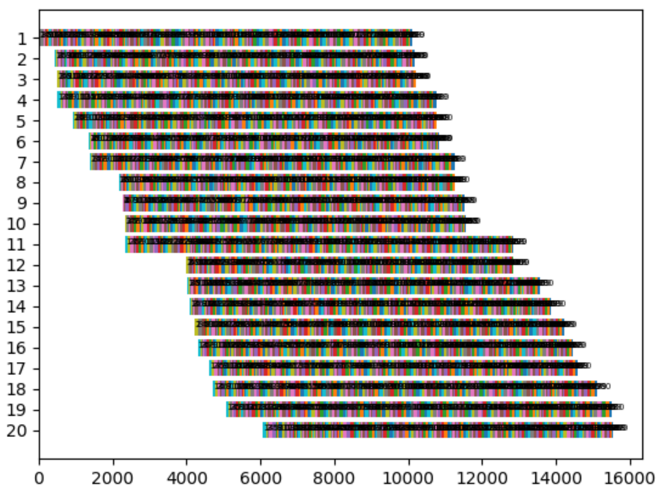

Figure 2.33: Matrix 101, FSSNW

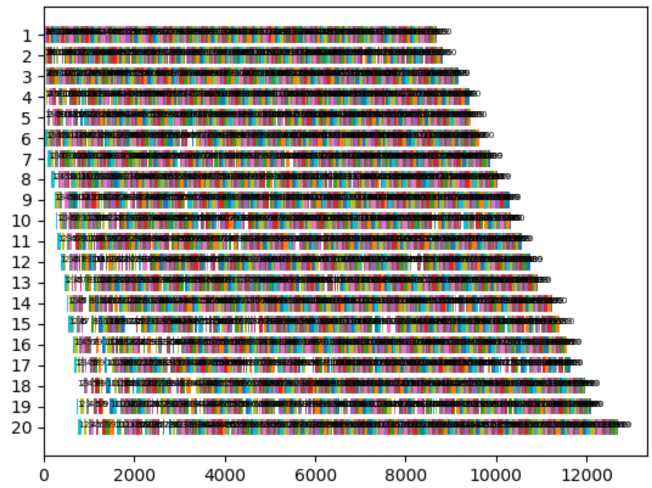

Figure 2.34: Matrix 114, FSS

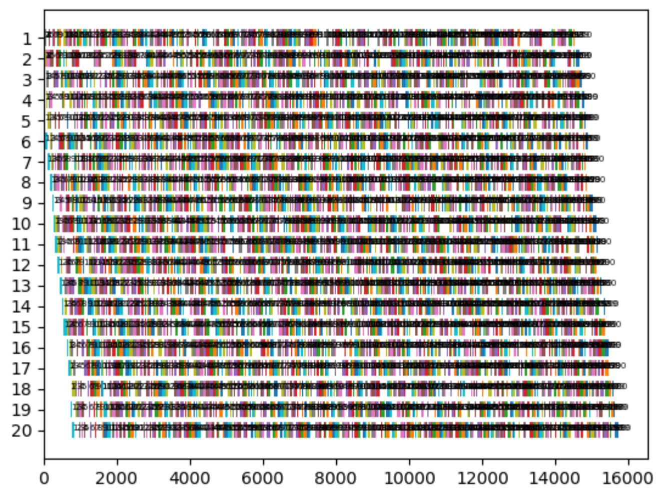

Figure 2.35: Matrix 114, FSSB

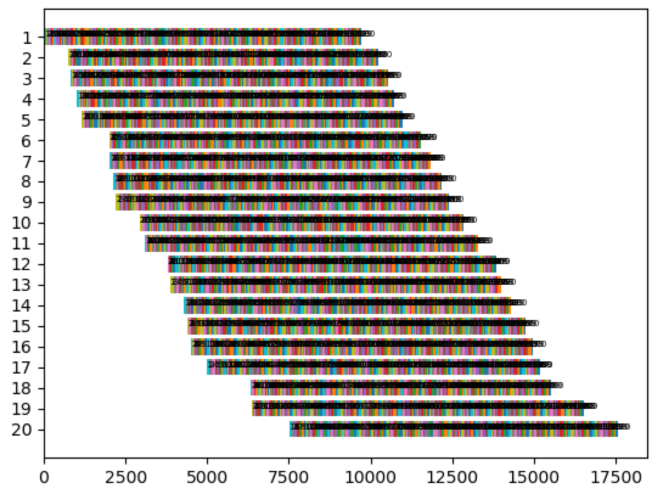

Figure 2.36: Matrix 114, FSSNW

Few optimum solutions

	20	5																			
ta001	1278	3	17	15	8	9	6	5	14	16	7	11	13	18	19	1	4	2	10	20	12
ta002	1359	6	10	17	7	18	15	12	3	1	9	11	8	19	4	13	16	14	5	20	2
ta003	1081	3	4	16	14	13	19	18	20	12	11	10	5	1	7	17	9	6	8	15	2
ta004	1293	13	17	19	9	16	11	15	2	12	20	7	4	10	1	5	8	6	14	3	18
ta005	1235	5	12	3	13	19	9	4	6	17	16	2	10	11	15	14	7	18	1	20	8
ta006	1195	11	14	13	16	12	17	6	15	19	20	4	7	9	1	18	10	8	2	5	3
ta008	1206	17	2	16	9	19	12	18	3	8	14	5	7	13	10	1	20	4	15	6	11
ta009	1230	4	12	10	2	8	1	11	18	17	6	20	3	15	7	16	13	19	14	9	5
ta010	1108	11	7	6	10	16	8	12	19	18	13	14	1	2	5	20	3	15	17	4	9

Figure 2.37: optimal and sequence of 1-10 matrix from the website

3 ANALYSIS

Comparing 3 objective functions for the same matrix, we can see that FSSB always have more unused spaces than FSS, and FSSNW has more unused spaces in the front of machines. The experiment result table shows that in the FSS situation, the NEH heuristic generates optimal values close to the actual optimal value. The difference between them is about 3%. However, none of the optimal value from NEH heuristic is equal to the actual value. It's normal in optimization problems. The actual optimal value can only be retrieved from Greed Algorithm, which needs to permutate all the possible order and sharing an operation time of O(n!), n is the number of jobs. If we use NEH heuristic to optimize an FSS problem, the worst time is O(n3). Although it still requires a lot of time when the number of jobs increases, NEH saves a huge amount of time than the greedy algorithm and still gets a satisfying result. Also, comparing Figure 1.37 and Figure 1.38, the sequnces of optimized matrixes from result and actual are similar each array. That means NEH huresitic behaves good enough. The time for lower matrixes are low. So using NEH to optimize small FSS problems are recommended. The time increase a lot when jobs increases. So, to optimize big FSS problems requires large amount of time, but the output is still satisfying because the percentage is not seems to reduce when the Matrixes becomes larger.

Table 2.1: Experiment Result 1-16

No.	F	Call	t	Cmax	Op	Percentage	Seqence
1	1	210	7.0	1286	1278	0.00622083981337481	3 17 9 8 15 14 11 16 13 19 6 4 5 18 1 2 10 7 20 12
1	2	210	3.0	1435			17 9 11 15 13 14 16 8 19 6 5 4 18 2 1 10 7 20 12 3
1	3	210	3.0	6284			11 17 13 9 8 15 6 16 4 5 18 10 7 20 19 14 1 3 12 2
2	1	210	2.0	1365	1359	0.0043956043956044	19 14 6 20 3 10 17 9 18 12 7 15 1 16 4 2 13 5 11 8
2	2	210	1.0	1461			20 9 12 7 1 13 5 11 16 3 19 4 17 10 6 15 2 8 18 14
2	3	210	4.0	5762			3 17 20 15 12 2 7 9 1 5 13 16 11 6 8 18 4 19 14 10
3	1	210	1.0	1159	1081	0.0672993960310613	16 3 20 18 7 1 12 10 5 2 9 4 19 14 17 6 13 11 8 15
3	2	210	2.0	1353			16 19 13 4 8 10 3 20 18 11 14 7 1 12 5 17 9 6 15 2
3	3	210	2.0	5585			3 16 14 19 8 18 10 20 5 12 7 17 6 11 2 9 1 4 15 13
4	1	210	6.0	1325	1293	0.0241509433962264	13 16 9 17 11 19 10 6 7 15 1 12 5 20 2 3 8 14 4 18
4	2	210	6.0	1552			13 9 16 17 4 8 20 11 15 10 2 7 1 5 19 12 14 3 6 18
4	3	210	4.0	6228			13 16 2 11 3 9 10 14 7 1 5 12 15 8 17 19 6 20 4 18
5	1	210	2.0	1305	1235	0.053639846743295	5 3 12 10 20 19 9 18 7 17 15 13 4 16 6 2 14 11 8 1
5	2	210	1.0	1398			3 12 10 19 9 20 18 7 15 4 13 17 16 6 2 14 11 8 5 1
5	3	210	1.0	6325			3 20 8 5 10 7 15 13 17 4 9 16 2 11 18 14 12 1 19 6
6	1	210	1.0	1228	1195	0.0268729641693811	11 5 20 13 8 17 6 16 1 7 12 14 2 18 10 15 9 4 19 3
6	2	210	1.0	1464			14 2 20 11 13 6 8 1 12 5 17 18 10 16 7 15 9 4 19 3
6	3	210	1.0	6242			11 13 20 5 8 17 1 16 15 9 6 19 3 12 7 4 18 10 2 14
7	1	210	0.0	1279	1234	0.0351837372947615	5 16 15 11 1 2 14 20 13 8 6 9 7 17 19 12 4 3 18 10
7	2	210	1.0	1450			5 16 1 9 11 20 8 6 3 18 7 4 2 13 17 19 15 12 14 10
7	3	210	1.0	5363			10 15 13 11 8 14 2 19 12 6 1 20 3 5 16 7 4 18 17 9
8	1	210	2.0	1235	1206	0.0234817813765182	17 12 9 2 14 4 10 18 16 19 7 8 6 5 20 15 13 1 3 11
8	2	210	0.0	1456			12 17 6 9 1 3 2 14 5 16 18 19 10 20 8 7 13 4 15 11
8	3	210	2.0	5976			17 11 12 2 6 4 10 14 9 5 16 19 8 20 7 13 15 3 18 1
9	1	210	1.0	1291	1230	0.0472501936483346	4 2 20 18 17 15 1 10 7 9 16 13 8 3 5 12 6 14 11 19
9	2	210	1.0	1457			4 10 2 15 12 16 7 20 18 17 3 8 1 6 19 14 11 9 13 5
9	3	210	1.0	6216			4 15 16 12 10 7 17 20 9 11 1 6 18 3 5 8 14 13 19 2
10	1	210	1.0	1151	1108	0.0373588184187663	7 19 11 12 16 6 1 13 10 15 2 8 3 4 18 14 17 5 20 9
10	2	210	0.0	1349			11 7 19 12 15 16 6 10 13 1 2 8 3 4 18 14 17 9 5 20
10	3	210	1.0	5536			11 7 12 16 1 19 20 8 13 10 3 14 6 5 2 9 18 15 17 4
11	1	210	2.0	1680	1582	0.0583333333333333	18 5 2 17 3 6 12 9 15 10 20 13 8 14 19 11 4 7 1 16
11	2	210	1.0	1767			18 5 9 15 12 10 17 13 14 3 6 20 7 2 8 19 11 4 1 16
11	3	210	3.0	16299			17 4 2 3 5 14 12 7 9 6 15 18 8 20 16 10 11 1 13 19
12	1	210	1.0	1786	1659	0.0711086226203807	17 9 12 5 15 7 11 3 10 1 20 2 16 4 19 6 13 8 14 18
12	2	210	1.0	1903			12 13 17 9 19 5 8 4 20 11 15 7 10 1 2 16 14 3 6 18
12	3	210	1.0	15283			17 20 18 7 2 12 11 15 5 16 6 1 10 13 8 14 9 19 3 4
13	1	210	1.0	1557	1496	0.0391779062299294	4 9 16 7 2 5 12 13 11 15 1 20 6 14 17 10 3 18 19 8
13	2	210	1.0	1772			4 7 9 16 12 13 18 5 1 2 11 14 6 17 15 10 20 19 3 8
13	3	210	1.0	15340			19 3 1 7 9 16 10 5 2 12 13 8 11 20 14 6 18 15 17 4
14	1	210	1.0	1450	1377	0.0503448275862069	3 9 20 18 13 4 16 1 10 15 2 11 7 6 19 8 12 14 17 5
14	2	210	1.0	1625			18 11 20 3 12 9 6 4 16 1 10 2 15 5 13 7 14 17 8 19
14	3	210	1.0	13316			18 11 3 12 20 16 19 15 5 4 6 14 10 1 2 7 13 9 17 8
15	1	210	1.0	1502	1419	0.0552596537949401	16 8 4 20 18 14 15 13 9 6 1 7 3 17 2 5 19 12 11 10
15	2	210	3.0	1722			16 8 18 20 14 15 1 7 2 19 9 6 13 10 12 4 5 3 17 11
15	3	210	8.0	12649			16 4 17 18 5 6 7 15 8 12 19 13 1 2 20 9 10 3 14 11
16	1	210	1.0	1453	1397	0.0385409497591191	20 8 19 18 16 11 14 3 6 15 13 4 5 7 12 17 10 9 2 1
16	2	210	1.0	1679			18 16 8 19 14 6 3 7 13 4 5 17 11 15 10 20 9 2 1 12
16	3	210	1.0	14295			10 9 8 17 11 4 18 14 20 5 3 16 7 1 12 19 6 15 2 13

Table 2.2: Experimental Result 17-30

17	1	210	1	1562	1484	0.0499359795134443	19 6 7 10 3 17 1 4 8 20 18 9 2 5 16 14 15 13 11 12
17	2	210	0	1738			19 4 6 1 9 17 16 8 18 20 3 2 7 14 11 5 13 12 10 15
17	3	210	1	13498			7 4 19 10 17 16 3 9 6 15 2 11 14 12 5 13 18 20 8 1
18	1	210	1	1609	1538	0.0441267868241144	8 7 18 20 14 4 17 2 5 9 19 3 6 11 1 13 15 10 16 12
18	2	210	0	1814			7 17 8 18 14 2 1 9 6 11 10 3 13 20 19 15 5 4 16 12
18	3	210	0	15501			7 20 4 17 13 18 2 9 12 6 11 1 16 19 15 14 8 3 5 10
19	1	210	1	1647	1593	0.0327868852459016	14 12 8 17 4 11 2 20 1 19 16 3 13 18 7 15 10 5 6 9
19	2	210	0	1832			14 12 8 17 4 2 20 19 3 1 6 7 15 13 18 16 10 5 11 9
19	3	210	3	14415			8 13 6 9 17 14 2 5 12 7 10 19 3 4 20 15 18 16 11 1
20	1	210	1	1653	1591	0.0375075620084694	5 13 17 9 19 4 7 8 16 6 20 2 10 3 18 1 15 14 11 12
20	2	210	0	1854			5 16 14 13 19 4 17 9 7 8 2 20 10 18 15 6 1 11 12 3
20	3	210	1	15185			12 19 14 5 17 4 9 6 2 7 10 13 8 1 18 20 11 15 16 3
21	1	210	1	2410	2297	0.0468879668049793	16 15 10 8 9 12 13 11 5 1 20 14 17 2 18 6 7 19 3 4
21	2	210	0	2530			16 18 5 14 8 10 3 15 1 12 13 9 11 20 2 6 4 17 7 19
21	3	210	1	45856			15 4 20 18 8 9 6 14 7 3 17 11 2 13 12 1 10 16 5 19
22	1	210	1	2150	2099	0.0237209302325581	18 3 11 4 5 13 1 12 16 19 15 6 14 10 20 17 7 9 8 2
22	2	210	1	2285			18 3 11 4 15 12 6 16 14 13 5 19 8 10 20 1 7 17 9 2
22	3	210	1	42543			17 15 1 18 16 11 6 14 10 4 3 2 12 8 20 5 19 13 7 9
23	1	210	1	2411	2326	0.0352550808793032	5 19 4 20 11 13 1 9 16 8 15 17 18 3 12 2 10 14 6 7
23	2	210	1	2564			2 19 4 5 9 16 1 12 3 17 11 15 13 18 10 14 6 8 20 7
23	3	210	1	44749			4 16 20 15 6 8 3 7 11 17 14 19 9 18 12 13 1 10 5 2
24	1	210	1	2262	2223	0.0172413793103448	14 3 18 5 2 8 12 4 6 11 20 15 13 1 7 19 16 10 17 9
24	2	210	1	2399			14 3 18 8 6 20 2 4 12 15 1 7 5 13 19 16 10 9 17 11
24	3	210	1	42010			14 8 3 4 5 2 20 18 7 15 12 10 17 9 19 1 13 6 16 11
25	1	210	0	2397	2291	0.0442219440967877	10 9 19 11 15 13 5 2 3 18 17 4 20 12 14 1 16 6 8 7
25	2	210	1	2538			10 5 9 4 2 7 15 3 17 20 19 1 13 11 12 16 18 14 6 8
25	3	210	2	47793			14 16 13 20 19 6 8 3 10 5 17 2 1 15 12 11 7 9 4 18
26	1	210	1	2349	2226	0.0523627075351213	18 6 8 20 16 9 17 4 13 15 10 2 14 5 1 3 7 12 11 19
26	2	210	2	2472			18 6 8 20 1 2 13 16 9 5 17 15 10 4 3 7 12 14 11 19
26	3	210	2	42584			13 14 4 6 11 15 18 10 7 8 1 12 5 9 2 17 3 19 20 16
27	1	210	1	2362	2273	0.0376799322607959	10 12 16 14 5 19 18 6 7 17 4 2 11 15 20 8 9 3 1 13
27	2	210	1	2498			17 5 12 14 4 10 6 16 18 2 11 8 19 20 1 3 9 7 15 13
27	3	210	1	44607			4 10 8 6 16 11 17 9 7 15 3 14 19 12 13 2 20 5 1 18
28	1	210	1	2249	2200	0.0217874610938195	4 2 16 10 20 5 1 14 6 7 3 11 17 19 13 12 8 18 15 9
28	2	210	0	2411			4 10 14 11 7 20 2 5 13 17 3 12 19 8 6 18 15 16 1 9
28	3	210	1	45279			15 6 13 2 20 10 7 18 14 17 5 12 3 4 19 8 11 16 9 1
29	1	210	8	2320	2237	0.0357758620689655	1 17 11 8 2 13 14 18 7 3 16 9 10 6 15 4 12 19 20 5
29	2	210	1	2421			1 8 17 13 6 11 2 14 4 7 18 10 12 3 20 9 15 16 19 5
29	3	210	1	43521			13 1 8 16 15 20 11 18 17 2 6 14 10 5 9 12 3 7 4 19
30	1	210	1	2277	2178	0.0434782608695652	6 3 17 8 19 15 12 9 10 16 1 2 13 5 7 18 4 11 20 14
30	2	210	1	2425			6 3 17 8 7 19 15 12 5 9 10 16 2 1 11 13 18 4 20 14

Table 2.3: Experimental Result 31-47 (without sequence)

31	1	1275	6	2733	2724	0.00329308452250274
31	2	1275	7	3321		
31	3	1275	8	14299		
32	1	1275	7	2843	2834	0.00316567006683081
32	2	1275	5	3550		
32	3	1275	7	13927		
33	1	1275	11	2640	2621	0.0071969696969697
33	2	1275	10	3223		
33	3	1275	13	12800		
34	1	1275	13	2782	2751	0.0111430625449317
34	2	1275	5	3393		
34	3	1275	5	13712		
35	1	1275	7	2868	2863	0.00174337517433752
35	2	1275	4	3431		
35	3	1275	6	14621		
36	1	1275	5	2850	2829	0.00736842105263158
36	2	1275	5	3413		
36	3	1275	10	13947		
37	1	1275	5	2776	2725	0.018371757925072
37	2	1275	6	3262		
37	3	1275	9	14135		
38	1	1275	5	2703	2683	0.00739918608953015
38	2	1275	5	3328		
38	3	1275	7	13495		
39	1	1275	5	2574	2552	0.00854700854700855
39	2	1275	7	3118		
39	3	1275	7	12748		
40	1	1275	5	2789	2782	0.00250986016493367
40	2	1275	4	3451		
40	3	1275	6	13459		
41	1	1275	5	3146	2991	0.0492689129052765
41	2	1275	6	3967		
41	3	1275	8	30999		
42	1	1275	10	3032	2867	0.0544195250659631
42	2	1275	11	3795		
42	3	1275	12	29606		
43	1	1275	13	3021	2839	0.0602449520026481
43	2	1275	11	3819		
43	3	1275	15	29175		
44	1	1275	12	3198	3063	0.0422138836772983
44	2	1275	8	3938		
44	3	1275	12	30451		
45	1	1275	10	3128	2976	0.0485933503836317
45	2	1275	5	3897		
45	3	1275	9	29111		
46	1	1275	9	3178	3006	0.0541220893643801
46	2	1275	5	3828		
46	3	1275	10	33665		
47	1	1275	7	3277	3093	0.0561489166920964
47	2	1275	8	3960		
47	3	1275	8	32192		

Table 2.4: Experimental Result 48-65 (without sequence)

48	1	1275	6	3193	3037	0.048856874412778
48	2	1275	6	3802		
48	3	1275	10	32194		
49	1	1275	9	3002	2897	0.0349766822118588
49	2	1275	9	3887		
49	3	1275	8	30299		
50	1	1275	6	3257	3065	0.0589499539453485
50	2	1275	6	3943		
50	3	1275	10	32652		
51	1	1275	19	4038	3850	0.0465577018325904
51	2	1275	9	4850		
51	3	1275	15	81695		
52	1	1275	11	3921	3704	0.0553430247385871
52	2	1275	7	4597		
52	3	1275	13	84963		
53	1	1275	9	3890	3640	0.06426735218509
53	2	1275	6	4653		
53	3	1275	13	73715		
54	1	1275	10	3987	3723	0.0662151993980436
54	2	1275	6	4714		
54	3	1275	14	83445		
55	1	1275	8	3822	3611	0.0552066980638409
55	2	1275	9	4475		
55	3	1275	13	76327		
56	1	1275	11	3918	3681	0.060490045941807
56	2	1275	7	4566		
56	3	1275	14	86122		
57	1	1275	12	3952	3704	0.062753036437247
57	2	1275	7	4575		
57	3	1275	15	85334		
58	1	1275	9	3955	3691	0.0667509481668774
58	2	1275	6	4721		
58	3	1275	13	76897		
59	1	1275	9	3952	3743	0.0528846153846154
59	2	1275	8	4575		
59	3	1275	14	82990		
60	1	1275	10	4016	3756	0.0647410358565737
60	2	1275	6	4802		
60	3	1275	15	85765		
61	1	5050	39	5567	5493	0.0132926172085504
61	2	5050	36	6798		
61	3	5050	48	28136		
62	1	5050	36	5284	5268	0.00302800908402725
62	2	5050	36	6596		
62	3	5050	39	25810		
63	1	5050	39	5241	5175	0.0125930165998855
63	2	5050	38	6363		
63	3	5050	41	25392		
64	1	5050	39	5023	5014	0.00179175791359745
64	2	5050	37	6216		
64	3	5050	45	25389		
65	1	5050	39	5266	5250	0.00303835928598557
65	2	5050	36	6455		
65	3	5050	50	26781		

Table 2.5: Experimental Result 66-82 (without sequence)

66	1	5050	37	5139	5135	0.000778361548939482
66	2	5050	34	6404		
66	3	5050	42	25717		
67	1	5050	37	5266	5246	0.00379794910748196
67	2	5050	36	6452		
67	3	5050	64	27468		
68	1	5050	36	5129	5094	0.00682394228894521
68	2	5050	35	6277		
68	3	5050	43	24893		
69	1	5050	38	5489	5448	0.00746948442339224
69	2	5050	37	6672		
69	3	5050	40	26791		
70	1	5050	34	5354	5322	0.00597683974598431
70	2	5050	35	6681		
70	3	5050	44	26582		
71	1	5050	50	5848	5770	0.0133378932968536
71	2	5050	44	7607		
71	3	5050	70	60045		
72	1	5050	46	5444	5349	0.0174504041146216
72	2	5050	43	7247		
72	3	5050	76	54792		
73	1	5050	50	5801	5676	0.0215480089639717
73	2	5050	39	7463		
73	3	5050	128	60196		
74	1	5050	53	5973	5781	0.0321446509291813
74	2	5050	42	7761		
74	3	5050	70	59465		
75	1	5050	47	5669	5467	0.0356323866643147
75	2	5050	43	7398		
75	3	5050	77	57332		
76	1	5050	55	5373	5303	0.0130281034803648
76	2	5050	59	7180		
76	3	5050	83	56389		
77	1	5050	58	5739	5595	0.0250914793518035
77	2	5050	58	7347		
77	3	5050	87	59861		
78	1	5050	52	5790	5617	0.0298791018998273
78	2	5050	55	7369		
78	3	5050	127	60700		
79	1	5050	56	5983	5871	0.018719705833194
79	2	5050	44	7568		
79	3	5050	86	63969		
80	1	5050	67	5927	5845	0.0138349924076261
80	2	5050	41	7561		
80	3	5050	72	60561		
81	1	5050	75	6661	6202	0.0689085722864435
81	2	5050	65	8406		
81	3	5050	153	140658		
82	1	5050	93	6446	6183	0.0408004964318957
82	2	5050	58	8342		
82	3	5050	126	136385		

Table 2.6: Experimental Result 83-100 (without sequence)

83	1	5050	102	6691	6271	0.0627708862651323
83	2	5050	68	8286		
83	3	5050	158	140906		
84	1	5050	78	6554	6269	0.0434848947207812
84	2	5050	70	8332		
84	3	5050	148	146955		
85	1	5050	98	6692	6314	0.0564853556485356
85	2	5050	86	8222		
85	3	5050	196	157682		
86	1	5050	98	6761	6364	0.0587191243898832
86	2	5050	96	8336		
86	3	5050	186	136958		
87	1	5050	106	6659	6268	0.058717525153927
87	2	5050	99	8421		
87	3	5050	155	140982		
88	1	5050	95	6824	6401	0.0619871043376319
88	2	5050	73	8602		
88	3	5050	157	140755		
89	1	5050	88	6664	6275	0.0583733493397359
89	2	5050	72	8347		
89	3	5050	151	141297		
90	1	5050	95	6671	6434	0.0355269075101184
90	2	5050	54	8523		
90	3	5050	140	142717		
91	1	20100	450	10992	10862	0.0118267831149927
91	2	20100	394	14458		
91	3	20100	902	111992		
92	1	20100	616	10751	10480	0.0252069574923263
92	2	20100	545	14529		
92	3	20100	857	116426		
93	1	20100	374	11027	10922	0.00952208216196608
93	2	20100	313	14462		
93	3	20100	673	116643		
94	1	20100	380	11057	10889	0.0151939947544542
94	2	20100	320	14449		
94	3	20100	575	106190		
95	1	20100	371	10631	10524	0.0100649045245038
95	2	20100	324	14280		
95	3	20100	622	110456		
96	1	20100	378	10445	10329	0.0111057922450933
96	2	20100	324	14390		
96	3	20100	616	111325		
97	1	20100	420	10950	10854	0.00876712328767123
97	2	20100	380	14583		
97	3	20100	480	107544		
98	1	20100	368	10827	10730	0.00895908377205135
98	2	20100	308	14548		
98	3	20100	594	115245		
99	1	20100	375	10634	10438	0.0184314463043069
99	2	20100	310	14288		
99	3	20100	517	104710		
100	1	20100	407	10765	10675	0.00836042731072922
100	2	20100	319	14507		
100	3	20100	636	114339		

MX(FSS)	Optimal	Sequence
1	1278	3 17 9 8 15 14 11 16 13 19 6 4 5 18 1 2 10 7 20 12
2	1359	19 14 6 20 3 10 17 9 18 12 7 15 1 16 4 2 13 5 11 8
3	1081	16 3 20 18 7 1 12 10 5 2 9 4 19 14 17 6 13 11 8 15
4	1293	13 16 9 17 11 19 10 6 7 15 1 12 5 20 2 3 8 14 4 18
5	1235	5 3 12 10 20 19 9 18 7 17 15 13 4 16 6 2 14 11 8 1
6	1195	11 5 20 13 8 17 6 16 1 7 12 14 2 18 10 15 9 4 19 3
7	1234	5 16 15 11 1 2 14 20 13 8 6 9 7 17 19 12 4 3 18 10
8	1206	17 12 9 2 14 4 10 18 16 19 7 8 6 5 20 15 13 1 3 11
9	1230	4 2 20 18 17 15 1 10 7 9 16 13 8 3 5 12 6 14 11 19
10	1108	7 19 11 12 16 6 1 13 10 15 2 8 3 4 18 14 17 5 20 9

Figure 2.38: optimal and sequence of 1-10 matrix from the website

4 SUMMARY

This experiment is using NEH heuristics to solve FSS, FSSB and FSSNW problems. It proves the effectiveness that NEH owns in combinatorial optimization. The way to use NEH, to separate an array and insert it back is easy to code and to understand. Also, the experimental result shows that the optimal value of NEH is extremely close to the actual optimal value from the greedy algorithm in FSS. Although NEH heuristic shares the worst time of O(n3), which increases quickly for a large n, it is one of the fast algorithms among the metaheuristics. From what we get in the result the conclusion can be drawn that NEH is good when solving FSS, FSSB and FSSNW problems without doubts.

Comparing FSS, FSSB, and FSSNW, FSS has the least spare time of each machine. FSSB has a larger time, and they are separated between jobs. FSSNW has the largest spare time, are they are distributed in front of the machines. After that, FSSNW machines won't be stopped during half of the works.