Name: _____

1. You determine the acceleration due to gravity by switching off an electromagnet to release a ball-bearing and measuring the time t it takes to fall a distance d, so that $d = gt^2/2$. The distances are measured with negligible uncertainty; the times are measured to the nearest 10 ms.

Time (s)	0.16	0.40	0.58	0.72	0.97
Distance (m)	0.20	1.00	2.00	3.00	5.00

   Analytically derive a least-squares estimate for g with appropriate uncertainties assuming:

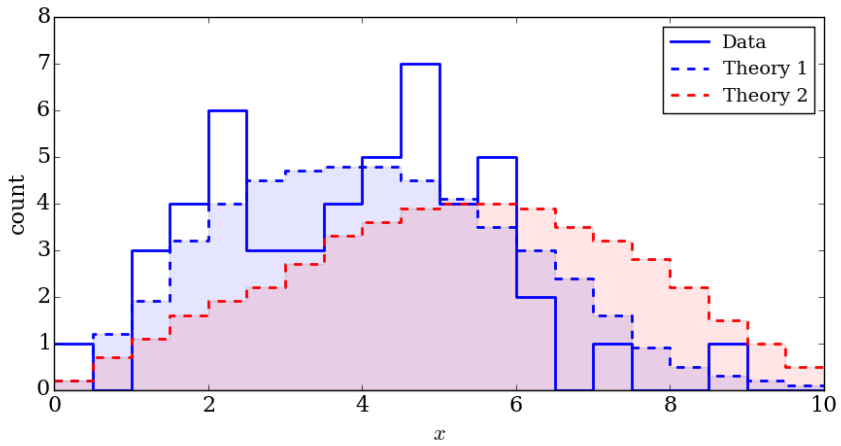
   (a) (5 points) The times are as given.

   (b) (5 points) The field in the magnet takes an unknown but constant time to decay and release the ball-bearing.

   Hint: express your χ^2 with t as the dependent variable and $x = \sqrt{d}$ as the independent variable, then minimize.

   Is your χ^2 reasonable or not for each case?

2. The file `data.txt` contains a histogram with data; the first two columns are the bin boundaries, and the third column contains the numbers of entries $\{n_i\}$, $i = 1, \ldots, 20$, which we treat as Poisson random variables. The files `theory1.txt` and `theory2.txt` give predictions for the expectation values $\{\nu_i\}$, and are shown with the data in the figure below.



   (a) (5 points) Write a program to read in the files and calculate Pearson's χ^2 statistic for each of the two theories.

   (b) (5 points) Write a program to check the distribution of Pearson's χ^2 by generating 10,000 random data sets from the two theory predictions. What are the p-values of the two theories when you use the test statistic computed from the data in (a)? What would the p-values be if one assumed the test statistic followed the usual χ^2 distribution?

3. Charged particles traversing a volume of gas produce ionization such that the mean amount of ionization depends on particle type. Suppose a test statistic t based on ionization measurements is constructed so that it follows a Gaussian of mean 0 for electrons and Gaussian of mean 2 for pions, with a standard deviation of 1 for both hypotheses. A test is constructed to select electrons by requiring $t < 1$.

   (a) (5 points) Given the test threshold, what is the probability of selecting electrons (i.e., accepting a particle that is an electron)?

   (b) (5 points) What is the probability that a pion will be mistakenly accepted as an electron?

   (c) (5 points) Suppose a sample of particles is known to consist of 99% pions and 1% electrons. What is the purity of the electron sample selected by $t < 1$ (i.e., the probability that a particle is an electron given that it was selected by requiring $t < 1$)?