

From Chaos to Order

The Fractal Geometry of Our World

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Abstract

This paper is the result of my burning curiosity about Chaos Theory and Fractals and acts as an introduction to both, in hopes that it sparks some curiosity for you as well. I aim to explore the fascinating connection between Chaos Theory and Fractal Geometry in nature.

I will begin by providing the background you will need to understand the mathematics involved to follow along. Then, I will demonstrate the incredible power of mathematics to understand the complexities of the natural world, illustrated by the complex patterns we see in coastlines, trees, and mountains. I will conclude the paper with some philosophical insights—that there is no order without chaos—and two perspectives to consider when reflecting on the unpredictable beauty of nature.

Contents

1	Background	3
2	Chaos Theory	3
2.1	What is Chaos Theory?	3
2.2	What is the Butterfly Effect?	3
2.3	What is the role of strange attractors in chaotic systems?	3
2.4	Why is nonlinearity important for chaotic behavior?	3
2.5	How can complex patterns emerge from simple systems?	3
2.6	What is the relationship between chaos and fractals?	3
3	Fractals	4
3.1	What are fractals?	4
3.2	What is self-similarity in fractals?	4
3.3	How do fractals exhibit infinite detail?	4
3.4	What is fractal dimension?	4
3.5	What are the mathematical tools used in fractal geometry?	4
3.6	How are iterated maps used to generate fractals?	4
3.7	What were Mandelbrot's contributions to fractal geometry?	4
3.8	How does bifurcation theory explain the transition from regular to chaotic behaviour?	5
4	Fractals in Nature	5
4.1	How do chaotic processes contribute to fractal patterns in nature?	5
4.2	How do coastlines display fractal patterns?	5
4.3	How do trees display fractal patterns?	5
4.4	How do mountains display fractal patterns?	5
5	Conclusion	5

1 Background

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2 Chaos Theory

2.1 What is Chaos Theory?

Chaos Theory is a branch of mathematics focusing on the behavior of dynamical systems that are highly sensitive to initial conditions. This phenomenon is popularly referred to as the butterfly effect.

2.2 What is the Butterfly Effect?

TODO

2.3 What is the role of strange attractors in chaotic systems?

TODO

2.4 Why is nonlinearity important for chaotic behavior?

TODO

2.5 How can complex patterns emerge from simple systems?

TODO

2.6 What is the relationship between chaos and fractals?

TODO

3 Fractals

3.1 What are fractals?

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3.2 What is self-similarity in fractals?

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3.8 How does bifurcation theory explain the transition from regular to chaotic behaviour?

TODO

4 Fractals in Nature

4.1 How do chaotic processes contribute to fractal patterns in nature?

TODO

4.2 How do coastlines display fractal patterns?

TODO

4.3 How do trees display fractal patterns?

TODO

4.4 How do mountains display fractal patterns?

TODO

5 Conclusion

The dynamics of a system at each moment of time can be in one of these two states:

- Chaos (unstable)
- Order (stable)

At either of those states you also need a perspective to be able to maximize your effectiveness and live optimally. These perspectives are Zooming out and Zooming in.

You zoom out when the system is in a state of chaos. What that means is you try to grasp the bigger picture and understand why things unfold in the long run.

You zoom in when the system is in a state of order. What that means is you bring yourself to the present moment and try to take it in as much as possible. This includes when the economy is stable, when routines are predictable, and when life feels steady.

Chaos theory teaches us that there is no certainty in life, only possibility and patterns, and that is enough.