After Mid

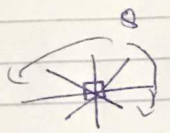
Lecture#1

All the curves which have equation can be detect from hough transform.

Lets say circle:

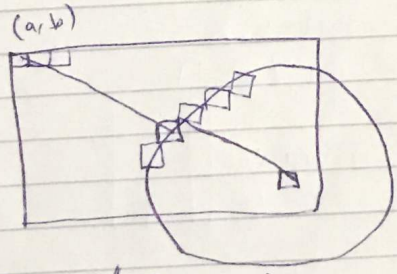
Unit circle: $x^2+y^2=r^2$

Circle: at point (x,y) $(x-a)^2+(y-b)^2=r^2$



for every θ we find r.

- If a point is Edge point that many circle can pass through it.



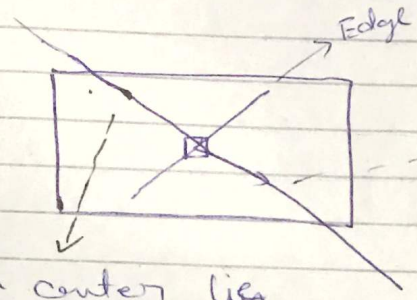
- find the b & r, by assuming every point/pixel as the origin.

Vote at $A(a, b, r)$ ++

For every edge pixel
for every (a, b) in image

Compute r
increment A (a, b, r) by 1

Slide 14



gradient direction

than center lies
on this line

Now loop can run in 1D

Slide 15

- Circle Parameters
  x_0, y_0, r
  ↓
  origin

- Every circle is an ellipse

Slide 16

We will

- Affine

- Corre
  cont

- So,
  Trans

Scaling

Rotati

"Spatial Transformation"

We will study 2D spatial Transformation

$$T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

- Affine transformation include:
  - Scaling
  - Rotation
  - Shear
  - Translation
- carried out via matrix-vector multiplication
- So, Affine Transformation is Linear Transformation.

$$\vec{x}' = M\vec{x}$$

M - Matrix

Scaling:

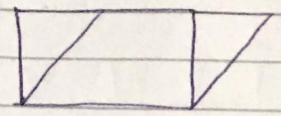
$$\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

$$\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xS_x \\ yS_y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

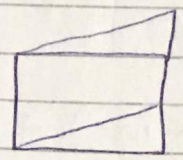
Rotation:

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Shear



OR



$$\begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix}$$

Translation:

Slide 5:

This Translation is not be written in matrix-vector form.

but

Can be done if you think outside the box

$$x' = x + 10$$
$$y' = y$$

$$\begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1/0 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

So,

$$x' = x + 10$$
$$y' = y$$

Generic:

$$\begin{bmatrix} 1 & 0 & T_{rx} \\ 0 & 1 & T_{ry} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Now wh
go to

Q:

This is

If

Now when we add coordinate 1 in Matrix we go to projection space

$$\mathbb{R}^2 \longrightarrow \mathbb{P}^2$$

This projection space corresponds to $\mathbb{R}^2$

Q: $\mathbb{P}^N$ So, its space dimension is $N+1$

This is homogenous coordinates.

$$\underset{\mathbb{R}^2}{\begin{bmatrix} x \\ y \end{bmatrix}} \longrightarrow \underset{\mathbb{P}^2}{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}$$

Here $z = 1/0$

Homogenous Coordinates

If we want to go Back

$$\mathbb{P}^2 \longrightarrow \mathbb{R}^2$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \xrightarrow{①} \begin{bmatrix} x/w \\ y/w \\ w/w \end{bmatrix} \xrightarrow{②} \begin{bmatrix} x/w \\ y/w \end{bmatrix}$$

Throw away the 3rd coordinates

$\mathbb{P}^2$

$$\begin{matrix} x \\ y \\ 0 \end{matrix} \longrightarrow \begin{bmatrix} \infty \\ \infty \\ NaN \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} NaN \\ NaN \\ NaN \end{bmatrix}$$

$$\mathbb{P}^2 = \mathbb{R}^3 \setminus \vec{0} \,|\, [0; 0; 0]$$

$\downarrow$

Set difference Operator

Quiz of Slide: 16

Why R is written like this

$[x ; y]$ to $[x+10 ; y]$

7-12-16

$$\begin{bmatrix} 1 & \cdot \\ & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

?? Not possible

Now,

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x+10 \\ y \\ 1 \end{bmatrix}$$

Scaling Example:

a) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$

c) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$

Rotation Matrix:

In Linear Algebra, a rotation matrix is a matrix that is used to perform a rotation in Euclidean space. For example the matrix

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

rotates points in the xy-cartesian plane counter-clockwise through an angle θ about the origin of cartesian coordinate system. To perform the rotation using a Rotation Matrix R, the position of each must be represented by a column vector $\vec{v}$. A rotated vector is obtained by using matrix multiplication

$$R\vec{v}$$

- It is the arbitrary rotation of the axes about the origin.

$$\cos\theta \cancel{\sin\theta} - \sin\theta \cancel{\cos\theta}$$

$$=$$

$$\cancel{V = \cos x + i\sin x}$$

$$V = \cos\theta + i\sin\theta$$

- Cameras
space
- There
space.
- Reflecti
- Reflect

- Mirro

- Orth
- Slide
- Bcz
rota
cha

Translat

- W
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- So
sc
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$\begin{bmatrix} \cos \\ \sin \\ 0 \end{bmatrix}$

Lec#2

- Cameras all modeling is projective space
- There is no origin in Projective space.
- Reflection is also linear transformation.
- Reflection is -ve scaling.

- Mirror Matrix is $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

- Orthogonal Matrix ??
- Slide 17 prove ??
- Bcz of this property, after rotation its magnitude does not change

Orthonormal

Translation:

- We have to go to $\mathbb{P}^2$ every time

- So, we have to write all scaling, shear, translation and rotation in $\mathbb{P}^2$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & e_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ e_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse Transform:

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad S^{-1} = \begin{bmatrix} 1/S_x & 0 & 0 \\ 0 & 1/S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S S^{-1} = I$$

- Inverse of scaling is also scalling

Rotation Matrix

$\rightarrow R R^{-1} = I$

Slide 22 Imp

$\rightarrow$ If we want to rotate w.r.t (a, b) not w.r.t origin.

$$\begin{bmatrix} e_x & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

① Translate to keep in to origin (0,0)

Translate by $-a, -b$

② Now perform simple rotation

③ Now Translate every result by $\{a, b$.

$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & 1/s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Linear Transformation can be concatenated.

$$\vec{P'} \quad \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Here we are working in $\mathbb{P}^2$ from now on

also

$$\left[S_{(s_x, s_y)}\left[R_\theta \vec{P'}\right]\right] T_{[dx \ dy]}$$

- First P rotate, than scale and Translate.

Now first compute

We have (A B C)

(S R T)

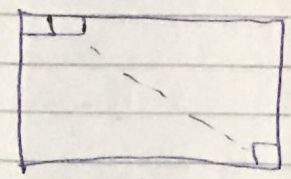
w.r.t

A(BC)

Call it M

(AB) C

We have image



Before:

① We apply rotation at every pixel

② " " Scaling " " "

③ " " Translation " " "

After:

When we multiply ~~both~~ all of these "R S T" $\Rightarrow$ then apply on each pixel.

Order of Transformation:

① Rotation / Scaling / Shear, followed by Translation.

② Translation followed, R/S/S

- Affine Transformation have 6 parameters

$$\begin{bmatrix} a_1 & a_2 & b_1 \\ a_3 & a_4 & b_2 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{Always there}$$

- Parallel lines are preserved [proof?]

<u>C.V Problem:</u>

We have two images (Satellite Images)

- How do we register two images together.
- If you given the correspondences $\rightarrow$ Transformation

$$P_1 \xrightarrow{T} P_1'$$

$$P_2 \xrightarrow{T} P_2'$$

$$P_3 \xrightarrow{T} P_3'$$

Now,

find T?

Lec#3

Recovering Best Affine Transformation:

Input: Some correspondant points

Output: find $a_1 - a_6$

$$\underset{2\times1}{\vec{x}_1'} = \underbrace{\underset{6\times2}{T}\ \underset{2\times1}{\vec{x}_1}}_{6\times1}$$

Problem.

So,

$$= \underbrace{\underbrace{\underset{2\times2}{T}\ \underset{2\times1}{\vec{x}_1}}_{2\times1}}$$

But

For Translation.

$$\underset{3\times1}{\vec{x}_1'} = \underset{3\times3}{T}\ \underset{3\times1}{\vec{x}_1}$$

Now,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

We kno

We ha
unknow
as w

$\begin{bmatrix} x_1' \\ y_1 \end{bmatrix}$

Here

Now,

$\rightarrow [$

If
the
poss

ns formation:

points

a_6

$$= \begin{bmatrix} a_1 x + a_2 y + a_3 \\ a_4 x + a_5 y + a_6 \\ 1 \end{bmatrix}$$

We know,

$$A\vec{x} = b$$
$$\vec{x} = A^{-1}\vec{b}$$

We have T as matrix of unknown, but we can write it as ~~matrix~~ vector of unknown.

$$\begin{bmatrix} x_1' \\ y_1' \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}$$

Here unknowns are in vector.

Now,

$$\rightarrow \begin{bmatrix} 13 & 17 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -7$$

If no. row's ~~[illegible]~~ are less than the unknown the solution is not possible.

Now,

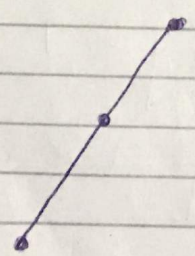
$$\underbrace{\begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}}_{x}$$

$$A\vec{x} = \vec{b}$$
$$\vec{x} = A^{-1} b$$

-> We require only 3 corrus-pondance to find the transfor-mation.

If the points are in line than they are linearly dependant or in dependant.

??



- No. of independant rows are the rank.

-> Yes, t
and
full. S
not
the

-> In a
are

Now

We stu

We can't

$\begin{bmatrix} 6.8 \\ 4.7 \end{bmatrix}$

$$\begin{bmatrix} 0 & 0 & 1 \\ x_1 & y_1 & 1 \\ 0 & 0 & 0 \\ 0 & y_2 & 1 \\ 0 & 0 & 0 \\ x_3 & y_3 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}$$

x

3 corres-
transfor-

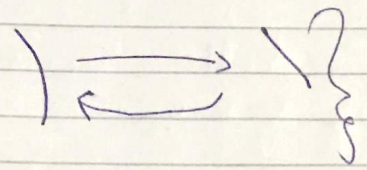
line
rly dependent

??

are

→ Yes, they are linearly dependent and their rank is not full. So, their inverse is not possible and we cant find the

→ In affine transformation lines are preserve



No Possible

Now

$$\vec{x}_1' = T\,\vec{x}_1$$

We studied

$$\begin{bmatrix} 7 \\ 9 \end{bmatrix} \longleftarrow \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

T - (There is always noise in our measurement So, this measurement holds only partially)

We can't find it as:

$$\begin{bmatrix} 6.8 \\ 4.7 \end{bmatrix} \xleftarrow{T^*} \underbrace{\begin{bmatrix} 3.6 \\ 2.9 \end{bmatrix}}$$

because these are not integer

- It is also possible that the T become singular.

If we have n-correspondance

$$\underbrace{T/A}_{2N \times 6} \underbrace{\vec{v}}_{6 \times 1} = \underbrace{\vec{b}}_{2N \times 1}$$

- If we have more correspondance we have better answer which have less noise.

- A is non-square matrix (more than 3-correspondance)

- Inverse is of only sq. matrix

$$d > k$$

$$[k \times d]$$

$$\mathbb{R}^d \rightsquigarrow \mathbb{R}^k$$

- higher to lower So, you loose some info.

- ~~If~~ Now to resolve this If we multiply both sides by A^T

$$\underbrace{[A^T A]}_{6 \times 6} \underbrace{\vec{v}}_{6 \times 1} = \underbrace{A^T b}_{6 \times 1}$$

$$\vec{v}^* = \underbrace{(A^T A)^{-1} A^T}_{A^+} \vec{b}$$

$A^+ \vec{b}$

This is called Pseudo Inverse

$$\vec{v}^* = A^+ \vec{b}$$

$$A^+ = (A^T A)^{-1} A^T \quad \text{// Pseudo Inverse}$$

$$= \underbrace{(A^T)^{-1} A^{-1}}_{A^{-1}(A^T)^{-1}} A^T$$

$$= I A^{-1}$$

$$= A^{-1}$$

- If A is invertible than Pseudo inverse is inverse.
- If A is not invertible than Pseudo inverse give the best approximation
- We move towards A^+ (Pseudo Inverse)
  - If measurements are noisy.
  - If A is non sq. matrix.

$$v^* = \underbrace{\arg\min_v}_{\downarrow} \|Av - b\|^2$$

Quiz

Its derivative w.r.t 'v

In matlab,

$$A\vec{v} = \vec{b}$$

$$\vec{v} = A \backslash b \xrightarrow{\text{invertable}} A^{-1}b$$

$$\vec{v} = A \backslash b \xrightarrow{\text{non-invertable}} A^{+}b$$

2D Displacement Models:

Translation:

Rigid:

Rotation, Translation

Affine:

Shear, Rotation, Scalling & Translation.

Projective:

$Av-b\|^2$

$\Rightarrow$

able $A^{-1}b$

$\Rightarrow A^{+}b$

vertable

on

ng

Projective

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \underbrace{\begin{bmatrix} a_1 & a_2 & b_1 \\ a_3 & a_4 & b_2 \\ 0 & 0 & 1 \end{bmatrix}}_{\boxed{c_1 \quad c_2 \quad 1}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

If

Now,

$$w' = c_1 x + c_2 y + 1$$

So,

$$x' = \frac{a_1 x + a_2 y + b_1}{w'}$$

$$y' = \frac{a_3 x + a_4 y + b_2}{c_1 x + c_2 y + 1/w'}$$

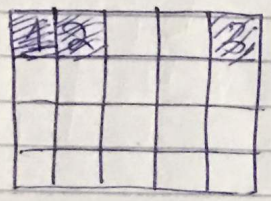
This is not a linear transformation because x, y are in both denominator & numerator.

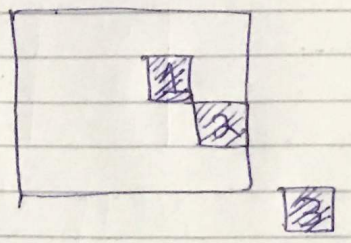
- If we will be in projective space.

Warping

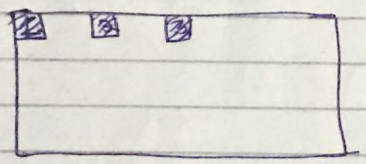
$x' = A\vec{x}$

X





Now, Real Issue



There will be holes in the image

T

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scaling in Y

- Now, Make empty transform-image, than see its each pixel, from where it comes

Bilinearly Interpolation

Do it on your own.

Recovering Best

We ku

We want t
that $A\vec{v}'^*$
to $\vec{b}$.

To find $\vec{v}'^*$,
first derival
eq ① an

$\nabla_v \{ \| A\vec{v}' -$

$\nabla_v \{ (A\vec{v}'$

$(A\vec{v} - \vec{b})^T [A$

$A(A\vec{v}'$

W

$A^T(A$

Recovering Best Affine Transformation

We know, $A\vec{v} = \vec{b}$

$$A\vec{v} - \vec{b} = 0 \quad \text{— (1)}$$

We want to find the v^*, ~~which~~ So that $A\vec{v}^*$ is as close as possible to $\vec{b}$.

$$A\vec{v} \approx \vec{b}$$

To find $\vec{v}^*$, we have to find the first derivative w.r.t 'v' of squared norm of eq (1) and equals it to 0.

$$\nabla_v \{ \| A\vec{v} - \vec{b} \|^2 \} = 0$$

$$\nabla_v \{ (A\vec{v} - \vec{b})^T (A\vec{v} - \vec{b}) \} = 0$$

$$(A\vec{v} - \vec{b})^T [A - 0] + [A - 0]^T (A\vec{v} - \vec{b}) = 0$$

$$A (A\vec{v} - \vec{b})^T + A^T (A\vec{v} - \vec{b}) = 0$$

We know $AA^T = A^TA$

$$A^T (A\vec{v} - \vec{b}) + A^T (A\vec{v} - \vec{b}) = 0$$

$$2A^T (A\vec{v}^* - \vec{b}) = 0$$

$$A^T (A\vec{v}^* - \vec{b}) = 0$$

$$A^TA\vec{v}^* - A^T\vec{b} = 0$$

$$A^TA\vec{v}^* = A^T\vec{b}$$

$$\vec{v}^* = (A^TA)^{-1} A^T\vec{b}$$

$A^{+} = (A^{T}A)^{-1}A^{T}$ is known as pseudo-inverse of A.

$$\vec{v}^{*} = A^{+}\vec{b}$$

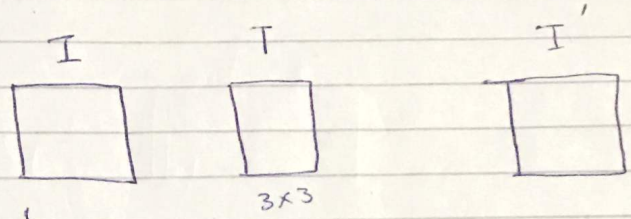
own as
of A.

Lec#4

Warping:

- image is given
- Transformation matrix is given

Solution



Take its location

$I(x, y)$ Multiply with $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$

$I'(x', y') = I(x, y)$

Not possible if they are float

$I'(\underbrace{x', y'}) = I(x, y)$

Make them round / Floor

One more problem

x', y' may be out of bound

To resolve this,

Make the transformed Image I' much bigger.

- Affine Transformation are invertable

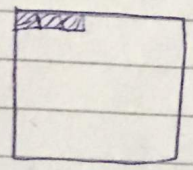
- If don't want to bigger the size of transform image than use the 'if' statement.

On Slide '37'

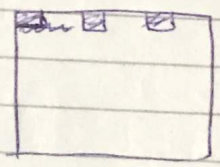
- The reason of blurring of image.

There is a problem in this algo

$$I \qquad \overset{T}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 \\ \cancel{1}\,3 \\ 1 \end{bmatrix} \quad I'$$



Holes are Made



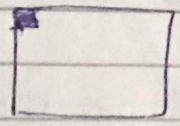
To resolve this Apply loop on the locations of I'

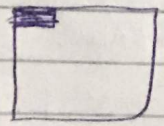
$$x = A^{-1} x'$$

Now apply

$$\overset{T^{-1}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{x'}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

I $\qquad$ I'





at (1,1) Round

$$\begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

at (1,2) Round

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

We are rounding float but is there is more sophisticated problem.

One solution
By Hisham:

① 1, 1.5

So, $\frac{1,1 + 1,2}{2}$

but it fail in

1.3, 1.5

1 2
70% 30%
(1−0.3) (0.3)

Now, we will do averaging in both direction and it is weighted

$x, y = 17.6, 4.8$

For top left:

By taking the floor

$\underline{x}, \underline{y}$

For top Right:

$\underline{x}, \underline{y}+1$

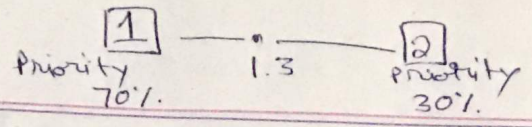
For bottom left:

$\underline{x}+1, y$

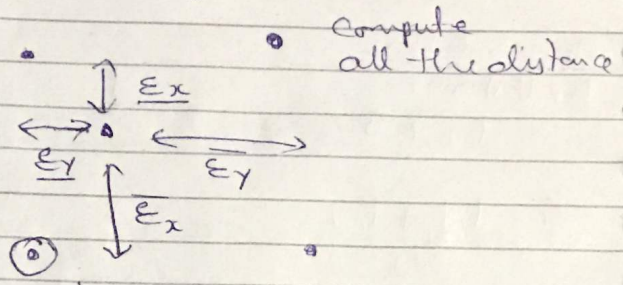
For bottom Right:

$\underline{x}+1, \underline{y}+1$

Ceiling will fail some time when values will be integer.



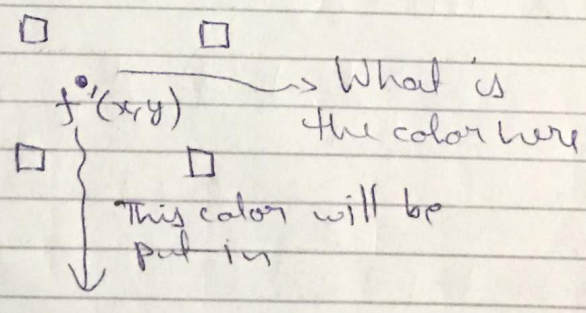
To Find, کس کا کتنا حق ہے / کون زیادہ قریب ہے



For his right
Multiply it by $\varepsilon_x \, \varepsilon_y$
Do it for other.

Slide 40

Slide is Telling:



$I'(x', y')$

This is called Bilinear Interpolation

Now, We

This

→

→ To di
See p

→ P.T
when
projec
remain

Now

- It i
colli

Now,

We change T to

$$\begin{bmatrix} a_1 & a_2 & b_1 \\ a_3 & a_4 & b_2 \\ c_1 & c_2 & 1 \end{bmatrix}$$

This is projective Transformation

→ Affine Transformation is the subset of projective Transformation. (P.T)

→ To distinguish Affine and Projective see parallel lines.

→ P.T is not linear because when we come back from projective space it will not remain linear. $\mathbb{P}^2$

Now

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ c_1 & c_2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$\mathbb{R}^2$

$$x' = \frac{a_1 x + a_2 y + a_3}{c_1 x + c_2 y + 1}$$ // Non Linear

- It is also called homography or collineation.

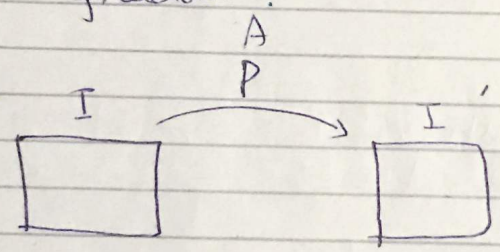
- Collineation is called because it still preserve lines, not parallel lines.

- In this we have 8 degree of freedom.

- Affine Transformation have 6-degree of freedom.

- P.T have 8-degree of freedom.

Now



you just have to add one line

$$c_1 x + c_2 y + 1$$

We have studied.

$$x_1 \xrightarrow{A} x_1'$$

$$\vdots$$

$$x_N \longrightarrow x_N'$$

ANS $\quad v^* = A^+ b$

Now,

- Now h
- The pro

- We
three
- Cross

- Its
area

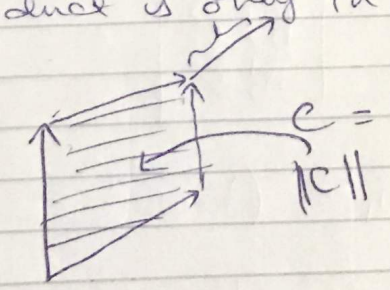
Now,

$$\begin{array}{ccc} & H/P? & \\ x_1 & \longrightarrow & x_1' \\ & \vdots & \\ x_N & \longrightarrow & x_N' \end{array}$$

- Now have to find eight parameter.
- The property we use are:

$$\underbrace{X_i'}_{3\times1} = \underbrace{\underbrace{H}_{3\times3}\ \underbrace{x_i}_{3\times1\ \text{Projective coordinates}}}_{3\times1}$$

- We say LHS & R.H.S as three no's vector.
- Cross product is only in 3D space.



- Its norm represent this area.

  Its like parallelogram, so Area of parallelogram.

- If vel
than H
product

ection
s -

- 3 eq.
- We need

possible

So,

$$x_1' \times H = \vec{0}$$
$$x_2' \times H = \vec{0}$$
$$x_3' \times H = \vec{0}$$

If $N = 3$

Now,

$$H\vec{x_i} = \begin{bmatrix} h_1^T \vec{x} \\ h_2^T \vec{x} \\ h_3^T \vec{x} \end{bmatrix}$$

In vector

$$h_1 \begin{bmatrix} = \\ = \\ = \end{bmatrix}$$

So

$$h_1^T : (\text{Row-1 of } H)^T$$

$$h_1 \begin{bmatrix} - & - & - \\ & & \\ & & \end{bmatrix}$$

but as vector $\begin{bmatrix} - \\ - \\ - \end{bmatrix}$

"Est

direction
cross -

x1
possible

n vector

T

see expression of Cross product.

"Est. of Homography: DLT"

Lec # 5

→ If two vectors are in same direction than their cross product Mag. is zero. So, Its area is zero.

In $\mathbb{R}^2$

$$\vec{a} = \vec{b}$$

$$\begin{bmatrix} - \\ - \\ - \end{bmatrix} = \begin{bmatrix} - \\ - \\ - \end{bmatrix}$$

In $\mathbb{R}^2$

$$\vec{a} = \vec{b}$$ (Same direction same magnitude)

$$\vec{a} \equiv \vec{b}$$ (Same direction but mag. can be diff.)

$$\Downarrow$$

$$\vec{a} = K\vec{b}$$

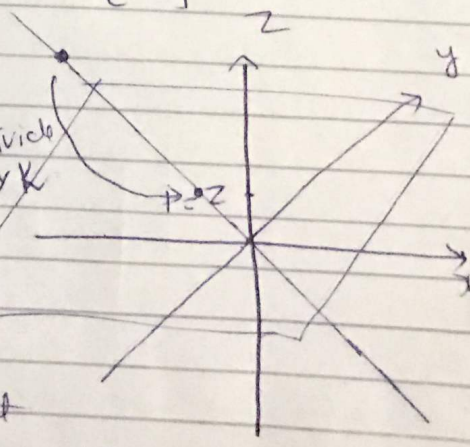
$$\begin{bmatrix} Kb_1 \\ Kb_2 \\ Kb_3 \end{bmatrix} \qquad \begin{bmatrix} b_1 \\ b_2 \\ 1 \end{bmatrix}$$

In $\mathbb{R}^2$

divide by K

$$\vec{a} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \qquad \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \vec{b}$$

divide by K



There is a chance that $z \neq 1$ than you will go out of the bound

There is a plane at $z = 1$

V.V.V Imp Projective equivalent Slide

$$\mathbb{R}^2 \rightarrow \text{inf. } \mathbb{P}^2$$

Proof:

Degree of freedom is 9. But its not 9, its 8 lets see

Now,

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}$$

In $\mathbb{P}$ space

$$K(H\vec{v}) \equiv H\vec{v} \quad \text{(projectively equivalent)}$$

$$(KH)\vec{v} = H\vec{v}$$

So,

$$KH \equiv H$$

can be any value of H but for convention $\frac{1}{H(3,3)}$ (last value)

$$\frac{1}{H(3,3)} H \equiv H$$

So,

$$\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & 1 \end{bmatrix} \equiv H$$

The transformation which is achieved by H (9-degree of freedom) can also be achieved by L.H.S using (8-degree of freedom)

- When we have to find homography we have to find 8 parameter.

Now

$$\vec{x}_i' \equiv H\vec{x}_i \quad \text{(both point in same direction)}$$

So,

$$\vec{x}_i' \times H\vec{x}_i = \vec{0}_{3\times1}$$

- $a \times b$ is a linear transformation so we can write it in Matrix vector form.

- The rank is 2. And the matrix is a skew-symmetric matrix

$$\underbrace{\begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}}_{[\vec{a}]_\times} \Bigg\} \text{ these are linearly independent}$$

a representing cross product.

Now

$$\vec{V} = \begin{bmatrix} - \\ - \\ - \\ - \end{bmatrix} \quad \text{column vector}$$

(Superscript)

$$\begin{array}{c} h^{1T} \\ h^{2T} \\ h^{3T} \\ h^{4T} \end{array} \begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

$h_1 \quad h_2 \quad h_3 \quad h_4$ (Subscript)

homography
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direction)

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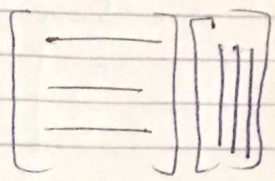
independa

We have to write its skew symmetric matrix $\vec{x_i}' \times H\vec{x_1}$

$$\underbrace{\vec{x_i}'}_{a} \times \underbrace{H\vec{x_1}}_{b}$$

Than,

$$H\vec{x_i} = \begin{bmatrix} h^{1T}\vec{x_i} \\ h^{2T}\vec{x_i} \\ h^{3T}\vec{x_i} \end{bmatrix} \Longrightarrow$$



Here,

H.W verify

$$x_i' \times H x_i = \begin{bmatrix} y_i h^{3T} x_i - w h^{2T} x_i \\ \vdots \end{bmatrix}$$

$\rightarrow h^{1T} x_i$ size is 1×1 (a scalar)

We have $\vec{a}^T \vec{b} = b^T a$

$$\vec{h}^{1T} \vec{x_i} = \vec{x_i}^T h^1 \quad \text{(In 2nd Step)}$$

$$\Rightarrow \begin{bmatrix} h^1 \\ h^2 \\ h^3 \end{bmatrix} \approx \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ \vdots \\ h_9 \end{bmatrix}$$

All unknowns are in vector.

Now we need ~~a~~ Matrix A so that

$$A\vec{h} = \vec{0}$$

3 Numbers

For A:

$$\begin{bmatrix} 0_{1\times3} & — & — \\ — & 0_{1\times3} & — \\ — & — & 0_{1\times3} \end{bmatrix} \begin{bmatrix} \vec{h}^1 \\ \vec{h}^2 \\ \vec{h}^3 \end{bmatrix}$$

3 Numbers each.

We have written

$$\boxed{\vec{x}_i' \times H \vec{x}_i = \vec{0}_{3\times1}}$$

$$\downarrow$$

$$\boxed{A_i \vec{h} = \vec{0}} \longrightarrow$$

3 eq.

but 9 parameter

not possible but if you give three correspondance we can find.

A_1, A_2, A_3 → give 6 eq

Rank is 2 but we need 8.

Linearly independent

So it give 2 linearly independent eq.

Now,

you need atleast 4-correspondance which used to find 8 degree of freedom.

→ So the N correspondance must not be co-linear.

$$\underset{2N \times 9}{\begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_N \end{bmatrix}} \underset{9 \times 1}{\begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_9 \end{bmatrix}} \Rightarrow 2N \times 1$$

-> We have

$$\underset{2N \times 9}{A} \; \underset{9 \times 1}{\vec{h}} = \underset{2N \times 1}{\vec{0}}$$

Minimum 4 correspondance

$$\underbrace{8 \times 9}_{\text{Rank } 8} \; \underbrace{9 \times 1} = \underbrace{8 \times 1}_{\text{zeros}}$$

$$\underline{\text{Rank}} + \underbrace{\text{nullity}}_{\text{No of vectors gone to zero}} = \text{dimension}$$

$$8 + 1 = 9$$

So, In this case there must be one vector which maps to zero vector. So, the solution must be unique.

Now $N \geq 4$

Rank is never > 8 bcz it has degree of freedom : 8.

There is a solution:

$$\underbrace{S \quad V \quad d}_{\text{Single value decomposition}} (A)$$

$$U \quad \underset{\text{diagonal}}{D} \quad V$$

Orthognal Matrix

Orthogonal میں Row/Colum باقی Colum/Row جیسی کی ہیں)
(یہ

All rows ~~se~~ colums are unit vector / Normalized.

So, we can write Original matrix as

$$A = UDV$$

We have borrowed that V (matrix last Colum)

$$\left[\quad \right] \rightarrow \text{is } b^*$$

This is called direct linear transformation.

Tips:
1

Tips:

① x_i x_i'

It talks about its normalization.

(جمس)

unit

matrix

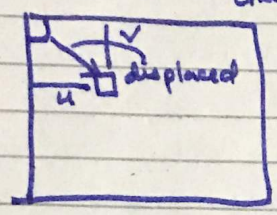
lum)

b^*

Lec#6

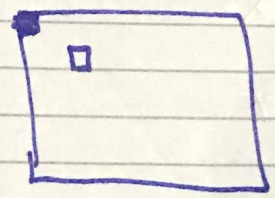
Optic ~~Flow~~ Flow

$I(\overrightarrow{x, y}, \underset{time}{z})$ pixel



the second Image is the sequence of image I

$I(x, y, z+1)$



you have to tell how much it displace in x ~~and~~ y direction

We have to find that vector.

We have to do it on every pixel.

This is call Optic flow

If we know the displacement vector than we ~~can~~ find

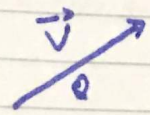
$$\vec{x}' = \vec{x} + \begin{bmatrix} u \\ v \end{bmatrix}$$

-> To find correspondance usually the optic flow is not used.

-> We are only interested in displacement vector which means translation.

- What is optic flow good for?

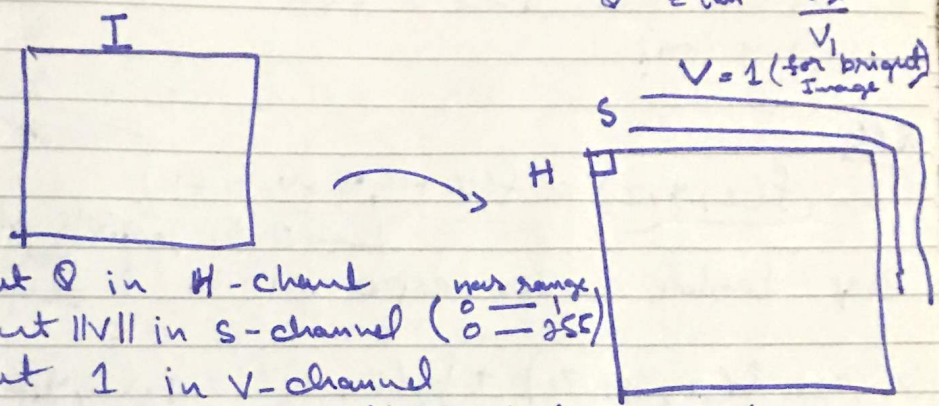
Lets say:



It also has

$\|\vec{V}\| = \sqrt{V_1^2 + V_2^2}$

$\theta = \tan^{-1} \frac{V_2}{V_1}$



put θ in H-chanel

put $\|V\|$ in S-channel (nor range $0 - 1$, $0 - 255$)

put 1 in V-channel

→ So, we have to normalize $|V|$ before assigning.

→ To make it Colour coded displacement.

Slide: 4 is Imp for assignment.

Now see how to find $\vec{U}$ & $\vec{V}$:

$f(x, y, z)$ In first frame

↳ grey value

In next frame

$f(x+u, y+v, z+1)$;

both of these are same so,

$$f(x, y, z) = f(x+u, y+v, z+1)$$

Assumption:

Lets say their color is not changed

this is called Grey value Constancy Assumption.

- Can we find u ~~&~~ v from these assumption?

~~Ans~~

$$\underbrace{f(x,y,z) = f(x+u, y+v, z+1)}$$ — 1st Assumption

↳ Only possible if it close to x, y, z

By taylor series expansion.

$$f(x,y,z) = f(x,y,z) + f_x(x,y,z)\,u + f_y(x,y,z)\,v + f_z(x,y,z) + O(f^2)$$

So, u, v value must be small, otherwise it will not be used in taylor series.

- We have avoid higher order derivative because:
  - We need linear in our equation
  - u, v are small so, u^2 & v^2 are more small that's why we ignore.
  - If the denominator become very large so we ignore.

t changed

Constancy

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1st Assumption

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it close to
x, y, z

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me very

$$f(x,y,z) + f_x(x,y,z) + f_y(x,y,z) + f_z(x,y,z)$$
$$\rightarrow \cancel{f(x,y,z)} = 0$$

So,

$$f_x u + f_y v + f_z \Big|_{(x,y,z)} = 0$$ — 2nd Assumption

1 eq. two unknown we can't solve.

Now

- 1st assumption violat if, it will not be a huge violation. It would be small.

- In 2nd " , if it violat it will not be valid for taylor series any more
  It is normally violated in normal cameras.

- Conventional video cameras suffer from temporal undersampling.

- It can be solved by downsampling slightly after that displacement become small and its size become small and it will be computed early.

spatial downsampling:
gaussian smoothing +
sub sampling

Aperture Problem

Suppose we have white piece of paper having hole so you can see only from that.

Slide 9: ??

In B

actual movement is ↑

but we will see

↖

bcz of line on sheet

Actual

↖↑

Movement:

- Along the edge did not show
- but across the edge will be visible

This is called Aperture Problem.

Why we study this:

We want to find

$\square \longrightarrow \begin{bmatrix} u \\ v \end{bmatrix}$??

but we can only find normal flow $\longrightarrow \begin{bmatrix} u_n \\ v_n \end{bmatrix}$

At the
Slide 11

Now see
We have pixe
D
x_0, y_0

→ x_0, y_0
the sa

all neig
It will be
true when
correct
u, v are
given.

E (u, v

The be
that

00349587

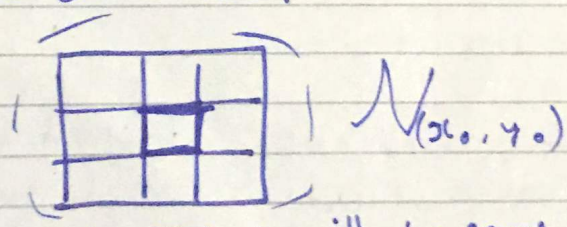
At the movement we are skipping Slide 11.

But we may have to consult this for assignment.

Now see the method to get 2nd eq:
We have pixel

D
x_0, y_0 we want to find u & v

→ x_0, y_0 neighbouring pixel will also go in the same direction.



So, the u, v will be same for all neighbouring pixel.

It will be true when correct u, v are given. // $f_x u + f_y v + f_z = 0$ for one

$$E(u,v) = \sum_{N(x_0,y_0)} \underbrace{(f_x u + f_y v + f_z)^2}_{\text{LOF}}$$

The best u^*, v^* is that that minimize $E(u,v)$

$$u^*, v^* = \arg\min \left[E(u,v)\right]$$

for fixed x_0, y_0

We have use Σ insted of $\int$

Why $1/2$: 2 will come down ~~Ec~~ it will be cancelled out.

These two eq are linear in our unknown u, v

On slide 16:

A may be invertable in many ~~some~~ cases and can be non-invertable in some cases

- If A rank is zero then we can't say any thing.
- If A rank is 1 we can't find inverse ~~so~~ we can find normal flow.

Lec #6

Computer Ethics:

It is like other ethics

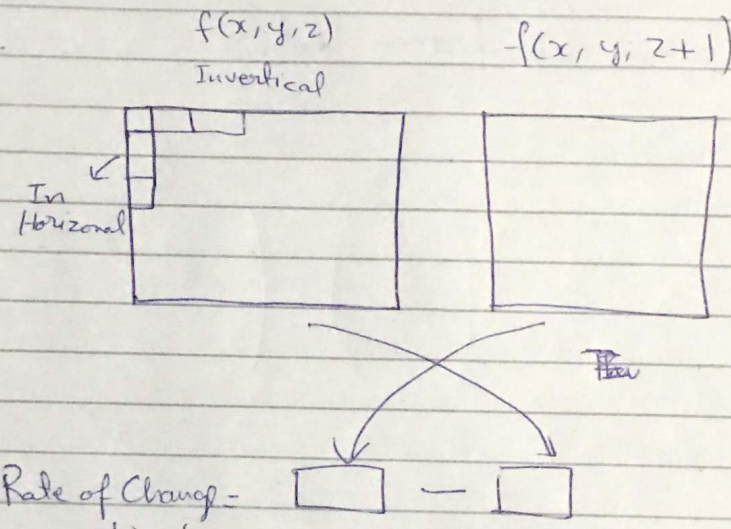
from slide

C.V

Lec#7

Assumption

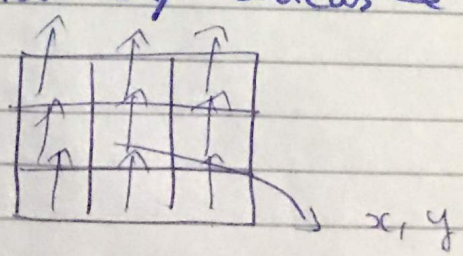
$$f(x+u, y+v, z+1) - f(x, y, z) = 0$$



Do Your Own Eq. of "The Normal Flow"
- Normal flow don't give good result.

New Assumption by Lucas & other

Their Assumption



All have the same displacement vector.

find u, v displacement vector.

- Ya 50 (min) in Batting
- Ya 1 fifer in Match
- ~~Ya 6 wickets in Match~~

Before we have 1 fixed x, y eq.

$$f_x u + f_y v + f_z \Big|_{x,y} = 0$$

but Now we have 4 eq. ~~for~~ with same u, v

This is 4 over constraint system

So,

$$E(u,v) = \frac{1}{2} \sum_N \underbrace{\left(\overbrace{f_x u}^{f_x(x,y,z)u} + f_y v + f_z \right)^2}_{\text{error}}$$

Because value become bigger & we are going away from the 0.

So, this is an error function.

Now what will be u^* & v^* is that minimises the error so,

$$u^*, v^* = \arg\min E(u, v)$$

Now,

$$\frac{\partial E}{\partial u} = \sum_N (f_x u + f_y v + f_z) \cdot f_x = 0$$

$$\frac{\partial E}{\partial v} = \sum_N (f_x u + f_y v + f_z) f_y = 0$$

$\pm u + \pm v$

Both Eq. are linear so, ye can write its in Matrix-Vector form.

Now, we have

- Unknown vector
- Structure tensor like matrix

From Slide "Spatial Approach of Lucas and Kanade"

$$\begin{pmatrix} - & - \\ - & - \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \quad \\ \quad \end{pmatrix}$$

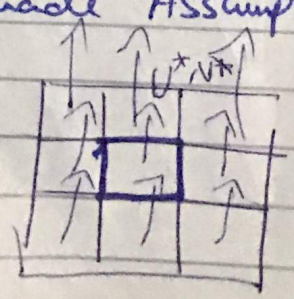
$$A \qquad x = b$$

$$x = A^{-1} b$$

- A^{-1} can be invertible if Rank is 2 than Optic flow is done.
- If A's rank is 1 than Normal flow
- If its rank is 0 than its derivative is zero.

So, there is no colour change in that region.

Lucas & Kanade Assumption:



the u^*, v^* which is found are on the pixel only

Trace of a Matrix tell about the ~~trace~~ eigen values

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\lambda_1 + \lambda_2 \left(\text{Eigen Values}\right) = \text{trace}(M) = a + d$$

$$\lambda_1 * \lambda_2 = \det(M) = ad - bc$$

Rank(J) = 0

Criteria

$a + d \leq \varepsilon$

threshold

$ad - bc < \varepsilon$

Criteria

trce(J) > ε

If trace value is very small

- Positive definite is that which have eigen values > 0
- Positive semidefinite is that which have eigen values ≥ 0

To visualize Optic flow

Now

① $m = \sqrt{u^2 + v^2}$

② $\theta = \arctan \frac{v}{u}$

$H(x, y) = \theta$

$S(x, y) = m$

$V(x, y) = 1$

Advantages / Disadvantages

Beigner local Approach X

- Global optic flow find u^*, v^* on the basis of whole image.

"Horn is the pioneer of C.V"

- From now on 'u^* & v^*' is a matrix of whole image size

$u(x, y)$ 'u' component of x, y
$v(x, y)$ 'v' component of x, y

Now again write error function for whole image.

$$E(\underbrace{u, v}_{\text{Matrix}}) = \frac{1}{2} \int_{\Omega} (f_x u + f_y v + f_z)^2$$

Ω: whole Img domain (pixels)

$f(x, y, z)\, u(x, y)$

When you minimize this you will get result of Normal flow

- Both of them say

$$E(u,v) = \frac{1}{2}\sum_{\substack{\text{all}\\ \text{pixel}}} (f_x u + f_y v + f_z)^2 + \alpha \underbrace{\left(u_x^2 + u_y^2 + v_x^2 + v_y^2\right)}$$

these are the derivatives

we want these derivative small so that this term will be small & error will be small So, it is called smoothness term.

→ Lucas, Kanade have hard constraint of derivative is 'zero'

- Horn & Schunck have soft constraint of derivative is 'small'

Quiz on Error function Understanding

If data term is small & smoothness term is big

Example

R.T	S.T
≈ 10	≈ 1000 ~~million~~

the min value is maximize by 1 million term

If we divide 1000 by 100 than value is 10 so these values are comparable.

x^2
$2x \cdot 1$

Lucas & Kanade

In this we have:

$$E(u,v) = \frac{1}{2}\int_{B_p(x_0,y_0)} (f_x u + f_y v + f_z)^2 \, dx\,dy$$

$(x+1)^2$

$2(x+1)$

u^2

$2u^{2-1}\left(\frac{du}{dy}\right)$

$(u^*, v^*) = \arg\min E(u,v)$

So,

$$\frac{\partial E}{\partial u} = \int_{B_p(x_0,y_0)} f_x (f_x u + f_y v + f_z)\, dx\,dy = 0 \quad (1)$$

$$\frac{\partial E}{\partial v} = \int_{B_p(x_0,y_0)} f_y (f_x u + f_y v + f_z)\, dx\,dy = 0 \quad (2)$$

Eq. (1)

$$\int_{B_p(x_0,y_0)} f_x^2 u \, dx\,dy + \int f_x f_y v \, dx\,dy + \int f_x f_z \, dx\,dy = 0$$

Eq (2)

$$\int_{B_p} f_y f_x u \, dy\,dx + \int_{B_p} f_y^2 v \, dy\,dx + \int_{B_p} f_y f_z \, dx\,dy = 0$$

∂x^2
$2x \cdot 1$

$dxdy$

$D)^2$

$)$

$= 0$
①

$y = 0$
②

$y = 0$

f

eq① $\rightarrow \int_{B_p} f_x^2 u \, dx\, dy + \int_{B_p} f_x f_y v \, dx\, dy = -\int_{B_p} f_x f_z \, dx\, dy$

eq② $\rightarrow \int_{B_p} f_y f_x u \, dx\, dy + \int_{B_p} f_y^2 v \, dx\, dy = -\int_{B_p} f_y f_z \, dx\, dy$

In matrix Vector form.

$$\underbrace{\begin{bmatrix} \int_{B_p} f_x^2 dxdy & \int_{B_p} f_x f_y dxdy \\ \int_{B_p} f_y f_x dxdy & \int_{B_p} f_y^2 dxdy \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -\int_{B_p} f_x f_z dxdy \\ -\int_{B_p} f_x f_z dxdy \end{bmatrix}}_{b}$$

- Multiply Both sides by $1/|B_p|$, so integral converts to averaging
- Averaging to weighted averaging
- Weighted averaging is Gaussian smoothing.

$$\begin{bmatrix} \frac{1}{|B_p|}\int_{B_p} '' & \frac{1}{|B_p|}\int_{B_p} '' \\ \frac{1}{|B_p|}\int_{B_p} '' & \frac{1}{|B_p|}\int_{B_p} '' \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \frac{1}{|B_p|}\int_{B_p} '' \\ \frac{1}{|B_p|}\int_{B_p} '' \end{bmatrix}$$

Now, $\frac{1}{|B_p|}\int_{B_p} \approx K_p$ (Gaussian convolution)

So,

$$\overbrace{\begin{bmatrix} K_p * (f_x^2) & K_p * (f_x f_y) \\ K_p * (f_x f_y) & K_p * (f_y^2) \end{bmatrix}}^{J} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -K_p * (f_x f_z) \\ -K_p * (f_y f_z) \end{bmatrix}$$

Horn & Schunck

$$\frac{1}{2}\int_{\Omega}\Big[\underbrace{(f_x u + f_y v + f_z)^2}_{\text{data term}} + \alpha\underbrace{(|\nabla u|^2 + |\nabla v|^2)}_{\text{Smoothness Term}}\Big]\,dx\,dy$$

Has Unique Solution that depend Continuously on image data.

Lec#8

$$\underbrace{E(u,v) = \frac{1}{2}\sum_{\substack{\text{all}\\\text{pixel}}} (f_x u + f_y v + f_z)^2}_{\text{Only Normal flow}} + \alpha(|\nabla u|^2 + |\nabla v|^2)$$

So,

$$= \frac{1}{2}\sum_{\substack{\text{all}\\\text{pixel}}} \underbrace{f_x(x,y,z)u(x,y) + f_y(x,y,z)v(x,y) + f_z(x,y,z)}$$

$$+ \alpha\underbrace{\left[u_x^2 + u_y^2 + v_x^2 + v_y^2\right]}$$

F (Functional

$$u^*, v^* = \arg\min E(u, v)$$

Calculus:

$$f(x) \to \mathbb{R}$$
$$x \in \mathbb{R}$$

$$\frac{d}{dx} f(x) = 0$$

$I(x,y)$ // We can treat it as function

So, we can treat $U(x,y)$ & $V(x,y)$ also as function.

So, E is a function of 'functions'
So, E is called functional.

Now Calculus of **Variation:**

$F(f) \to \mathbb{R}$

In functional,

f^* is the minimizing function

find f^*, best function

This is called Euler-Lagrange Equation

So,

We can take here it as scalar for 1 pixel
$U(x,y), V(x,y)$

$$F(x, y, U, V, U_x, U_y, V_x, V_y) \Big\} \text{ Integrand}$$

$U_x(x,y)$ - - - -

Now In case of functional for minimizing Euler-langrange eq. must be satisfied.

$$\partial_x F_{U_x} + \partial_y F_{U_y} - F_U = 0,$$

$$\partial_x F_{V_x} + \partial_y F_{V_y} - F_V = 0$$

Here

$$F_U = (f_x U + f_y V + f_z) f_x$$

$$F_{U_x} = 2\alpha U_x(x,y)$$

$$\partial_x F_{U_x} = 2\alpha U_{xx}$$

etc !

∇ (gradient operator)

Δ (laplasian operator)

ΔU means

ΔU = Sum of 2nd derivatives

$= U_{xx} + U_{yy}$

We have skip the boundary conditions equation.

→ Now

Slide 41

$$0 = \frac{\alpha}{h^2} \sum_{j \in N(i)} (U_j - U_i) - f_{xi}(f_{xi} U_i + f_{yi} U_i + f_{zi})$$

$$0 = \quad '' \quad - f_{yi}(f_{xi} U_i + f_{yi} V_i + f_{zi})$$

Now

$$\frac{\alpha}{h^2} \sum_{j \in N(i)} (U_j - U_i) = f_{xi}(\quad '' \quad)$$

$$\frac{\alpha}{h^2} \sum_{''} U_j - \frac{\alpha}{h^2} W_i U_i - f_{x_i}^2 = \quad ''$$

C.V

Lec#9

Motivation:

Slides

Camera

Summary:

The whole process can be encoded in a 3x4 Camera Projection matrix

- It is a linear transformation
- 'X' point in world
- 'x' point in the image

Pinhole Camera:

Pinhole Camera Model (Imp for Paper)

- **'C' focal Point:** This is the location of pinhole. OR Centre of lens. The ray pass and make image.

- The ray passes from focal point & make picture.
  scene point **'M'** (Inworld) connect it to focal Point **'C'** and connect it to Image plane at **'m'** image point.

- 3D coordinate system is extracting from focal point 'C' is X, Y & Z.

Z-axis:

- The camera is oriented where

Crucial Point

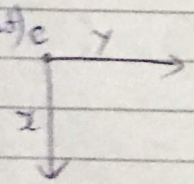
- 'M' is world point is represented in this coordinate system.

-> Pinhole & Image plane (distance b/w them is focal distance

-> X, Y plane are called focal Plane

-> The ray which pass through focal point unaffected through optical axis is at 'c' on image plane and we can place x,y plane at this point.

(principal point) c



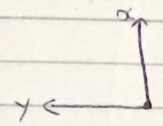
we invert them bcz of inversion in camera

-> So, Now image has mapped the 3D world into a 2-D image.

-> ray from 'm' to 'M' is called optical ray.

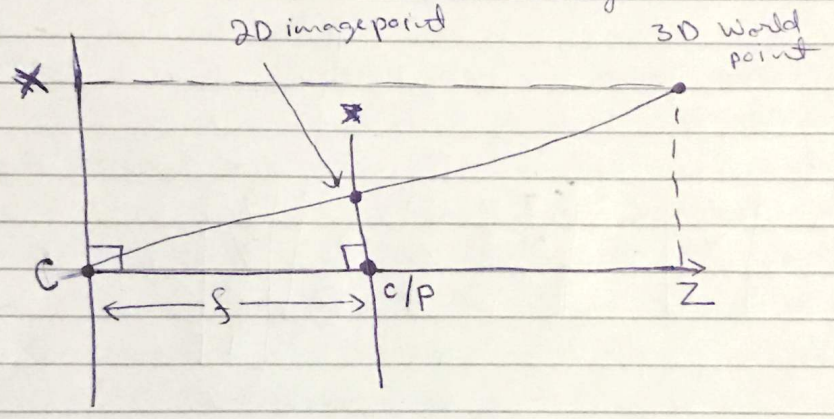
Drawback:

Simplification: If we keep image plane in front of focal plane 'f' distance



Axis are same.

- but it makes virtual Image.



→ See it from Upside

- Similar triangle are those which have same opening angel.

So, $\frac{X}{Z} = \frac{x}{f}$ (these ratio's are same)

$$\left.\begin{aligned} \Rightarrow x &= \frac{fX}{Z} \\ \Rightarrow y &= \frac{fY}{Z} \end{aligned}\right\} \text{These are image point mapping.}$$

These are not linear bcz of division by Z.

<u>Now</u>, for this we will use projective space/coordinates.

- Any scalling of 'm' lies on the same optic ray.

Homogenous Coordinates:

homos = same
genos = kind

- Depth info is lost.

(i) take world point to homogenous space

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{Projection Matrix}} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\tilde{m} = P\tilde{M}$$

→ We are working in Projective space
→ & We are also working in projection

$\tilde{v}$ means it is in projective space

Lec#10

Matlab:

tan	tand		atan	atan2
(Angle in Rad) $\frac{\pi}{2}$	(Angle in θ°) 90°		(Range: $-\frac{\pi}{2}$ to $\frac{\pi}{2}$)	(Range: $-\pi$ to π)

atan2d
(Range: -180° to 180°)

Back to Lecture:

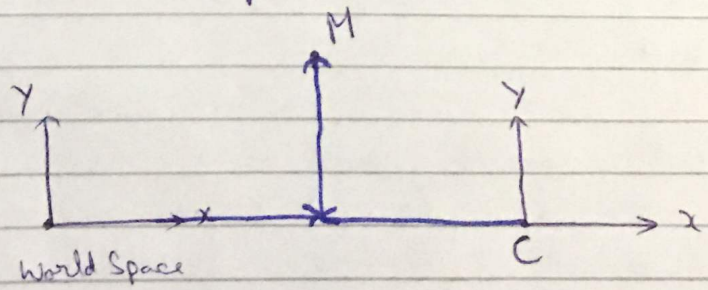
$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\tilde{m} \simeq P\tilde{M}$$

Notation of projective equation:

$$\underbrace{w}_{\text{Scalling}}\tilde{m} = P\tilde{M}$$

Example



If we apply some translation on M we will get the location of M according to C.

$$\underbrace{TM}_{\downarrow}$$

$$T = \begin{bmatrix} 1 & 0 & -C_X \\ 0 & 1 & -C_Y \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

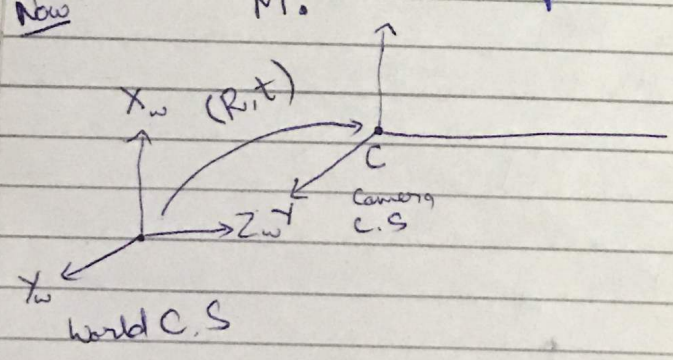
$$M_c = \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 1 \end{bmatrix}$$

$$M_c = \begin{bmatrix} 5-10 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 10 \\ 1 \end{bmatrix}$$

Extrinsic & Intrinsic Camera parameter

Now

M_o



To convert world point into camera point we can do it by some rotation & translation.

$$M_c = [R \quad T] \, M_w$$

first apply rotation then translation

→ Any po
→ Rotation
it
→ focal le
→ So, 4
but
→ We
conve
Scalli

Intrinsic:

→ Repres
w.r.t
transla

- hu &
- hu Co

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} [R \quad T] \, M_W$$

Extrinsic Camera Parameters

→ Any point in the world have a unit.
→ Rotation & Translation does not change ~~each~~ its units. (Assume Unit: inches)
→ focal length is also in inches
→ So, the image location is in inches but this is not pixel location
→ We want inches to pixel conversion. We can do it by ~~[illegible]~~ ?? Scalling.

Intrinsic:

N°
c

→ Representing 'm' w.r.t 'N' not with w.r.t 'c' we can do it by some translation in image plane.

$$H = \begin{bmatrix} h_u & -h_u \cot\theta & u_0 \\ 0 & h_v/\sin\theta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

- h_u & h_v converting it to pixel
- $h_u \cos\theta$ is telling about the shear.

- Normally in cameras pixel are square but there is possibility that pixel can be rectangle.

- In worst case, θ may be $\neq 90^\circ$, so we calculate shear.

After that we will get 'm' in pixel.

<u>Question Paper</u>:

They are going from which space to which.

Extrinsic

→ To do translation in 3D, degree of freedom. 3

→ " " Rotation " " , degree of freedom 3

Outside the camera

So, extrinsic degree of ~~rotation~~ Fredom is 6.

Projection Matrix, D.O.F : 1 (Included in Intrinsic)

Intrinsic " , D.O.F : 5

Total 12 parameters.

square but
can be

$= 90°$, so

in

ace to

ee of

at

Fredom

1 (Included in Intrinsic)

Extrinsic : 4×4 in $\mathbb{P}^3$
Projection : 3×4
Intrinsic : 3×3
So, final result is 3×4
D.O.F is 12. (That is in Correct)
D.O.F is 11

→ After scalling, ~~of~~ It don't effect the functionality

Paper Imp

→ Projectively equivalent Matrix.

Example

$$m = PM$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = P.M \xrightarrow{\text{Scalling}} m \begin{bmatrix} ka \\ kb \\ kc \end{bmatrix} \Rightarrow \begin{bmatrix} ka/kc \\ kb/kc \\ kc/kc \end{bmatrix}$$

$$\begin{bmatrix} a/c \\ b/c \\ 1 \end{bmatrix} = P.M \qquad \approx \qquad = \begin{bmatrix} a/c \\ b/c \\ 1 \end{bmatrix}$$

⇒ The scalling of P do the same work as of P.

→ So, we have 1 free scalling parameter.
Degree of Freedom = 11

What is the scalling factor in all of the above:

'f' focal distance

- Picture big or small they have to go to same place.

Thursday, Camera Slides Quiz

Lec#11

Anatomy of P

1st Slide:

Camera Centre:

We have

$$P = \begin{bmatrix} \boxed{\begin{matrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{matrix}} & \begin{matrix} P_{14} \\ P_{24} \\ P_{34} \end{matrix} \end{bmatrix}$$

M

Max Rank : 3, 2, 1 Dimentionality = 4

If Rank = 3:

Rank + nullity = dim

$\underset{(\text{No. of rows})}{3} + 1 = \underset{(\text{No. of cols})}{4}$

Null space represented by how many vectors $P\vec{v} = \vec{0}$

If we multipy P matrix by $\vec{v}$ & its answer is null vector than $\vec{v}$ lies in the null space of P.

One such vector exist by which we multiply $\vec{v}$ by P than answer is $\vec{0}$.

$$\underset{3\times4}{P}\ \underset{4\times1}{\vec{v}} = \vec{0}$$

$\mathbb{P}^3$

In matlab:

null(P) this is the camera centre C location.

Join of A & C

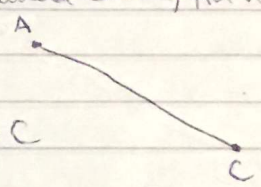
$$X(\lambda) = (1-\lambda)\underbrace{C}_{\text{Camera Centre/Pin hole}} + \lambda \overbrace{A}^{\text{Point}}$$

$\lambda = 0$, $X(\lambda) = C$

$\lambda = 1$, $X(\lambda) = A$

$\lambda = 0.5$, In middle of A & C

$\lambda = 0.75$, More towards A



→ All point ~~same as~~ on the line ~~A~~ on from A to C will map on the same image point

So,

project of A = Projection of $X(\lambda)$

$$PA \approx PX(\lambda) \longrightarrow \text{In rand} \equiv$$

$$\begin{bmatrix} - \\ - \\ - \end{bmatrix} \quad \begin{bmatrix} - \\ - \\ - \end{bmatrix}$$

These are projectively equivalent.

$$\lambda(PA) = \cancel{(\lambda)}\ P[(1-\lambda)C + \lambda A]$$

$$\cancel{\lambda PA} = (1-\lambda)PC + \cancel{\lambda PA}$$

$$(1-\lambda)PC = 0$$

$$\Rightarrow PC = \vec{0}$$

Paper Must

- Prove Camera ~~Space~~ Centre lies at null space

$$\underset{\mathbb{P}^2}{\underset{\sim}{PC}} = \underset{\mathbb{P}^2}{\underset{\sim}{0}}$$

Wrong: Projective space have all point but not have $(0,0,0)$

So, camera cant take picture of camera centre.

If you say, you want to capture point 'C' than it will not intersect and there is no line.

- You can do it in both Mathematically and geometrically.

- 'P' Rank is same as 'M'
- **If Rank(M) = 2**, than camera centre lies at infinity point.

It is called orthographic camera.

A camera can be looking at realy far away we can you the model of Rank = 2, In which camera centre lies at infinity.

This is **camera at Infinity** model.

<u>Note</u>

We have point in:

$$\mathbb{P} \qquad \mathbb{R}$$

$$\begin{bmatrix} a \\ b \\ 1 \end{bmatrix} \qquad \begin{bmatrix} a/1 \\ b/1 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ 1/2 \end{bmatrix} \qquad \begin{bmatrix} 2a \\ 2b \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ 1/100 \end{bmatrix} \qquad \begin{bmatrix} 100\,a \\ 100\,b \end{bmatrix}$$

So,

$$\lim_{w \to 0} \begin{bmatrix} a \\ b \\ w \end{bmatrix} \longrightarrow \left\{ \text{at point at infinity in direction of } \begin{bmatrix} a \\ b \end{bmatrix} \right.$$

It approaches to Infinity in the direction of a & b.

If we are in 3D:

$$\begin{bmatrix} a \\ b \\ c \\ w/0 \end{bmatrix} \longrightarrow \text{at point in infinity in direction of } \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

→ In non-homogenous space $\mathbb{R}^2, \mathbb{R}^3$ ---
we can define infinity, so we use symbol.
But in projective space, we can define infinity

- Q: What is the representation of x-axis in $\mathbb{R}^3$.

point at infinity in direction of x-axis $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

in y-axis. $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

in z-axis. $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

Now, $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ this is $\mathbb{R}^3$ origin in $\mathbb{P}^3$.

In $\mathbb{R}^3$ if you convert ~~to~~ $\mathbb{P}^3$ to $\mathbb{R}^3$

$$\begin{bmatrix} 1/0 \\ 0/0 \\ 0/0 \end{bmatrix} \begin{matrix} \infty \\ \text{NAN} \\ \text{NAN} \end{matrix}$$

this is wrong. so always use the limiting case

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ \to 0 \end{bmatrix} \to \begin{bmatrix} \to \infty \\ 0 \\ 0 \\ \cdot \end{bmatrix}$$

So, camera matrix P:

$$\mathbb{P}^2 \xrightarrow{P} \mathbb{P}^3$$

$$P\begin{bmatrix}1\\0\\0\\0\end{bmatrix} \longrightarrow \begin{bmatrix}P_{11}\\P_{21}\\P_{31}\end{bmatrix}$$

So, we can tell this is the image of infinity in the x-direction.

$$P\begin{bmatrix}0\\1\\0\\0\end{bmatrix} \longrightarrow \begin{bmatrix}P_{12}\\P_{22}\\P_{32}\end{bmatrix}$$

→ this is the image of infinity in y-direction.

$$P\begin{bmatrix}0\\0\\1\\0\end{bmatrix} \longrightarrow \begin{bmatrix}P_{13}\\P_{23}\\P_{33}\end{bmatrix}$$

→ this is the image of infinity in z-direction

$$P\begin{bmatrix}0\\0\\0\\1\end{bmatrix} \longrightarrow \begin{bmatrix}P_{14}\\P_{24}\\P_{34}\end{bmatrix}$$

it is the image of world origin.

Exam Question: Why are these point of infinity.

-> Infinity picture is called ~~infinity~~ vanishing point (P)

Question in Paper:

In this image.

Pixel 'p' in camera Matrix is the third column.

Question at Slide V. Imp for paper.

Column wise

$$\begin{array}{cccc} P_1 & P_2 & P_3 & P_4 \end{array}$$
$$\begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix}$$

Row wise

$$P^i = \begin{bmatrix} P_{i1} \\ P_{i2} \\ P_{i3} \\ P_{i4} \end{bmatrix}$$

Other Representation

$$P = \begin{bmatrix} P^{1T} \\ P^{2T} \\ P^{3T} \end{bmatrix}$$

point

is the

line eq: $y = mx + c$

$y - mx - c = 0$

We can also write it as

$-mx + 1y - c = 0$

$$\begin{bmatrix} -m & 1 & -c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

Now,

$$aX + bY + cZ + d = 0$$

X, Y, Z which satify this equation lies in plane.

So,

$$\begin{bmatrix} \underbrace{\overbrace{a \quad b \quad c}^{\text{represent plane normal vector.}} \quad \overbrace{d}^{\text{plane distance from origin.}}}_{\text{these define the plane}} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = 0$$

-> There 4 - number representing the plane

-> So, If we given four number we represent p^{1T}

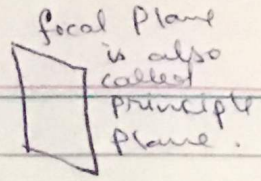
-> With another given " " "

" p^{2T}

same for p^{3T}

Now, If this point $\vec{X}$ satisfy

$$p^{1T} \vec{X} = 0$$



$$p^{2T}\vec{X} = 0$$

$$p^{3T}\vec{X} = 0$$

Now, lets consider p^{3T} plane.

If

$$p^{3T}X = 0 \qquad p\vec{X}$$

So,

its image is of form $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

So, p^{3T} is principle plane.

al Plane is also called principle plane.

Lec#12

- Vanishing point is the image of infinity in the image.

Slide: 33

- The 2nd row representing is the plane defined by the camera centre and image x-axis.

- With two points we canonly tell about line but not about the plane.
- For plane, we need three points
- There are three points which are representing the unique plane.

Ex There are 4-numbers which are defining the plane.

- **Principle Point:**

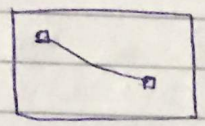
Vector of the form

$$\begin{bmatrix} - \\ - \\ - \\ 0 \end{bmatrix}$$ for infinity

C.V

Lec#13

$\mathbb{P}^2$



$\mathbb{R}^2$

$\vec{a}$ $\vec{b}$

$y = mx + c$

$-mx + y - c = 0$

So,
this is the representation in $\mathbb{P}^2$

$-m \quad 1 \quad -c$

<u>Now</u>

$x = 2$

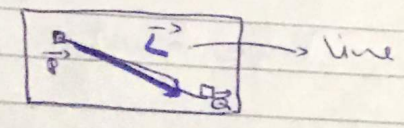
$1x + 0y - 2 = 0$

$$\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

we can represent it as line.

So,
There is a duality b/w pts and line.

In $\mathbb{P}^2$ space:



$\vec{p}$ is passing through the point

$$\begin{matrix} p_x \\ p_y \\ p_w \end{matrix} \qquad \begin{matrix} L_x \\ L_y \\ L_w \end{matrix}$$

→ So if this point lie on L than its eq. must equal to zero.

$$L_x x + L_y y + L_w w = 0$$

$$\vec{L}^T \vec{p} = 0 \qquad \because \vec{p} \text{ lies on } \vec{L}$$

So,

$$\vec{L}^T \vec{q} = 0$$

Now, look at it with pure linear algebra:

$$\vec{L}^T \cdot \vec{p} = 0 \qquad \text{it means } \vec{L} \perp \vec{p}$$

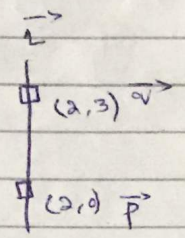
$$\text{&} \quad \vec{L} \perp \vec{q}$$

→ The cross product is orthognal to both the vector so,

$$\vec{L} = \vec{p} \times \vec{q}$$

bcz $\vec{L}$ is orthogonal to both $\vec{p}$ & $\vec{q}$.

Paper question must



Do it on your own

It is "$\vec{L}$" is obtained by the cross product of Both

Its answer would be $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} i & j & k \\ p_x & p_y & p_w \\ q_x & q_y & q_w \end{vmatrix}$$

So,

$$\vec{p}, \vec{q} \longrightarrow \vec{L} = \vec{p} \times \vec{q} \longrightarrow \text{Case I}$$

Flip the roles: Duality Property

Now, If we have two lines than when we take their cross product it gives the point from which both lines are passing

$$\vec{L_1}, \vec{L_2} \longrightarrow \vec{p} = \vec{L_1} \times \vec{L_2} \longrightarrow \text{Case II}$$

'p' is intersection point so, it lies on both lines

$\vec{p}$ lies on $\vec{l_1}$ $\longrightarrow$ $\vec{p}^T \cdot \vec{l_1} = 0$

$\vec{p}$ lies on $\vec{l_2}$ $\longrightarrow$ $\vec{p}^T \cdot \vec{l_2} = 0$

$\Rightarrow \vec{p} \perp l_1$

$\Rightarrow \vec{p} \perp l_2$

$\therefore$ $\vec{p}$ must be the crossproduct of $\vec{l_1}$ & $\vec{l_2}$

All of the above are exam questions.
100%

Remember All these facts for remaining Lecture.

<u>Last lecture:</u>

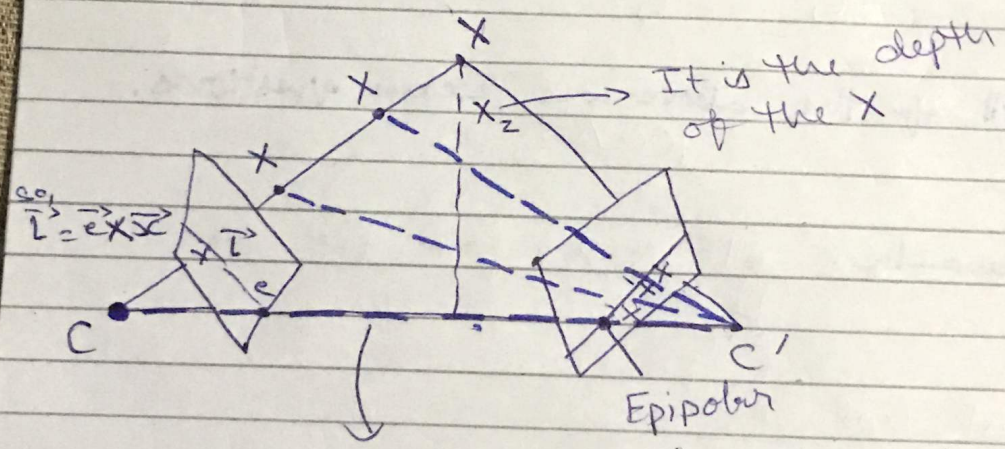
two cameras with centre C & C' capturing the point X

This whole diagram is called epipolar plane

Can we take picture of camera two with camera One

C & C' are on same line so, the intersection of that line with the image plane of C is image of C' (centre of camera two) and vice versa for second camera two thats why we call it epipolar plane.

- These two points lie on line & this line lie on that plane that's why it is called epipolar plane



As X prove along the line than the x' also move alon the line. So, if X move the its x' also move on the epipolar line. So, all lines necessarily pass through this.

- If I give you any pixel in the first plane. You will get line in the 2nd plane.

$$\left.\begin{array}{l} x_1 \longrightarrow l_1 \\ x_2 \longrightarrow l_2 \\ \quad\vdots \\ x_n \longrightarrow l_n \end{array}\right\} \text{all these lines pass through the epipole}$$

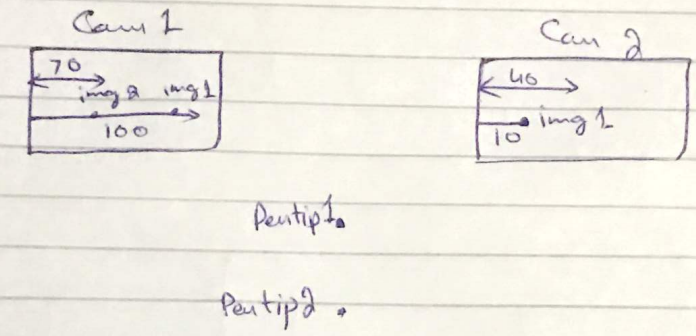
So,

$$e' \Longrightarrow l_1 \times l_2$$

Similarly

Lec#14

disparity

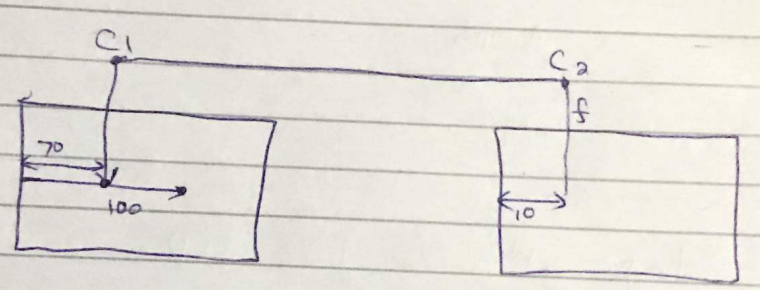


When take pic of pen tip from cam 1 & cam 2
The distance at x-axis at point where img is from cam 1 at 100 and at cam 2 = 10

Now if we increase.

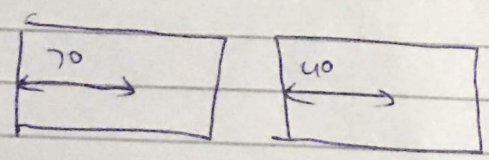
depth

$$Z \propto \frac{1}{\text{disparity}} \quad ; \quad Z \propto \frac{1}{x_1 - x_2}$$



$$\Rightarrow \quad Z \propto \frac{1}{100-10} \Rightarrow Z \propto \frac{1}{90}$$

Now



$$Z \propto \frac{1}{70-40} \Rightarrow Z \propto \frac{1}{30}$$

Now

$$Z = \frac{bf}{x_1 - x_2}$$

Find

Let $x_{x_1} \xrightarrow{?} x'_{x^2}$

x' will always lies on the epipole line

$$\vec{e}' \times \vec{x}' = \vec{l}'$$

$\vec{l}'$ is line passing through $\vec{x}'$

Since $x' = H\vec{x}$ } Homography mapping

So,

$$\vec{l}' = \underbrace{[\vec{e}']_x \times H}_{\text{Fundamental Matrix say } F}\vec{x}$$

Slide of Epipolar Constraint $\Leftarrow$ Fundamental Matrix.

where $F = [e']_x H$

So,

$$l' = F x$$

$$\text{Rank}(A \times B) = \min(\text{Rank}(A), \text{Rank}(B))$$

$$\vec{l}' = F\vec{x}$$

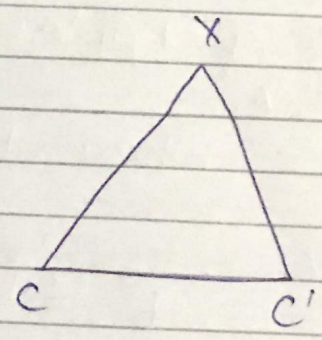
$\text{Rank}(F) = 2$ b/c $\vec{e}$ rank 2 and H rank is 3

$$\vec{l}' = F\vec{x}$$

get a point $\vec{x}$ in camera 1

→

- We don't know any thing about the plane
- don't know about X, C, C' or any epipolar.



→ If we have some correspondances than we can find the matrix 'F'

→ we know x' lie on l'

$$\underbrace{\vec{x}'^T l'}_{0} = \vec{x}'^T F x$$

So it is the epipolar constraint

If $\vec{x}$ & $\vec{x}'$ are correspondand points than R.H.S would be 0.

→ F is 3x3 matrix & its rank is 2 & its in projective space

→ F: degree of Freedom
9 − 2(Rank)
F(DOF): 7

$$\vec{x}'^{T} l' = \underbrace{\vec{x}'^{T} F x}_{\text{Scalar}}$$

(1 Equation/Constraint)

F

$x_1 \longrightarrow x_1'$

⋮

$x_n \longrightarrow x_n'$

This type of stereo matrix is called calibrated matrix.

$F = ?$

Constraint

$$x_2^T F x_1 = 0$$

So,

$$(x_2, y_2, 1) \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Result would be linear

We want to write our unknowns in vector form.

Estimat of fundamental matrix (imp)

~~$x_1 F x_2$~~

$$x_1 x_2 f_{11} + y_1 x_2 f_{12} + x_2 f_{13} + \cdots$$

= 9 terms

each term will be linear

So its a dot product of

$$\underbrace{\begin{bmatrix} x_1 x_2 & y_1 x_2 & \cdots \end{bmatrix}}_{S_1^T} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ \vdots \end{bmatrix}$$

So,

$$\vec{S_1}^T \vec{F} = 0$$

$$\vec{S_2}^T \vec{F} = 0$$

$$\vdots$$

$$S_n^T F = 0$$

// For 2nd corresponding term

So,

$$\begin{bmatrix} S_1^T \\ S_2^T \\ \vdots \\ S_n^T \end{bmatrix} \underset{9\times1}{\vec{f}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$N \times 9$ $\qquad$ $N \times 1$

$S_1^T f = 0$ $\qquad$ if F is correct than whole eq gives zero.

- $\vec{f}$ will be given we put it in eq. closer to zero ans means f is the best Ans.

So, f^* will be that which minimizes our error.

$$F(f) = \sum_{N=1}^{N} (S_N^T f)$$

We want to find f^*, which minimize the eq.

$$f^* = \arg\min$$

$$\sum_n (S_n^T f)^T (S_n^T f)$$

$$\sum_n f^T S_n S_n^T f$$

$$f^T \underbrace{\left(\sum_N S_n S_n^T\right)}_{A} f$$

$$= f^T A f \longrightarrow \text{eq } 5$$

eq 5 is similar to eq given in corner detection.

f^A = eigen vector of A

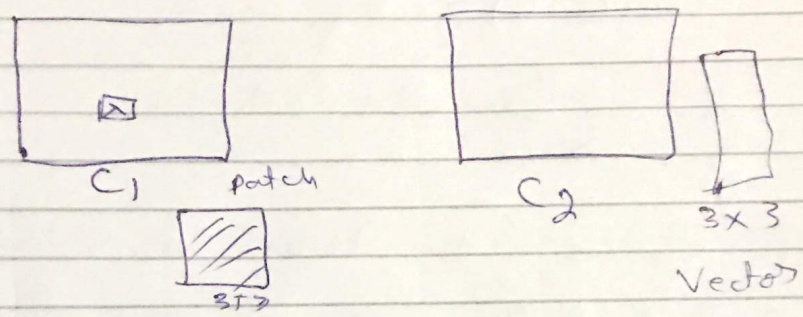
corresponding to smallest non zero.

Two eigen vector.

??

Find f^*, 90% chance in paper.

Now how we find corresponding point in cam 2 (x')



We will compare the value of pixels in region & not compare direct values in both cams.

Subtract the Avrg value from the vectors.

$$\vec{a} = \frac{\begin{bmatrix} - \\ - \\ - \\ - \\ - \\ - \\ - \end{bmatrix} - \text{avrg}(a)}{\left\| \begin{bmatrix} - \\ - \\ - \\ - \\ - \\ - \\ - \end{bmatrix} - \text{avg} \right\|}$$

$$b = \frac{\begin{bmatrix} - \\ - \\ - \\ - \\ - \\ - \\ - \end{bmatrix} - \text{avg}(b)}{\left\| \begin{bmatrix} - \\ - \\ - \\ - \\ - \\ - \\ - \end{bmatrix} - \text{avrg}(b) \right\|}$$

Co-relation Co-efficient:-

If patches are similar.

$$\left.\begin{array}{l} \max(a^T b) = 1 \\ \min(a^T b) = -1 \end{array}\right\} \text{Range}$$

$\vec{a} \longrightarrow$ Same direction
$\vec{b} \longrightarrow$

If value = 1 then it means a & b both are in same direction

Basic Stereo Algorithm Outline: