*International Journal of Mathematical Sciences and Applications*
*Vol. 1 No. 2 (May, 2011)*

*www.ijmsa.yolasite.com*

# ASSOCIATORS IN THE NUCLEUS OF LIE ADMISSIBLE RINGS

*Dr.D. Prabhakara Reddy,*
*Assistant Professor, Humanities and Sciences Department, CMR Institute of Technology, Kandlakoya, Medchal Road,Hyderabad-501401, Andhra Pradesh, INDIA.*

## ABSTRACT

In this paper we consider a simple Lie admissible third power associative ring $R$ satisfying $(y,x,x)=k\,(x,x,y)$ for all $x, y \in R$, $k \neq 0,-1$ and $(k+1)^2 + 2 \neq 0$ and prove that the nucleus is equal to the center in $R$. Also we show that if $R$ is a prime third power associative antiflexible ring, then $R$ is Lie admissible. Using this, it is proved that $R$ is either associative or the nucleus is equal to the center.

**Mathematics Subject Classification:** Primary 17A30

**Keywords:** Nonassociative ring, Lie admissible ring, antiflexible ring,simple ring,prime ring, characteristic and associator.

## 1. INTRODUCTION

E. Kleinfeld and M. Kleinfeld **[2]** studied a class of Lie admissible rings. Also in **[3]** they have proved some results of a simple Lie admissible third power associative ring $R$ satisfying an equation of the form $(x,y,x) = k\,(x,x,y)$ for all $x,y \in R$, $k \neq 0, -1$, and $k^2 + 2 \neq 0$. In this paper we consider the same ring $R$ satisfying $(y,x,x)=k(x,x,y)$ for all $x, y \in R$, $k \neq 0,-1$ and $(k+1)^2 + 2 \neq 0$ and prove that the nucleus is equal to the center in $R$. Also we show that if $R$ is a prime third power associative antiflexible ring, then $R$ is Lie admissible. Using this, it is proved that $R$ is either associative or the nucleus is equal to the center.

## 2. PRELIMINARIES

Let $R$ be a nonassociative ring. We shall denote the commutator and the associator by $(x,y) = xy-yx$ and $(x,y,z) = (xy)z-x(yz)$ for all $x,y,z$ in $R$ respectively. A ring $R$ is called antiflexible if $(x,y,z) = (z,y,x)$ for all $x,y,z$ in $R$. The nucleus $N$ of a ring $R$ is defined as $N = \{n \in R \,/\, (n,R,R) = (R,n,R) = (R,R,n) = 0\}$.The center $C$ of $R$ is defined as $C = \{ c \in N \,/\, (c, R) = 0\}$. A ring $R$ is said to be of characteristic $\neq n$ if $nx = 0$ implies $x = 0$, for all $x \in R$ and $n$ is a natural number. A ring $R$ is of characteristic $\neq n$ is simply denoted by char.$\neq n$. A ring $R$ is called simple if $R^2 \neq 0$ and the only nonzero ideal of $R$ is itself. Since $R^2$ is a non-zero ideal of $R$, we have $R^2 = R$. A ring $R$ is called prime if whenever $A$ and $B$ are ideals of $R$ such that $AB = 0$, then either $A = 0$ or $B = 0$.

Throughout this paper $R$ represents a nonassociative ring of char. $\neq 2, 3$.

## 3. MAIN RESULTS

If $R$ is Lie admissible, then it satisfy the identity

$$(x,y,z) + (y,z,x) + (z,x,y) = 0, \qquad 3.1$$

for all $x,y$ and $z \in R$.

We use the following two identities which hold in all rings:

$$P(w,x,y,z) = (wx,y,z) - (w,xy,z) + (w,x,yz) - w(x,y,z) - (w,x,y)z = 0 \qquad 3.2$$

and $D(x,y,z) = (xy,z) - x(y,z) - (x,z)y - (x,y,z)$

$$- (z,x,y) + (x,z,y) = 0. \qquad 3.3$$

The first is known as the Teichmuller identity and the second is called the semi Jacobi identity.

We denote $S(x,y,z) = (x,y,z) + (y,z,x) + (z,x,y)$. 3.4

Then from $D(x,y,z) - D(y,x,z)$, we obtain

$$((x,y),z) + ((y,z),x) + ((z,x),y) = S(x,y,z) - S(x,z,y). \qquad 3.5$$

As was observed by Maneri in [4], in an arbitrary ring with elements $w, x, y, z$ we have

$$0 = P(w,x,y,z) - P(x,y,z,w) + P(y,z,w,x) - P(z,w,x,y)$$
$$= S(wx,y,z) - S(xy,z,w) + S(yz,w,x) - S(zw,x,y) - (w,(x,y,z))$$
$$+ (x,(y,z,w)) - (y,(z,w,x)) + (z,(w,x,y)).$$

We now assume that $R$ satisfies identity 3.1, $S(a,b,c) = 0$, for all $a,b,c \in R$.

So the above equation imply

$$(w,(x,y,z)) + (x,(y,z,w)) - (y,(z,w,x)) + (z,(w,x,y)) = 0. \qquad 3.6$$

Let $N$ be the nucleus of $R$, and let $n \in N$. By substituting $n$ for $w$ in 3.6, we get

$$(n,(x,y,z)) = 0. \qquad 3.7$$

i.e., $n$ commutes with all associators.

The combination of 3.7 and 3.2 yields

$$(n,w(x,y,z)) = -(n,(w,x,y)z). \qquad 3.8$$

If $u$ and $v$ are two associators in $R$, then substituting

$z = n$, $x = u$, $y = v$ in 3.3, we get

$$(uv, n) = 0. \qquad 3.9$$

If $u = (a,b,c)$, then $((a,b,c)v,n) = 0$. Using 3.8, we have $-(a(b,c,v),n) = 0$.

From this and 3.3, we obtain $-a((b,c,v),n) - (a,n)(b,c,v) = 0$.

Using 3.7, we have $(a,n)(b,c,v) = 0$. 3.10

Now we prove the following theorem:

***Theorem 3.1:*** *If $R$ is semiprime third power associative ring of char. $\neq 2$ which satisfies 3.1, then either $N = C$ or $R$ is associative.*

***Proof:*** If $N \neq C$, then there exist $n \in N$ and $a \in R$ such that $(a,n) \neq 0$. Hence from 3.10 we have $(b,c,v) = 0$, for all associators $v$, and $b,c \in R$. We can write this as $(R,R,(R,R,R)) = 0$.

By putting $v = (q,r,s)$ in 3.9, we get

$(u(q,r,s),n) = 0$. Using 3.8 this leads to $-((u,q,r)s,n) = 0$. From this and 3.3, we obtain $(u,q,r)(s,n) = 0$. Since $N \neq C$, we have $(u,q,r) = 0$, for all associators $u$, and $q, r \in R$. We can write this as $((R,R,R),R,R) = 0$. Using $(R,R,(R,R,R)) = 0 = ((R,R,R),R,R)$, identity 3.1 gives $(R,(R,R,R),R) = 0$.

Thus $(R,R,R) \subset N$. Since $R$ is semiprime, we use the result in [1] to conclude that $R$ must be associative. This completes the proof of the theorem. $\square$

Henceforth we assume that $R$ satisfies an equation of the form

$$(y,x,x) = k\,(x,x,y) \qquad 3.11$$

for all $x, y \in R$, $k \neq 0, -1$, and $(k+1)^2 + 2 \neq 0$.
Using 3.1, identity 3.11 implies

$$(x,y,x) = -(k+1)\,(x,x,y) = -\frac{(k+1)}{k}\,(y,x,x). \qquad 3.12$$

Now we prove the following Lemma:

***Lemma 3.1:*** *Let* $T = \{t \in R/(t,N) = 0 = (tR,N) = (Rt,N)\}$. *Then* $T$ *is an ideal of* $R$.

***Proof:*** Let $t \in T$, $n \in N$, and $x, y, z \in R$. Then $(tx\cdot y,n) = (t\cdot xy,n) = 0$, using 3.7 and the definition of $T$. Also 3.3 implies $(y\cdot tx,n) = (y,n)\cdot tx$. But 3.3 also yields $(yt,n) = (y,n)t = 0$, since $(yt,n) = 0$.

Now $((y,n),t,x) = ((y,n)\cdot t)x - (y,n)\cdot tx$

$= -(y,n)\cdot tx$

$= -(y\cdot tx,n)$.

or $\quad (y\cdot tx,n) = -((y,n),t,x)$. 3.13

Now consider $((y,n),x,x) = (yn,x,x) - (ny,x,x)$. 3.14

Using 3.12 $(yn,x,x) = k(x,x,yn)$, while $0 = P(x,x,y,n) = (x,x,yn) - (x,x,y)n$ and $0 = P(n,y,x,x) = (ny,x,x) - n(y,x,x)$.

Substituting this in 3.14 and using 3.12 and 3.7 gives

$$\begin{aligned} ((y,n),x,x) &= k(x,x,yn) - n(y,x,x)\\ &= k(x,x,y)n - kn(x,x,y)\\ &= k((x,x,y),n)\\ &= 0. \end{aligned}$$

Linearizing the above identity, we get

$$((y,n),x,z) = -((y,n),z,x). \qquad 3.15$$

Again consider $((x,n),y,x) = (xn,y,x) - (nx,y,x)$. 3.16

From $P(x,n,y,x) = 0$ it follows that $(xn,y,x) = (x,ny,x)$.

From 3.12 it follows that $(x,ny,x) = -\frac{(k+1)}{k}(ny,x,x)$

and from $P(n,y,x,x) = 0$ we have $(ny,x,x) = n(y,x,x)$.

Thus we have $\quad (x,ny,x) = -\frac{(k+1)}{k}(ny,x,x)$ 3.17

From $P(n,x,y,x) = 0$ it follows that $(nx,y,x) = n(x,y,x)$,

while from 3.12 we have $n(x,y,x) = -\frac{(k+1)}{k}n(y,x,x)$.

Therefore $(nx,y,x) = n(x,y,x) = -\frac{(k+1)}{k}n(y,x,x)$. 3.18

Substituting 3.17 and 3.18 in 3.16, we get

$((x,n),y,x) = 0$ . 3.19

By linearizing 3.19, we get

$((x,n),y,z) = -((z,n),y,x)$. 3.20

Combining 3.15 and 3.20 we have

$((\Pi(x),n), \Pi(y), \Pi(z)) = sgn\,(\Pi)\,((x,n),y,z)$ 3.21

for every permutation $\Pi$ on the set $\{x, y, z\}$.

Applying 3.21 we see that

$$\begin{aligned}((y,n),t,x) &= ((y,n),x,t)\\ &= -((t,n),x,y)\\ &= ((t,n),y,x)\\ &= 0,\end{aligned}$$

using 3.15, 3.20, 3.15 and definition of $T$. From this combined with 3.13 we obtain $(y\cdot tx,n) = 0$. At this point $T$ is a right ideal of $R$. By going to the anti-isomorphic ring we similarly prove that $T$ is a left ideal of $R$. Therefore $T$ is an ideal of $R$. This completes the proof of the Lemma. □

***Theorem 3.2:*** *A ring R which satisfies 3.1 and 3.11, is simple and of char. ≠ 2,3 is either associative or satisfies nucleus equals center, N = C.*

***Proof:*** Simplicity of R implies either that $T = R$, in which case $N = C$ and we are done $T = 0$. Hence assume that $T = 0$. Let $u = (a,b,c)$ be an arbitrary associator with elements $a, b, c \in R$. We have already observed that for every associator $v$, we have $(uv,n) = 0$. Now using $(P(u,x,x,y),n) = 0$ and 3.7 gives $((u,x,x)y,n) = -(u(x,x,y),n) = 0$. Using $(P(y,x,x,u),n) = 0$ gives $(y(x,x,u),n) = -((y,x,x)u,n) = 0$. Also 3.12 implies that $y(u,x,x) = ky(x,x,u)$. So $(y(u,x,x),n) = 0$.

Since $((u,x,x),n) = 0$ by 3.7, we have $(u,x,x) \in T$. Since we are assuming $T = 0$, we have $(u,x,x) = 0$, for all $x \in R$. Using this in 3.12, we get $(x,u,x) = 0$ and $(x,x,u) = 0$. Thus

$(x,u,x) = (x,x,u) = (u,x,x) = 0$. 3.22

For $a, b \in R$, we define $a \equiv b$ if and only if $(a - b, n) = 0$ for all $n \in N$.

Let $\alpha = x(y,x,z)$. Because of 3.7, all associators are congruent to zero. Thus $P(x,y,x,z) = 0$ implies $\alpha \equiv -(x,y,x)z$. Now 3.12 implies $\alpha \equiv -(x,y,x)z \equiv -k(x,x,y)z$. Continuing to use $P$ and 3.12 yields

$$\alpha = x(y,x,z) \equiv -(x,y,x)z \equiv (k+1)(x,x,y)z$$

$$\equiv -(k+1)x(x,y,z) \equiv (k+1)(x,x,y)z \equiv \frac{(k+1)}{k}(y,x,x)z$$

$$\equiv -\frac{(k+1)}{k}\,y(x,x,z) \equiv \frac{1}{k}y(x,z,x) \equiv -\frac{1}{k}(y,x,z)x$$

$$\equiv \frac{1}{k}y(x,z,x) \equiv -\frac{(k+1)}{k^2}y(z,x,x) \equiv \frac{(k+1)}{k^2}(y,z,x)x. \qquad 3.23$$

By permutting $y$ and $z$ in 3.23, we get

$$\beta = x(z,x,y) \equiv -(x,z,x)y \equiv (k+1)(x,x,z)y$$

$$\equiv -(k+1)\,x(x,z,y) \equiv (k+1)(x,x,z)y \equiv \frac{(k+1)}{k}(z,x,x)y$$

$$\equiv -\frac{(k+1)}{k} z(x,x,y) \equiv \frac{1}{k} z(x,y,x) \equiv -\frac{1}{k} (z,x,y)x$$

$$\equiv \frac{1}{k} z(x,y,x) \equiv -\frac{(k+1)}{k^2} z(y,x,x) \equiv \frac{(k+1)}{k^2} (z,y,x)x. \qquad 3.24$$

From identity 3.1 we obtain

$$x(x,y,z) + x(z,x,y) = -x(y,z,x). \qquad 3.25$$

using 3.23 and 3.24 in 3.25, we get

$$-\frac{\alpha}{k+1} + \beta \equiv -x(y,z,x). \qquad 3.26$$

However $P(x,y,z,x) = 0$ gives $-x(y,z,x) \equiv (x,y,z)x$.

Thus $-\frac{\alpha}{k+1} + \beta \equiv (x,y,z)x$. 3.27

However using 3.12 and $P(z,x,x,x) = 0$ we have $(x,x,z)x \equiv \frac{1}{k}(z,x,x)x \equiv \frac{-1}{k} z(x,x,x)$. Since $(x,x,x) =$ 0, we have $(x,x,z)x = 0$. Linearization of this gives

$$(x,y,z)x + (y,x,z)x + (x,x,z)y \equiv 0, \text{ or}$$

$$(x,y,z)x \equiv -(y,x,z)x - (x,x,z)y. \qquad 3.28$$

Using 3.27, 3.23, 3.24 in 3.28, we get

$$-\frac{\alpha}{k+1} + \beta \equiv k\alpha - \frac{1}{k+1}\beta. \text{ So } \frac{(k^2+k+1)}{k+1}\alpha \equiv \frac{(k+2)}{k+1}\beta.$$

Using 3.23 and 3.24 to substitute for $\frac{\alpha}{k+1}$ and $\frac{\beta}{k+1}$ in the above equation gives

$$-(k^2+k+1)\, x(x,y,z) \equiv -(k+2)\, x(x,z,y). \text{ So}$$

$$(k^2+k+1)\, x(x,y,z) \equiv (k+2)\, x(x,z,y). \qquad 3.29$$

Linearizing 3.29, we obtain

$$(k^2+k+1)\, (w(x,y,z) + x(w,y,z)) \equiv (k+2)\, (w(x,z,y) + x(w,z,y)). \qquad 3.30$$

By substituting $w = u = (a,b,c)$ in 3.30 and using 3.9, we get

$$(k^2+k+1)\, x(u,y,z) \equiv (k+2)\, x(u,z,y). \qquad 3.31$$

Linearizing 3.22, we have $(u,z,y) = -(u,y,z)$. Using this in 3.31, we obtain

$$(k^2+k+1)\, x(u,y,z) \equiv -(k+2)\, x(u,y,z)$$

or $(k^2+2k+3)\, x(u,y,z) \equiv 0$.

i.e., $((k+1)^2+2)\, x(u,y,z) \equiv 0$. Thus if $(k+1)^2 + 2 \neq 0$, we have $x(u,y,z) \equiv 0$, or $(x(u,y,z),n) = 0$ for all $n \in N$.

From $P(u,y,z,x) = 0$ and $u(y,z,x) \equiv 0$, we obtain $(u,y,z)x \equiv 0$, or $((u,y,z)x,n) = 0$ for all $n \in N$. Thus $(u,y,z) \in T$. Since $T = 0$, we have $(u,y,z) = 0$.

Similarly $\frac{(k^2+k+1)}{k+1}\alpha \equiv \frac{(k+2)}{k+1}\beta$ also yields

$$\frac{(k^2+k+1)}{k+1}(y,z,x)x \equiv \frac{(k+2)}{k^2}(z,y,x)x.$$

Linearizing the above equation, we get

$$(k^2+k+1)\,((y,z,x)w + (y,z,w)x) \equiv (k+2)\,((z,y,x)w + (z,y,w)x).$$

By putting $w = u = (a,b,c)$ in the above and using 3.9, we get

$(k^2+k+1)\,(y,z,u)x \equiv (k+2)\,(z,y,u)x.$

Linearizing 3.22, we have $(z,y,u) = -(y,z,u)$. Using this in the previous equation, we obtain

$$(k^2+k+1)\,(y,z,u)x \equiv -(k+2)\,(y,z,u)x$$

or $((k+1)^2+2)\,(y,z,u)x \equiv 0$. Thus if $(k+1)^2 + 2 \neq 0$, we have $(y,z,u)x \equiv 0$, or $((y,z,u)x,n) = 0$ for all $n \in N$.

Now using $P(x,y,z,u)=0$ and $(x,y,z)u \equiv 0$, we get $x(y,z,u)\equiv 0$, or $(x(y,z,u),n)=0$, for all $n \in N$. Thus $(y,z,u) \in T$. Since $T = 0$, we have $(y,z,u) = 0$.

Now we have both $(y,z,u) = 0$ and $(u,y,z) = 0$. Using these two equations in 3.1, we get $(z,u,y) = 0$. Now we are in the situation where all associators are in the nucleus, i.e., $(R,R,R) \subset N$.

We use result in [1] to conclude that $R$ must be associative. This completes the proof of the theorem. $\square$

We now consider a prime ring with the antiflexible identity

$$(x,y,z) = (z,y,x). \qquad 3.32$$

Now we prove the following theorem:

***Theorem 3.3:*** *If $R$ is a prime third power associative antiflexible ring of char. $\neq 2,3$, then $R$ is either associative or satisfies nucleus equals center, $N=C$.*

***Proof:*** Using the identity 3.32 in the linearization of $(x,x,x) = 0$, we obtain the identity 3.1. i.e., $(x,y,z) + (y,z,x) + (z,x,y) = 0$.

For every $n \in N$ and $x, y, z \in R$, we obtain from 3.2 that

$$(nx,y,z) = n\,(x,y,z).$$

Now using 3.32, 3.2 and 3.7, we have

$$(xn,y,z) = (z,y,xn) = (z,y,x)n = (x,y,z)n = n(x,y,z).$$

Consequently $((n,x),y,z) = 0$. 3.33

Using 3.32 and 3.1, 3.33 allows us to conclude

$$(N,R) \subset N. \qquad 3.34$$

Using 3.32 and 3.1 in semi-Jacobi identity 3.3 which holds in any ring, we have

$$(xy,z) = x(y,z) + (x,z)y - 2\,(x,z,y), \text{ or}$$

$$(z,xy) = x(z,y) + (z,x)y + 2(x,z,y).$$

Replacing $z$ by $n \in N$ in the above equation, we obtain

$$(n,xy) = x(n,y) + (n,x)y. \qquad 3.35$$

Clearly $\sum (N,R) + R(N,R) = S$ is an ideal of $R$ by 3.34 and 3.35.

Now we define $V = \{u \in R/\ (N,R)u = 0\}$.

For $u \in V$, $r \in R$, we have

$$(n,x)\cdot ur = ((n,x)u)r - ((n,x),u,r).$$

Using definition of $V$ and 3.33 in the above equation, we get $(n,x)\cdot ur = 0$. So $ur \in V$. At this point $V$ is a right ideal of $R$.

Also we have $(n,x)\cdot ru = (n,x)r\cdot u - ((n,x),r,u) = (n,x)r\cdot u$.

Using 3.35 and definition of $V$ in the above equation, we get

$$(n,x)\cdot ru = ((n,xr) - x(n,r))\,u$$
$$= -(x(n,r))u$$

$= -(x,(n,r),u) + x((n,r)u).$

Using 3.34 and definition of $V$, we obtain $(n,x)\cdot ru = 0$. Thus $ru \in V$. Therefore $V$ is an ideal of $R$. Hence $SV = 0$. Since $R$ is prime, we have either $S = 0$ or $V = 0$. By assuming that $N \neq C$ we have $S \neq 0$ and therefore $V = 0$.

We have $(n,y)\,(x,y,z) = ny(x,y,z) - yn\,(x,y,z)$

$= ny\,(x,y,z) - y(x,y,z)n$

$=(n,y(x,y,z)).$

However 3.2 implies $x(y,y,z) = (xy,y,z) - (x,yy,z) + (x,y,yz) - (x,y,y)z.$

Using 3.7, 3.32 and 3.2, we have the following.

$(n,x(y,y,z))= -(n,(x,y,y)z)= -(n,(y,y,x)z)=(n,y(y,x,z))=(n,y(z,x,y)).$ 3.36

Using 3.36, 3.32, 3.2 and 3.1, we obtain

$(n,y(y,z,x)) = (n,z(x,y,y)) = (n,z(y,y,x)) = (n,y(x,z,y)) = -(n,(y,x,z)y) = -(n,(z,x,y)y) = (n,((x,y,z)+(y,z,x))y) = -(n,x(y,z,y)+ y(z,x,y)) = (n,2x(z,y,y) - y(z,x,y)) = (n,2x(y,y,z) - y(z,x,y)) = (n,2y(z,x,y) - y(z,x,y)) = (n,y(z,x,y)).$

i.e., $(n,y(y,z,x)) = (n,y(z,x,y)).$ 3.37

Similarly, we obtain

$(n,y(x,y,z)) = (n,y(z,x,y)).$ 3.38

From the identity 3.1, we have

$y(x,y,z) + y(y,z,x) + y(z,x,y) = 0$. So

$(n,y(x,y,z)) + (n,y(y,z,x)) + (n,y(z,x,y)) = 0.$

Using 3.37 and 3.38 in the above equation, we get $3(n,y(x,y,z)) = 0$.

Since $R$ is of char. $\neq 3$, we have $(n,y(x,y,z)) = 0$. So

$(n,y)\,(x,y,z) = 0,$ 3.39

since $(n,y)\,(x,y,z) = (n,y(x,y,z)).$

Using 3.2, 3.34, and 3.39 gives

$((n,y)x,y,z) = 0.$ 3.40

From 3.35 we have $(n,y)x = (n,yx) - y(n,x).$

Using 3.40 and 3.34, we have

$-(n,x)\,(y,y,z) = 0.$ 3.41

From 3.41 it is clear that $(y,y,z) \in V = 0$. So $R$ is left alternative. Using 3.32 we have $(z,y,y) = 0$. Thus R must be alternative. So 3.1 gives

$3(R,R,R) = 0.$ 3.42

By linearizng 3.39, we get

$(n,R)\,(R,(R,R,R),R) = -(n,(R,R,R))\,(R,R,R) = 0$ because of 3.7. Thus $(R,(R,R,R),R) \subset V = 0$. Therefore

$(R,R,R) \subset N.$ 3.43

In **[1]** equation 3.39 is shown to imply

$2(R,R,R)\,(R,R,R) = 0.$ 3.44

But then 3.42 and 3.44 imply

$(R,R,R)\,(R,R,R) = 0.$ 3.45

At this point as in **[1]**, we have that the associator ideal squares to zero. So $R$ must be associative. This completes the proof of the theorem. □

## REFERENCES

**[1]** E. Kleinfeld, A class of rings which are very nearly associative,
Amer. Math. Monthly, 93(1965), 720-722.

**[2**] E. Kleinfeld and Margaret Kleinfeld, A class of Lie admissible rings, Comm in.Algebra.13(1985), 465-477.

**[3]** E. Kleinfeld and Margaret Kleinfeld, On the nucleus of certain Lie admissible rings, Comm.in.Algebra.27(3) (1999),1313-1320.

**[4]** C. Maneri, Simple (-*1*,*1*) rings with an idempotent, Proc. Amer. Math. Soc.Vol.14(1963), 110-117.