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AN UNSTEADY RADIAL FLOW OF A VISCOUS INCOMPRESSIBLE FLUID IN A POROUS MEDIUM AROUND A SPHERE OF VARIABLE RADIUS

MOHD. NAHEED[1] & N. CH. PATTABHIRAMACHARYULU[2]

[1]Department of Mathematics, Muffakham Jah College of Engineering & Technology, Hyderabad, Andhra Pradesh, India

[2]Former Faculty, Department of Mathematics and Humanities, National Institute of Technology, Warangal, Andhra Pradesh, India

ABSTRACT

This paper deals with an unsteady radial flow of a viscous incompressible fluid in a porous medium around a time dependent spherical surface. The momentum equation considered for the flow through a porous medium takes care of the fluid- inertia and the Newtonian stresses in addition to the classical Darcy's friction. Expression for the pressure distribution has been derived in terms of the expansion rate of the sphere -radius. Two special cases: (A) Exponentially decaying sphere radius and (B) When the sphere-radius decays exponentially and asymptotically approaches to a constant value are discussed and the variation of the pressure with time and Darcian parameter are illustrated and conclusions are drawn in each of the two cases considered.

KEYWORDS: Porous Medium, Pressure, Darcy's Number, Expansion Factor, Radius Decay- Parameter

INTRODUCTION

Studies on Radial flows of a viscous fluid were initiated in the year 1915 by Jeffery G.B [5] and was followed later by Hamael G [4] and Harrison W. J, Such flows are discussed at length by Dryden H.L, Murnaghan F.D and Batemen H [3] in their classical work on Hydrodynamics. Recently Raisinghania M.D [8] in his treatises Fluid Dynamics discussed several radial flows of viscous fluids in a clear medium. Study of flows through porous media has been a subject of considerable research activity for the last one and half centuries, because of their wide range of application in diverse fields of science, engineering and technology. Serious studies in this area were initiated in 1856 by Darcy H [2] based on a series of experiments on flows of slurry fluids through channels. Darcy formulated an empirical law for fluid flows through porous media : The total volume of the fluid percolating in unit time is proportional to the hydraulic head and inversely proportional to the distance between the inlet and outlet.

This law was generalized by Brinkman H.C [1] , taking in to account for the stresses viscous generated in the flow region. Later, Yamamoto K and Yoshida Z [9] further generalized the basic equations by the inclusion fluid inertia in addition to the Newtonian-Stresses developed in fluids in motion. Later Pattabhi Ramacharyulu.N.Ch [7] examined several flow problems through straight tubes of diverse cross sections. A general solution for an incompressible flow through porous media has been obtained by Narasinhacharyulu.V and Pattabhi Ramachryulu.N.Ch [6]. The present investigation is on an unsteady radial flow of a viscous incompressible fluid through a porous medium surrounding a sphere whose radius (r) is time dependent. A generalized momentum equation given by Yamamoto K and Yoshida Z [9] for the flows through porous medium has been solved for the radial flow. It is noticed that the flow is independent of a Newtonian viscous stresses. However the flow depends on Darcian friction. Expression for the pressure distribution has been obtained in terms of the radial velocity on the sphere- surface. The cases

(A) $r = e^{-\alpha t}$ and

(B) $r = \frac{1}{1+\varepsilon}(1+\varepsilon e^{-\alpha t})$ are discussed in detail

In this 'r' is the radius of the surface of the sphere at time 't'. The variation of the pressure for different values of the flow parameter 'α' and the Darcy number characteristic of the medium position at different instants of time in each of the cases has been discussed and illustrated.

BASIC EQUATIONS FOR THE RADIAL FLOWS THROUGH POROUS MEDIA

Consider a spherical co-ordinate system R,θ,ϕ with a origin 'O' at the center of the sphere which is fixed where R is the radial distance from the origin, θ is the polar angle, ϕ is the azimuthal angle. The flow of a viscous incompressible fluid through a porous medium is governed by the modified Navier-Stokes equations suggested by Yamamoto K and Yoshida Z [9]:

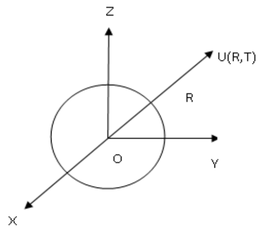

Figure 1: Flow- Sketch

$$\rho \frac{d\vec{q}}{dt} = -\nabla p + \mu\nabla^2\vec{q} - \frac{\mu}{k}\vec{q} \tag{1}$$

together with the equation of continuity

$$\rho \frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = 0 \tag{2}$$

In the above equations '$\vec{q}$' represents the fluid velocity and 'p' is the fluid pressure. Further ρ = the fluid mass density, μ = coefficient of Newtonian of viscosity and 'k' is the coefficient of Darcian porosity of the medium and these are assumed to be constants. The term $\mu\nabla^2\vec{q}$ on the R.H.S of (1) represents the contribution of the Newtonian Viscous-Stress to the momentum and $-\frac{\mu}{k}\vec{q}$ is the classical Darcy -resistance to the flow. For the unsteady radial flow under investigation, the velocity field can be taken as

$$\vec{q} = U((R,T),0,0) \tag{3}$$

By radial and axi-symmetries

$$\frac{\partial\vec{q}}{\partial\theta} = 0 \quad \text{and} \quad \frac{\partial\vec{q}}{\partial\phi} = 0 \tag{4}$$

The continuity equation (2) now reduced to

$$\frac{1}{R^2}\frac{\partial(R^2U)}{\partial R} = 0 \tag{5}$$

Momentum equation in the R-direction

$$\frac{\partial U}{\partial T}+U\frac{\partial U}{\partial R}=-\frac{1}{\rho}\frac{\partial P}{\partial R}-\nu^{*}\frac{U}{R} \tag{6}$$

Momentum equation in the θ and ϕ -direction

$$\frac{\partial P}{\partial \theta}=0 \text{ and } \frac{\partial P}{\partial \phi}=0 \tag{7}$$

where $\nu^{*}=\frac{\mu}{\rho}$

It can be noted from the equation (7) that

- The pressure (P) is a function of R and T only. (i.e independent of θ and ϕ)
- It is also independent of Newtonian viscous stresses and
- The viscous resistance on the porous media ($-\frac{\mu}{k}\vec{q}$) influences the pressure distribution.

For simplicity the following non dimensional quantities are introduced in the foregoing analysis.

$$R=R_0 r\,;\; T=\frac{R_0}{U_0}t;\; U(R,T)=U_0 u(r,t)\,;\; P=\rho U_0^{\,2} p\,;\; K=\frac{U_0\upsilon}{R_0} \tag{8}$$

Where $R_0 =$ initial radius of the sphere, U_0 =initial radial velocity.

By definition , the radial velocity ‘ u ’ on the sphere of the surface is given by

$$u=\frac{dr}{dt} \quad \text{On the sphere-surface} \tag{9}$$

The following are the basic equations written in the non-dimensional form

Continuity Equation

$$\frac{1}{r^2}\frac{\partial (r^2 u)}{\partial r}=0 \tag{10}$$

Momentum equation in the radial direction

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}=-\frac{\partial p}{\partial r}-Du \tag{11}$$

From (10), we get

$$u=\frac{f(t)}{r^2} \tag{12}$$

Here $f(t)=\frac{dr}{dt}$ is a suitably choose function of time. This depends on the expansion velocity of the sphere. From (9) and (12), we notice that

$$f(t) = r^2 \frac{dr}{dt} \quad \text{on } r = 1 \tag{13}$$

Substituting equation (12) in equation (11) and rearranging the terms, we get the equation for the determination of the fluid pressure (P):

$$-\frac{\partial p}{\partial r} = \frac{f^1(t)}{r^2} - 2\frac{(f(t))^2}{r^5} + D\frac{f(t)}{r^2} \tag{14}$$

Integrating with respect to $'r'$ we get :

$$p_\infty - p = -\frac{1}{r^2}(\frac{df}{dt}) + \frac{1}{2r^4}\frac{d^2 f}{dt^2} - \frac{D}{r}(f(t)) \tag{15}$$

Where D =Darcy number, p_∞ in the pressure at infinity i.e $\lim_{r\to\infty} p = p_\infty$ and the equation (15) now reduces to :

$$p = r\frac{d^2 r}{dt^2} + (1.5)(\frac{dr}{dt})^2 + Dr(\frac{dr}{dt}) \quad \text{where} \quad p = p - p_\infty \tag{16}$$

computed on the surface of the sphere

Case-A

when the sphere radius is exponentially decaying

$$r = e^{-\alpha t}$$

(A.1)

where α is a constant, Here

$$f(t) = r^2 = e^{-2\alpha t} \tag{A.2}$$

Also $$p = \frac{e^{-2\alpha t}}{2}[5\alpha^2 - 2D\alpha] \tag{A.3}$$

RESULTS AND DISCUSSIONS

It is noticed from the equation (A.3) that the pressure exponentially decays with the characteristics time $1/2\alpha$ which is half the time of decay of sphere-radius and depends on α and D in apart from time t. The Pressure (P) linearly decreases with Darcy number (D). The variation of pressure for a wide spectra of the values of D, α and t are illustrated in the figures (A.1)--(A.10). The variation of pressure on the surface $r = 1$ verses Radius decay-parameter (α) is illustrated at time instants t =0.3, 0.5, 0.7, 1.0, 1.5, 2.0 and for different values of the Porosity D =0, 0.5, 1.0, 1.5 are illustrated in the figures figure (A.1) to figure (A.6). It is noticed that the pressure steeply increases initially and critical, thereafter ‘ r ’ asymptotically decreases to zero this trend change occurs when α takes a value α_c . Further as the Darcy number increases the pressure linearly falls and this is illustrated by evident from figure (A.7). The exponential nature of the pressure variation verses Time is illustrated in figure (A.8)-(A.10) for a wide spectrum of values of the Darcy no (D).

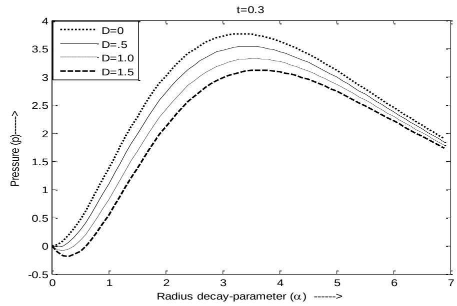

Figure (A.1): Variation of the Pressure Vs Alpha for Different 'D' at t=.3

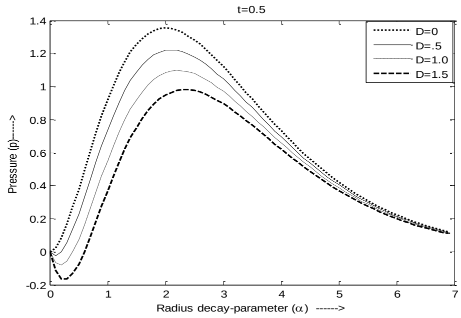

Figure (A.2): Variation of the Pressure Vs Alpha for Different 'D' at t=.5

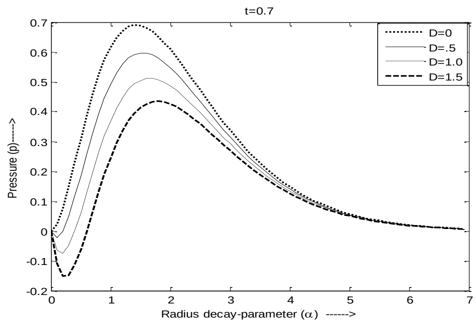

Figure (A.3): Variation of the Pressure Vs Alpha for Different 'D' at t=.7

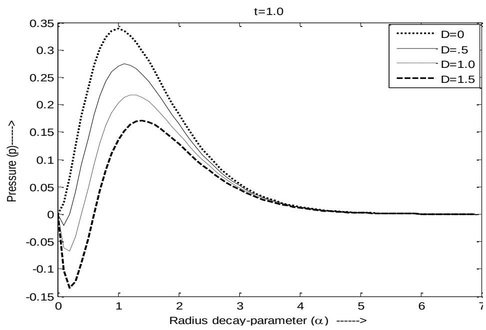

Figure (A.4): Variation of the Pressure Vs Alpha for Different 'D' at t=1.0

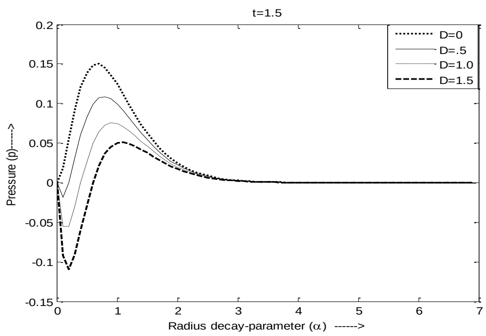

Figure (A.5): Variation of the Pressure Vs Alpha for Different 'D' at t=1.5

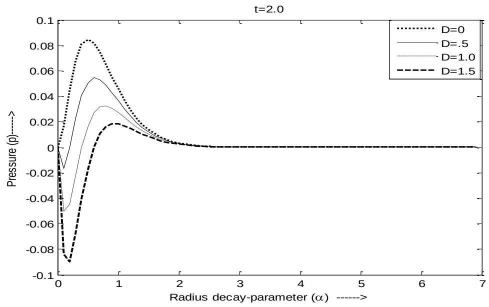

Figure (A.6): Variation of the Pressure Vs Alpha for Different 'D' at t=2.0

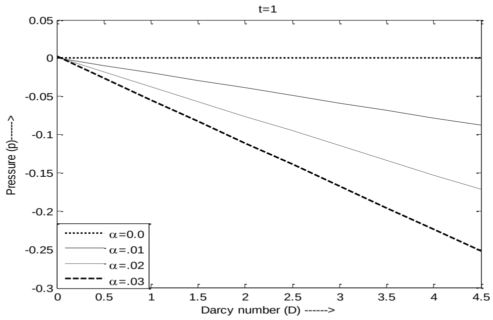

Figure (A.7): Variation of the Pressure Vs Darcy Number for Different 'α 'at t=1.0

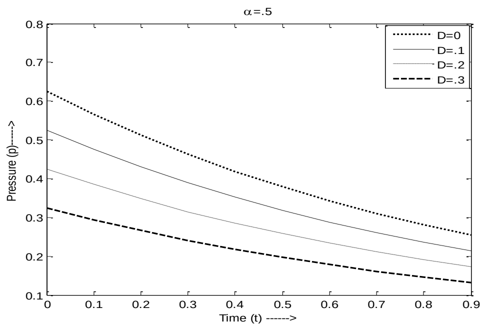

Figure (A.8): Variation of the Pressure Vs Time for Different 'D' at α=.5

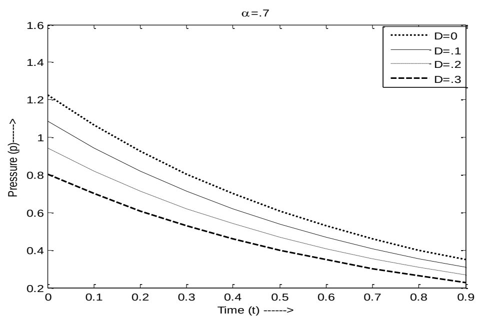

Figure (A.9): Variation of the Pressure Vs Time for Different 'D' at α=.7

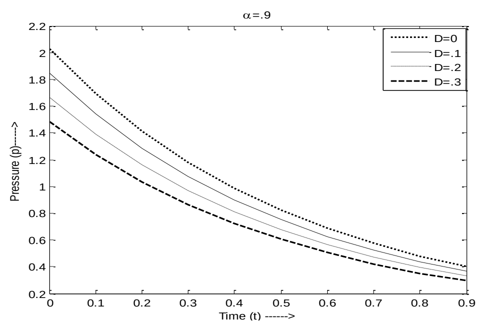

Figure (A.10): Variation of the Pressure Vs Time for Different 'D'at α=.9

Case-B

When case $r = \frac{1}{(1+\varepsilon)}(1+\varepsilon e^{-\alpha t}) = r^{*}(1+\varepsilon e^{-\alpha t})$ (B.1)

The sphere radius is exponentially decaying from its initial value $r^{*} = \frac{1}{(1+\varepsilon)}$ and α is a constant (i.e. Characteristic Radius decay-parameter), ε is the expansion factor. This is the case of superposition of a exponentially decrease spherical surface over a surface of constant radius. From equation (11) we have

$$f(t) = (r^{*})^{2}(1+\varepsilon e^{-\alpha t})^{2} \quad \text{(B.2)}$$

Also $p = \frac{e^{-\alpha t}}{2}\alpha\varepsilon[(5\alpha\varepsilon - 2D\varepsilon)e^{-\alpha t} + 2(\alpha - D)]$ (B.3)

RESULTS AND DISCUSSIONS

It is noticed from the equation (B.2) that the pressure exponentially decreases with characteristic time which depends on α and D in addition to time t. The pressure (p) linearly decreases with Darcy number (D). The variation of pressure for wide spectra of the values of α, D and t are illustrated figures (B.1)—(B.10). The variation of the pressure on the sphere surface $r = 1$ verses a Characteristic Radius decay-parameter (α) at different time instants t =1, 1.5, 2, 3, at ε =.1, ε =.4, ε =.7 for different values of the porosity D =0, D =0.5, D =1.0, D =1.5, D =1.0 are illustrated in the figures (B.1) to (B.10). It is noticed that the pressure steeply increases up to $\alpha = \alpha_c$ and thereafter asymptotically decreases to zero. Further the pressure linearly falls as Darcy number increases. This is evident from the figures (B.11) -- (B.12) it is illustrated in the figures (B.13) -- (B.16).

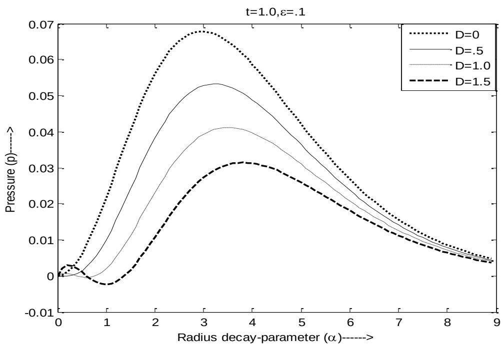

Figure (B.1): Variation of the Pressure Vs Alpha for Different 'D' at t=1.0, $\varepsilon = .1$

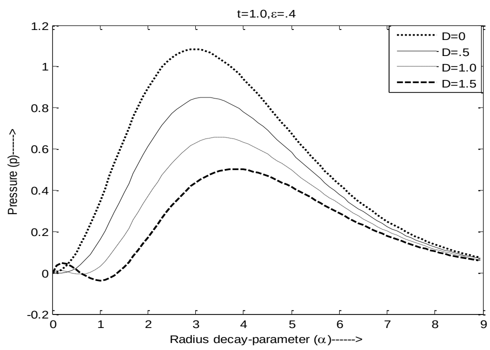

Figure (B.2): Variation of the Pressure Vs Alpha for Different 'D' at t=1.0, ε=.4

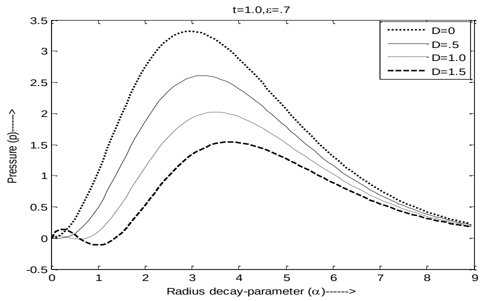

Figure (B.3): Variation of the Pressure Vs Alpha for Different 'D' at t=1.0, ε=.7

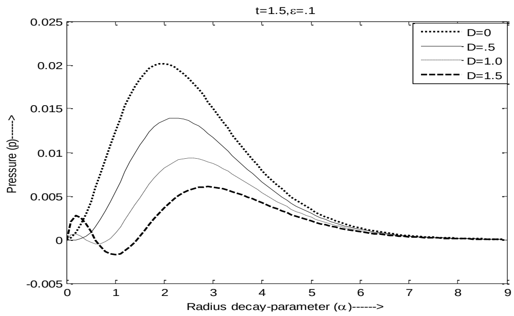

Figure (B.4): Variation of the Pressure Vs Alpha for Different 'D' at t=1.5, ε=.1

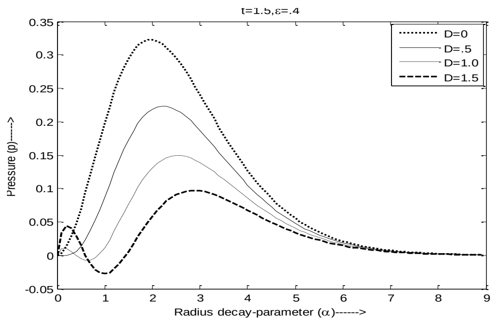

Figure (B.5): Variation of the Pressure Vs Alpha for Different 'D' at t=1.5, ε=.4

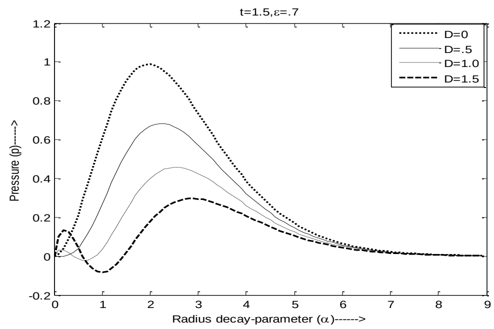

Figure (B.6): Variation of the Pressure Vs Alpha for Different 'D' at t=1.5, ε=.7

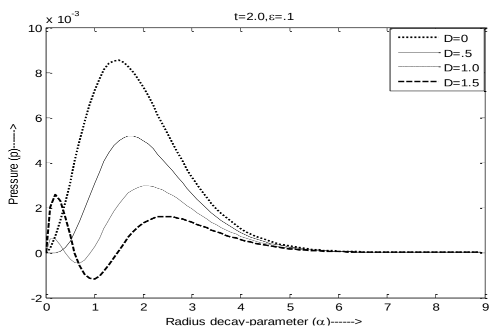

Figure (B.7): Variation of the Pressure Vs Alpha for Different 'D' at t=2.0, ε=.1

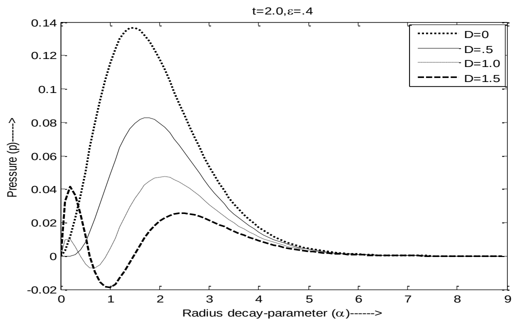

Figure (B.8): Variation of the Pressure Vs Alpha for Different 'D' at t=2.0, ε=.4

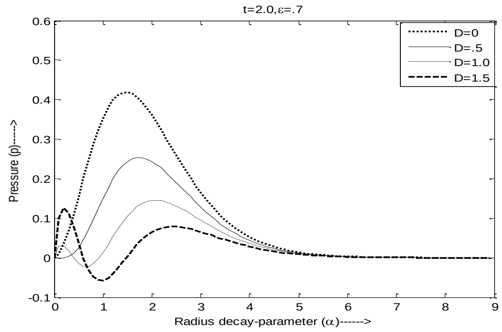

Figure (B.9): Variation of the Pressure Vs Alpha for Different 'D' at t=2.0, ε=.7

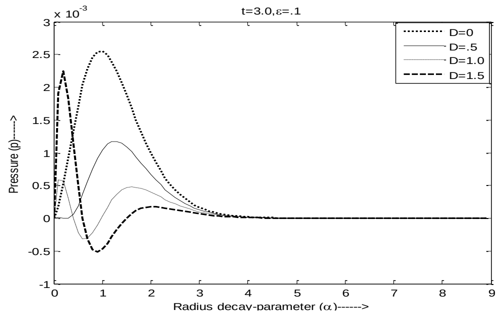

Figure (B.10): Variation of the Pressure Vs Alpha for Different 'D' at t=3.0, ε=.1

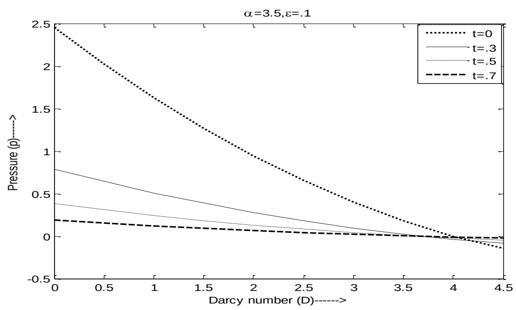

Figure (B.11): Variation of the Pressure Vs Darcy no for Different 't' at α=3.5, ε=.1

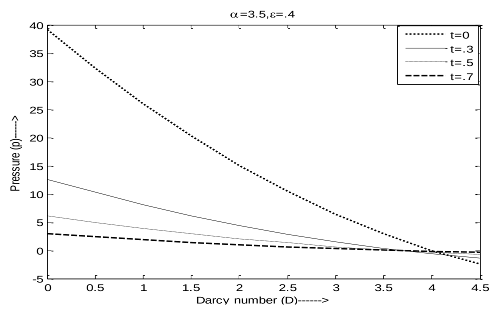

Figure (B.12): Variation of the Pressure Vs Darcy no for Different 't' at α=5, ε=.4

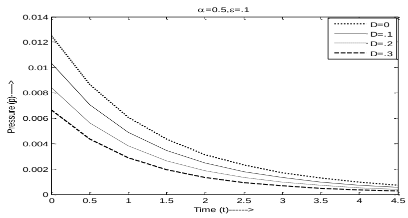

Figure (B.13): Variation of the Pressure Vs Time for Different 'D' at α=.5, ε=.1

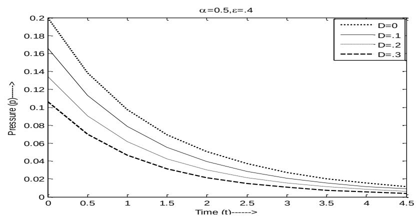

Figure (B.14): Variation of the Pressure Vs Time for Different 'D' at α=.5, ε=.4

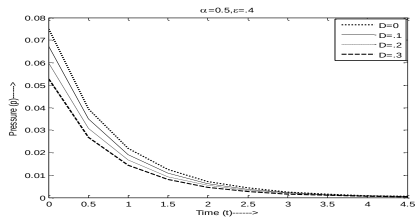

Figure (B.15): Variation of the Pressure Vs Time for Different 'D' at α=1.0, ε=.1

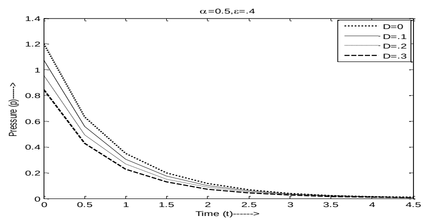

Figure (B.16): Variation of the Pressure Vs Time for Different 'D' at α=1.0, ε=.4

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