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RIGHT ALTERNATIVE RINGS WITH ASSOCIATORS IN THE MIDDLE NUCLEUS

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ABSTRACT

In this paper we study some properties of a nonassociative ring R with associators in the middle nucleus. If R is a semiprime right alternative ring of char. $\neq 2$ with $(R, R, R) + (R, (R, R, R))$ in the middle nucleus, then R is associative.

Keywords: Nonassociative ring, Right alternative ring, Simple ring, Semiprime ring, Lie ideal and characteristic.

1. INTRODUCTION

In [2] Kleinfeld proved the associativity of a nonassociative semiprime ring R of char. $\neq 2$ satisfying $(R, R, R) \subseteq N_l \cap N_m \cap N_r$. Yen [3] generalized Kleinfeld's result under the weaker hypothesis $(R, R, R) \subseteq$ two of the three nuclei. In this paper, we study some properties of a nonassociative ring R with associators in the middle nucleus and if R is a semiprime right alternative ring of char. $\neq 2$ with $(R, R, R) + (R, (R, R, R)) \subseteq N_m$, then R is associative.

2. PRILIMINARIES

Let R be a nonassociative ring. We shall denote the commutator and the associator by $(x, y) = xy - yx$ and $(x, y, z) = (xy)z - x(yz)$ for all x, y, z in R respectively. A ring R is called right alternative if $(y, x, x) = 0$ for all x, y in R. The left nucleus N_l of a ring R is defined as $N_l = \{n \in R/(n, R, R) = 0\}$. The middle nucleus N_m of a ring R is defined as $N_m = \{n \in R/(R, n, R) = 0\}$. The right nucleus N_r of a ring R is defined as $N_r = \{n \in R/(R, R, n) = 0\}$. The nucleus N of a ring R is defined as $N = \{n \in R/(n, R, R) = (R, n, R) = (R, R, n) = 0\}$ i.e., $N = N_l \cap N_m \cap N_r$. A ring R is said to be of characteristic $\neq n$ if $nx = 0$ implies $x = 0$, for all $x \in R$ and n is a natural number. A ring R is of characteristic $\neq n$ is simply denoted by char. $\neq n$. A ring R is called simple if $R^2 \neq 0$ and the only nonzero ideal of R is itself. Since R^2 is a non-zero ideal of R, we have $R^2 = R$. A ring R is called semiprime if the only ideal of R which squares to zero is the zero ideal. Clearly, every simple ring is a semiprime ring. An additive subgroup T of R is a Lie ideal of R if $(T, R) \subseteq T$. Let S be a nonempty subset of R. Then the ideal of R generated by S is denoted by $< S >$.