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AN ALGORITHM TO FIND OPTIMUM TIME COST TRADE OFF PAIRS IN A FIXED CHARGE LINEAR CAPACITATED TRANSPORTATION PROBLEM WITH ENHANCED FLOW

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***Abstract**: The present paper presents an algorithm to find optimum time- cost trade off pairs in a fixed charge linear capacitated transportation problem with enhanced flow. Sometimes, situations arise where either reserve stocks have to be kept at supply points say, for emergencies, or there may be extra demand in the markets. In such situations, the total flow needs to be controlled or enhanced. Moreover, sometimes a fixed charge (like set up cost for machines, landing fees at an airport, cost of renting a vehicle) is also associated with every origin that gives rise to fixed charge problem. In this paper a special class of transportation problem is studied, where the total transportation flow is enhanced to a specified level. A numerical example is given to illustrate the developed algorithm.*

***Keywords**: Transportation problem, capacitated transportation problem, trade off, optimum time cost trade off, fixed charge transportation problem, enhanced flow.*

1 Introduction

Sometimes there may exist emergencies such as fire services, ambulance services, police services etc when the time of transportation is more important than the cost of transportation. Arora and Ahuja [5] ;Garfinkel and Rao [6] and Hammer [7] have studied the time minimizing transportation problem which is a special case of bottleneck linear programming problems. Pandian and Natarajan [10] gave a new method namelyBlocking method for finding an optimal solution to bottleneck transportation problem.Basu ,Pal and Kundu [3] developed an algorithm for the Optimum time cost trade off in a fixed charge linear transportation problem giving some priority to cost and time

The fixed charge problem was originally formulated by Dantzig and Hirisch [8] in 1954.Then Murthy [4] solved the fixed charge problem by ranking the extremepoints.In real world situations, when a commodity is transported, a fixed charge is incurred in the objective function. The fixed cost may represent the cost of renting a vehicle, landing fees at an airport, set up cost for machines etc. sandrock [1] discussed fixed charge transportation problem in 1982.

Another important class of transportation problems consists of capacitated transportation problem. Many researchers, i.e. Arora and Gupta [13], Dahiya [2] have contributed in this field. Sometimes situations arise due to extra demand in the market that the total flow needs to be enhanced, compelling some factories to increase their productionin order to meet the extra demand. The total flow from the factories in the market is now increased by an amount of the extra demand. This motivated us to study enhanced flow in a capacitated fixed charge optimum time cost trade off pairs transportation problem.

In this paper we shall be discussing the case when the flow gets enhanced in an optimum time cost trade off capacitated transportation problem with fixed charge.

2 Problem Formulation:

The general model of the linear capacitated fixed charge Bi-criterion transportation problems with bounds on rim conditions is given below:

$$\min\{ \sum_{i\in I} \sum_{j\in J} C_{ij} X_{ij} + \sum_{i\in I} F_i, \max_{i\in I, j\in J} (t_{ij} / x_{ij} > 0) \}$$

Subject to

$$a_i \le \sum_{i\in I} X_{ij} \le A_i \ \forall \ i \in I \qquad 1.1$$

$$b_j \le \sum_{j\in J} X_{ij} \le B_j \ \forall j \in J \qquad 1.2$$

and integers $\forall i \in I, j \in J$ 1.3

I = {1,2,…….,m} is the index set of m origins.

J = {1,2,…….,n} is the index set of n destinations

X_{ij} = number of units transported from i^{th} origin to j^{th} destination.

C_{ij} = cost of transporting one unit of commodity from i^{th} origin to j^{th} destination.

l_{ij} and u_{ij} are the bounds on number of units to be transported from i^{th} origin to j^{th} destination.

a_i isthe availability at the i^{th} origin, $i \in I$

$b_j$is the bounds on the demand at the jth destination, $j \in J$

t_{ij} is the time of transporting goods from i^{th} origin to j^{th} destination.

$F_i$is the fixed cost associated with i^{th} origin.

Sometimes because of extra demand in the market,the total flow from the factories in the market is increased.Let P (> max ($\sum_{i\in I} a_i, \sum_{j\in J} b_j$)) be the enhanced flow.This flow constraint change the structure of the transportation problem.The resulting fixed charge bi-criterion capacitated transportation problem with enhanced flow is

$$(P_1): \quad \min\{ \sum_{i\in I} \sum_{j\in J} C_{ij}X_{ij} + \sum_{i\in I} F_i, \max_{i\in I, j\in J}(t_{ij} / x_{ij} > 0) \}$$

Subject to

$$\left.\begin{array}{l} \sum_{i\in I} X_{ij} \geq a_i \ \forall \ i \in I \\ \sum_{j\in J} X_{ij} \geq b_j \ \forall j \in J \\ l_{ij} \leq x_{ij} \leq u_{ij} \quad \text{and integers} \ \forall i \in I, j \in J \\ \sum_{i\in I} \sum_{j\in J} X_{ij} = \text{P} \ (> \max(\sum_{i\in I} a_i, \sum_{j\in J} b_j)) \\ x_{ij} \geq 0 \ i \in I, j \in J \end{array}\right\} \quad \ldots\ldots\ldots\ldots(1)$$

For the formulation of F_i (I = 1,2,….,m), we assume that F_i (I = 1,2,….m) has p number of steps so that

$$F_i = \sum_{l=1}^{p} F_{il}\partial_{il}, \text{I} = 1,2,\ldots..\text{m} \quad , \text{l} = 1,2,\ldots.,\text{p}$$

where $\partial_{il} = \begin{cases} 1 & \text{if} \sum_{j=1}^{n} x_{ij} > a_{il} \\ 0 & \text{otherwise} \end{cases}$ for l = 1,2,….,p, I = 1,2,…m

Here , $0 = a_{i1} < a_{i2} < \ldots\ldots\ldots < a_{ip}$, $a_{i1}, a_{i2}, \ldots\ldots, a_{ip}$ (I = 1,2,….,m) are constants and F_{il} are the fixed costs ∀ i = 1,2,...,m and l = 1,2,.....,p

The problem (P_1) is solved in the following way:

(1) First, we minimize cost without considering time and then minimize time with respect to the minimum cost obtained.
(2) Secondly, after defining a new cost as follows with respect to minimum time obtained in the last result,we minimize cost.Then we minimize time with respect to the minimum costof last result.Step (2) is repeated until the solution is infeasible. This is known as re-optimisation procedure.

$$C_{ij}^{1} = \begin{Bmatrix} M & if t_{ij} \geq T^1 \\ c_{ij} & if t_{ij} < T^1 \end{Bmatrix}$$

The above problem (P_1) is separated into two problems (P_2) and (P_3) for solving it by re-optimisation procedure, where

(P_2): $\min(\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{i \in I} F_i)$ subject to (1.1), (1.2), (1.3) and

(P_3): $\max(t_{ij} / x_{ij} > 0)$ ∀ I = (1,2,...m) and j = (1,2....n) subject to (1.1), (1.2) and (1.3)

To solve the problem (P_2), we first convert it into related problem (P'_2) given below.

(P'_2): $\min(\sum_{i \in I'} \sum_{j \in J'} c_{ij}' y_{ij}' + \sum_{i \in I'} F_i'$ subject to

$$\sum_{j \in J'} y_{ij} = a_i' \forall i \in I'$$

$$\sum_{i \in I'} y_{ij} = b_j' \forall j \in J'$$

$$l_{ij} \leq y_{ij} \leq u_{ij} \forall i \in I, j \in J$$

$$0 \leq y_{m+l,j} \leq \sum_{i \in I} u_{ij} - b_j ; \forall j \in J$$

$$0 \leq y_{i,n+1} \leq \sum_{j \in J} u_{ij} - a_i \forall i \in I$$

$y_{m+1,n+1} \geq 0$ and integers

where $a_i' = \sum_{j \in J} u_{ij} \forall i \in I, a'_{m+l} = \sum_{j \in J} \sum_{i \in I} u_{ij} - P = b'_{n+l} ; b_j' = \sum_{i \in I} u_{ij} \forall j \in J$

$$c'_{ij} = c_{ij}, \forall i \in I, j \in J, c'_{m+l,j} = c'_{i,n+l} = 0, \forall i \in I, j \in J, c_{m+1,n+1} = M$$
$$F'_i = F_i \forall i = (1,2,......,\mathrm{m}), \mathrm{F}'_{m+1} = 0$$
$$I' = \{1,2,...,m,m+1\}, J' = \{1,2,....,n,n+1\}$$

To solve the problem(P_3), we convert it into related problem (P_3’) given below.

(P_3’): $\min \mathrm{T} = \max(\mathrm{t}'_{ij} / x'_{ij} > 0) \forall i \in I', j \in j'$

Subject to

$$\sum_{j \in J'} y_{ij} = a_i' \forall i \in I'$$

$$\sum_{i\in I'} y_{ij} = b_j{}' \forall\, j \in J'$$

$$l_{ij} \le y_{ij} \le u_{ij} \; \forall\, i \in I,\, j \in J$$

$$0 \le y_{m+l,j} \le \sum_{i\in I} u_{ij} - b_j ; \forall j \in J$$

$$0 \le y_{i,n+1} \le \sum_{j\in J} u_{ij} - a_i \forall i \in I$$

$y_{m+1,n+1} \ge 0$ and integers

where $a_i{}' = \sum_{j\in J} u_{ij} \forall i \in I, \mathrm{a}'_{m+l} = \sum_{j\in J}\sum_{i\in I} u_{ij} - P = b'_{n+l}; b_j' = \sum_{i\in I} u_{ij} \forall j \in J$

$$\mathrm{t}'_{ij} = t_{ij}, \forall i \in I,\, j \in J, \mathrm{t}'_{m+l,j} = t'_{i,n+l} = 0, \forall i \in I,\, j \in J, \mathrm{t}_{m+1,n+1} = M$$
$$I' = \{1, 2, ..., m, m+1\},\, J' = \{1, 2, ..., n, n+1\}$$

To obtain the set of efficient time cost trade off pairs,we first solve(P_2’) and read the time with respect to the minimum cost Z where time T is given by problem (P_3’). At the first iteration, let Z_1^* be the minimum total cost of the problem (P_2’), find all alternate solutions i.e. solutions having the same value of Z = Z_1^*. Let these solution be X_1,X_2,………,X_n.

Corresponding to these solutions, find the T_1^* = $\min_{X_1, X_2, ...Xn} \{ \max_{i\in I', j\in J'} (\mathrm{t}_{ij} / \mathrm{x}_{ij} > 0)\}$. The ($Z_1^*$,$T_1^*$) is called the first cost time trade off pair.Modify the cost with respect to the time so obtained i.e. define C_{ij} = $\begin{Bmatrix} M & if t_{ij} \ge T^* \\ C_{ij} & if t_{ij} < T^* \end{Bmatrix}$ and form the new problem and find its optimal solution and all feasible alternate solutions. Let the new value of Z be Z_2^* and the corresponding time is T_2^*, then (Z_2^*,T_2^*) is the second cost time trade off pair. Repeat this process. Suppose that after q^{th} iteration, the problem becomes infeasible. Thus ,we get the following complete set of cost-time trade off pairs. (Z_1^*,T_1^*), (Z_2^*,T_2^*),………..(Z_q^*, T_q^*) where $Z_1^* \le Z_2^* \le$ ……..$\le Z_q^*$ and

$T_1^* > T_2^* >$ ………$> T_q^*$.The pairs so obtained are pareto-optimal solution of the given problem. Then we identify the minimum cost Z_1^* and minimum time T_q^* among the above trade off pairs. The pair(Z_1^*, T_q^*) with minimum cost and minimum time is termed as the ideal pair which cannot be achieved in practical situations.

3 Theoretical development:

Definition: ***Corner feasible solution:*** A basic feasible solution $\{y_{ij}\}\ i \in I', j \in J'$ to problem (P_2) is calleda corner feasible solution (cfs) if $y_{m+1,n+1} = 0$

Theorem 1: A non corner feasible solution of problem (P_2) cannot provide a basic feasible solution to problem (P_1).

Proof: Let $\{y_{ij}\}_{I'\times J'}$ be a non corner feasible solution to problem (P_2).Then, $y_{m+1,n+1} = \lambda(>0)$

Thus, $\sum_{i\in I'} y_{i,n+1} = \sum_{i\in I} y_{i,n+1} + y_{m+1,n+1}$

$$= \sum_{i\in I} y_{i,n+1} + \lambda$$

$$= \sum_{i\in I} \sum_{j\in J} u_{ij} - P$$

Therefore, $\sum_{i\in I} y_{i,n+1} = \sum_{i\in I} \sum_{j\in J} u_{ij} - (\mathrm{P}+\lambda)$

Now, for $i \in I$,

$$\sum_{j\in J'} y_{ij} = a_i = \sum_{j\in J} u_{ij}$$

$$\sum_{i\in I} \sum_{j\in J'} y_{ij} = \sum_{i\in i} \sum_{j\in J} u_{ij}$$

$$\sum_{i\in I} \sum_{j\in J} y_{ij} + \sum_{i\in I} y_{i,n+1} = \sum_{i\in I} \sum_{j\in J} u_{ij}$$

$$\sum_{i\in I} \sum_{j\in J} y_{ij} + \sum_{i\in I} \sum_{j\in J} u_{ij} - (\mathrm{P}+\lambda) = \sum_{i\in I} \sum_{j\in J} u_{ij}$$

Therefore, $\sum_{i\in I} \sum_{j\in J} y_{ij} = P + \lambda$

This implies that total quantity transported from the sources in I to the destinations in J is $P + \lambda$ >P, a contradiction to assumption that total flow is P and hence $\{y_{ij}\}_{I'\times J'}$ cannot provide a feasible solution to problem(P_1)

Lemma 1: There is one to one correspondence between a feasible solution of problem (P_2) and a corner feasible solution of problem (P_2').

Proof: Let $\{x_{ij}\}_{I\times J}$ be a feasible solution of problem (P_2).

So by relation (1), we have $x_{ij} \le u_{ij}$ which implies $\sum_{j\in J} x_{ij} \le \sum_{j\in J} u_{ij}$ (1.4)

By relation (1) and (1.4), we get

$a_i \le \sum_{j\in J} x_{ij} \le \sum_{j\in J} u_{ij} = a_i'$

Similarly, $b_j \le \sum_{i\in I} x_{ij} \le \sum_{i\in I} u_{ij} = b_j'$

Define $\{y_{ij}\}_{I'\times J'}$ by the following transformation

$$y_{ij} = x_{ij}, i \in I, j \in J \tag{1.5}$$

$$y_{i,n+1} = \sum_{j\in J} u_{ij} - \sum_{j\in J} x_{ij}; \forall i \in I \tag{1.6}$$

$$y_{m+1,j} = \sum_{i\in I} u_{ij} - \sum_{i\in I} x_{ij}; \forall j \in J \tag{1.7}$$

$$y_{m+1,n+1} = 0 \tag{1.8}$$

It can be shown that $\{y_{ij}\}$ so defined is a cfs to problem (P_2')

Relation (1) and (1.5) imply that $l_{ij} \le y_{ij} \le u_{ij}; \forall i \in I, j \in J$

Relation (1) and (1.6) imply that $0 \le y_{i,n+1} \le \sum_{j\in J} u_{ij} - a_i; \forall i \in I$

Relation (1) and (1.7) imply that $0 \le y_{m+1,j} \le \sum_{i\in I} u_{ij} - b_j; \forall j \in J$

Relation (1.8) implies that $y_{m+1,n+1} \ge 0$

Also for $i \in I$,relation (1.5) and (1.6) imply that

$$\sum_{j\in J'} y_{ij} = \sum_{j\in J} y_{ij} + y_{i,n+1} = \sum_{j\in J} x_{ij} + \sum_{j\in J} u_{ij} - \sum_{j\in J} x_{ij} = \sum_{j\in J} u_{ij} = a_i$$

For $i = m+1$

$$\sum_{j\in J'} y_{m=1,j} = \sum_{j\in J} y_{ij} + y_{m+1,n+1} = \sum_{j\in J} \left(\sum_{i\in I} u_{ij} - \sum_{i\in I} x_{ij} \right)$$

$$= \sum_{i\in I} \sum_{j\in J} u_{ij} - \sum_{i\in I} \sum_{j\in J} x_{ij}$$

$$= \sum_{i\in I} \sum_{j\in J} u_{ij} - P = a'_{m+1}$$

Therefore, $\sum_{j\in J'} y_{ij} = a'_i; \forall i \in I'$

Similarly, it can be shown that $\sum_{i\in I} y_{ij} = b'_j; \forall j \in J'$

Therefore, $\{y_{ij}\}_{I'\times J'}$ is a cfs to problem ($P_2^{'}$).

Conversely, let $\{y_{ij}\}_{I'\times J'}$ be a cfs to problem ($P_2^{'}$). Define $x_{ij}\ , i \in I, j \in J$ by the following transformation.

$$x_{ij} = y_{ij}, i \in I, j \in J \tag{1.9}$$

It implies that $l_{ij} \le x_{ij} \le u_{ij}, i \in I, j \in J$

Now for $i \in I$,the source constraints in problem ($P_2^{'}$) imply

$$\sum_{j\in J'} y_{ij} = a_i^{'} = \sum_{j\in J} u_{ij}$$

$$\sum_{j\in J} y_{ij} + y_{i,n+1} = \sum_{j\in J} u_{ij}$$

$$\Rightarrow a_i \le \sum_{j\in J} y_{ij} \le \sum_{j\in J} u_{ij} \qquad (\text{ since } 0 \le y_{i,n+1} \le \sum_{j\in J} u_{ij} - a_i; \forall i \in I\ )$$

Hence, $\sum_{j\in J} y_{ij} \ge a_i, i \in I$ and subsequently, $\sum_{j\in J} x_{ij} \ge a_i, i \in I$

Similarly, for $j \in J, \sum_{j\in J} y_{ij} \ge b_j; \forall j \in J$ and subsequently, $\sum_{i\in I} x_{ij} \ge b_j; \forall j \in J$

For $i = m+1$

$$\sum_{j\in J'} y_{m+1,j} = a'_{m+1} = \sum_{i\in I}\sum_{j\in J} u_{ij} - P$$

$$\Rightarrow \sum_{j\in J} y_{m+1,j} = \sum_{i\in I}\sum_{j\in J} u_{ij} - P \text{ because } y_{m+1,n+1} = 0 \tag{1.10}$$

Now for $j \in J$ the destination constraints in problem ($P_2^{'}$) give

$$\sum_{i\in I} y_{ij} + y_{m+1,j} = \sum_{i\in I} u_{ij}$$

Therefore, $\sum_{i\in I}\sum_{j\in J} y_{ij} + \sum_{j\in J} y_{m+1,j} = \sum_{i\in I}\sum_{j\in J} u_{ij}$

By relation (1.10), we have

$$\sum_{i\in I}\sum_{j\in J} y_{ij} = \sum_{i\in I}\sum_{j\in J} u_{ij} - \sum_{j\in J} y_{m+1,j} = P$$
$$\Rightarrow \sum_{i\in I}\sum_{j\in J} x_{ij} = P$$

Therefore, $\{x_{ij}\}_{I\times J}$ is a feasible solution to problem (P_2).

Remark 1: If problem (P_2') has cfs, then since $c'_{m+1,n+1} = M$ and $d'_{m+1,n+1} = M$,it follows that non corner feasible solution can not be an optimal solution to problem (P_2).

Lemma 2 : The value of the objective function of problem (P_2) at a feasible solution $\{x_{ij}\}_{I\times J}$ is equal to the value of the objective function of problem (P_2') at its corresponding cfs $\{y_{ij}\}_{I'\times J'}$ and conversely.

Proof: The value of the objective function of problem (P_2) at a feasible solution $\{x_{ij}\}_{I\times J}$ is

$$\sum_{i\in I'}\sum_{j\in J'} c'_{ij} y_{ij} + \sum_{i\in I'} F_i'$$
$$= \sum_{i\in I}\sum_{j\in J} c'_{ij} y_{ij} + \sum_{i\in I} c'_{i,n+1}, y_{i,n+1} + \sum_{j\in J} c'_{m+1,j}, y_{m+1,j} + c'_{m+1,n+1}\, y_{m+1,n+1} +$$
$$\sum_{i\in I} F_i' + F'_{m+1}$$

$$= \sum_{i\in I}\sum_{j\in J} c_{ij} x_{ij} + \sum_{i\in I} F_i \text{ because} \left\{ \begin{array}{c} c'_{ij} = c_{ij}, \forall i\in I, j\in J \\ x_{ij} = y_{ij}, \forall i\in I, j\in J \\ c'_{i,n+1} = c'_{m+1,j} = c_{m+1,n+1} = 0 \\ F'_{m+1} = 0, F'_i = F_i, \forall i\in I \end{array} \right\}$$

= objective function value of problem (P_2) at $\{x_{ij}\}$ .Converse can be proved in a similar way.

Lemma 3:: There is a one to one correspondence between the optimal solution among the corner feasible solution to problem (P_2') and the optimal solution to problem(P_2).

Proof : Let $\{\hat{x}_{ij}\}_{I\times J}$ be an optimal solution to problem (P_2) with the value of objective function as Z^0 .Since $\{\hat{x}_{ij}\}_{I\times J}$ is an optimal solution, $\therefore \{x_{ij}\}$ is a feasible solution to problem (P_2) . Then by lemma 1, there exist a corresponding feasible solution $\{\hat{y}_{ij}\}_{I'\times J'}$ is Z^0 [refer to lemma 2]

we will show that $\{\hat{y}_{ij}\}_{I'\times J'}$ is the optimal solution to problem (P_2').

Now,Let if possible, $\{\hat{y}_{ij}\}$ be not an optimal solution to problem ($P_2^{'}$).Therefore there exist a feasible solution $\{y'_{ij}\}$ say to problem ($P_2^{'}$) having the value of objective function $Z' < Z^0$ .Let $\{x'_{ij}\}$ be the corresponding feasible solution to problem (P_2). Then by theorem 2,

$$Z' = \sum_{i \in I} \sum_{j \in J} c_{ij} x'_{ij} + \sum_{i \in I} F_i < Z^0$$

Which contradicts that $\{\hat{x}_{ij}\}$ is an optimal solution to problem (P_2).

Similarly, starting from an optimal feasible solution to problem ($P_2^{'}$), one can derive an optimal corner feasible solution to problem (P_2) having the same objective function value.

Theorem 2: Optimizing problem ($P_2^{'}$) is equivalent to optimizing problem (P_2) provided problem (P_2) has a feasible solution.

Proof: As problem (P_2) has a feasible solution, by lemma 1, there exists a cfs to problem ($P_2^{'}$).Thus by remark 1, an optimal solution to problem ($P_2^{'}$) will be a cfs. Hence, by lemma 3 , an optimal solution to problem (P_2) can be obtained.

4 Algorithm

Step 1.starting from the given linear capacitated transportation problem (P_1) with enhanced flow, form a related transportation problem (P_2) by introducing a dummy source and a dummy destination with

$$a_i^{'} = \sum_{j \in J} u_{ij}; \forall i \in I \ , \ a^{'}_{m+1} = \sum_{i \in I} \sum_{j \in J} u_{ij} - P = b^{'}_{n+1} \ , \ b_j^{'} = \sum_{i \in I} u_{ij}; \forall j \in J \ ,$$

$$c^{'}_{ij} = c_{ij} \forall i \in I, j \in J \ , \ c^{'}_{m+1,j} = c^{'}_{i,n+1} = 0; \forall i \in I, j \in J \ , \ c^{'}_{m+1,n+1} = M$$

Step 2 : Find an initial basic feasible solutionto (P_2) with respect to variable cost only. Let B be its corresponding basis.

Step 3 : Calculate the fixed cost of the current basic feasible solution and denote it by F(current), where F(current) $= \sum_{i=1}^{m} F_i$

Step 4(a) : Find $\Delta F_{ij} = F(\mathrm{NB}) - \mathrm{F(current)}$ where F(NB) is the total fixed cost obtained when some non basic cell (i, j) undergoes change.

Step 4(b) : Calculate $\theta_{ij}, (\mathrm{c}_{ij} - \mathrm{z}_{ij})$ for all non basic cells such that

$$u_i + v_j = c_{ij}; \forall (\mathrm{i, j}) \in \mathrm{B}$$
$$\mathrm{u}_i + v_j = z_{ij}; \forall (\mathrm{i, j}) \in \mathrm{N}_1 \& N_2$$

Θ_{ij} = level at which a non basic cell (I,j) enters the basis replacing some basic cell of B.

N_1 and N_2 denotes the set of non basic cells (I,j) which are at their lower and upper bounds respectively.

Note : u_i, v_j are the dual variables which are determined by using above equations and taking one of the u_i 's or v_j 's as zero.

Step 4(c) : Find $R^1_{ij}; \forall (\mathrm{i,j}) \in \mathrm{N}_1$ and $R^2_{ij}; \forall (\mathrm{i,j}) \in \mathrm{N}_2$ where $R^1_{ij} = \theta_{ij}(\mathrm{c}_{ij} - \mathrm{z}_{ij}) + \Delta\,\mathrm{F}_{ij} \geq 0; \forall (\mathrm{i,j}) \in \mathrm{N}_1$ and $R^2_{ij} = -\theta_{ij}(\mathrm{c}_{ij} - \mathrm{z}_{ij}) + \Delta\,\mathrm{F}_{ij} \geq 0; \forall (\mathrm{i,j}) \in \mathrm{N}_2$

Step 5 : If $R^1_{ij} \geq 0; \forall (\mathrm{i,j}) \in \mathrm{N}_1$ and $R^2_{ij} \geq 0; \forall (\mathrm{i,j}) \in \mathrm{N}_2$ then the current solution so obtained is the optimal solution to ($P_2^{'}$) ,Go to step 5. Otherwise some (I,j) ∈ N_1 for which $R^1_{ij} < 0$ or some (I,j) ∈ N_2 for which $R^2_{ij} < 0$ will undergo change. Go to step 3.

Step 6 : Let Z^1 be the optimal cost of ($P_2^{'}$) yielded by the basic feasible solution $\{y'_{ij}\}$ .Find all alternate solutions to the problem ($P_2^{'}$) with the same value of objective function. Let these solutions be $X_1, X_2, \ldots\ldots, X_n$ and $T^1 = \min_{X_1, X_2, \ldots\ldots, X_n} \{ \max_{i \in I', j \in J'} (t_{ij} / x_{ij} > 0)\}$. Then the corresponding pair (Z^1 ,T^1) will be the first time cost trade off pair for the problem (P_1).To find the second cost-time trade off pair, go to step 7.

Step 7 : Define $c^1_{ij} = \begin{cases} M & if t_{ij} \geq T^1 \\ c_{ij} & if t_{ij} < T^1 \end{cases}$ where M is a sufficiently large positive number .From the corresponding capacitated fixed charge transportation problem with variable cost c^1_{ij} .Repeat the above process till the problem becomes infeasible. The complete set of time cost trade off pairs of (P_1) at the end of q^{th} iteration are given by $(\mathrm{Z}^1, \mathrm{T}^1), (\mathrm{Z}^2, \mathrm{T}^2), \ldots\ldots\ldots\ldots (\mathrm{Z}^q, \mathrm{T}^q)$ where $Z^1 \leq Z^2 \leq \ldots\ldots\ldots \leq Z^q$ and $T^1 > T^2 > \ldots\ldots\ldots > T^q$.

Remark : The pair (Z^1, T^q) with minimum cost and minimum time is the ideal pair which cannot be achieved in practice except in some trivial case.

Convergence of the algorithm: The algorithm will converge after a finite number of steps because we are moving from one extreme point to another extreme point and the problem becomes infeasible after a finite number of steps.

5. Numerical Illustration:

Consider the following 2 x 3 capacitated fixed charge transportation problem with bounds on rim conditions. Table 1 gives the values of c_{ij}, A_i ,B_j for i=1,2 and j=1,2,3.Table 2 gives values of t_{ij} for i =1,2 and j =1,2,3

Table 1:cost matrix of problem (P1)

	D_1	D_2	D_3	A_i
O_1	5	9	9	30
O_2	4	6	2	40
B_j	30	20	30	

Table 2 :Time matrix of problem (P1)

	D_1	D_2	D_3
O_1	15	8	13
O_2	10	13	11

Note:O_1 and O_2 are origins.D_1,D_2,D_3 are the destinations . c_{ij} is the cost mentioned in table 1 at the upper left corner of each cell and t_{ij} is the time in table 2.

$$5 \le \sum_{j=1}^{3} x_{1j} \le 30 \ , \quad 10 \le \sum_{j=1}^{3} x_{2j} \le 40 \ , \ 10 \le \sum_{i=1}^{2} x_{i1} \le 30 \ , \quad 7 \le \sum_{i=1}^{2} x_{i2} \le 20 \ , \ 5 \le \sum_{i=1}^{2} x_{i3} \le 30$$

$1 \le x_{11} \le 10$, $2 \le x_{12} \le 10$, $0 \le x_{13} \le 5$,$0 \le x_{21} \le 15$, $3 \le x_{22} \le 15$, $1 \le x_{23} \le 20$

$F_{11} = 150$, $F_{12} = 50$, $F_{13} = 50$, $F_{21} = 200$, $F_{22} = 100$, $F_{23} = 50$

$F_i = \sum_{l=1}^{2} F_{il}\delta_{il}$ for l=1,2,3 where

$$\delta_{i1} = \begin{Bmatrix} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 0 \\ 0 & \text{otherwise} \end{Bmatrix}$$

$$\delta_{i2} = \begin{Bmatrix} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 10 \\ 0 & \text{otherwise} \end{Bmatrix}$$

$$\delta_{i3} = \begin{Bmatrix} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 20 \\ 0 & \text{otherwise} \end{Bmatrix}$$

Introduce a dummy origin and a dummy destination in Table 1 with $c_{i4} = 0$ for all i = 1,2 and $c_{3j} = 0$ for all j = 1,2,3 Also we have $0 \le x_{14} \le 25$, $0 \le x_{24} \le 30$, $0 \le x_{31} \le 20$, $0 \le x_{32} \le 13$, $0 \le x_{33} \le 25$, $0 \le x_{34} \le M$ and $F_{3j} = 0$ for j=1,2,3,4 In this way , we form the problem (P2´).Similarly on introducing a dummy origin and a dummy destination in Table 2 with $t_{i4} = 0$ for i=1,2 and $t_{3j} = 0$ for j=1,2,3,4, we form problem (P3´) .Find an initial basic feasible solution of problem (P2´) which is given in table 3 below.

Table 3: A basic feasible solution of problem (P2´)

	D_1	D_2	D_3	D_4	u_i
O_1	5 **10**	9 $\underline{2}$	9	0 **18**	0
O_2	4 **0**	6 **5**	2 **5**	0 $\overline{30}$	-1
O_3	0 $\overline{20}$	0 $\overline{13}$	0 $\overline{25}$	0 **22**	0
v_j	5	7	3	0	

Note: Values in the upper left corner of each cell in table 3 are c_{ij} ’s and entries of the form $\underline{a}$ and $\overline{b}$ in the upper right corner represent non basic cells which are at their lower bounds and upper bounds respectively. Entries in bold at the upper right corner represent basic cells.

F (current) = 200 + 200 + 0 = 400

Table 4: Optimality condition of problem (P2´)

NB	O_1D_2	O_1D_3	O_2D_4	O_3D_1	O_3D_2	O_3D_3
$(c_{ij}-z_{ij})$	2	6	1	-5	-7	-3
θ_{ij}	2	4	7	0	0	0
$\theta_{ij}(c_{ij}-z_{ij})$	4	24	7	0	0	0
F(NB)	400	400	450	400	400	400
ΔF_{ij}	0	0	50	0	0	0
R^1_{ij}	4	24				
R^2_{ij}			43	0	0	0

Since $R_{ij}^{1} \geq 0\ ;\forall(i,j) \in N_1$ and $R_{ij}^{2} \geq 0; \forall(i,j) \in N_2$, the solution given in table 3 is an optimal solution of problem (P2´) and hence yields an optimal solution of (P2) with minimum cost Z^1 = 508 and the corresponding time T^1=15.Therefore the first time cost trade off pair is (508,15).

Define $c_{ij}^{1} = \begin{Bmatrix} M & \text{if } t_{ij} \geq 15 \\ c_{ij} & \text{if } t_{ij} < 15 \end{Bmatrix}$

A basic feasible solution to the new cost problem is given in table 5 below.

Table 5:A basic feasible solution to the new cost problem

	D_1	D_2	D_3	D_4	u_i
O_1	M $\underline{1}$	9 **4**	9	0 $\overline{25}$	3
O_2	4 **9**	6 **3**	2 **5**	0 **23**	0
O_3	0 $\overline{20}$	0 $\overline{13}$	0 $\overline{25}$	0 **22**	0
v_j	4	6	2	0	

F(current) = 150 + 300 + 0 = 450

Table 6: optimality condition of the new cost problem

NB	O_1D_3	O_1D_4	O_3D_1	O_3D_2	O_3D_3
$(c_{ij}-z_{ij})$	4	-3	-4	-6	-2
θ_{ij}	2	0	6	12	15
$\theta_{ij}(c_{ij}-z_{ij})$	8	0	-24	-72	-30
F(NB)	450	450	500	500	500
ΔF_{ij}	0	0	50	50	50
R_{ij}^{1}	8				
R_{ij}^{2}		0	74	122	80

Since $R_{ij}^{1} \geq 0$; $\forall(i, j) \in N_1$ and $R_{ij}^{2} \geq 0; \forall(i, j) \in N_2$,the solution given in table 5 is an optimal solution with minimum cost Z^{2} =555 and the corresponding time T^{2} =13.Therefore the second time cost trade off pair is (555,13).

Proceeding like this, the time cost trade off pairs are (508,15), (555,13), (555,11).If we proceed further, the problem becomes infeasible.

6 Conclusion

In this paper , we have proposed an algorithm to find optimum time – cost trade off pairs in a capacitated fixed charge transportation problem with bounds on total availabilities at sources and total destination requirements. We separated the problem in to two problems and formed the related fixed charge capacitated transportation problem by introducing a dummy source and a dummy destination to find the optimum time cost trade off pairs.

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