NON-STEADY RADIAL FLOW OF A VISCOUS INCOMPRESSIBLE LIQUID IN THE POROUS MEDIUM AROUND A RADIALLY OSCILLATING SPHERICAL SURFACE

By

MOHAMMED NAHEED

Department of Information and Technology (Mathematics Section), Nizwa College of Technology (Nizwa), Sultanate of Oman.

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ABSTRACT

This paper deals with non-steady radial flow of a viscous, incompressible liquid in the porous medium around a radially oscillating time dependent spherical surface. The momentum equation considered for the flow through the porous medium takes care of the fluid inertia and the Newtonian stresses in addition to the classical Darcy's friction. Expressions for the pressure and velocity distributions have been derived in terms of the expansion rate of sphere radius using analytical method and effects of variation of pressure, velocity of a viscous, incompressible, and homogeneous fluid flow in a porous medium are reported and the results are presented graphically for the two special cases of radius of the sphere.

Keywords: Pressure, Darcy's Number, Porous Medium, Expansion Factor, Radius Oscillating Parameter.

INTRODUCTION

Fluid flows through porous medium have been attracting the attention of applied mathematicians and engineers for the last one and half centuries because of their wide range of applications in diverse fields of science and engineering technology. Notably in the flows of oils through porous rocks, drug permeation through human skin, extraction of energy through geothermal regions, and filtration of solids from liquids. Knowledge of the flow through porous medium is immensely useful in the efficient recovery of crude oil from the porous reservoir rocks by the displacement with immiscible water.

Fluid flows through porous medium occur in the groundwater hydrology, irrigation, drainage problems, and also in absorption and filtration processes in chemical engineering. The subject of fluid flows through porous medium has widespread applications to problems encountered in the civil engineering and agricultural engineering and several other areas of importance in industries. Diffusion and flow of fluids through ceramic materials as bricks and porous earthenware has long been problems of the ceramic industry. The construction of filter beds for municipal water system and water seepage through, around and beneath dams, earthen reservoirs, the scientific treatment of problems of irrigation, soil erosion, and tile drainage are some of the present day developments of flows through porous media.

This paper deals with non-steady radial flow of a viscous incompressible fluid flow through porous medium around a radially oscillating time dependent spherical surface and the fluid is assumed to be viscous, incompressible, and homogeneous. The problem of viscous incompressible fluid flow in the existence of porous spherical bodies has been a subject of broad study, not only for their theoretical interest, but also on account of their possible industrial applications in Tribology like linear bearings, sintered gas bearings, and porous bearings. Spherical air bearings are used for reduction of friction and application in physiological situations such as flow in the vicinity of glands in various parts of living bodies.

Studies on radial flows of a viscous fluid were initiated in the year 1915 by (Jeffery,1915) and these were later followed by (Hamel, 1917) and suchradial flows are discussed at length by (Dryden, Murnaghan, & Batemen, 1956) in their classical work on Hydrodynamics. Several types of radial flows of a viscous liquid in a clear medium are discussed by (Rayleigh, 2006; Raisinghania, 1982), in their treatise, Incompressible flow and fluid dynamics, respectively.

Studies on this were initiated in 1856 by Darcy based on a series of experiments on flows of slurry fluids through the channels (Darcy, 1856). Darcy formulated an empirical law for fluid flows through the porous media (Darcy, 1856). The total volume of the fluid percolating in unit time is proportional to the hydraulic head and inversely proportional to the distance between the inlet and outlet. This law was later generalized by Brinkman by taking into account the stresses generated in the flow section (Brinkman,1949). Later, Yamamoto, and Yoshida further, generalized the basic governing equations by the inclusion of fluid inertia in addition to the Newtonian-stresses developed in fluids in motion (Yamamoto & Yoshida, 1974).

Pattabhi (1976) examined different flow problems through straight tubes of various cross sections and also Narasimhacharyulu and Pattabhi (1980) have obtained a general solution for an incompressible flow through porous media. Islam, Mohyuddin, and Zhou (2008) have examined few exact solutions of the non-Newtonian fluid in a porous medium and Mohyuddin (2006a, 2006b) has obtained solutions of non-linear equations arising in Rivli-Ericksen fluids and apart from that examined resonance and viscoelastic poiseuille flow in a porous medium. Recently, the author Naheed and Pattabhi (2013) have investigated an unsteady radial flow of a viscous, incompressible fluid through a porous medium around a radially oscillating spherical surface with exponential radius of the sphere.

The present study is an another class of non-steady radial flow of a viscous, incompressible liquid in the porous medium around a radially oscillating spherical surface with a sinusoidal radius of the sphere. Yamamoto and Yoshida (1974) have generalized a momentum equation for the flows through the porous medium, which has been solved for the radial flow. It is observed that the flow is independent of the Newtonian viscous stresses. However, the radial flow depends on Darcian resistance. Expression for the pressure and velocity distributions has been obtained on the surface of the sphere using analytical method. The two cases of the sphere radius in the non-dimensional form is,

$$r = \sin\alpha t$$

$$r = \frac{1}{(1+\varepsilon)}(1-\varepsilon \sin\alpha t)$$

1. Mathematical Formulation and Solution of the Problem

Referred to a spherical co-ordinate frame (R, θ, ϕ) with an origin O fixed at the center of the sphere (Figure 1). R is the radial distance from the origin, θ is the polar angle, and ϕ is the azimuthal angle. The flow of a viscous, incompressible fluid through the porous medium is governed by the modified Navier-Stokes equations recommended by (Yamamoto & Yoshida, 1974):

$$\frac{d\vec{q}}{dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{q} - \frac{\nu}{k}\vec{q}, \tag{1}$$

together with the equation of continuity,

$$\nabla . \vec{q} = 0. \tag{2}$$

Here, $\vec{q}$ represents the fluid velocity and p is the fluid pressure. Further, ρ is the fluid mass density, μ is the coefficient of Newtonian of viscosity and k is the coefficient of Darcian porosity of the medium and all these coefficients are assumed to be constants. The term $\mu\nabla^2\vec{q}$ on the R.H.S of equation (1) represents the contribution of the Newtonian viscous-stress and $-\frac{\mu}{k}\vec{q}$ is the classical Darcy -resistance to the flow.

By radial and axis-symmetries,

$$\frac{\partial\vec{q}}{\partial\theta} = 0 \quad \text{and} \quad \frac{\partial\vec{q}}{\partial\phi} = 0. \tag{3}$$

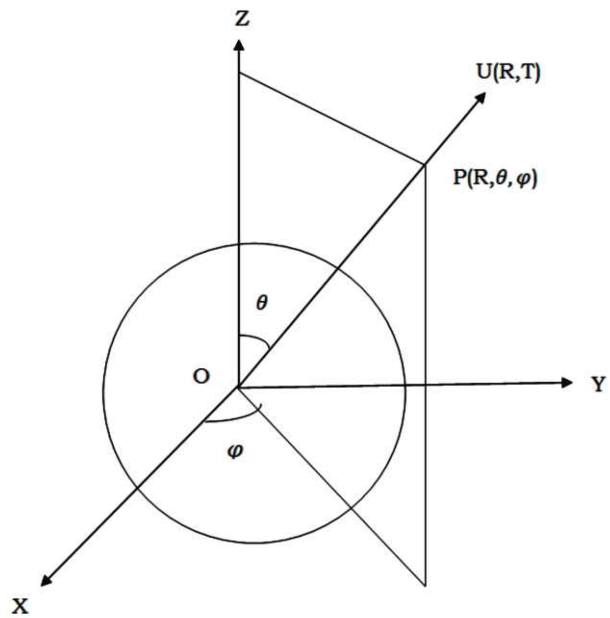

Figure 1. Flow Configuration

For the non-steady radial flow under investigation, the velocity field can be taken as,

$$\bar{q} = U((R,T),0,0) \tag{4}$$

The continuity equation (2) in dimensional form is given as,

$$\frac{1}{R^2}\frac{\partial (R^2U)}{\partial R} = 0 \tag{5}$$

The momentum equation in the radial direction (dimensional form) is given as,

$$\frac{\partial U}{\partial T} + U\frac{\partial U}{\partial R} = -\frac{1}{\rho}\frac{\partial P}{\partial R} + \nu\left(\frac{\partial^2 U}{\partial R^2} + \frac{2}{R}\frac{\partial U}{\partial R} - \frac{2U}{R^2}\right) - \frac{\nu}{k}U \tag{6}$$

The momentum equation in the θ and ϕ-direction are,

$$\frac{\partial P}{\partial \theta} = 0 \text{ and } \frac{\partial P}{\partial \phi} = 0 \tag{7}$$

It can be noted from equation (7) that

- The pressure P is the function of R and T only, i.e., independent of θ and ϕ
- It is also independent of Newtonian viscous stresses, and
- The Newtonian viscous resistance on the porous media $-\frac{\mu}{k}\bar{q}$ influences the pressure distribution.

For simplicity, the following non-dimensional quantities are presented.

$$P = \frac{\mu^2 p}{\rho r_0^2},\ U = \frac{\mu u}{\rho r_0},\ R = r_0 r,\quad D = \frac{r_0^2}{k},\ T = \frac{\rho r_0^2 t}{\mu} \tag{8}$$

where D is the Darcy porosity coefficient and R_0 is the initial radius of the sphere. By definition, u is the radial velocity on the sphere of the surface which is given by,

$$u = \frac{dr}{dt} \tag{9}$$

Now, the basic governing equations in the non-dimensional form are given as:

The continuity equation in non-dimensional form

$$\frac{1}{r^2}\frac{\partial (r^2 u)}{\partial r}=0 \tag{10}$$

The momentum equation in the radial direction (non-dimensional form),

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}=-\frac{\partial p}{\partial r}-Du \tag{11}$$

From equation (10) we get,

$$u=\frac{f(t)}{r^2} \tag{12}$$

It follows from equations (9) and (12) that

$$f(t)=r^2u \quad on \quad r=1 \tag{13}$$

Using equation (12) in equation (11) we get,

$$-\frac{\partial p}{\partial r}=\frac{f^1(t)}{r^2}-2\frac{(f(t))^2}{r^5}+D\frac{f(t)}{r^2} \tag{14}$$

Integrating this with respect to r yields,

$$P=-\frac{1}{r}\frac{df}{dt}+\frac{1}{2r^4}\frac{d^2 f}{dt^2}-\frac{D}{r}f(t) \tag{15}$$

where p_∞ is the pressure at infinity i.e., $\lim_{r\to\infty} p = p_\infty$. This is computed on the sphere surface and is given as,

$$P=r\frac{d^2r}{dt^2}+\frac{3}{2}\left(\frac{dr}{dt}\right)^2+Dr\frac{dr}{dt} \quad \text{on } r=1 \tag{16}$$

Computed on the surface of the sphere

1.1 Case-A

$$r=\sin\alpha t \tag{17}$$

From equations (16) and (12), the pressure and velocity distributions are respectively,

$$p=\alpha^2-\alpha^2\cos 2\alpha t+2D\alpha\sin 2\alpha t \tag{18}$$

$$u=\alpha\cos\alpha t \tag{19}$$

From the equation (18), the maximum and minimum pressures on the sphere surface are

$$\left.\begin{aligned} p_{\max}&=\frac{\alpha}{4}(\alpha+\sqrt{\alpha^2+4D^2}) \\ p_{\min}&=\frac{\alpha}{4}(\alpha-\sqrt{\alpha^2+4D^2}) \end{aligned}\right\} \tag{20}$$

where p_{max} is always positive and p_{min} is positive whenever $\alpha^2>(\alpha^2+4D^2)$.

1.2 Case - B

$r=\frac{1}{(1+\varepsilon)}\left(1-\varepsilon\sin\alpha t\right)$, where α is the sphere radius oscillation parameter, which is a constant.

From equations (16) and (12) the pressure and velocity distributions respectively are,

$$p=\varepsilon\sin\alpha t+\frac{1}{2}\varepsilon^2(3-5\sin^2\alpha t)+D\left(\frac{1}{2}\varepsilon^2\sin 2\alpha t-\varepsilon\alpha\cos\alpha t\right) \tag{21}$$

$$u=-\frac{\varepsilon\cos\alpha t}{1+\varepsilon} \tag{22}$$

2. Results and Discussions

In this paper, analytical solutions for effects of variation of pressure, velocity of a viscous, incompressible, and homogeneous fluid flow in a porous media are reported and the results are presented graphically from Figures 2-11.

2.1 Case - A

It is observed from Figures 2-5, the variation of the pressure on the spherical surface versus radius oscillation parameter for distinct values of Darcy number at distinct instants of time, the number of oscillation beats increases as radius oscillation parameter increases. All these variations of the pressure may be attributed due to the increase in the internal Darcian resistance due to the porosity of the medium.

Also it is observed from Figures 6-7, the variation of the pressure on the spherical surface versus Darcy number for distinct values of radius oscillation parameter at distinct instants of time, the pressure gradually increases this may be associated due to the decrease in internal resistance in the porous medium.

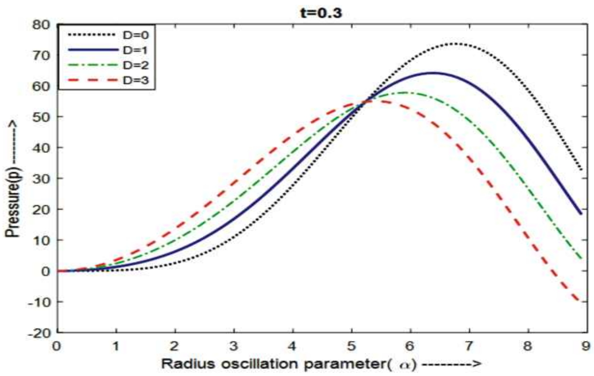

Figure 2. Variation of the Pressure with Radius Oscillation Parameter for Different D when t=0.3

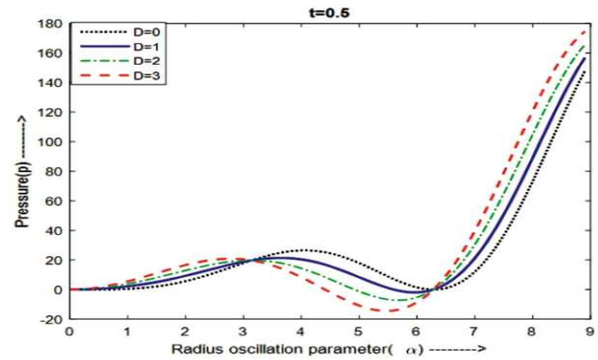

Figure 3. Variation of the Pressure with Radius Oscillation Parameter for Different D when t=0.5

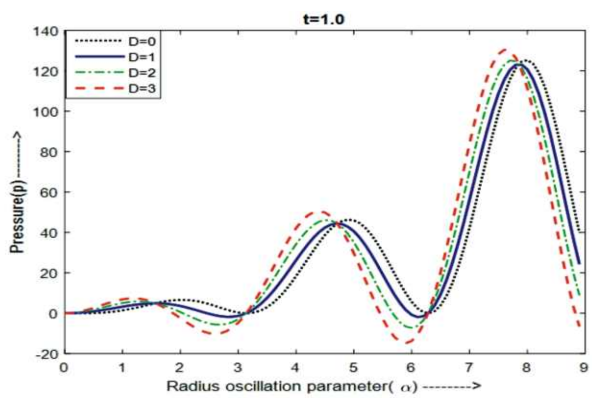

Figure 4. Variation of the Pressure with Radius Oscillation Parameter for Different D when t=1.0

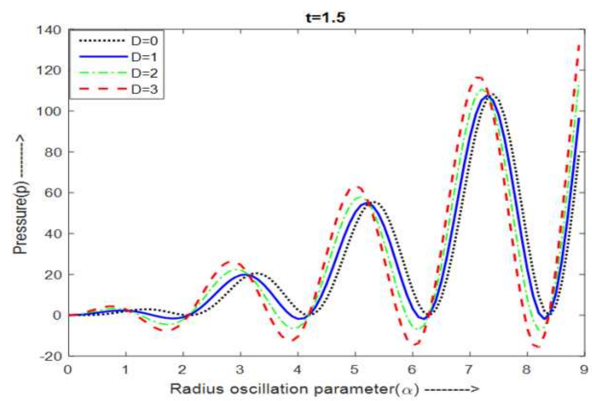

Figure 5. Variation of the Pressure with Radius Oscillation Parameter for Different D when t=1.5

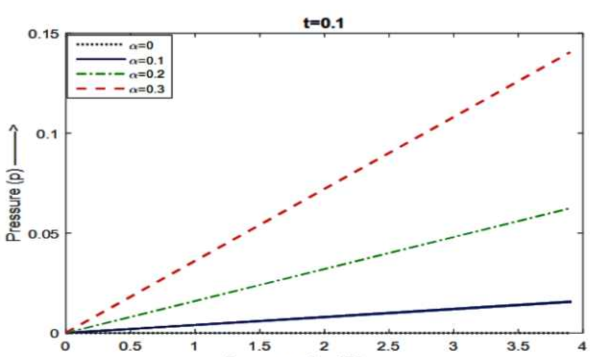

Figure 6. Variation of the Pressure with Darcy Number for Different Oscillation Parameter when t=0.1

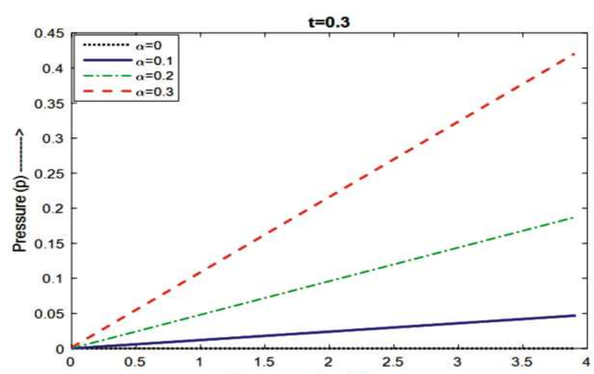

Figure 7. Variation of the Pressure with Darcy Number for Different Oscillation Parameter when t=0.3

2.2 Case - B

It is observed from Figures 8-9, the variation of the pressure on the spherical surface versus radius oscillation parameter for distinct values of Darcy number at distinct instants of time, the number of oscillation beats increases as radius oscillation parameter increases. All these variations of the pressure may be attributed to the increase in the internal Darcian resistance due to the porosity of the medium.

Also it is observed from Figure 10, the variation of the pressure on the spherical surface versus Darcy number for distinct values of radius oscillation parameter at distinct instants of time, the pressure gradually increases this may be associated due to the decrease in internal resistance in the porous medium.

3. Validations

It is noted that, if we put D=0 in the equation (21), the result coincides with (Rayleigh, 2006) and the results are compared with Figure 11 and Table 1.

Conclusion

The problem considered in this work is to study the effect of pressure and velocity on flow through porous medium of a non-steady radial flow a viscous incompressible flow around a sphere of variable radius.

The pressure distribution has been numerically computed from the obtained solution for different values of radius oscillation parameter, Darcy number, exponential-decay parameter at distinct expansion factor at different instants of time and the velocity distribution have also been numerically computed from the obtained solution. On the boundary, the graphs for pressure for the surface of the sphere are shown.

Current work in the absence of the Darcy number term coincides with (Rayleigh, 2006) and compared the results with Figure 11.

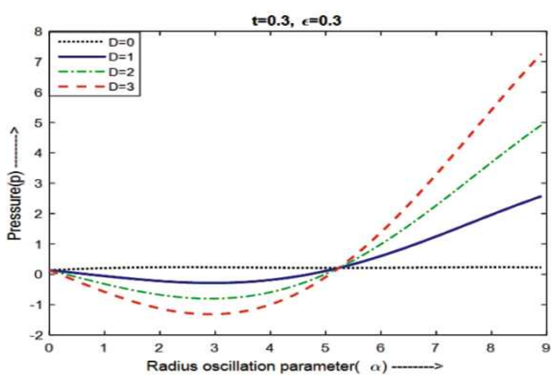

Figure 8. Variation of the Pressure with Radius Oscillation Parameter for Different D when t=0.3, ∈=0.3

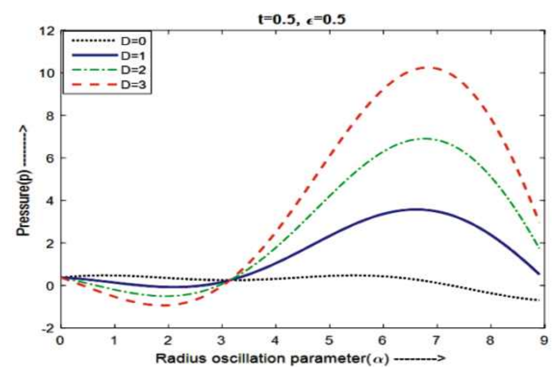

Figure 9. Variation of the Pressure with Radius Oscillation Parameter for Different D when t=0.5, ∈=0.5

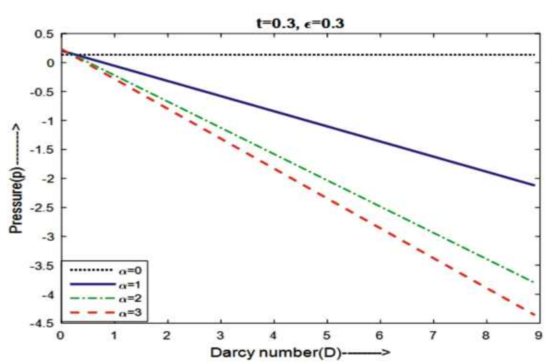

Figure 10. Variation of the Pressure with Darcy Number Time at Different Radius Oscillation Parameter when t=0.3, ∈=0.3

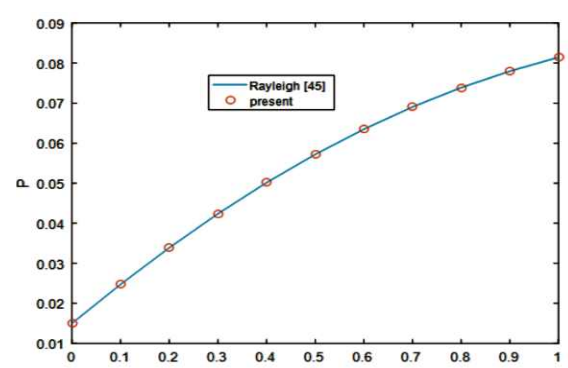

Figure 11. Variation of the Pressure Vsat Darcy Number D=0

l	Result of the Work	Rayleigh (2006) Result
0.0	0.015000	0.015000
0.1	0.024734	0.024734
0.2	0.033880	0.033880
0.3	0.042368	0.042368
0.4	0.050150	0.050150
0.5	0.057196	0.057196
0.6	0.063493	0.063493
0.7	0.069046	0.069046
0.8	0.073870	0.073870
0.9	0.077992	0.077992
1.0	0.081445	0.081445

Table 1. Comparison of Analytical Results for Pressure Distribution (Case: A) when $\in=0.1$, $\alpha=1$

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ABOUT THE AUTHOR

Dr. Mohammed Naheed is working as a Lecturer in the Department of Information Technology (Math Section: Nizwa College of Technology, Nizwa, Sultanate of Oman). He received his Ph.D. in Mathematics from the Jawaharlal Nehru Technology University, India.He has held faculty positions at several engineering institutions, including Osmania University and Jawaharlal Nehru Technology University, India. His areas of Interest includes Fluid Dynamics and Mathematical Modeling and published and presented research papers in National and International Conferences in India.