A TIME DEPENDENT RADIAL FLOW OF A VISCOUS LIQUID IN A POROUS MEDIUM CONTAINING A CONTRACTING SPHERE

MOHD. NAHEED

Deptt. of Mathematics, Muffakham Jah College of Engineering &Technology, Hyderabad-500034 (A. P.), India

N. CH. PATTABHIRAMACHARYULU

Former faculty, Deptt. of Mathematics and Humanities, National Institute of Technology, Warangal-506004 (A.P.), India

AND

SHAHNAZ BATHUL

Former faculty, Deptt. of Mathematics and Humanities, Jawaharlal Nehru Technological University Hyderabad-500086 (A.P.), India

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This paper deals with a radial flow of a viscous liquid in a porous medium around a spherical surface of radius '*r*' which varies with time (*t*). The momentum equation considered for the flow through a porous medium takes care of the fluid-inertia and the Newtonian stresses in addition to the classical Darcy's friction. Expression for the pressure distribution has been derived in terms of the Contraction rate of the sphere -radius.

Two special cases: (A) $r = (1 + \varepsilon t e^{-\alpha t})$ (B) $r = (1 + \varepsilon t^2 e^{-\alpha t})$ are discussed in detail. In this 'α' and 'ε' is the sphere radius contracting parameters. The variation of the pressure for different values of 'α' for different Darcy number [*D*] at different instants of time '*t*' and for different 'ε' are in each of the cases has been discussed and illustrated and conclusions are drawn. A common observation : The pressure drops steeply as the Darcy number increases this can be attributed to the greater resistance offered by the porous medium to the flow.

KEYWORDS: Pressure, Darcy's number, Porous Medium, Expansion factor, Radius decay-parameter.

INTRODUCTION

Studies on radial flows of a viscous fluid were initiated in the year 1915 by Jeffery G.B. [5] and these were followed later by Hamael G. [4] and Harrison W. J. Such flows are discussed at length by Dryden H.L., Murnaghan F.D. and Batemen H. [3] in their classical work on Hydrodynamics. Recently Raisinghania M.D. [9] in his treatise on Fluid Dynamics discussed several types of radial flows of viscous fluids in a clear medium.

Flows through porous media have been a subject of considerable research activity for over the last one and half centuries, because of their wide range of application in diverse fields of science, engineering and technology.

Studies on flows through porous media were initiated in 1856 by Darcy H. [2] based on a series of experiments on flows of slurry fluids through channels. Darcy formulated an empirical law for fluid flows through porous media : The total volume of the fluid percolating in unit time is proportional to the hydraulic head and inversely proportional to the distance between the inlet and outlet. This law was later generalized by Brinkman H.C. [1] by, taking into account for the Newtonian-stresses generated in the flow region. Later, Yamamoto K. and Yoshida Z. [10] further generalized the basic equations by the inclusion of fluid inertia in addition to the Newtonian-Stresses developed in fluids in motion. Later Pattabhi Ramacharyulu. N.Ch. [8] examined several flow problems through straight tubes of diverse cross sections. A general solution for an incompressible flow through porous media has been obtained by Narasinhacharyulu V. and Pattabhi Ramachryulu N.Ch. [6]. Recently, the present authors [7] investigated a class of unsteady radial flows of a viscous incompressible fluid through a porous medium around a sphere of time dependent radius (r). The present investigation is on another class of unsteady radial flows of a viscous incompressible flow through a porous medium around a sphere whose surface is contracting exponentially with time. A generalized momentum equation given by Yamamoto K. and Yoshida Z. [10] for the flows through porous medium has been solved for the radial flow. It is noticed that the flow is independent of a Newtonian viscous stresses. However the flow depends on Darcian friction. Expression for the pressure distribution has been obtained in terms of the radial velocity on the sphere- surface. The cases of the sphere radius at time are

(A) $r = (1 + \varepsilon t e^{-\alpha t})$ and (B) $r = (1 + \varepsilon t^2 e^{-\alpha t})$ in the non dimensional form

MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

Consider a spherical co-ordinate system (R, θ, ϕ) with a origin 'O' fixed at the center of the sphere. R is the radial distance from the origin, θ the polar angle, ϕ the azimuthal angle. The flow of a viscous incompressible fluid through a porous medium is governed by the modified Navier-Stokes equations suggested by Yamamoto K. and Yoshida Z. [10] :

$$\rho \frac{d\vec{q}}{dt} = -\nabla p + \mu \nabla^2 \vec{q} - \frac{\mu}{k}\vec{q} \qquad \ldots(1)$$

together with the equation of continuity

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\,\vec{q} = 0 \qquad \ldots(2)$$

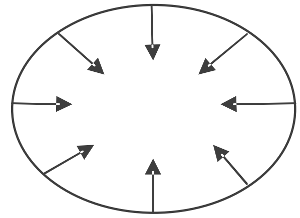

Fig.1: Flow- Sketch

where '$\vec{q}$' represents the fluid velocity and 'p' is the fluid pressure. Further ρ is the fluid mass density, μ is the coefficient of Newtonian of viscosity and 'k' is the coefficient of Darcian porosity of the medium and these are assumed to be constants. The term $\mu\nabla^2\vec{q}$ on the R.H.S. of (1) represents the contribution of the Newtonian Viscous-Stress and $-\frac{\mu}{k}\vec{q}$ is the classical Darcy -resistance to the flow.

By radial and axis-symmetries

$$\frac{\partial\vec{q}}{\partial\theta}=0 \quad \text{and} \quad \frac{\partial\vec{q}}{\partial\phi}=0 \qquad \ldots (3)$$

For the unsteady radial flow under investigation, the velocity field can be taken as

$$\vec{q}=U((R,T),0,0) \qquad \ldots (4)$$

The continuity equation (2) now reduces to

$$\frac{1}{R^2}\frac{\partial(R^2U)}{\partial R}=0 \qquad \ldots (5)$$

Momentum equation in the Radial-direction (R)

$$\frac{\partial U}{\partial T}+U\frac{\partial U}{\partial R}=-\frac{1}{\rho}\frac{\partial P}{\partial R}-\left(\frac{\nu}{k}\right)U \qquad \ldots (6)$$

where $$\nu=\frac{\mu}{\rho}$$

Momentum equation in the θ and ϕ-direction

$$\frac{\partial P}{\partial\theta}=0 \quad \text{and} \quad \frac{\partial P}{\partial\phi}=0 \qquad \ldots (7)$$

It can be noted from the equation (7) that

(I) The pressure (P) is a function of (R) and (T) only. (*i.e.* independent of θ and ϕ)

(II) It is also independent of Newtonian viscous stresses and

(III) The Darcian viscous resistance on the porous media $\left(-\frac{\mu}{k}\vec{q}\right)$ only influences the pressure distribution.

For simplicity the following non dimensional quantities are introduced in the foregoing analysis

$$R=R_0r;\ U=\frac{\mu u}{\rho R_0};\ T=\frac{\rho R_0{}^2 t}{\mu};\ P=\frac{\mu^2 p}{\rho R_0{}^2};\ D=\frac{R_0{}^2}{k} \qquad \ldots (8)$$

where 'D' is the non-dimensional Darcy porosity coefficient and where R_0 is the initial radius of the sphere. By definition, the radial velocity 'u' on the sphere of the surface is given by

$$u = \frac{dr}{dt} \quad \text{on the sphere-surface} \qquad \dots (9)$$

The following are non-dimensional form of basic equations

Continuity Equation

$$\frac{1}{r^2}\frac{\partial (r^2 u)}{\partial r} = 0 \qquad \dots(10)$$

Momentum equation in the radial direction

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{\partial p}{\partial r} - Du \qquad \dots (11)$$

From (10), we get

$$u = \frac{f(t)}{r^2} \qquad \dots (12)$$

It follows from (9) and (12) that

$$f(t) = r^2 \frac{dr}{dt} \quad \text{on } r = 1 \qquad \dots (13)$$

and from (11) and (12) we get the equation for the determination of the fluid pressure (P):

$$-\frac{\partial p}{\partial r} = \frac{f^1(t)}{r^2} - 2\frac{(f(t))^2}{r^5} + D\frac{f(t)}{r^2} \qquad \dots (14)$$

Integrating with respect to 'r' we get :

$$p_\infty - p = -\frac{1}{r^2}\left(\frac{df}{dt}\right) + \frac{1}{2r^4}\frac{d^2 f}{dt^2} - \frac{D}{r}(f(t)) \qquad \dots (15)$$

where p_∞ in the pressure at infinity *i.e.* $\lim_{r\to\infty} p = p_\infty$ .This when computed on the sphere surface yield

$$P = r\frac{d^2 r}{dt^2} + \left(\frac{3}{2}\right)\left(\frac{dr}{dt}\right)^2 + Dr\left(\frac{dr}{dt}\right) \quad \text{on } r = 1 \qquad \dots (16)$$

where $P = p - p_\infty$

Case-A :

In this case we have

$$r = (1 + \varepsilon t e^{-\alpha t}) \qquad \dots (A.1)$$

where α is a parameter characteristic of the decay of the sphere-radius

$$f(t) = (1 + \varepsilon t e^{-\alpha t})^2 (\varepsilon e^{-\alpha t}(1 - \alpha t)) \qquad \dots (A.2)$$

Also from equation (16) we get

$$P = e^{-2\alpha t}\varepsilon^2\left(\frac{3+\alpha t(5\alpha t-5)}{2}+Dt(1-D\alpha t)\right)+e^{-\alpha t}\varepsilon(\alpha(\alpha t-2))+D(1-\alpha t)) \qquad \ldots \text{(A.3)}$$

RESULTS AND DISCUSSION

The variation of the pressure vrs radius contraction parameter (α) for different values of porosity parameter (D) is illustrated in figures (A.1)-(A.9). It is noticed that the pressure decreases up to with some 'α' critical (α_c) and there after raises slowly as 'D' increases the pressure increases with D. All these variations of the pressure may be attributed to the increase in the internal Darcian resistance due to the porosity of the medium. The variation of (α_c) and pressure for different 'ε' and time (t) is tabulated in table number (1).

Table 1. Showing values of 'α_c' and pressure for different 'ε' and 't'

t ↓ / ∈ →	.1		.5		.9	
	α_c	Pressure	α_c	Pressure	α_c	Pressure
.1	10	− 0.3692	10	− 1.873	10	− 3.421
	Fig. 1		Fig. 2		Fig. 3	
.2	5	− 0.1853	5	− 0.9535	5	− 1.765
	Fig.4		Fig.5		Fig.6	
.3	3.3	− 0.123	3.3	− 0.642	3.3	− 0.933
	Fig. 7		Fig.8		Fig. 9	

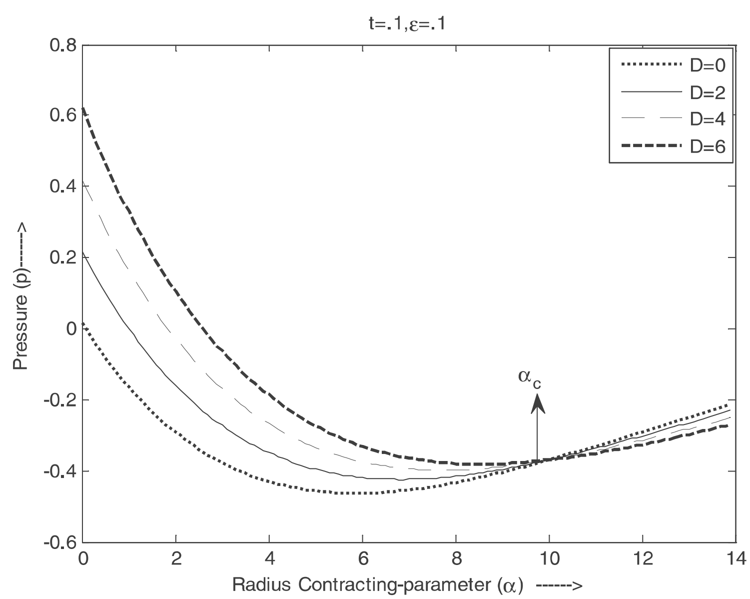

Fig. (A.1). Variation of the Pressure vs. alpha for different 'D' at t = 0.1, ε = 0.1

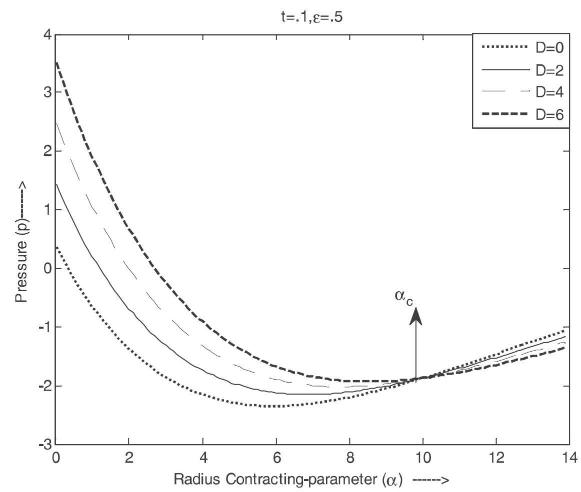

Fig. (A.2). Variation of the Pressure vs alpha for different '*D*' at *t* = 0.1, ε = 0.5

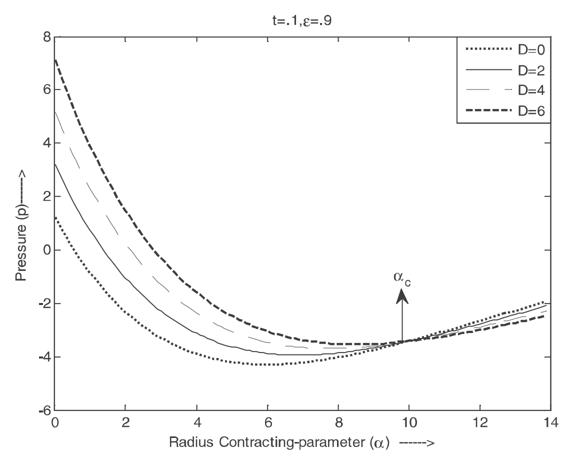

Fig. (A.3). Variation of the Pressure vs alpha for different '*D*' at *t* = 0.1, ε = 0.9

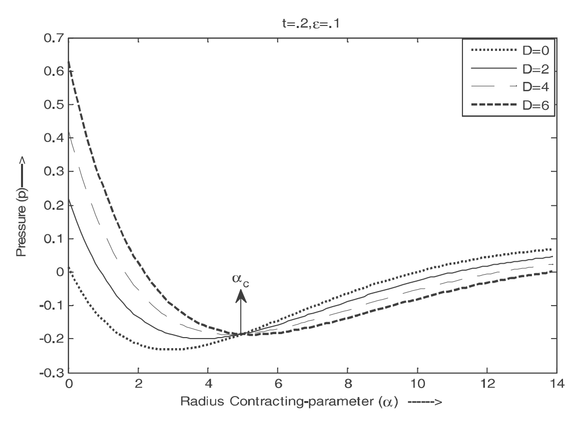

Fig. (A.4). Variation of the Pressure vs alpha for different '*D*' at *t* = 0.2, ε = .01

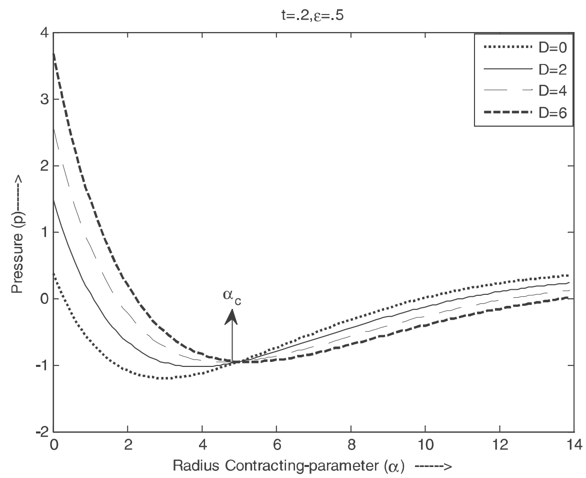

Fig. (A.5). Variation of the Pressure vs alpha for different '*D*' at *t* = 0.2, ε = 0.5

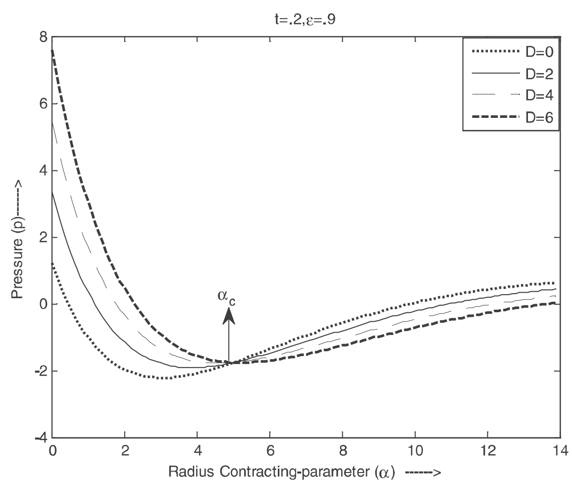

Fig. (A.6). Variation of the Pressure vs alpha for different '*D*' at *t* = 0.2, ε = 0.9

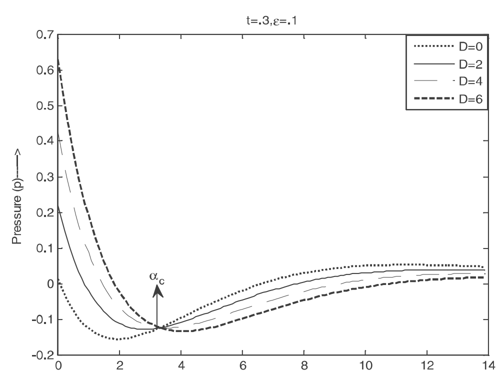

Fig. (A.7). Variation of the Pressure vs alpha for different '*D*' at *t* = 0.3, ε =0.1

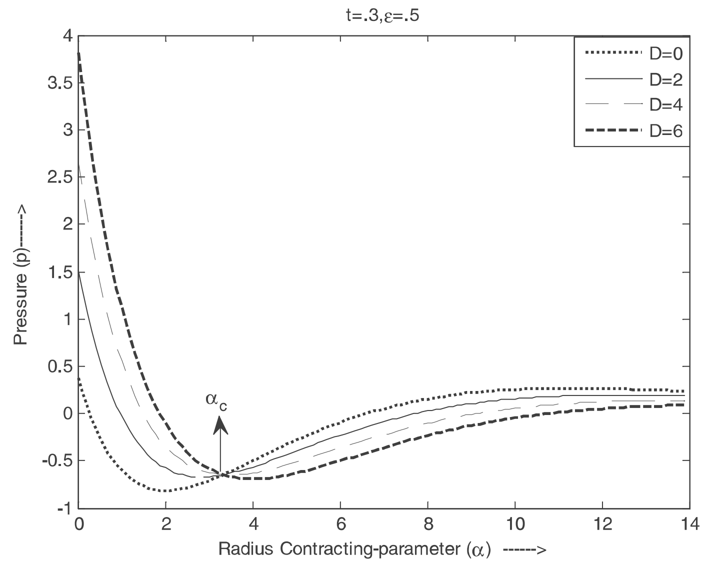

Fig. (A.8). Variation of the Pressure vs alpha for different '*D*' at *t* = 0.3, ε = 0.5

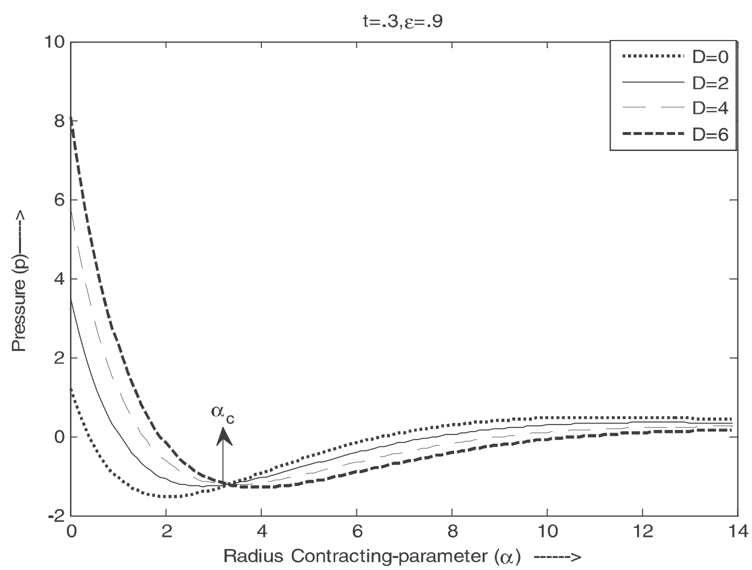

Fig. (A.9). Variation of the Pressure vs alpha for different '*D*' at *t* = 0.3, ε = 0.9

The variation of the pressure vrs porosity parameter (D) for different values of α, ε and t are illustrated in figures (A.10)-(A.15). It is noticed that the pressure increases as the Darcy number increases the pressure almost linearly raises as the Darcy number increases and decreases with α.

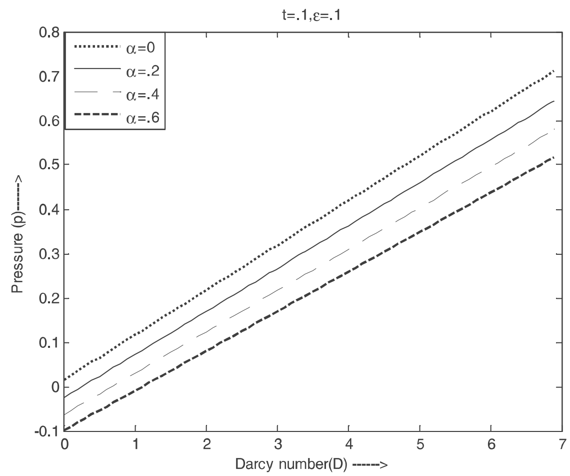

Fig. (A.10). Variation of the Pressure vs Darcy number for different time at $t = 0.1$, $\varepsilon = 0.1$

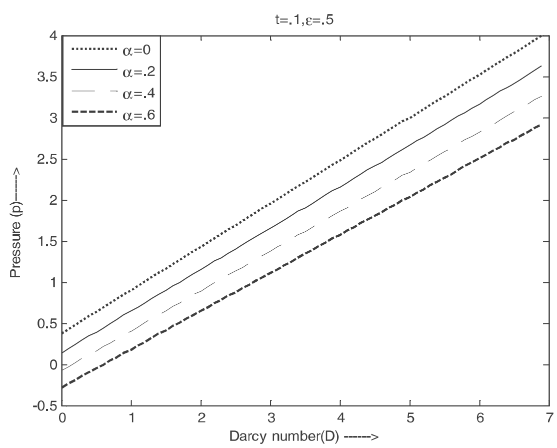

Fig. (A.11). Variation of the Pressure vs Darcy number for different time at $t = 0.1$, $\varepsilon = 0.5$

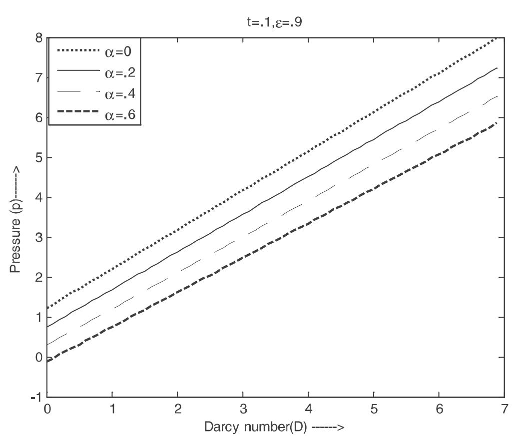

Fig. (A.12). Variation of the Pressure vs Darcy number for different time at $t = 0.1$, $\varepsilon = 0.9$

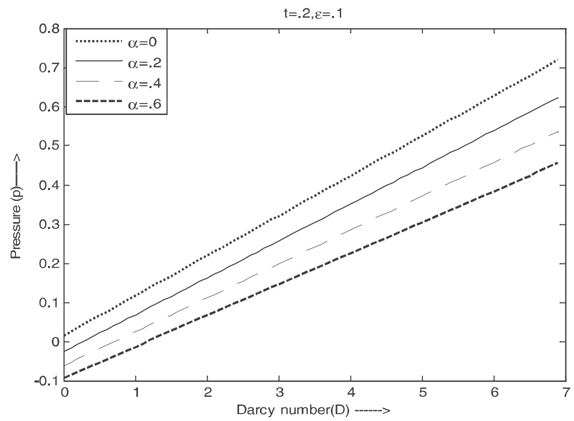

Fig. (A.13). Variation of the Pressure vs Darcy number for different time at $t = 0.2$, $\varepsilon = 0.1$

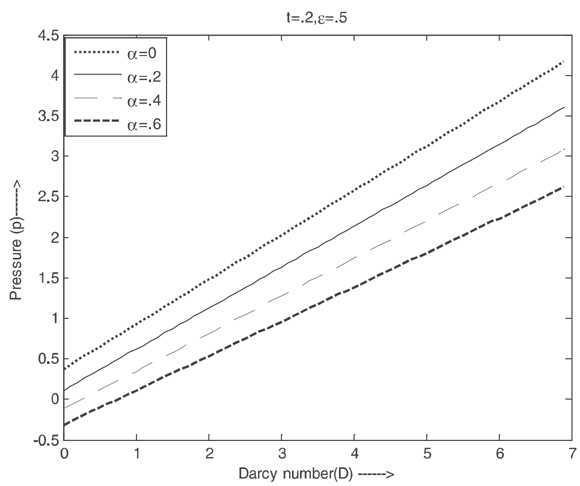

Fig. (A.14) Variation of the Pressure vs Darcy number for different time at $t = 0.2$, $\varepsilon = 0.5$

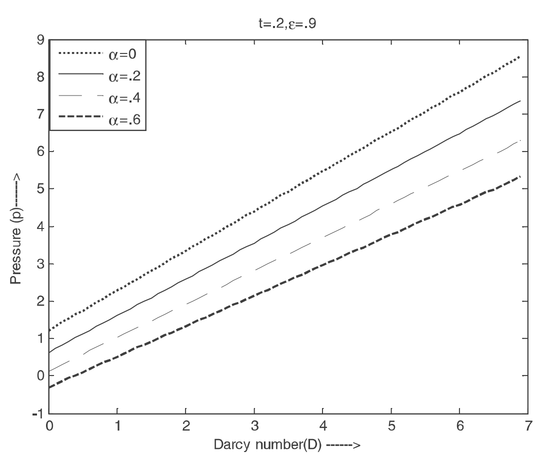

Fig. (A.15). Variation of the Pressure vs Darcy number for different time at $t = 0.2$, $\varepsilon = 0.9$

The variation of the pressure vrs time (t) for different values of 'D' are illustrated in figures (A.16)-(A.24). for large value of 'D' the pressure decreases up to a critical time (t_c) and later on the pressure decreases there exist a critical time (t_c) for all values of 'D' and for each set of values of ε and α as ε increases the pressure initially raises before it starts decreasing.

Table 2. Showing values of 't_c' and pressure for different 'ε' and 'α'

$\in \rightarrow$ / $\alpha \downarrow$	.1		.5		.9	
	t_c	Pressure	t_c	Pressure	t_c	Pressure
.1	9.9	– 0.0036	10	– 0.0522	10	– 0.1427
	Fig. 16		Fig. 17		Fig. 18	
.2	5	– 0.0087	5	– 0.0518	5	– 0.1756
	Fig. 19		Fig. 20		Fig. 21	
.3	3.3	– 0.0113	3.3	– 0.1029	3.3	0.192
	Fig. 22		Fig. 23		Fig. 24	

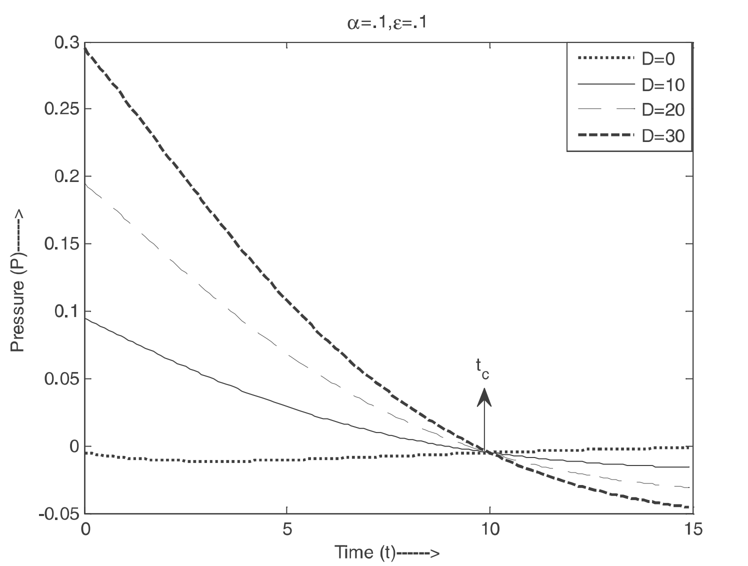

Fig. (A.16). Variation of the Pressure vs Time for different 'D' at $\alpha = 0.1$, $\varepsilon = 0.1$

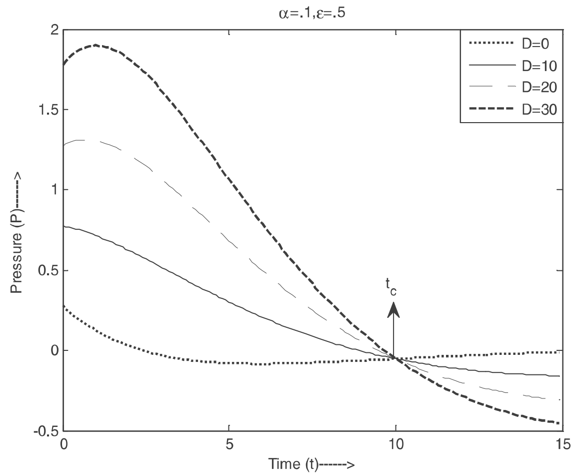

Fig. (A.17). Variation of the Pressure vs Time for different '*D*' at $\alpha = 0.1$, $\varepsilon = 0.5$

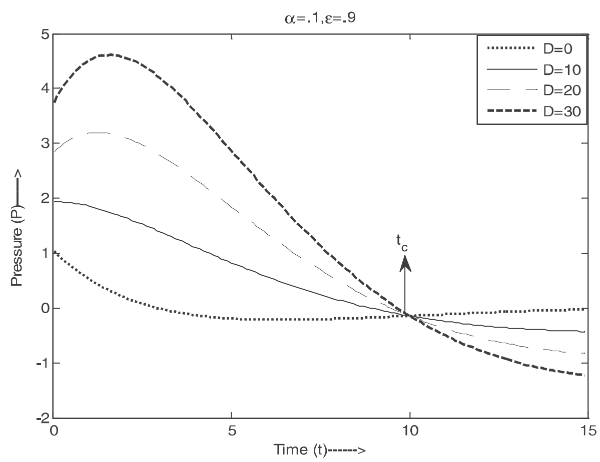

Fig. (A.18). Variation of the Pressure vs Time for different '*D*' at $\alpha = 0.1$, $\varepsilon = 0.9$

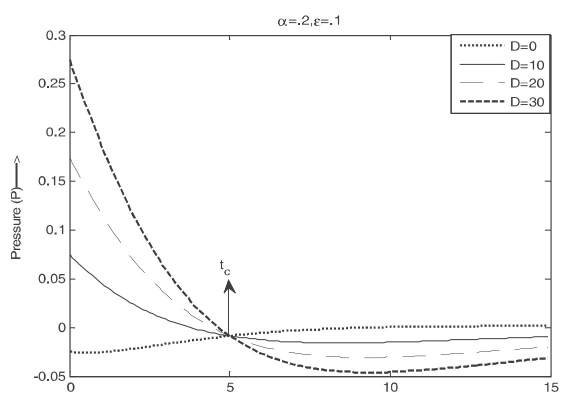

Fig. (A.19). Variation of the Pressure vs Time for different '*D*' at $\alpha = 0.2$, $\varepsilon = 0.1$

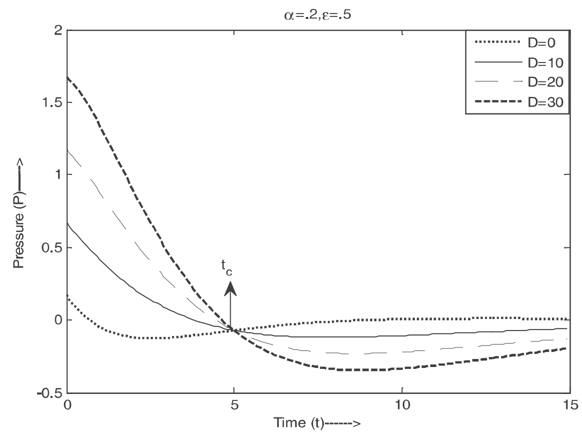

Fig. (A.20). Variation of the Pressure vs Time for different '*D*' at $\alpha = 0.2,\ \varepsilon = 0.5$

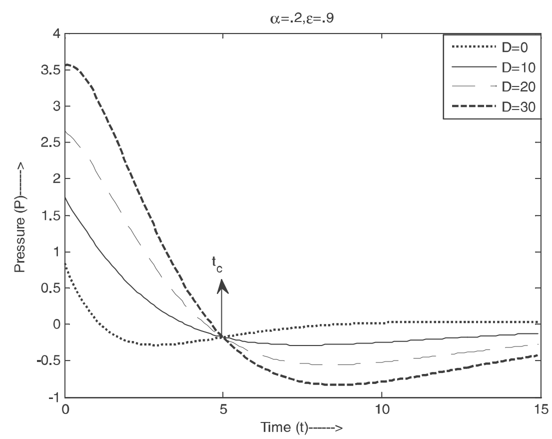

Fig. (A.21). Variation of the Pressure vs Time for different '*D*' at $\alpha = 0.2,\ \varepsilon = 0.9$

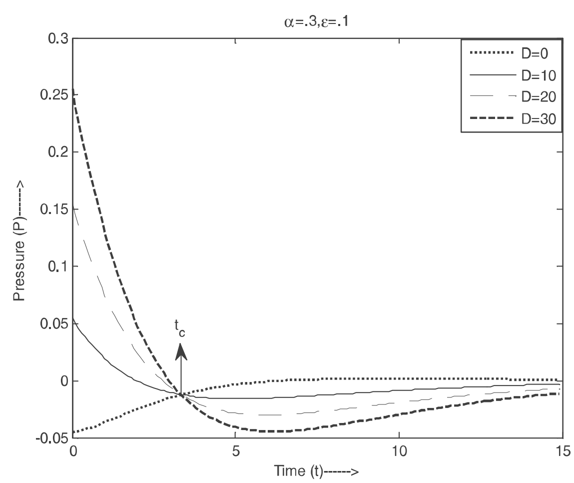

Fig. (A.22) Variation of the Pressure vs Time for different '*D*' at $\alpha = 0.3,\ \varepsilon = 0.1$

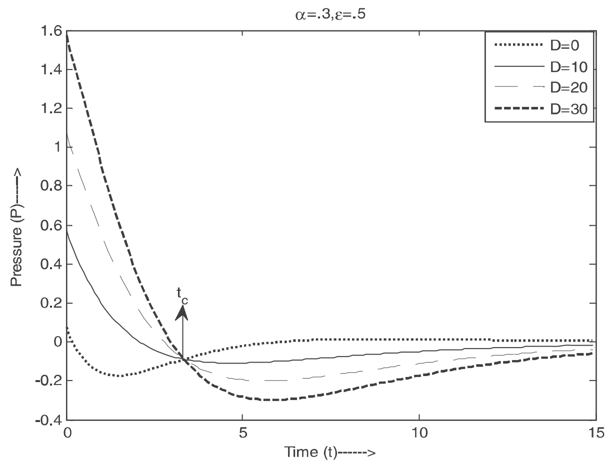

Fig. (A.23). Variation of the Pressure vs Time for different '*D*' at $\alpha = 0.3,\ \varepsilon = 0.5$

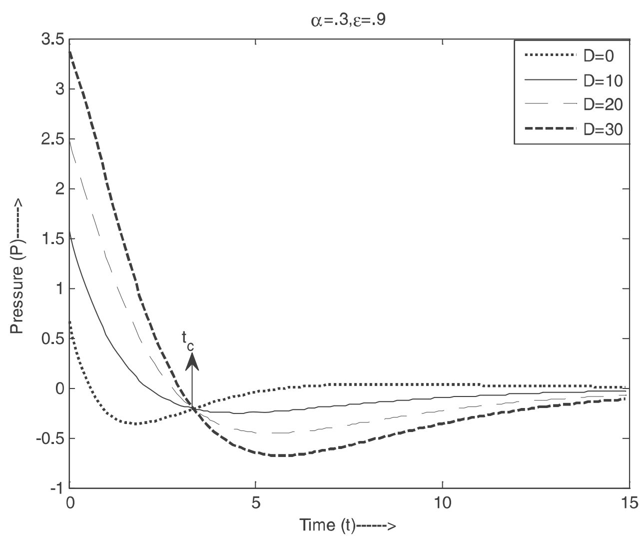

Fig. (A.24). Variation of the Pressure vs Time for different '*D*' at $\alpha = 0.3,\ \varepsilon = 0.5$

Case-B

In this case $$r = (1+\varepsilon t^2 e^{-\alpha t}) \quad \text{… (B.1)}$$

$$f(t) = (1+\varepsilon t^2 e^{-\alpha t})^2(-\varepsilon\alpha t^2 e^{-\alpha t} + 2t\varepsilon e^{-\alpha t}) \quad \text{… (B.2)}$$

then from equation (16) we get

$$P = e^{-2\alpha t}\varepsilon^2\left(\frac{3}{2}(2t - t^2\alpha)^2 + 2t^2 - 4t^3\alpha + t^4\alpha^2 + 2Dt^3\alpha^2\varepsilon^{-2} - Dt^4\alpha\right)$$
$$+e^{-\alpha t}\varepsilon(2 - 4t\alpha + t^2\alpha^2 + 2Dt - Dt^2\alpha) \quad \text{… (B.3)}$$

RESULTS AND DISCUSSION

The variation of the pressure vrs radius contraction parameter (α) for different values of porosity parameter (D) for diverse 'ε' and time instant (t) are illustrated in figures (B.1)-(B.9). It is noticed that the pressure raises steeply for small values of 'α', reaching a max value and slowly reduces to reach a zero value asymptotically the greatest peak to which the pressure raises also increases with D the correspondingly contracting values of 'α' to be closer to the origin as 't' and 'ε' increases.

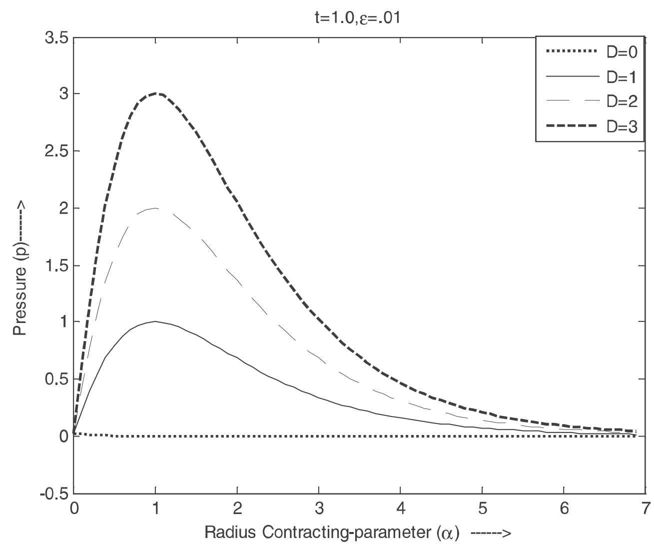

Fig. (B.1). Variation of the Pressure vs alpha for different 'D' at $t = 1.0$, $\varepsilon = 0.01$

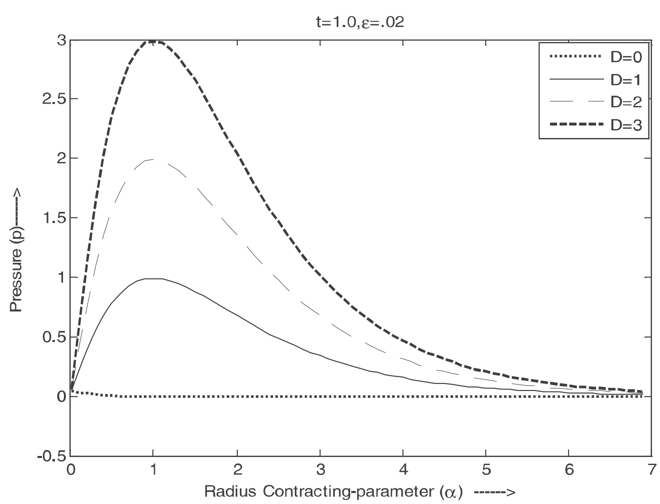

Fig. (B.2). Variation of the Pressure vs alpha for different 'D' at $t = 1.0$, $\varepsilon = 0.02$

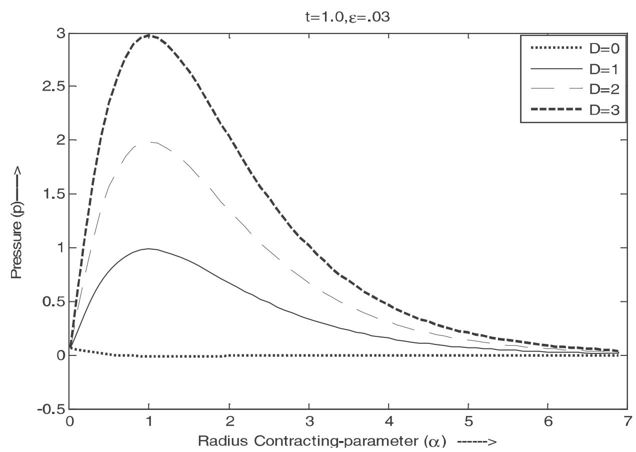

Fig. (B.3). Variation of the Pressure vs alpha for different '*D*' at *t* = 1.0, ε = 0.03

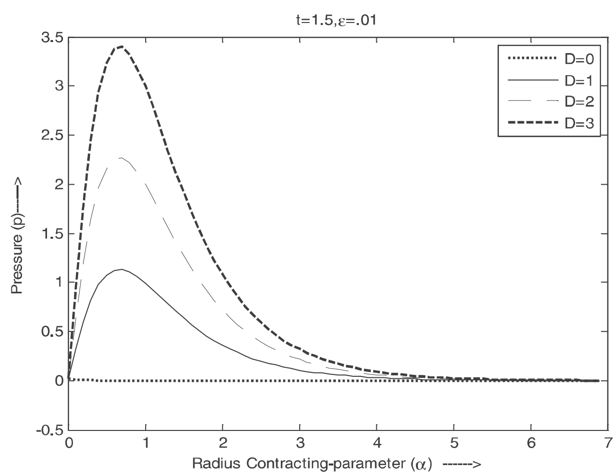

Fig. (B.4). Variation of the Pressure vs alpha for different '*D*' at *t* = 1.5, ε = 0.01

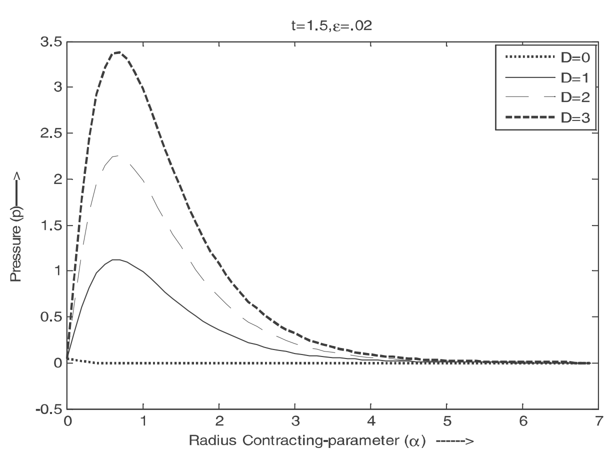

Fig. (B.5). Variation of the Pressure vs alpha for different '*D*' at *t* = 1.5, ε = 0.02

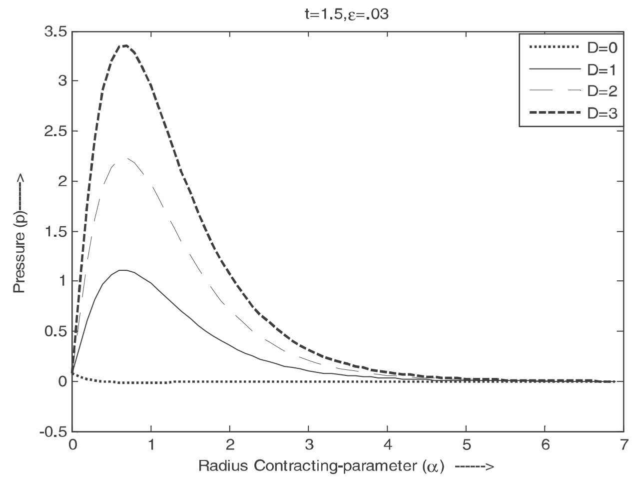

Fig. (B.6). Variation of the Pressure vs alpha for different '*D*' at *t* = 1.5, ε = 0.03

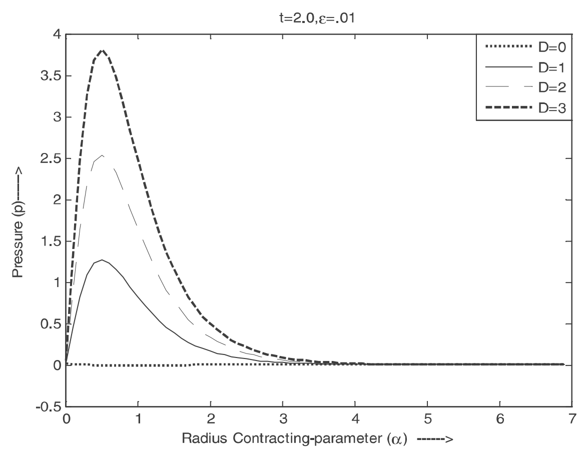

Fig. (B.7). Variation of the Pressure vs alpha for different '*D*' at *t* = 2.0, ε = 0.01

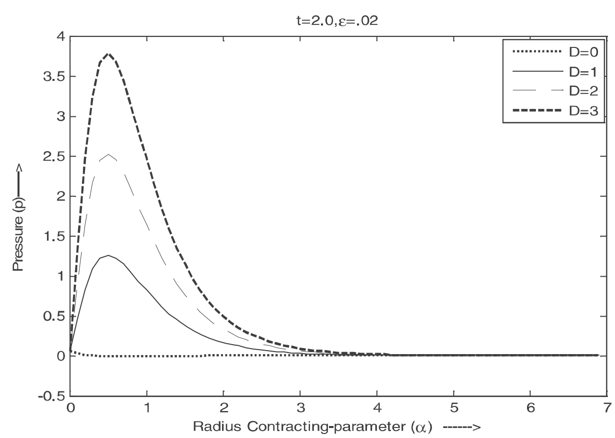

Fig. (B.8). Variation of the Pressure vs alpha for different '*D*' at *t* = 2.0, ε = 0.02

The variation of the pressure vrs porosity parameter (D) for different values of α, ε and t are illustrated in figures (B.9)-(B.15). It is noticed that the pressure variation is almost linear and starting from the origin and the slope of which increases with α.

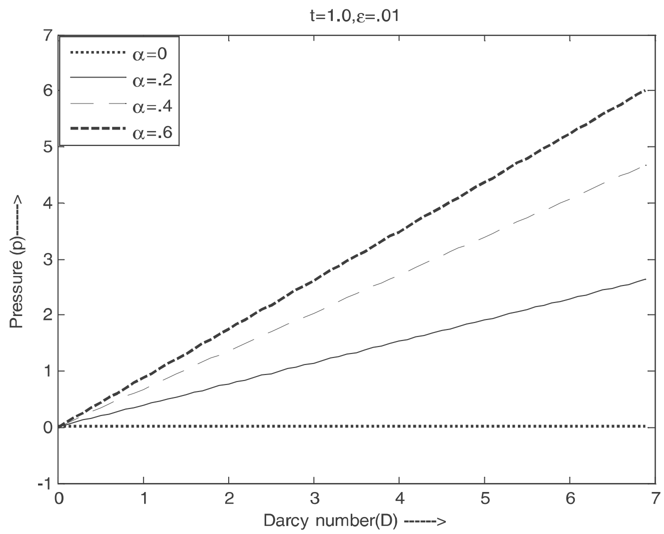

Fig. (B.9). Variation of the Pressure vs Darcy for different 'α' at t = 1.0, ε = 0.01

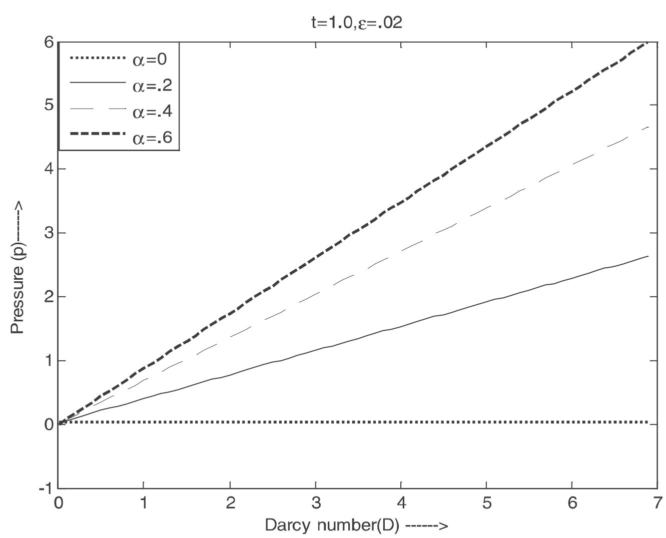

Fig. (B.10). Variation of the Pressure vs Darcy for different 'α' at t = 1.0, ε = 0.02

The variation of the pressure vrs time (t) for different values of porosity parameter (D) for diverse ε and time instant (t) are illustrated in figures (B.16)-(B.25). It is noticed that the pressure raises steeply for small values of 'α', reaching a max value and slowly reduces to reach a zero value asymptotically the greatest peak to which the pressure raises also increases with D the correspondingly contracting values of 'α' to be closer to the origin as 't' and 'ε' increases.

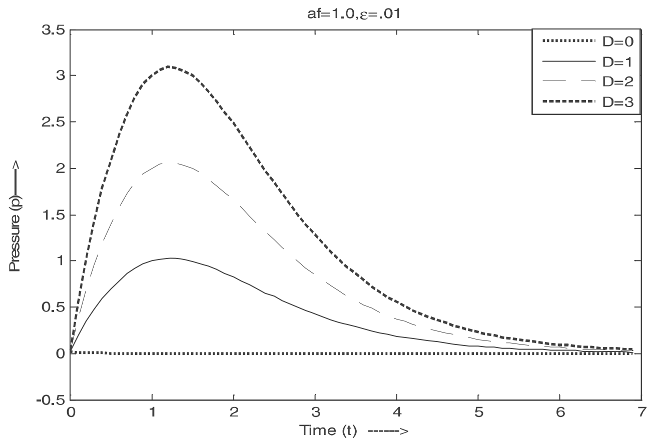

Fig. (B.13). Variation of the Pressure vs Time for different '*D*' at $\alpha = 1.0$, $\varepsilon = 0.01$

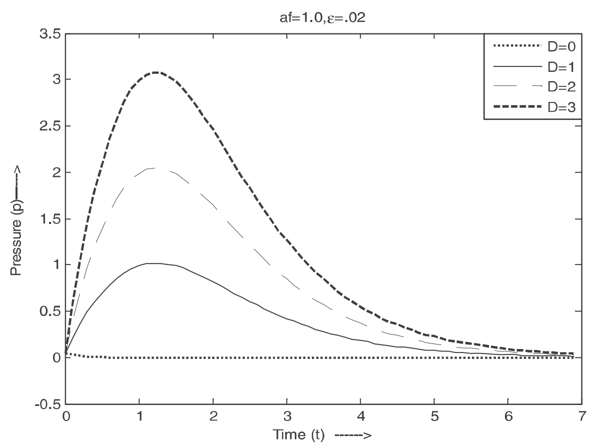

Fig. (B.14). Variation of the Pressure vs Time for different '*D*' at $\alpha = 1.0$, $\varepsilon = 0.02$

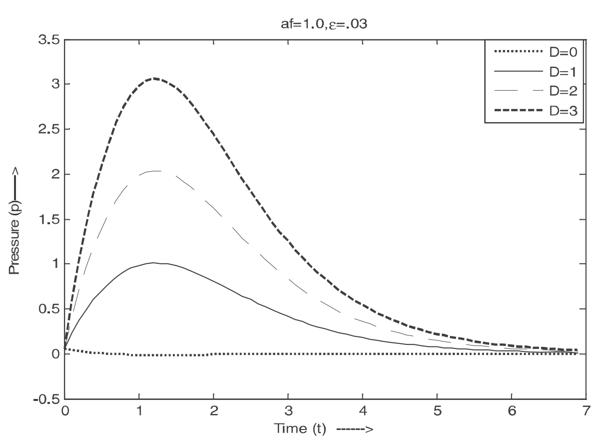

Fig. (B.15). Variation of the Pressure vs Time for different '*D*' at $\alpha = 1.0$, $\varepsilon = 0.02$

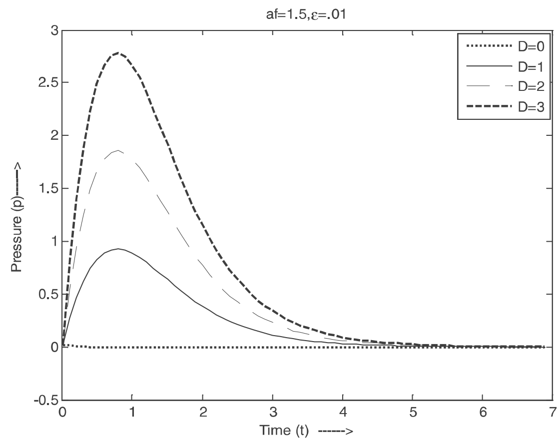

Fig. (B.16). Variation of the Pressure vs Time for different '*D*' at $\alpha = 1.5, \varepsilon = 0.01$

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