

Research Proposal

Submitted to Open Research Grant Program

Project Title: Applications and Classical stabilities of Various Multiplicative Inverse Functional Equations

Project Description

The aim of this research project is to introduce many multiplicative inverse functional equations in the theory of functional equations. Then we associate the significance of these functional equations in different areas such as other branches of Mathematics, Physics and Medicine. We also solve these functional equations and their fundamental stability problems pertinent to the renowned Ulam stability ([4], [12]) in various spaces.

The investigation of stability of functional equations is a hot and emerging field in the present scenario of research. In recent times, there are many research articles concerning stability problems of different forms of functional equations. The motivation behind for the growth of this field is due to the significant role of functional equations in many problems when a variable does not change with respect to another variable. In such situations, modelling functional equations and their solutions play noteworthy responsibility in analyzing the nature of solution of the problems.

Since the field of functional equations is being progressing day by day, their stunning applications are employed not only in mathematics but also in several other areas. Lot of applications of functional equations arise in many fields such as Classical mechanics, Decision theory, Dynamic programming, Fuzzy set theory, Game theory, Geometry, Group Theory, Inequalities, Inner product space, Measure Theory, Mechanics, Multivalued logic, Polynomials, Cluster analysis, Stochastic process, Physics, Astronomy, Reproducing scoring system, taxation, Population ethics, Applied Science, Computer graphics, Artificial intelligence, Neural networks, Digital image processing and many other fields.

So far in the literature, the investigation is carried out to solve many functional equations and their stability problems which are derived from additive, quadratic, cubic, quartic, quintic, sextic functions.

But for the first time, the authors in [7] obtained the general solution and discussed the stability results for a new multiplicative inverse functional equation and associated it with the well-known “Reciprocal formula” in determining the equivalent resistance of an electric circuit with a couple of resistors in parallel. Also the authors discussed that it holds good in a geometric construction in [8].

The study of multiplicative inverse functional equation motivated many mathematicians to solve stability problems of various forms of multiplicative inverse functional equations, such as reciprocal-quadratic, reciprocal-cubic, reciprocal-quartic, etc., (one can refer to [1], [2], [3], [6]).

The authors in [9] dealt a new multiplicative inverse functional equations to remove noise in an image by filtering techniques. Also, there are some other applications of multiplicative inverse functional equations in Physics, Electrical Engineering (details available in [10], [11]).

There are only few mathematicians have studied the stabilities of multiplicative inverse functional equations in the literature. We have been working with various forms of reciprocal type functional equations since 2010. We find that it would be worth and ground breaking if the theory of multiplicative inverse functional equations is further developed by means of applications in other fields. Hence we concentrate on studying multiplicative inverse functional equations.

Since many rational functions arise in Physics and Medicine, in this project, we model functional equations whose solution is a rational function. Using classical methods, we achieve the solution of reciprocal type functional equations. Hence such type of rational functions could be modelled to investigate various phenomena arising in several areas. For the purpose of mathematical analysis, we also investigate their classical stability results like Ulam-Hyers stability and other related stability results in various spaces.

Project Significance

Currently, the theory of functional equations is a developing branch of mathematics with vast relevances. Day by day it has been increasingly used to study problems in mathematical analysis, combinatorics, biology, information theory, statistics, physics, behavioural sciences, and engineering.

The beauty of functional equation can be realized only if common people understand how far the functional equation can be applied to various sciences. For that, we need articles which relate and discuss the applications of functional equations. Throughout the world, mathematicians from only few countries like Greece, India, Iran, Iraq, Portugal, China, Korea are currently contributing to this area of research. By carrying out this project, this may be a motivation for mathematicians in Oman and other Gulf Countries to work in this field of research. In the recent years, I have published some research articles on applications of rational functional equations in digital image enhancement, electric circuits, physics.

The intention of this project is to concentrate more on application of rational functional equations and their stability by publishing research articles in reputed journals, so that research work on this area will be attracted by many mathematicians and can be popularized about the functional equations and also this theory of functional equations will have exceptional growth in future.

Methodology

We find only a very few articles deal with this topic. So I am interested in working some new type rational functional equation which will cater the needs and applications of the other fields.

We apply anyone of the following methods to achieve the solution of rational functional equation depending upon the nature of the equation:

1. Substitution method
2. Mathematical Induction method
3. Method of Transformation of variables.

To investigate generalized Hyers-Ulam stability of rational form of functional equations, we apply the following methods:

1. Direct method introduced by D. H. Hyers [4]
2. Fixed Point method devised by L. Cadariu and V. Radu [12]

Using the above two methods, we construct an approximate rational function near to the exact solution of the multiplicative inverse functional equation. By obtaining this approximate rational function, we prove that the Hyers-Ulam stability results also hold good for rational functional equations.

Timeline

During First 6 Months:

- (a) To model multiplicative inverse functional equations with solution in rational form.
- (b) To investigate generalized Hyers-Ulam stability of the rational type functional equations using direct method and fixed point method.

During Second 6 Months :

- (a) To extend the stability results to new modern spaces like Non-Archimedean spaces, Paranormed spaces, Quasi-Beta normed spaces, etc.,
- (b) To find an application of the rational type functional equations and their results in other fields of mathematics and science.
- (c) To publish the results obtained as research articles in reputed referred journals.

Bibliography

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Roles and Responsibilities of PI & Co-Investigator:

To find the application of functional equation in various other fields, PI & Co-Investigator will collect materials from various sources like published papers in reputed journals, textbooks, magazines, research articles, monographs and work out to obtain novel rational functional equations which will be needed for this project. PI will consult experts in this field throughout the world to bring out this project effectively and successfully. PI will prepare the research papers as per the reputed journals templates and submit for publication.

Budget & Budget Justification

#	Description	Category	Justification	Amount (OMR)
1	Collection of materials from published papers	Data Collection & Analysis	For subscription fees to Journals	500
2	Purchase of textbooks on functional equations published recently	Equipment and Facilities	To have thorough literature survey	500
3	Travelling expenses for consultation	Travel (conferences)	To consult with experts in this field	300
4	Postal/Courier Expenses	Materials & Supplies	To meet out unexpected expenses	100
Total				1400