Prime Assosymmetric Rings

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Abstract: In this paper we see that commutator and associator is in the nucleus of assosymmetric ring. Also we prove that in a prime assosymmetric ring associator is in the center. Using this it is shown that a prime assosymmetric ring of characteristic ≠2,3 is third power associative. At the end of this paper we present some examples of assosymmetric rings which are neither flexible nor power associative.

Keywords: Nonassociative ring, Assosymmetric ring, Prime ring, Nucleus, Center, Characteristic and Associator.

1. Introduction

Kleinfeld [3] defined a class of non associative rings called as assosymmetric rings in which the associative law of multiplication has been weakened to the condition that $(P(x), P(y), P(z)) = (x, y, z)$, for every permutation P of x, y and z. These rings are neither flexible nor power associative. But the commutator and the associator are in the nucleus of this ring.

2. Priliminaries

Let R be a nonassociative ring. We shall denote the commutator and the associator by $(x,y) = xy-yx$ and $(x,y,z) = (xy)z-x(yz)$ for all x,y,z in R respectively. A ring R is called antiflexible if $(x,y,z) = (z,y,x)$ for all x,y,z in R. The nucleus N of a ring R is defined as $N = \{n \in R / (n,R,R) = (R,n,R) = (R,R,n) = 0\}$[1].The center C of R is defined as $C = \{ c \in N / (c, R) = 0\}$. A ring R is said to be of characteristic $\neq n$ if $nx = 0$ implies $x = 0$, for all $x \in R$ and n is a natural number. A ring R is of characteristic $\neq n$ is simply denoted by char.$\neq n$. A ring R is called simple if $R^2 \neq 0$ and the only nonzero ideal of R is itself. Since R^2 is a non-zero ideal of R, we have $R^2 = R$. A ring R is called prime if whenever A and B are ideals of R such that $AB = 0$, then either $A = 0$ or $B = 0$. A ring R is called Semiprime if A is an ideal of R such that $A^2 = 0$, then $A = 0$. Throughout this paper R denotes an assosymmetric ring of char. ≠2, 3.

3. Main Results

An assosymmetric ring R is one in which

$(P(x), P(y), P(z)) = (x, y, z)$. 3.1.1

where P is a permutation of x, y, z in R.

We have the following identities in any ring R.

$(wx, y, z) - (w, xy, z) + (w, x, yz) = w(x, y, z) + (w, x, y)z$ 3.1.2

$(xy, z) = x(y,z) + (x, z)y + (x, y, z) + (z, x, y) - (x, z, y)$ 3.1.3

and $(x,y,z) + (y,z,x) + (z,x,y) = (xy,z) + (yz,x) + (zx,y)$, 3.1.4

for all w, x, y, z in R.

Now we present some properties of assosymmetric ring [2].

Lemma 3.1.1: An assosymmetric ring R is not flexible.

Proof: Let R be a flexible assosymmetric ring. Then $(x,y,z) + (z,y,x) = (x,y,z) + (x,y,z) = 2(x,y,z) = 0$ and hence $(x,y,z) = 0$ for all $x, y, z \in R$. So R is associative, a contradiction. Hence R is not flexible.

Corollary 3.1.1: An assosymmetric ring R is not commutative.