

# Mathematical Modeling of Solar PV Panels

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**Abstract** -Both research and technological development in the area of renewable energy sources are necessary to account for the increase in energy demand and environment problems in the world. Stand-alone photovoltaic systems are the best solutions such as communication system, water pumping and low power appliances in rural area. Such systems are consisting of a PV generator, energy storage devices, AC or DC consumers and elements for power conditions. PV module represents the fundamental power conversion unit of a PV generator system. The output characteristic of PV module depending on the irradiance intensity and the cell's temperature is nonlinear, so it is necessary to model it for the simulation of maximum power point tracking for stand-alone PV systems. One-diode equivalent circuit is employed to investigate I-V and P-V characteristics of a typical solar module.

**Keywords**—Solar PV; Modeling; MATLAB;SIMULINK

## I. INTRODUCTION

Among the renewable energy resources, the energy due to the photovoltaic (PV) effect can be considered the most essential and prerequisite sustainable resource because of the ubiquity, abundance, and sustainability of solar radiant energy. Regardless of the intermittency of sunlight, solar energy is widely available and is free. Recently, photovoltaic system is recognized to be in the forefront in renewable electric power generation. It can generate direct current electricity without environmental impact and contamination when exposed to solar radiation. Being a semiconductor device, the PV system is static, quiet, free of moving parts, and has little operation and maintenance costs. PV module represents the fundamental power conversion unit of a PV generator system. The output characteristics of a PV module depend on the solar insolation, the cell temperature and the output voltage of the PV module. Since PV module has nonlinear characteristics, it is necessary to model it for the design and simulation of maximum power point tracking (MPPT) for PV system applications. Mathematical modeling of PV module is being continuously updated to enable researcher to have a better understanding of its working. Due to reserve of fossil fuel dwindling and the global warming effect looming large, alternative energies become popular. The most attention of alternative energies is solar energy. There are two types of technology that employed solar energy, namely solar thermal and solar cell. A PV cell (solar cell) converts the sunlight into the electrical energy by the photovoltaic effect. Energy from PV modules offers several advantages, such as, requirement of little maintenance and no environmental pollution. Recently, PV arrays are used in many applications, such as, battery chargers, solar powered water pumping systems, grid connected PV systems, solar hybrid vehicles and satellite power systems. PV module represents the fundamental power conversion unit of a PV

generator system. The output characteristic of PV module depends on the solar insolation and the cell temperature. Since PV module has nonlinear characteristics, it is necessary to model it for the design and simulation of maximum power point tracking (MPPT) for PV system applications. A PV module typically consists of a number of PV cells in series. The conventional technique to model a PV cell is to study the p-n junction physics. A PV cell has a non-linear voltage-current ( $V$ - $I$ ) characteristic which can be modeled using current sources, diode(s) and resistors. Single-diode and double-diode models are widely used to simulate PV characteristics. The single-diode model emulates the PV characteristics fairly and accurate.

Mathematical modeling of PV module is being continuously updated to enable researcher to have a better understanding of its working. [1]- [6]

## II. MODELLING OF PHOTOVOLTAIC SYSTEM

### A. Modelling of PV Module:

The model does not take into account the internal losses of the current. A diode is connected in anti-parallel with the light generated current source (Fig. 1a). The output current  $I$  is obtained by Kirchhoff law:

$$I = I_{ph} - I_d \quad (1)$$

( $I_{ph}$ ) is the photocurrent, ( $I_d$ ) is the diode current which is proportional to the saturation current and is given by the equation

$$I_d = I_0 \left[ \exp \left( \frac{V}{A N_s V_T} \right) - 1 \right] \text{ and} \quad (2)$$

$$V_T = K \cdot T_c / q \quad (3)$$

( $I_0$ ) is the reverse saturation or leakage current of the diode (A),  $V_{Tc} = 26$  mV at 300 K for silisium cell,  $T_c$  is the actual cell temperature (K),  $k$  Boltzmann constant  $1.381 \times 10^{-23}$  J/K,  $q$  is electron charge ( $1.602 \times 10^{-19}$  C).  $V_T$  is called the thermal voltage because of its exclusive dependence of temperature.

$N_s$  is the number of PV cells connected in series.  $A$  is the ideality factor. It depends on PV cell technology and can be chosen in Table 1. It is necessary to underline that  $A$  is a constant which depends on PV cell technology. All the terms by which,  $V$  is divided in equation (2) under exponential function are inversely proportional to cell temperature and so, vary with varying conditions. In this work, this term is designed by 'a' and called the thermal voltage ( $V$ ), the ideality factor, is considered constant and is chosen in Table 1 according to technology of the PV cell. The thermal voltage "a" is presented by the below equation [1].

$$a = \frac{N_s \cdot A \cdot K \cdot T_c}{q} = N_s \cdot A \cdot V_T \quad (4)$$

where 'a' is called "the modified ideality factor" and is considered as a parameter to determine, while A is the diode ideality.

In reality, it is impossible to neglect the series resistance  $R_s$  and the parallel resistance  $R_p$  because of their impact on the efficiency of the PV cell and the PV module. When  $R_s$  is taken into consideration, equation (2) should take the next form:

$$I_d = I_0 \left[ \exp \left( \frac{V + I \cdot R_s}{a} \right) - 1 \right] \quad (5)$$

Of course, Fig. 1b is a simplified form, easy to implement in simulators. But Fig. 1c is the most representative of the PV cell.

By applying Kirchhoff law, current will be obtained by the equation:

$$I = I_{ph} - I_d - I_p \quad (6)$$

Where  $I_p$  is the current leak in parallel resistor.

The output current of a module containing  $N_s$  cells in series will be:

$$I = I_{ph} - I_0 \left[ \exp \left( \frac{V + I \cdot R_s}{a} \right) - 1 \right] - \frac{V + R_s \cdot I}{R_p} \quad (7)$$

It is not easy to determine the parameters of this transcendental equation. But this model offers the best match with experimental values.

#### B. Determination of ( $I_{ph}$ ):

According to Fig. 1a, the output current at the standard test conditions (STC) is

$$I = I_{ph,ref} - I_{0,ref} \left[ \exp \left( \frac{V}{a_{ref}} \right) - 1 \right] \quad (8)$$

This equation allows quantifying  $I_{ph,ref}$  which cannot be determined otherwise. When the PV cell is short-circuited:

$$I_{sc,ref} = I_{ph,ref} - I_{0,ref} \left[ \exp \left( \frac{0}{a_{ref}} \right) - 1 \right] = I_{ph,ref} \quad (9)$$

But this equation is valid only in ideal case. So, the equality

is not correct. And then, equation has to be written as:

$$I_{ph,ref} \approx I_{sc,ref} \quad (10)$$

The photocurrent depends on both irradiance and temperature.

$$I_{ph} = \frac{G}{G_{ref}} (I_{ph,ref} + \mu_{sc} \cdot \Delta T) \quad (11)$$

G: Irradiance ( $W/m^2$ ),  $G_{ref}$ : Irradiance at STC= 1000  $W/m^2$ ,  $\Delta T = T_c - T_{c,ref}$  (Kelvin),  $T_{c,ref}$ : Cell temperature at STC = 25+ 273 = 298 K,  $\mu_{sc}$ : Coefficient temperature of short circuit current (A/K), provided by the manufacturer,  $I_{ph,ref}$ : Photocurrent (A) at STC.

#### C. Determination of ( $I_0$ ):

The shunt resistance  $R_p$  is generally regarded as great, so the last term of the relationship (8) should be eliminated for the next approximation. By applying equation (8) at the three most remarkable points at standard test condition: the voltage at open circuit ( $I=0, V=V_{oc,ref}$ ), the current at short circuit ( $V=0, I=I_{sc,ref}$ ), and the voltage ( $V_{mp,ref}$ ) and current ( $I_{mp,ref}$ ) at maximum power, the following equations can be written:[2]

$$I_{sc,ref} = I_{ph,ref} - I_{0,ref} \left[ \exp \left( \frac{I_{sc,ref} \cdot R_s}{a_{ref}} \right) - 1 \right] \quad (12)$$

$$0 = I_{ph,ref} - I_{0,ref} \left[ \exp \left( \frac{V_{oc}}{a_{ref}} \right) - 1 \right] \quad (13)$$

$$I_{mp,ref} = I_{ph,ref} - I_{0,ref} \left[ \exp \left( \frac{V_{mp,ref} + I_{mp,ref} \cdot R_s}{a_{ref}} \right) - 1 \right] \quad (14)$$

The (-1) term has to be neglected because it is very smaller than the exponential term. According to equation (11), and by substituting ( $I_{ph,ref}$ ) in equation (14):

$$0 \approx I_{sc,ref} - I_{0,ref} \left[ \exp \left( \frac{V_{oc,ref}}{a_{ref}} \right) \right] \quad (15)$$

$$\text{So } I_{0,ref} = I_{sc,ref} \left[ \exp \left( \frac{-V_{oc,ref}}{a} \right) \right] \quad (16)$$

The reverse saturation current is defined by:

$$I_0 = DT^3 C \left[ \exp \left( \frac{-q \epsilon_g}{A \cdot K} \right) \right] \quad (17)$$

where  $\epsilon_g$ : Material band gap energy (eV), (1.12 eV for Si).

D= diode diffusion factor.

In order to eliminate the diode diffusion factor, equation (18) is computed twice; at  $T_c$  and at  $T_{c,ref}$ . Then, the ratio of the two equations is written in the next expression:

$$I_0 = I_{0,ref} \left( \frac{T_c}{T_{c,ref}} \right)^3 \exp \left[ \left( \frac{q \epsilon_g}{A \cdot K} \right) \left( \frac{1}{T_{c,ref}} - \frac{1}{T_c} \right) \right] \quad (18)$$

$$I_0 = I_{sc,ref} \left[ \exp \left( \frac{-V_{oc,ref}}{a} \right) \left( \frac{T_c}{T_{c,ref}} \right)^3 \times \exp \left[ \left( \frac{q \epsilon_g}{A \cdot K} \right) \left( \frac{1}{T_{c,ref}} - \frac{1}{T_c} \right) \right] \right] \quad (19)$$

#### D. Determination of ( $R_p$ and $R_s$ ):

In order to make the proposed model more credible,  $R_p$  and  $R_s$  are chosen so that the computed max power  $P_{mp}$  is equal to the experimental one  $P_{mp,ex}$  at STC conditions. So it is possible to write the next equation

$$I_{mp,ref} = \frac{P_{mp,ref}}{V_{mp,ref}} = \frac{P_{mp,ex}}{V_{mp,ref}} = I_{ph,ref} - I_{0,ref} \left[ \exp \left( \frac{V_{mp,ref} + I_{mp,ref} \cdot R_s}{a} \right) - 1 \right] - \frac{V_{mp,ref} + R_s \cdot I_{mp,ref}}{R_p} \quad (20)$$

$$R_p = \frac{V_{mp,ref} + I_{mp,ref} \cdot R_s}{I_{sc,ref} - I_{0,ref} \left[ \exp \left( \frac{V_{mp,ref} + R_s \cdot I_{mp,ref}}{a} \right) - 1 \right] + I_{sc,ref} \left[ \exp \left( \frac{-V_{oc,ref}}{a} \right) \right] - \left( \frac{P_{max,ex}}{V_{mp,ref}} \right)}$$

The iteration starts at  $R_s = 0$  which must increase in order to move the modeled Maximum Power Point until it matches with the experimental Maximum Power Point. The corresponding  $R_p$  is then computed. There is only one pair ( $R_p, R_s$ ) that satisfies this condition. The implementation presented was used to simulate the proposed model by incrementing  $R_s$  until matching  $P_{mp}$  with  $P_{mp,ex}$ . [4-6].

TABLE 1 PWX 500 PV MODULE (49 W) CHARACTERISTICS.

Parameters	Values
$P_{mp}$ (W)	49
$I_{mp}$ (A)	2.88
$V_{mp}$ (V)	17
$I_{sc}$ (A)	3.11
$V_{oc}$ (V)	21.8
$R_s$ (Ω)	0.55
$N_{oct}$ °C	45
$I_{sc}$ (K°)	1.3 Æ 10-3
$K_d$ (K°)	-72.5 Æ 10-3
$N_s$	36

### III. SIMULATION RESULTS

This Part presents the simulation results of mathematical modeling of photovoltaic system. This system has four subsystem, each subsystem defined equations of parameters. The parameters are :  $I_{ph}$ ,  $I_{0.ref}$ ,  $I_0$  and  $I_{pv}$ .

#### A. Model Of the Photovoltaic System in SIMULINK

Based on the equations discussed in previous section, the Simulink block diagrams (Fig 2,3,4 and 5) showing Modeling those equations.

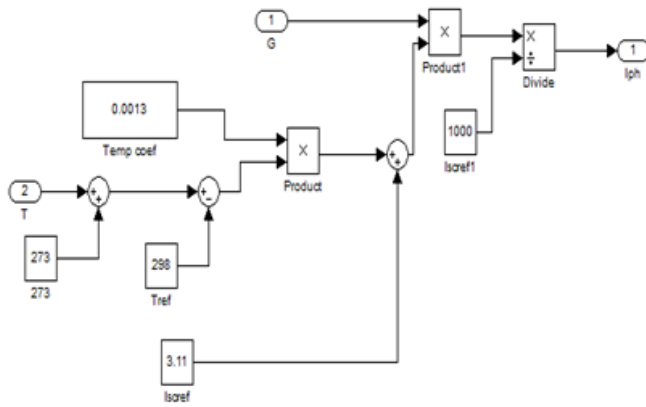


Fig 2. Modeling of  $I_{ph}$  equation

For our convenient, we converted them into a simple block called as subsystem (Fig 6,7,8 and 9).

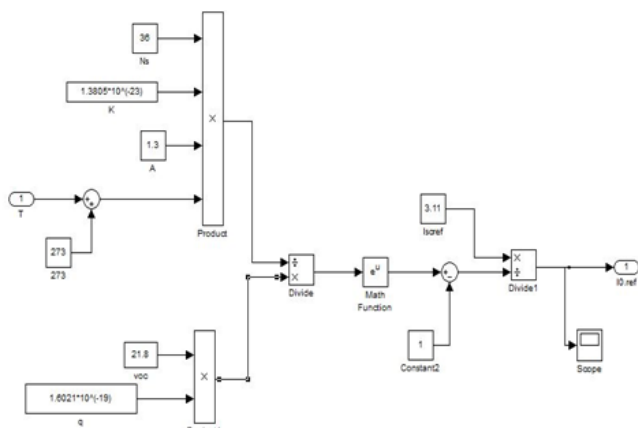


Fig 3 Modeling of  $I_{0.ref}$  equation

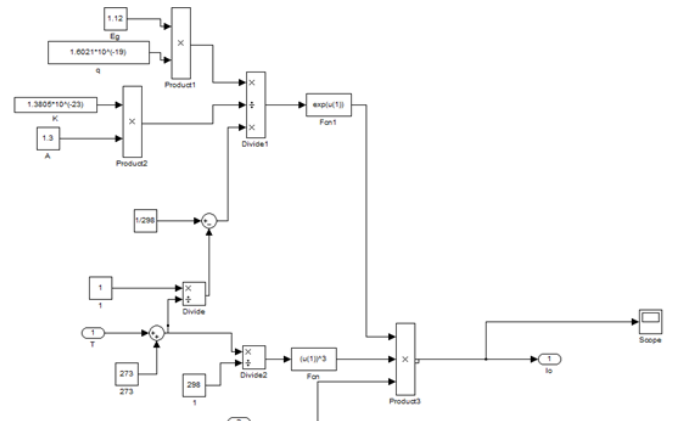


Fig 4 Modeling of  $I_0$  equation

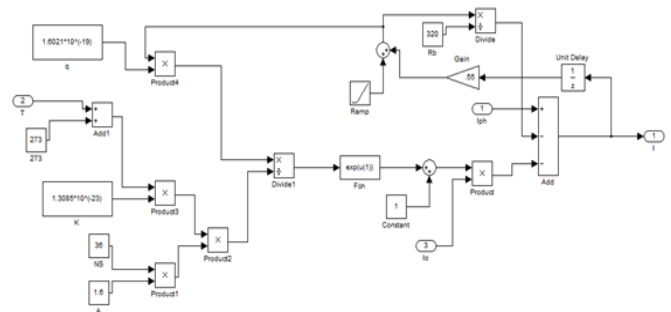


Fig 5 Modeling of  $I$  equation

For our convenient, we converted them into a simple block called as subsystem (Fig 6,7,8 and 9)

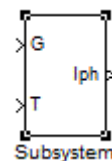


Fig 6 Subsystem Modeling of  $I_{ph}$  Equation

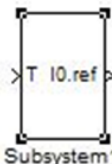
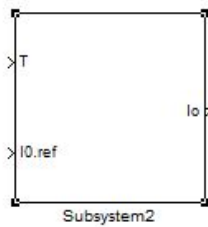
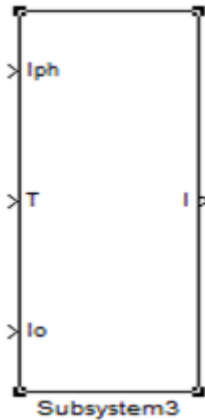


Fig 7 Subsystem Modeling of  $I_{0.ref}$  equation

Fig 8 Subsystem Modeling of  $I_0$  equationFig 9 Modeling of  $I$  equation

All blocks are combined to create the final model of PV system. (fig 10)

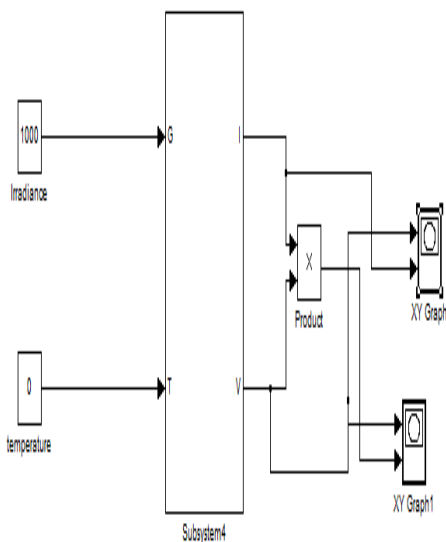


Fig 10 Modeling of PV System

The whole PV module with 5 units are given in fig 11.

### C. The characteristic of the photovoltaic system

The following graphs (Fig 12,13) show the relationship between voltage and current and the graphs (Fig 14,15,16 and 17) show relationship between voltage and power.

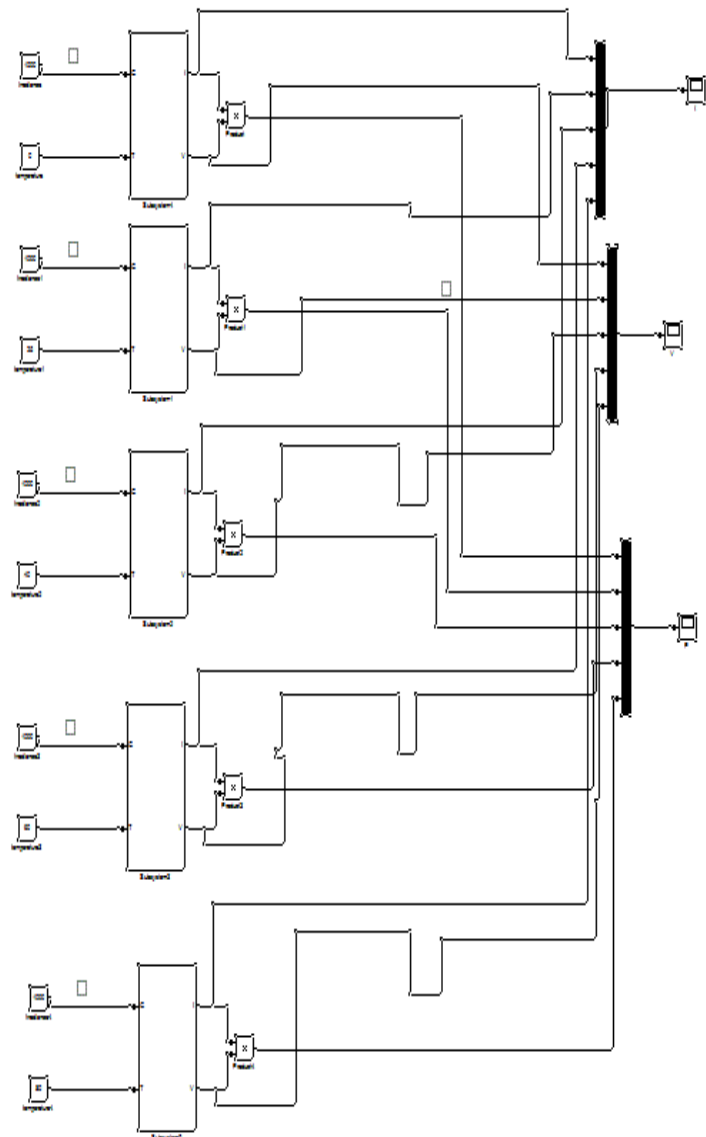


Fig 11 Whole PV System

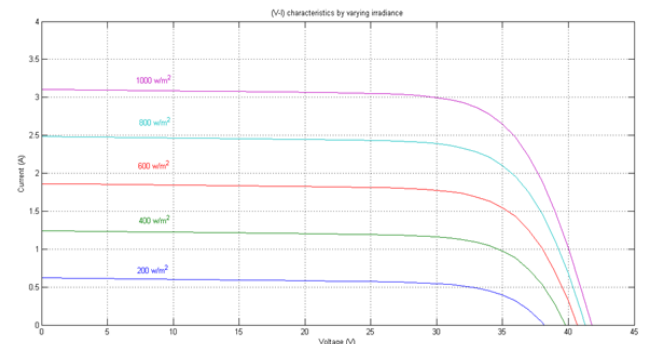


Fig 12 (V-I) Characteristic by Varying Irradiance

The above figure shows relation between voltage and current, when irradiance is varying. As shows in the graph, when the irradiance increase, the current increase, but the value of  $V_p$  is still constant during irradiance change.

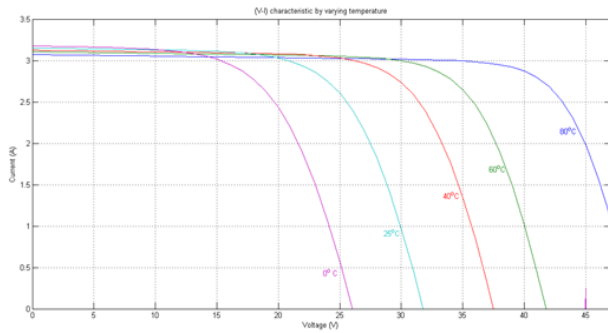


Fig 13 (V-I) Characteristic by Varying Temperature

The above figure shows relation between voltage and current when temperature is varying. As shows in the graph when the temperature increase, the current approximately constant, but the value of  $V_p$  is still change during temperature changes.

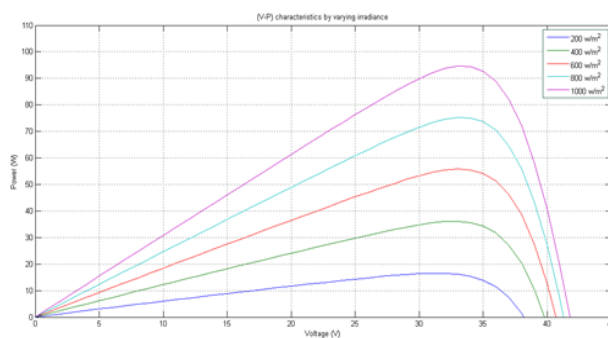


Fig 14 (V-P) Characteristic by varying irradiance

The above figure shows relation between voltage and power when irradiance is varying. As shows in the graph when the irradiance increase, the power increase, but the value of  $V_p$  is still constant during irradiance change.

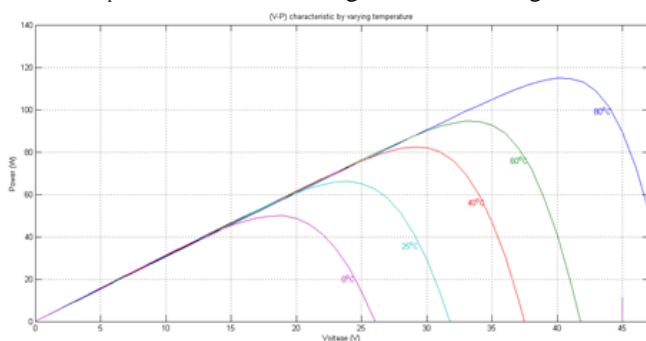
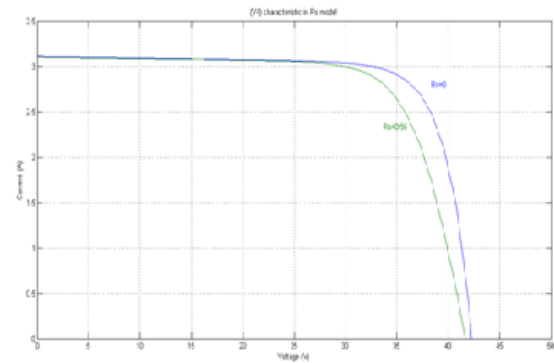
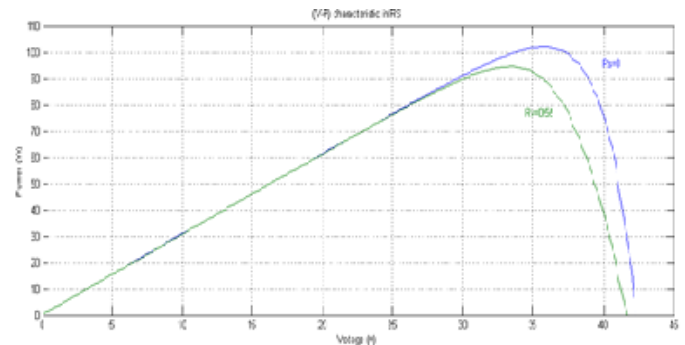


Fig 15 (V-P) Characteristic by varying temperature

The above figure shows relation between voltage and power when temperature is varying. As shows in the graph, when the temperature increase, the power increase also the value of  $V_p$  is increase; when temperature is increase. Next we are having the characteristics by varying the resistance  $R_s$ .

The below figures show relation between voltage and current (V-I) and voltage and power (V-P) when the series resistance ( $R_s$ ) is varying. As shows in the graph when the series resistance  $R_s$  increase the current and power decrease. Further more, when  $R_s$  decrease the peak of power move to the right and the V-I characteristic become more rectangular form. However, the value of  $R_s$  is given by manufacturer.

Fig 16 (V-I) Characteristic by varying  $R_s$ Fig 17 (V-P) characteristic by varying  $R_s$ 

#### IV CONCLUSION

The presented work is a detailed modeling and simulation of the PV cell and module. It is implemented under MATLAB/ Simulink environment; the most used software by researchers and engineers. This model is first drafted in accordance with the fundamentals of semiconductors and the PV cell technology. In other words, the PV module parameters have been selected according to their variation with illumination and temperature. It means that for any type of PV module, one can use this model and determine all the necessary parameters under any new conditions of irradiance and temperature and then, obtain the  $I(V)$  and  $P(V)$  characteristics. This model can be considered as a tool which can be used to study all types of PV modules available in markets, especially, their behavior under different weather data of standard test conditions (STC). It is important to compute  $R_s$ , even if it is given by a manufacturer because the experimental Maximum Power Point does not match with the computed one. For each iteration, a pair of ( $R_s$ ,  $R_p$ ) is obtained. But only one pair satisfies the condition of matching the modeled and the experimental peak power. So,  $R_s$  is iteratively increased until satisfying the condition. The proposed  $R_p$  model gave ( $R_s = 0.45 \Omega$ ,  $R_p = 310.0248 \Omega$ ) instead of ( $R_s = 0.55 \Omega$ , and  $R_p$  not provided). So it can be estimated to be more accurate in simulating the PV module. The proposed procedure provides an accurate, reliable and easy-to-tune model of photovoltaic array. Furthermore, it also robust advantageous in investigating the solar PV array operation from different physical parameters (series, shunt resistance, ideality factor, etc.) and working condition (varying temperature, irradiation and especially partial shadow effect) aspects

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### REFERENCES

- [1] Ranganath Muthu, Natarajan Pandiarajan, "Mathematical modeling of photovoltaic module with Simulink", International Conference on Electrical Energy Systems (ICEES 2011), 3-5 Jan 2011.
- [2] P.Sudeepika , G.Md. Gayaz Khan, "Analysis Of Mathematical Model Of PV Cell Module in Matlab/Simulink Environment", International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering, Vol. 3, Issue 3, March 2014.
- [3] J. Surya Kumari and Ch. SaiBabu, "Mathematical Modeling and Simulation of Photovoltaic Cell using Matlab-Simulink Environment" International Journal of Electrical and Computer Engineering (IJECE) Vol. 2, No. 1, February 2012, pp. 26~34 ISSN: 2088-8708 \_ 26.
- [4] S. Nema, R.K.Nema, and G.Agnihotri, "Matlab / simulink based study of photovoltaic cells / modules / array and their experimental verification," International Journal of Energy and Environment, pp.487500, Volume1,Issue3,2010.
- [5] S.Chowdhury, S.P.Chowdhury, G.A.Taylor, and Y.H.Song, "Mathematical Modeling and Performance Evaluation of a Stand-Alone Polycrystalline PV Plant with MPPT Facility," IEEE Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, July 20-24, 2008, Pittsburg, USA.
- [6] Jay Patell , Gaurag Sharma, "Modeling and simulation of solar photovoltaic module using MATLAB / SIMULINK" IJRET: International Journal of Research in Engineering and Technology, Vol2,2013.



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