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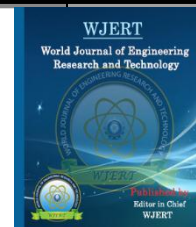


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## DESIGN OF PI CONTROLLER FOR A CONICAL TANK SYSTEM

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### ABSTRACT

Conical tanks are widely used in the process industries because of its shape that contributes better drainage for solid mixtures, slurries and viscous liquids. The cone is a well-known system, which is having high non linearity, due to the variation of the area with respect to height. Level control of conical tank is a challenging task due to its

non-linear shape and constantly varying cross-section. Thus automatic control of such nonlinear process is a challenging task. To recover the nonlinearity problem dynamics, those systems should be analyzed properly. This paper proposes the tuning of PI controller by Ziegler Nicholas and Cohen coon method. The controllers are designed and compared based on the performance analysis.

**KEYWORDS:** Ziegler Nicholas, Cohen Coon; Controller; Conical Tank; Simulation.

### INTRODUCTION

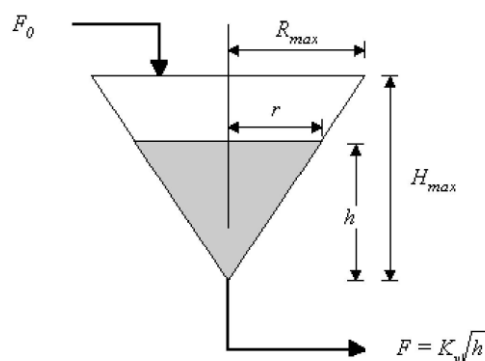
The common need for accurate and efficient control of today's industrial applications is driving the system identification field to face the constant challenges of providing better models of physical phenomena. Systems encountered in practice are often nonlinear or have time varying nature. So, it is difficult to identify accurate models of a nonlinear system. Approximate linear models are used in most of the industrial controllers, which may lead to lower the control performance. Hence, it is important to develop a simple and practical method for nonlinear process modeling and identification, and use that model for the control of nonlinear processes.

The control of liquid level in tank and flow in the tank is a basic problem in process industries. The process industries require the liquids to be pumped, stored in tanks and then pumped to another tank. Many times the liquid will be processed by chemical or mixing treatment in the tanks, but always the level of the fluid in the tanks must be controlled. Controlling of liquid level is an important and common task in process industries, in this level process, the tank is conical shape in which the level of liquid is desired to maintain at a constant value. The level control in the conical tank is a challenging problem because of its constantly changing its cross sectional area.<sup>[1]</sup>

This is achieved by controlling the input flow into the tank. The control variable is the level in a tank and the manipulated variable is the inflow to the tank. Conical tanks find wide applications in process industries, namely hydrometallurgical industries, food process industries, concrete mixing industries and wastewater treatment industries. Conical tank gives better drainage to the solid materials, semi solid materials as well as viscous fluids. The Proportional Integral Derivative (PID) Controllers have been widely used in the process industries for many years, due to their simplicity, flexibility and efficiency.<sup>[2]</sup> The tuning of a PID Controller is necessary, for the satisfactory operation of the process. Standard methods for tuning include, Zeigler-Nichol's (ZN) ultimate cycling method, and Cohen- Coon's (CC) open loop tuning method. In both these methods, the parameters of the controller are obtained for an operating point, when the plant or process model is linear. This paper focuses on the design of PI controllers for controlling the level of a conical tank process.

## 2. PROCESS DESCRIPTION

The conical tank system has single input and single output process. The output of this process is the level and the input to the process is the flow of liquid. The structure of the conical tank system is given in fig 1.



**Fig 1: Conical Tank System.**

A liquid of constant density is fed at a constant volumetric rate  $F_o$  into a conical tank of height  $H_{max}$  and maximum radius  $R_{max}$ . The outflow from the tank is  $K_v \sqrt{h}$ , where  $h$  is the height of the liquid in the tank and  $K_v$  is the valve coefficient.

The process has high nonlinearity due to the changes in Process gain and time constant with respect to the height of the liquid tank.<sup>[3,4]</sup>

Let the inflow ( $F_o$ ), outflow ( $F$ ). by the mass conservation law,

Mass accumulation = inflow – outflow

Accumulation manifests itself as an increase or decrease in volume.(i.e) Accumulation is the change in volume with respect to time.

$$A \frac{dh}{dt} = F_o - F \quad (1)$$

Right circular cone has volume

$$V = \frac{1}{3} \pi R_{max}^2 * H_{max} - \frac{1}{3} \pi r^2 * h \quad (2)$$

Consider

$$\frac{r}{R_{max}} = \frac{h}{H_{max}} \quad (3)$$

$$\text{Hence } r = \frac{R_{max}}{H_{max}} * h \quad (4)$$

Therefore

$$V = \frac{1}{3} \pi R_{max}^2 * H_{max} - \frac{1}{3} \pi * \left( \frac{R_{max}^2}{H_{max}^2} \right) * h^3 \quad (5)$$

$$V = \frac{1}{3} \pi R_{max}^2 (H_{max} - \left( \frac{1}{H_{max}^2} \right) * h^3) \quad (6)$$

$$dV(t) = \frac{1}{3} \pi r^2 dh = \frac{1}{3} \pi \left( \frac{R_{max}}{H_{max}} h \right)^2 dh \quad (7)$$

$$\text{So } \frac{dh}{dt} = \frac{F_o - K_v \sqrt{h}}{\frac{1}{3} \pi \left( \frac{R_{max}}{H_{max}} h \right)^2} \quad (8)$$

The above mathematical model is analyzed in MATLAB Simulink and the responses are obtained.

### 3. PROCESS OPERATING PARAMETERS

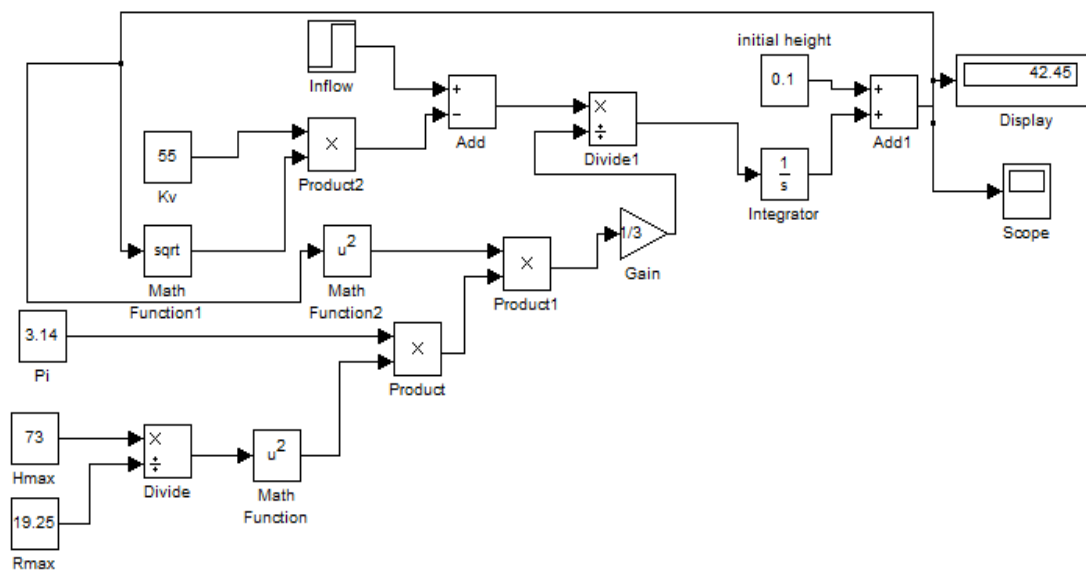
The various system parameters<sup>[5]</sup> obtained are tabulated as:

**Table 1: Parameters of Conical-tank system.**

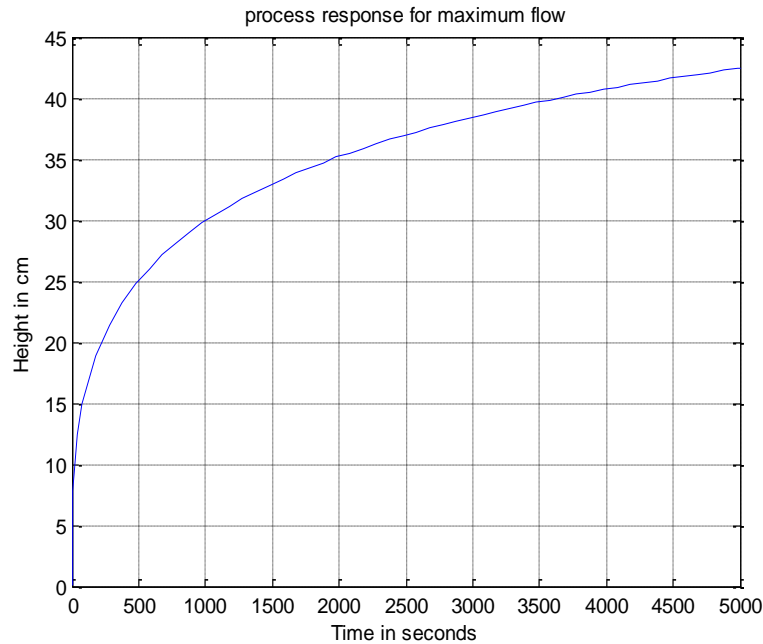
Parameter	Description	Value
$R_{\max}$	Total radius of the cone	19.25 cm
$H_{\max}$	Maximum total height of the tank	73 cm
$F_o$	Maximum inflow rate of the tank	400 LPH
$K_v$	Valve Coefficient	$55\text{cm}^2/\text{s}$
$h$	Height of the liquid level	Varied with the requirement
$F$	Outflow	Variable
$r$	Radius at the height $h$	Variable

### 4. SIMULATION RESULTS

Based on the dynamic equation derived in Mathematical Modelling section, a Simulink block diagram in figure 2 showing the nonlinear model of the plant is designed in MATLAB. Also the open loop response of the system is obtained as shown in figure 3.

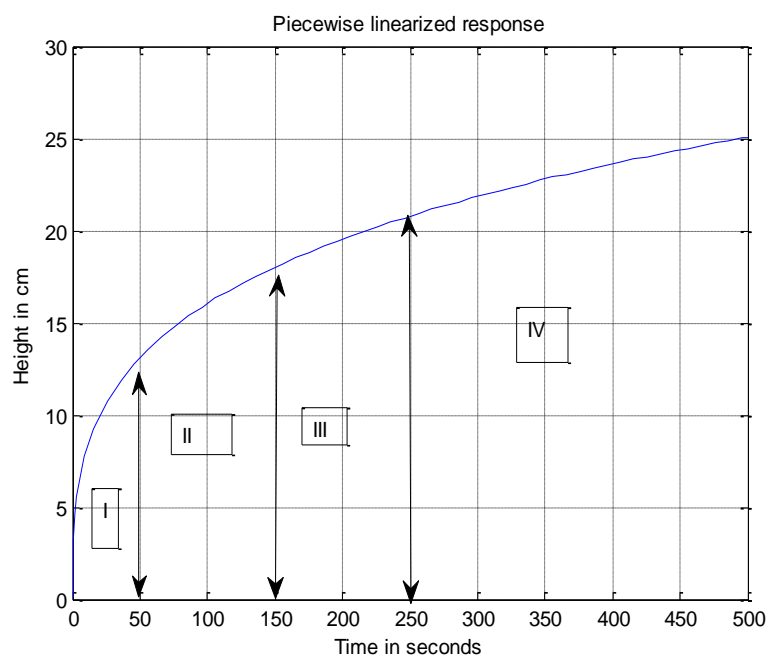


**Figure 2: Non-linear Model of Conical -Tank in Simulink.**



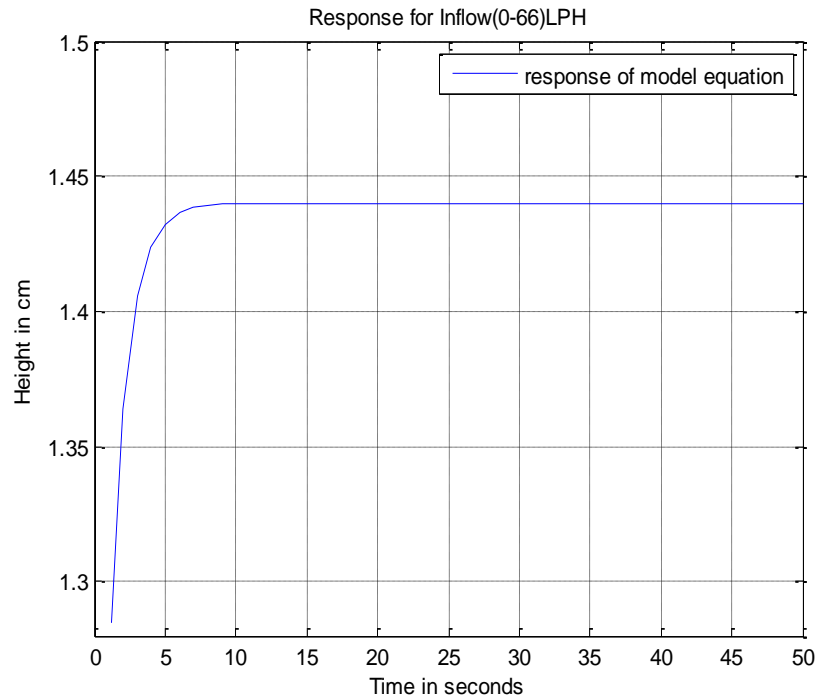
**Figure 3: Process input-output characteristic.**

To obtain a linear model the characteristic is divided into four different linear regions as shown in Figure 4.

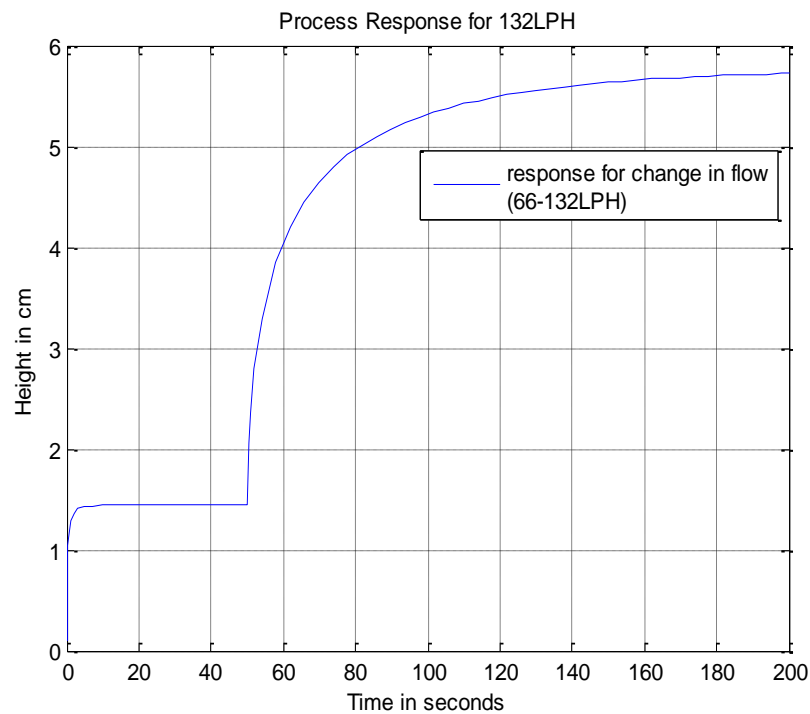


**Figure 4: Piecewise linearized process input- output characteristic.**

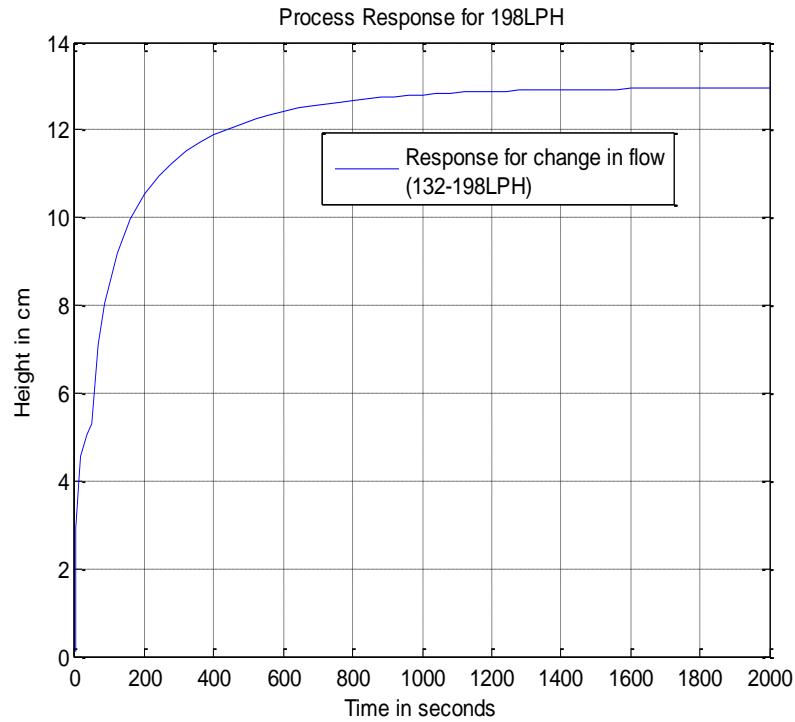
A first order mathematical model is then obtained for each region using process reaction curve method and the reaction curves for regions 1 to 4 are shown in Figures 5 to 8. The gain ( $K$ ), dead time ( $t_d$ ) and time constant ( $\tau$ ) are measured from the reaction curves and are given in the Table 2.



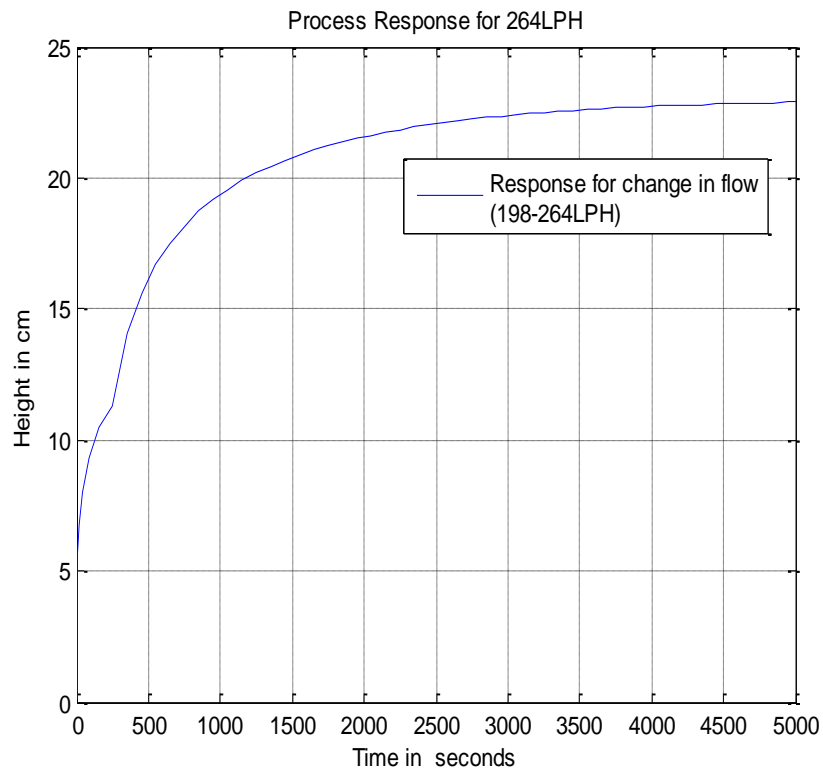
**Figure 5: Reaction curve for first region when step change in inflow (0-66)LPH.**



**Figure 6: Reaction curve for second region in inflow (66-132)LPH.**



**Figure 7: Reaction curve for 3<sup>rd</sup> region inflow(132-198)LPH.**



**Figure 8: Reaction curve for 4<sup>th</sup> region inflow (198-264) LPH.**



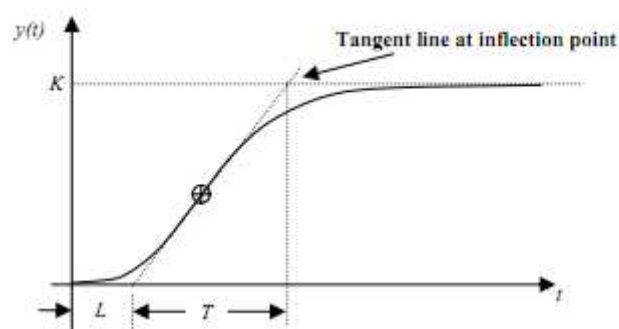
**Table 2: Process parameters obtained from the reaction curves.**

Inflow (LPH)	Time (Seconds)	Height/Level (cm)	Steady state gain	Time constant (Secs)
0-66	0-50	1.44	0.0218	0.002
66-132	50-150	5.76	0.087	0.24
132-198	150-250	12.81	0.194	1.97
198-264	250-500	23	0.348	11.75

## 5. CONTROLLER TUNING

### 5.1 Ziegler Nicholas Method

The Ziegler-Nichols design methods are the most popular methods used in process control to determine the parameters of a PID controller. Although these methods were presented in the 1940s, they are still widely used. The step response method is based on an open-loop step response test of the process. Hence requiring the process to be stable, the unit step response of the process is characterized by two parameters  $L$  and  $T$ . These are determined by drawing a tangent line at the inflection point, where the slope of the step response has its maximum value. The intersections of the tangent and the coordinate axes give the process parameters as shown in Figure 9, and these are used in calculating the controller parameters.<sup>[6]</sup>

**Figure 9: Response curve for ZN method.**

The parameters for PID controllers obtained from the Ziegler Nichols step response method are shown in Table 3.

**Table 3: Parameters tuned using Open-Loop ZN Method.**

PID Type	$K_p$	$T_i = K_p/K_i$	$T_d = K_d/K_p$
P	$T/L$	$\infty$	0
PI	$0.9(T/L)$	$L/0.3$	0
PID	$1.2(T/L)$	$2L$	$0.5L$

From the MATLAB Simulink model the ZN open loop response is observed and the value of  $T$ ,  $L$  are obtained and the parameters of  $K_p$  and  $K_i$  are calculated by using the above table. The calculated values are  $K_p=50$  and  $K_i=25000$ .

## 5.2 Cohen Coon Method

The Cohen-Coon method is a more complex version of the Ziegler-Nichols method. This method is more sensitive than the Ziegler-Nichols method. It is observed that the response of most of the processes under step change in input yields a sigmoidal shape (figure 10). The parameters for PID controllers obtained from the Cohen Coon step response method are shown in Table 4.

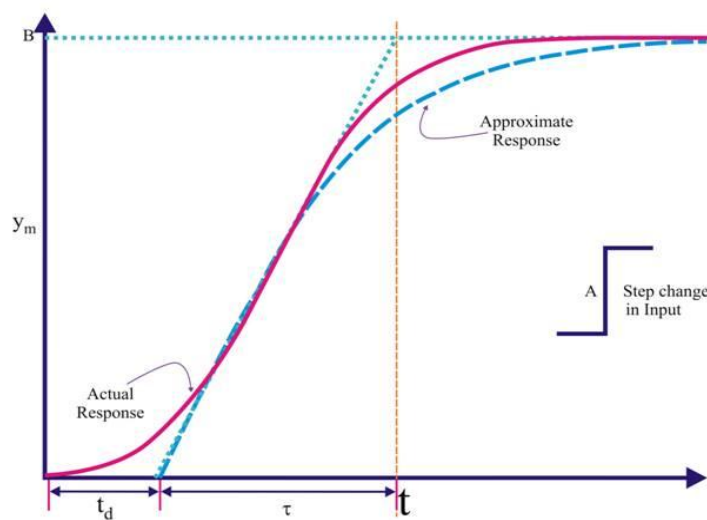


Figure 10: Response curve for CC method.

Table 4: Parameters tuned using Closed-Loop CC Method.

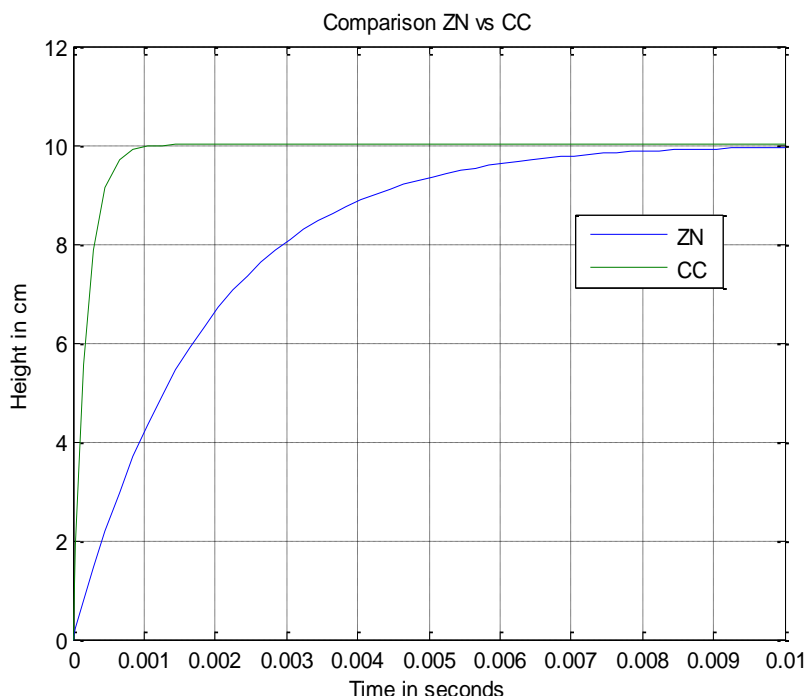
	$K_c$	$\tau_i$	$\tau_D$
P	$\frac{1}{K} \frac{\tau}{t_d} \left( 1 + \frac{t_d}{3\tau} \right)$	—	—
PI	$\frac{1}{K} \frac{\tau}{t_d} \left( 0.9 + \frac{t_d}{12\tau} \right)$	$t_d \frac{30 + 3 t_d / \tau}{9 + 20 t_d / \tau}$	—
PID	$\frac{1}{K} \frac{\tau}{t_d} \left( \frac{4}{3} + \frac{t_d}{4\tau} \right)$	$t_d \frac{32 + 6 t_d / \tau}{13 + 8 t_d / \tau}$	$t_d \frac{4}{11 + 2 t_d / \tau}$

From the MATLAB Simulink model the open loop response is observed and the value of  $K$ ,  $t_d$  and  $\tau_d$  are obtained as  $K=0.008135$ ,  $t_d=0.000373$  and  $\tau=0.001627$  and the parameters of

$K_p$  and  $K_i$  are calculated by using the above table. The calculated values are  $K_p=492.90$  and  $K_i = \frac{1}{\tau_I} = 407547.01$ .

## 6. RESULTS AND DISCUSSION

The response of Ziegler Nicholas method is compared with the Cohen Coon type tuning. The comparison graph is shown in figure 11.



**Figure 12: ZNT v/s CC Response.**

The graph shows that the Cohen Coon tuned controller is reaching the set point value in less time compared to the ZN. The performance of the controllers are analyzed by calculating the Integral Square Error (ISE), Integral time multiplied by Square Error (ITSE), Integral Absolute Error (IAE) and Integral time multiplied by absolute Error (ITAE). The obtained values are tabulated in Table 5.

**Table 5: Comparison of ZNT and CC.**

Parameter	ZNT	CC
ISE	0.09161	0.008973
IAE	0.01835	0.002165
ITAE	3.367e-005	1.194e-006
ITSE	8.369e-005	5.233e-007

## 7. CONCLUSION

The conical tank system is identified as a non-linear system. The model of conical tank system is implemented with the help of first principle differential equation. MATLAB ODE45 solver/Simulink is used to solve the differential equation. The results are validated by using the transfer function model and ODE response. The conventional PI controller is implemented to track the multi set point changes in level of the conical tank process by using different tuning rules.

The performance index of different tuning rules are also obtained. The simulation results proven that the Cohen Coon control method is an easy-tuning and more effective way to enhance stability of time domain performance of the conical tank system.

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