AN UNSTEADY RADIAL FLOW OF A VISCOUS INCOMPRESSIBLE FLUID FLOW IN A POROUS MEDIUM AROUND A SPHERE OF RADIUS WITH DAMPING.

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Abstract: This paper deals with an unsteady radial flow of a viscous incompressible fluid in a porous medium around an oscillatory spherical surface with damping. The momentum equation considered for the flow through a porous medium takes care of the fluid- inertia and the Newtonian stresses in addition to the classical Darcy's friction. Expression for the pressure distribution has been derived in terms of the expansion rate of the sphere -radius. Two special cases: (A) $r = e^{-\beta t}\cos\alpha t$ (B) $r = (1+\varepsilon e^{-\beta t}\cos\alpha t)$ are discussed in detail. In this, 'r' is the radius of the sphere at time 't', 'α' the radius oscillation parameter, 'β' the exponential - decay parameter and 'ε' is the sphere radius expansion factor parameter. The variation of the pressure with different values of 'α', Darcy number [D] and 'ε & β' at different time instants t has been discussed and illustrated followed by some conclusions.

Keywords: Pressure, Darcy's number, Porous Medium, Expansion factor, Radius oscillation and Exponential decay- parameters.

Introduction:

Studies on radial flows of a viscous fluid were initiated in the year 1915 by Jeffery G.B [3] followed later by Hamael G [2] and Harrison W. J. Such flows are discussed at length by Dryden H.L, Murnaghan F.D and Batemen H [1] in their classical work on Hydrodynamics. Recently Raisinghania M.D [6] in his treatise on Fluid Dynamics discussed several types of radial flows of viscous fluids in a clear medium.

Recently the present authors [4 & 5] investigated a class of unsteady radial flows of a viscous incompressible fluid through a porous medium around a sphere of variable and oscillating spherical surface with radius(r). The present investigation is on another class of unsteady radial flow of a viscous incompressible flow through a porous medium around a sphere whose surface is undergoing time dependent damped oscillations. A generalized momentum equation given by Yamamoto K and Yoshida Z [7] for the flows through a porous medium has been solved for the radial flow. It is noticed that the flow is independent of the Newtonian viscous stresses. However the flow depends on Darcian friction. Expression for the pressure distribution has been obtained in terms of the radial velocity on the sphere-surface. The cases of the sphere radius at time are

(A) $r = e^{-\beta t}\cos\alpha t$ and (B) $r = (1+\varepsilon e^{-\beta t}\cos\alpha t)$ in the non dimensional form

Mathematical Formulation and Solution of the Problem:

Consider a spherical co-ordinate system R,θ,ϕ with a origin 'O' fixed at the center of the sphere. R is the radial distance from the origin, θ is the polar angle and ϕ is the azimuthal angle. The flow of a viscous incompressible fluid through a porous medium is governed by the modified Navier-Stokes equations suggested by Yamamoto K and Yoshida Z [7]:

$$\frac{d\vec{q}}{dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{q} - \frac{\nu}{k}\vec{q} \qquad (where\ \nu = \frac{\mu}{\rho}) \tag{1}$$

together with the equation of continuity

$$\frac{\partial\rho}{\partial t} + (\nabla.\rho\vec{q}) = 0 \tag{2}$$

where '$\vec{q}$' represents the fluid velocity and 'p' is the fluid pressure. Further 'ρ 'is the fluid mass density, 'μ' is the coefficient of Newtonian of viscosity and 'k' is the coefficient of Darcian porosity of the medium through which fluid will flow and these are assumed to be constants. The term $\nu\nabla^2\vec{q}$ on the R.H.S of (1) represents the contribution of the Newtonian Viscous-Stress and $-\frac{\nu}{k}\vec{q}$ is the classical Darcy -resistance to the flow.

By radial and axial-symmetries

$$\frac{\partial\vec{q}}{\partial\theta} = 0 \text{ and } \frac{\partial\vec{q}}{\partial\phi} = 0 \tag{3}$$

For the unsteady radial flow under investigation, the velocity field can be taken as

$$\vec{q} = (U(R,T),0,0) \tag{4}$$

The continuity equation (2) now reduces to

$$\frac{1}{R^2}\frac{\partial(R^2U)}{\partial R} = 0 \tag{5}$$

Momentum equation in the Radial-direction(R):

$$\frac{\partial U}{\partial T} + U\frac{\partial U}{\partial R} = -\frac{1}{\rho}\frac{\partial P}{\partial R} - (\frac{\nu}{k})U \tag{6}$$

Momentum equations in the θ and ϕ -directions:

$$\frac{\partial P}{\partial\theta} = 0 \text{ and } \frac{\partial P}{\partial\phi} = 0 \tag{7}$$

It can be noted from the equation (7) that

I) The pressure (P) is a function of R and T only. (i.e independent of θ and ϕ)

II) It is also independent of Newtonian viscous stresses and

III) The Darcian viscous resistance on the porous media ($-\frac{\nu}{k}\vec{q}$) only influences the pressure distribution.

For simplicity the following non dimensional quantities are introduced in the foregoing analysis

$$R = R_0 r;\ U = \frac{\mu u}{\rho R_0};\ T = \frac{\rho R_0{}^2 t}{\mu};\ P = \frac{\mu^2 p}{\rho R_0{}^2};\ D = \frac{R_0{}^2}{k} \tag{8}$$

where 'D' is the non-dimensional Darcy porosity coefficient and where R_0 is the initial radius of the sphere. By definition the radial velocity 'u' on the sphere of the surface is given by

$$u = \frac{dr}{dt} \quad \text{on the sphere-surface} \tag{9}$$

The following are non-dimensional form of the basic equations

Continuity Equation:

$$\frac{1}{r^2}\frac{\partial(r^2 u)}{\partial r} = 0 \tag{10}$$

Momentum equation in the radial direction:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{\partial p}{\partial r} - Du \tag{11}$$

From (10), we get

$$u = \frac{f(t)}{r^2} \tag{12}$$

It follows from (9) and (12) that

$$f(t) = r^2\frac{dr}{dt} \quad \text{on } r=1 \tag{13}$$

And from (11) and (12) we get the equation for the determination of the fluid pressure (P):

$$-\frac{\partial p}{\partial r} = \frac{f^1(t)}{r^2} - 2\frac{(f(t))^2}{r^5} + D\frac{f(t)}{r^2} \tag{14}$$

Integrating with respect to $'r'$ we get:

$$p_\infty - p = -\frac{1}{r^2}\left(\frac{df}{dt}\right) + \frac{1}{2r^4}\frac{d^2 f}{dt^2} - \frac{D}{r}(f(t)) \tag{15}$$

$$P = r\frac{d^2 r}{dt^2} + \left(\frac{3}{2}\right)\left(\frac{dr}{dt}\right)^2 + Dr\left(\frac{dr}{dt}\right) \quad \text{on } r=1 \tag{16}$$

Case-A: Let

$$r = e^{-\beta t}\cos\alpha t \tag{A.1}$$

Now from (13) and equation (A.1) we have

$$f(t) = r^2\frac{dr}{dt}$$

$$= -e^{-3\beta t}(1+\cos 2\alpha t)(\alpha\sin\alpha t + \beta\cos\alpha t) \tag{A.2}$$

Also from equation (16) we get pressure equation**:**

$$P = \frac{e^{-2\beta t}}{4}[(5\beta^2 - 5\alpha^2 - 2D\beta)\cos 2\alpha t + 2\alpha(5\beta - D)\sin 2\alpha t + (5\beta^2 + \alpha^2 - 2D\beta)] \tag{A.3}$$

$$= \frac{e^{-2\beta t}}{4}(5\beta^2 + \alpha^2 - 2D\beta) + R^* \cos(2\alpha t - \phi)$$

where Amplitude and Phase lag respectively are

$$R^* = \frac{e^{-2\beta t}}{4}\sqrt{(5\beta^2 - 5\alpha^2 - 2D\beta)^2 + 4\alpha^2(5\beta - D)^2} \quad and \quad \phi = \tan^{-1}\left[\frac{2\alpha(5\beta - D)}{5\beta^2 - 5\alpha^2 - 2D\beta}\right]$$

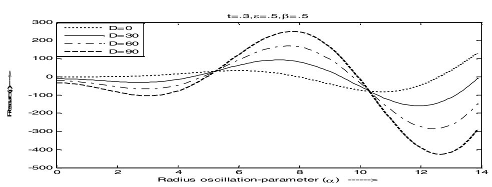
The maximum and minimum pressures on the surface respectively are

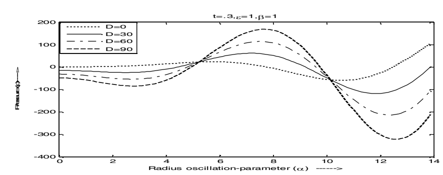
$$P_{\max} = \frac{e^{-2\beta t}}{4}[(5\beta^2 + \alpha^2 - 2D\beta) + \sqrt{(5\beta^2 - 5\alpha^2 - 2D\beta)^2 + 4\alpha^2(5\beta - D)^2}\,]$$

$$P_{\min} = \frac{e^{-2\beta t}}{4}[(5\beta^2 + \alpha^2 - 2D\beta) - \sqrt{(5\beta^2 - 5\alpha^2 - 2D\beta)^2 + 4\alpha^2(5\beta - D)^2}\,]$$

Results and Discussion:

It is noticed from the figures(A.1-A.2) the variation of the pressure on the spherical surface vrs Radius-oscillation parameter(α) for different values of porosity parameter(D) with different time instants (t =0.3, 0.6, 0.9) at different expansion factor (ε =0.5, 1.0) and at different exponential –decay factor(β =0.5, 1.0) the number of oscillation beats increases as Radius-oscillation parameter increases for a specific range of Radius-oscillation parameter.

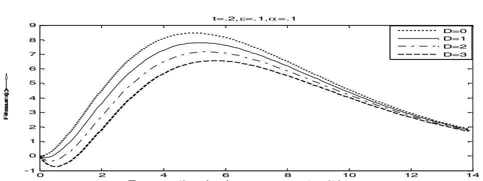


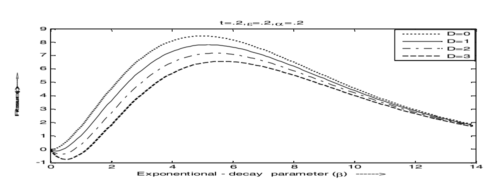


Fig(A.1) Variation of the Pressure vrs alpha for different 'D' at t=.3, ε =.5, β =.5

Fig(A.2) Variation of the Pressure vrs alpha for different 'D' at t=.3, ε =1.0, β =1.0

It is noticed from figures(A.3-A.4) that the variation of the pressure vrs Exponential –decay parameter (β) for different Darcy number with different time instants (t =0.2, 0.3, 0.4) at different expansion factor(ε =0.1, 0.2) and at different radius oscillation parameter (α =0.1,0.2) the pressure steeply increases initially and critical thereafter ' r ' asymptotically decreases to zero this trend change occurs when β takes a value β_c .





Fig(A.3) Variation of the Pressure vrs Beta for different 'D' at t=0.2, ε =0.1, α =0.1

Fig(A.4) Variation of the Pressure vrs Beta for different 'D' at t=0.2, ε =0.2, α =0.2

Case-B: Let

$$r = (1 + \varepsilon e^{-\beta t} \cos \alpha t) \tag{B.1}$$

Then $f(t) = r^2 \frac{dr}{dt}$ and $f(t)$ reduces to

$$= -\varepsilon e^{-\beta t}(1+\varepsilon e^{-\beta t}\cos\alpha t)^2(\beta\cos\alpha t+\alpha\sin\alpha t) \qquad \text{(B.2)}$$

Then from equation (16) we get the pressure distribution:

$$P=(2\beta-D\varepsilon)(\alpha e^{-\beta t}\sin\alpha t)+(\beta^2-\alpha^2-D\beta)(\varepsilon e^{-\beta t}\cos\alpha t)+(2\varepsilon^2\beta+3\beta e^{2\beta t}-2D\varepsilon^2)(\frac{\alpha}{2}e^{-2\beta t}\sin 2\alpha t)$$
$$+\frac{1}{4}((2\varepsilon\beta^2-2\varepsilon\alpha^2+3\varepsilon\beta^2-2D\beta)e^{-2\beta t}-3\alpha^2)\cos 2\alpha t+\frac{1}{4}[(2\varepsilon(\beta^2-\alpha^2)+3\varepsilon\beta^2-2D\beta)\varepsilon e^{-2\beta t}+3\alpha^2]$$

$$=\frac{1}{4}[(2\varepsilon(\beta^2-\alpha^2)+3\varepsilon\beta^2-2D\beta)\varepsilon e^{-2\beta t}+3\alpha^2]+R_1^*\cos(\alpha t-\phi_1)+R_2^*\cos(2\alpha t-\phi_2)$$

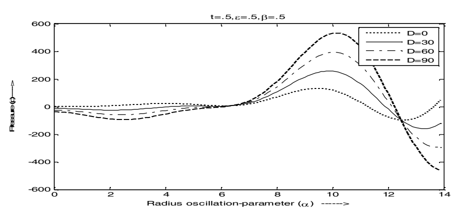
where Amplitudes and Phase lags respectively are given by

$$R_1^*=\sqrt{(\beta^2-\alpha^2-D\beta)^2+(2\beta-D\varepsilon)^2\alpha e^{-2\beta t}}$$
$$R_2^*=\sqrt{(.025)(2\varepsilon\beta^2-2\varepsilon\alpha^2+3\varepsilon\beta^2-2D\beta)e^{-2\beta t}-3\alpha^2)^2+(.25)((2\varepsilon^2\beta+3\beta e^{2\beta t}-2D\varepsilon^2)^2(\alpha e^{-4\beta t})}$$

$$\phi_1=\tan^{-1}\left[\frac{(2\beta-D\varepsilon)\alpha}{(\beta^2-\alpha^2-D\beta)\varepsilon}\right] \quad \textit{and} \quad \phi_2=\tan^{-1}\left[\frac{(2\varepsilon^2\beta+3\beta e^{2\beta t}-2D\varepsilon^2)\frac{\alpha}{2}e^{-2\beta t}}{\frac{1}{4}((2\varepsilon\beta^2-2\varepsilon\alpha^2+3\varepsilon\beta^2-2D\beta)e^{-2\beta t}-3\alpha^2)}\right]$$

Results and Discussion:

It is noticed from the figures(B.1-B.2) the variation of the pressure on the spherical surface vrs Radius-oscillation parameter(α) for different values of porosity parameter(D) with different time instants (t =0.5, 1.0) at different expansion factor (ε =0.5, 0.7) and at different exponential –decay factor(β =0.5, 0.7) the number of oscillation beats increases as Radius-oscillation parameter increases.



Fig(B.1) Variation of the Pressure vrs alpha for different 'D' at t=.5, ε =.5, β =.5

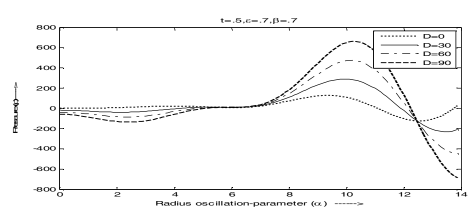
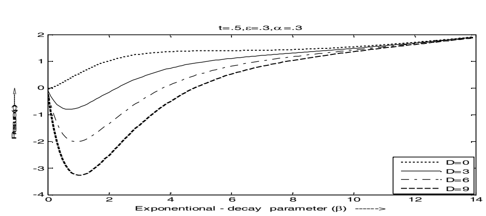
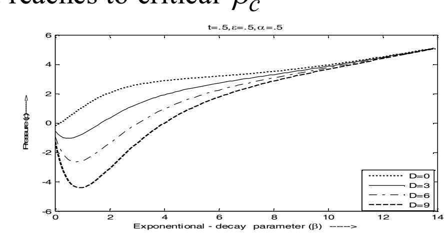


Fig (B.2) Variation of the Pressure vrs alpha for different 'D' at t=.5, ε =.7, β =.7

Also It is noticed from figures(B.3-B.4) that the variation of the pressure vrs Exponential –decay parameter (β) for different Darcy number with different time instants (t =0.5, 0.7) at different expansion factor(ε =0.3, 0.5) and at different radius oscillation parameter (α =0.3,0.5) the pressure exponentially increases and reaches to critical β_c



Fig(B.3) Variation of the Pressure vrs Beta for different (D) at t=0.5, ε =0.3, α =0.3



Fig(B.4) Variation of the Pressure vrs Beta for different (D) at t=.5, ε =.5, α =.5

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