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Applications Of Non Associativity In Different Disciplines

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Abstract: Non associativity is a fundamental property of nature. In this paper we present and prove the applications of non associativity in different fields with examples. The concept of non associativity has applications in many areas of mathematics like vector cross product, complex conjugate concept and practical octonions. In computer science floating point concept coding theory, cryptography, and polarized tree- based grammars. The gimbal lock situation in mechanical engineering is a famous significant example for non associativity originated from a singular matrix when it is represented by matrices coupled with trigonometric functions. The gimbal lock was experienced in Robotics in the name of wrist flip. In applied mathematics it appears in the form of Euler angles relates to Satellite engineering, computer graphics and animation. In physics there are so many approaches for non associatively in quantum mechanics connecting many areas of mathematics like lie algebras, octonion planes, In electronics logic NAND function is non associative. This function used a lot in computer programming, computer architecture and to construct Boolean logic circuits. In biology biologically motivated mathematics arises in genetics relates to non associatively.

Keywords: Octonions, Floting point, Gimbal lock, NAND function, genetics.

I. MATHEMATICS

In mathematics the non associativity exists in differential geometry, Riemannian symmetric spaces projective geometry, functional analysis, quantum logic, special relativity, super symmetry and in mathematical genetics. In mathematics we deal non associatively in abstract algebra. In general all rings are non associative except those satisfy associative law called as rings. Some of such non associative rings are Jordan ring (p q) p^2= p (qp^2) for every p, q ϵ R which is commute but not associative. Lie ring p^2=0 for every p ϵ R is associative but not commutative, whenever the associative property is replaced by Jacobi identity (p q)r +(q r)p+(r p)q=0 for every p,q,r,in R then Lie ring becomes non associative. The results and ideas relate to these Jordan and lie rings is used for solution to the Restricted Burnside problem is a famous significant example for non associativity in group theory. The Jacobi identity satisfied by vector cross product also. Next left alternative ring p^2q= p (p q) and right alternative ring qp2= (q p)p and those satisfy both are called alternative rings. The most practical example of non asociativity is octonions (O).

II. COMPUTER SCIENCE

In computer science floating – point numbers is a beautiful example for non associativity. It is curious but important practical fact in computer arithmetic that addition is non associative. This depends upon the memory of the device. This means that low –order digits on very large numbers can be lost to round- off error.

E.g.**:** $(10^{28} + (-10)^{28}) + 0.01 \neq 10^{28} + (-10^{28}+0.01)$

Really it is non associative. But practically we don't consider such numerical issue. Next non associative concept is grammar, a description of some computer language. A grammars consists of a sequence of symbols, they are bytes, characters or others from a fixed set. Thus the compact representation is called grammar.

III. MECHANICAL ENGINEERING

The concept of gimbal lock is non associative which have practical applications not only in applied mathematics but also in mechanical engineering, robotics satellite engineering, computer graphics and animation. A gimbal lock is a ring rotate about an axis through suspension. Thus gimbals are typically nested one with in another to accommodate rotation about multiple axes with three degrees of freedom. The three gimbals rotate independently with three mutually perpendicular gimble axes called roll, pitch, and yaw. Whenever these three gimbals rotate mutually perpendicular until there is no problem arises, but when a condition caused by collinear alignment of two or more gimbals. The system loss one degree of freedom and faces hazardous situation called gimble lock. At this stage the system faces dangerous situations particularly air force navigation system faces an unpredictable motion or velocities like in Apollo II moon mission.

Now updated techniques like fluid bearings or floating chambers were used in modern practice in the place of gimbals, now for inertial navigation system mounting inertial sensors give information about orientation and velocity digitally using quaternion methods. Rotating objects are probably quaternion numbers in most practical applications. Whenever we give mathematical formulation using matrix algebra, coupled with trigonometric functions sometimes we may get a singular matrix. This singular matrix can cause gimbals lock i.e. simply the system stops working because one degree of freedom among the three axes is lost. This was experienced in gyroscopes, tracking guided missiles and robot arms.

In robotics instead of gimbal lock it is refereed as “wrist flip” or wrist singularity contain three axes of wrist controlling yaw, pitch and roll, all these points passing through a common point. When we get a condition caused by the collinear alignments of two or more robot axes resulting in unpredictable velocities or motion. In applied mathematics the gimbal lock appears when one uses Euler angles, this can be avoid by taking care in software’s likes 3D computer programs, 3D modeling, embedded navigation system and videogames.

Let the three consecutive linear movements along three perpendicular axes X,Y and Z axes with translations described by three numbers x, y, and z. The complete rotation can be described using three numbers be α, β, and γ. These rotational movements are perpendicular one to the next. The comparison between angular co-ordinates and linear co-ordinates makes Euler angles very intuitive by suffer with gimbal lock problem unfortunately.

Numerically a rotation in 3D space can be represented by matrices coupled with trigonometric function in several ways one of these representation is

$$R=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\alpha & -\sin\alpha\\ 0 & \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\beta & 0 & \sin\beta\\ 0 & 1 & 0\\ -\sin\beta & 0 & \cos\beta\end{bmatrix}\begin{bmatrix}\cos\gamma & -\sin\gamma & 0\\ \sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1\end{bmatrix}$$

Put $\beta=\frac{\pi}{2}$

$$R=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\alpha & -\sin\alpha\\ 0 & \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ -1 & 0 & 0\end{bmatrix}\begin{bmatrix}\cos\gamma & -\sin\gamma & 0\\ \sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow R=\begin{bmatrix}0 & 0 & 1\\ \cos\alpha\sin\gamma+\sin\alpha\cos\gamma & \cos\alpha\cos\gamma-\sin\alpha\sin\gamma & 0\\ \sin\alpha\sin\gamma-\cos\alpha\cos\gamma & \sin\alpha\cos\gamma+\cos\alpha\sin\gamma & 0\end{bmatrix}$$

$$\Rightarrow R=\begin{bmatrix}0 & 0 & 1\\ \sin(\alpha+\gamma) & \cos(\alpha+\gamma) & 0\\ -\cos(\alpha+\gamma) & \sin(\alpha+\gamma) & 0\end{bmatrix} = -1\neq 0$$

In this case the rotation axis remains in Z –direction even through the rotation angle $(\alpha+\gamma)$ changes. And at the same time if we change the values of α and γ the matrix has the same effects. At this position the moving object rotated by the Euler angles X-Y-Z axes. The angle α represents pitch, yaw is set to π/2 and then the final rotation by γ is again represents the pitch whenever two gimbals aligned, it lost one degree of freedom and gimbal lock happened. At this stage the moving object movement is limited i.e an entire range of motion becomes impossible. At such situation pilots face very hazardous situation for an air force fighter navigation system.

This is due to singularities in Euler angles in matrix algebra. Moreover they produce a jerky and un natural type of movement, to overcome this now software engineers use quaternion tolls, which is four dimension with quaternion’s the rotations are smooth and natural and effective and efficient when compared to matrix algebra. For example a matrix rotation of two axes required several operations but in quaternions it requires only one operation. Thus the matrix algebra utilizes Euler angles work with three dimensions of height, length and width but using quaternions we produce a rotation on any of the infinite axes of a sphere. For this reason quaternion tool are also utilized in medical field like magnetic resonance images (MRI) and computed tomography (CT) to get clear idea of human body through cross-sectional slice images.

IV. PHYSICS

The application of non associativity in quantum theory of physics is not simple. There are so many approaches to non associativity in quantum physics. The concept of magnetic monopoles under research is non associative. A geometry relation between Heisenberg uncertaintainty and light cone [4] Lie group symmetries of the standard model [8] with gravity [1] and super symmetry with the standard model [2].Chirality and triality in fundamental particles [3] octonionic quantum theory and Dirac equation from both left and right associating operators [7].The Dirac equation with electromagnetic [5] Quark statistics in the strong force [6] spin operators and Lorentz lie algebra using Pauli matrices over complex numbers. Complex theories employ non associative elements for modeling physical properties which includes in division algebras like R, C, Q and O.

$$\text{Eg: } \int_v \Psi_n^* (h_n\Psi_n)\, dv \neq \int_v (\Psi_n^* h_n)\Psi_n dv.$$

Where Ψ is the unobservable wave function decomposed into orthogonal eigen function Ψ_n of operations. Unobservable properties are different from hidden variables.

H_n is the yield observable eigen values.

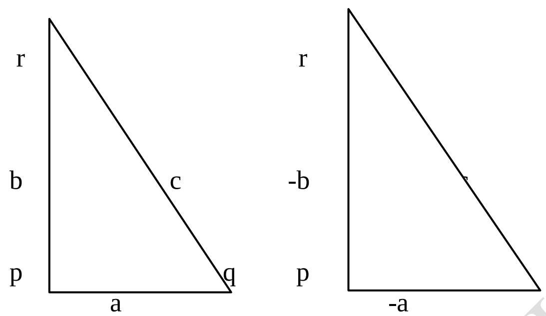
H is the expection value over some parameter space v determined through expression like

$$<\Psi/H/\Psi> = \sum_n h_n \int \Psi_n^* \Psi_n \, dv.$$

Non associativity of Lorentz transformation, in General Lorentz transformation is non associative it connects theory of relativity and quantum mechanics. Now we consider three non collinear moving bodies p, q, r with relative motion with velocities a, b, and c as shown

According to Lorentz-Einstein transformation

$$C_{L\text{-}E} = (-b)\bar{+}a = \frac{\sqrt{1-(b/c)^2 a} + \{[1-\sqrt{1-(b/c)^2}]a.b/b^2 - 1\}b}{1-a.b/c^2}$$



Here Lorentz-Einstein addition of velocities is represented by $\bar{+}$, when we swap the positions like in the above figure 2 the relative velocities.

$$C^1{}_{L\text{-}E} = (-a)\bar{+}b = \frac{\sqrt{1-(a/c)^2 b} + \{[1-\sqrt{1-(a/c)^2}]a.b/a^2 - 1\}a}{1-a.b/c^2}$$

Hence clearly $(a\bar{+}b)\bar{+}c \neq a\bar{+}(b\bar{+}c).$

V. ELECTRONICS

NAND is a logical operation denoted by a vertical bar or upward arrow operator i.e. p ↑ q is an example of non-associativity represented by Boolean algebra. The NAND or NOT AND function is a combination of the two separate logical functions. The AND function and the NOT function connected together in series.

(p↑ q) ↑r≠ p↑(q ↑ r) proved from the truth table as follows

P	↑	(q	↑	r)	(p	↑	q)	↑	R
F	T	F	T	F	F	T	F	T	F
F	T	F	T	T	F	T	F	F	T
F	T	T	T	F	F	T	T	T	F
F	T	T	F	T	F	T	T	F	T
T	F	F	T	F	T	T	F	T	F
T	F	F	T	T	T	T	F	F	T
T	F	T	T	F	T	F	T	T	F
T	T	T	F	T	T	F	T	T	T

VI. BIOLOGY

In mathematical genetics**,** biologically motivated algebra used to model inheritance in genetics, frequencies of gene expression and spreading factors rates of a diseases are non associative. Some variations of these algebras are special train and train algebras, Berstein algebras, copular algebras, gametic algebra, zygotic algebra and basic algebra. All these are used to encode the probabilities of producing off spring of various non associative types.

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