RINGS WITH (X, R, X) AND COMMUTATORS IN THE LEFT NUCLEUS

Dr. C. Jaya Subba Reddy[1], D. Prabhakar Reddy[2]

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Mathematics Subject Classification: Primary 17A30

Keywords: Non associative ring, Prime ring, Semiprime ring, Nucleus, Commutator, Associator, Characteristic.

1. INTRODUCTION

In [1] Albert used the identities consisting of the Jordan identity, flexible, Lie-admissible and commutators in the nucleus. His main result was that simple finite dimensional algebras are either associative algebras or Jordan. Next Kleinfeld [2] proved that semiprime rings without the Jordan identity are subdirect sums of associative and commutative rings, while prime are Commutative or associative. Then San Soucie [4] was able to drop the Lie-admissible hypothesis without losing the conclusions. More general result was obtained by Thedy [6]. In this direction Kleinfeld [3] by weakening the two remaining hypothesis of flexible and commutators in the nucleus proved the same results for semiprime and prime rings. In this section we consider a ring R with (x, y, x) and commutators in the left nucleus. We show that (x, y, x) and commutators are in the center. Using these properties, we prove that R must be a subdirect sum of a semiprime associative ring and a semiprime commutative ring.

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