

# **A Novel Approach for Allocation and Scheduling Problem with uncertainty by using Graph Theory**

**Title:** A Novel Approach for Allocation and Scheduling Problem with uncertainty by using Graph Theory

**Proposal ID:** BFP/RGP/CBS/19/226

**Type of project application:** Research Grant Program

**Current Status:** Approved For Funding

## **Sector Name**

Culture Humanities and Basic Sciences

## **Principal Investigator**

ABDUL SHUKOOR, NASEER AHMED (naseer.shukoor@nct.edu.om) (Natural science and engineering)

## **Co-Principal Investigator**

Al Shaqsi, Khalifa Zayid Khalifa (khalifa.alshaqsi@nct.edu.om)

## **Co-Investigators**

## **Other Team Members**

Gopal, Suganthi - Nizwa College of Technology (Active)

Jain, Vinesh - Nizwa College of Technology (Active)

K R, Ranjith Kumar - Nizwa College of Technology (Active)

Naheed, Mohammed - Nizwa College of Technology (Active)

Nair, Rajesh - Nizwa College of Technology (Active)

## **Technology Readiness Level (TRL)**

1 – 3: Knowledge Development (usually public organizations) e.g. basic research & lab-based experiments

## **Priority Area**

Other

## **Executive Summary**

All the real life decision making problems have three major constraints Quality, Cost, and Time, named triangular constraints. They are inter-related with each other. One of this cannot be changed without affecting others. The main objective of an allocation and scheduling problems is to optimize all the three constraints as much as possible. In addition to that, there are many other constraints, such as satisfaction, emotional intelligence, skill set etc., that affect the solution of allocation scheduling problems. In this research work, we are going to investigate the parameters of graph theory for which the possible outcomes that optimize all the constraints of the allocation and scheduling problem. Because, the real life problems that are related to allocation and scheduling can be easily converted as a vertex-edge model, it is easy to handle them by using graph theoretic approach. Out of several parameters in graph theory, domination number and chromatic number have many applications in allocation and scheduling problem respectively. Since there are many types of constraint involved in the problem, there are many types of parameters also exist in domination and coloring theory. To obtain the optimum solution by enhancing the idea of domination and coloring with more than one constraint, we define some new parameters that are associated with variants of domination and coloring of a graph. Due to the computational complexity of domination and coloring for general graph, their bounds will be found for several special graphs and family of graphs. Also, we characterize the graphs with the constraints of those parameters. The

necessary and sufficient condition is investigated to get a unique value for the parameter, which mean, unique solution for the given problem.

## **Introduction and Statement of the Problem / Project**

Graph theory is a delightful playground and its consequences have applications in many areas of the computing, computer science, social, natural sciences, and other scientific and not-so-scientific areas. Vertex-edge graphs are common data models that emerge in graph theory.

Without any doubt, the literature on domination and coloring in a graph is growing rapidly.

Most of the researchers are working on real life problems. To accomplish the optimal solution of real life problems that are related to allocation and scheduling with uncertainty, an application of fuzzy theory, graph theory and operation research are required to use together. Consequently, this research work objective to get in-depth knowledge of the parameters which are the joint application of fuzzy, optimization, domination and coloring in graph. Already, a significant amount of work has been dedicated on domination number and chromatic number of a graph. Still, there is lots of scope for doing research on it. If the additional constraints added based on the requirements in the real time problem with the already available various types of definitions of domination and coloring, different types of domination and coloring parameters might be obtained. Here the additional constraints referred for the external factors with fuzziness which give impact on the solution of the real time problem, for example, satisfaction of employee as well as employer, emotional intelligence, natural disasters etc. If the satisfaction of employee and employer is considered, then it is necessary to increase the satisfaction of both together in optimization process. Employee and employer also inter-related with each other. The techniques of fuzzy theory, graph theory and operation research provide an optimum solution to this kind of problems. They have witnessed a magnificent growth due to a number of applications in computer and communication, molecular physics and chemistry, social networks, biological sciences, computational linguistics and more other areas. Scheduling and allocation of the tasks/resources are direct applications of coloring and domination theory.

## **Literature Review and Analysis of Related Work**

The existing various types of decision making and problem solving with fuzziness are inevitable in all the scientific and non-scientific fields, even in day today life. The resources and time are the two major factors that we need to give much attention because improper utilization of resource and time will not meet the requirement and objective of the problem. Underutilization of resource and time also harm the decision maker. The optimum utilization of resource and time gives us the optimum solution in all kind of problem. This creates the attention towards resource allocation and time scheduling. There are several types of named allocation problem and scheduling problem like Job Shop Problem, Open Shop Problem, Nurse Rostering Problem etc. In project management, resource management is the scheduling of task and allocation of resources required by those tasks while taking into consideration both the resource availability and the time. The primary objective of this research work, obtain the optimum solution of the problem with fuzziness in which both allocation and scheduling taken place, is carried out from the beginning till the end of this thesis write up in terms of graph theory.

Due to this motivation, our attention seeks towards the parameters which are the joint application of domination theory and coloring theory with fuzziness in graphs. Since domination and coloring in graphs are two most important research area of graph theory, without a doubt, the literature on domination and coloring in a graph is growing rapidly, and a significant amount of work has been dedicated to find different types of parameters and characterization in domination and coloring and their lower and upper bounds. More over the fuzzy graph theory is also recently grown as an important research domain, because of the existence of fuzziness in almost all real time problems. Majority of fuzzy models are using fuzzy graph, intuitionistic fuzzy graph and vague graph in graph theory. Most of the researchers worked on the real life problems which are the applications of domination, coloring and both. But very few in the domination and coloring along with fuzzy. So the following literature review aims to get in-depth knowledge in the parameters in which both domination, coloring, and fuzzy related.

Since 1999, the research work progressively increased on the joint application of domination and coloring. The first paper which has combined idea of domination and coloring was

published by Irving and Manlove (1) in 1999. Moreover, they defined a parameter b-chromatic number by using coloring with the additional condition of adjacency of vertex (domination). As a continuation of that, Chinh T.Hoang and Mekia Kouider (2) in 2005 found some bounds on b-chromatic number. Then the next parameter which describes as a combined theory of domination and coloring is Dominator Chromatic number which was introduced by R. Gera et al. (3) in 2006. Further, R. Gera (4; 5), S.Hedetniemi et al. (6) and S. Arumugam (7), extend the work on dominator coloring. Recently Dominator chromatic number of Interval graphs (8), Central Graphs, Middle Graphs, Star and Double Star Graph (9), S.Balamurugan et al. (10) in 2011 defined and describe the results in Chromatic Strong Domination Number. Total Dominator Chromatic number was defined and its bounds also discussed by Vijayalekshmi in (11) in 2012.

Arumugam et al. determined the maximum number of maximal independent dominating sets in a minimum coloring of a graph, defined Dom Color Number (12) in 2008 later it named as Dominating Color number (13) in 2011. L.Benedict Michael Raj et al. (14) in 2009 defined chromatic transversal domatic number and motivated in (15). J.John Arul Singh and R. Kala (16) in 2010 proved certain necessary and sufficient conditions so that Min-Dom-Color number equals the chromatic number and characterized a family of graphs with dom color number is one (17) in 2011. In this sequence, Global Dominating Color number was introduced by I Sahul Hamid et al. (18) in 2014.

## Objectives

The main objective of this research work is

1. to examine the detailed review of the existing parameters in which both allocation and scheduling related.
2. to fix or define a parameter associated with both allocation and scheduling with fuzziness.
3. to approach the newly defined parameter scientifically.
4. to analyze and report the advantages of the proposed parameter.
5. to obtain the optimum solution for allocation and scheduling problems with various constraints that occur in real time situations. In particular, allocation and scheduling problems in logistics, Ad-hoc, Wireless and Mobile Network by using graph theory.

## Research Methodology [Describe your Implementation Plan,Time-line and Milestones]

The series of tasks for this research project involves seven major stages. The estimated total duration of the research project will be 22 months, including the contingency period of 2 months for extended tasks, if required. Time duration of each stage is roughly distributed to be:

1. Literature review - 4 months
2. Data collection and analysis - Site survey, study and review of the existing allocation and scheduling problems with fuzziness, Characteristics analysis and discussion, Study the related parameters allocation and scheduling problem with fuzziness - 4 months
3. Problem designing and modeling - Well define the problems in logistics and network in terms of graph theory - 4 months
4. Well define the parameters and Checking the existence - Define new parameters of graphs that represents the allocation and scheduling problem, Examine the existence of parameters in general graph - 4 months
5. Finding Bounds and Characterization - Find bounds for these parameters, Relation between other existing parameters and Characterization of graphs with these parameters - 6 months

## Benefits to Oman

Importance of graph theoretic model with fuzziness in decision making and problem solving is highlighted because its application and implementation of the proposed parameter in the real time situations and this will open the exposure for allocation and scheduling problems in

oil and gas industry, mobile, wireless and ad-hoc network, Airport, customer care, hospital, Electricity, ROP etc.,.

In general, the results can be utilized to get the optimum solution of numerous types of allocation and scheduling problems with fuzziness in the several service sectors in Oman.

The following are the expected economic benefits to the society after the successful completion of this research work

- In short term, the results of this research work can be applied in all real time project selections and in project management. This gives the profit for the projects which involves the joint application of allocation and scheduling.
- In day to day life, by knowing the results of this research work, people can take some good optimum decisions to save their money in all aspects.
- In long term, the money that is saved by using the research report in the present projects can be utilized for the future projects. Proper resource allocation and time scheduling stabilize the economy of the nation as well as every individual of the nation.

The following are the expected benefits after the successful completion of this research work in social, cultural, educational and welfare .

- The decision making is a skill set that is an important need for scientific and non-scientific fields as well as day to day life. Such skill can be improved systematically by this research.
- In educational aspect, this research work can published as a research journal paper and two research conference papers.
- Awareness of allocation and scheduling applications to student community by involving 5graduate level students in minor part of this project.
- Gives the motivation among the academicians towards research in discrete mathematics, in particular graph theory.
- Making optimum decisions for government and private projects lead to increase the comfort in the society in three aspects. Firstly, if the processing time is optimized society get the project benefits at right time some time earlier than the deadline. Second, if the cost of the project is optimized then the number of minor projects can be implemented by using the surplus money in the budget. Finally, if the quality is optimized then the beneficiary is satisfied in all the aspects for long term.

### **Academic, Scientific and/or Innovation Significance**

The following are the significance of this research work

- In depth research study is needed about the coherence, satisfactory, feasibility, cost and time effectiveness of the existing solution of allocation and scheduling problems with fuzziness.
- The discrete math course is offered for advanced diploma students. This research work seeks the attention of the students those studied or studying discrete math course in their advance diploma. Also, it motivates them to do their final year projects that are more related with discrete mathematics.
- This results the novel scientific approach to the entire decision making problem with fuzziness.
- The results of this research work can be applied to the problems in which fuzziness, allocation and scheduling together taken part.

### **Is this project going to result in a patent?**

No

### **Patent Review (e.g. any previous similar patents in literature, the potential of this project to result in a patent ...)**

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### Budget Summary:

Administration cost	250	250	500
Travel (Conference)	1,000	1,000	2,000
Dissemination	500	0	500
Equipment and Facilities	400	0	400
Research Assistants (Post Graduate)	600	600	1,200

**Duration in months**

24

**Overall TRC Requested Funding (OMR)**

4,600.00