1805004

To prove, TSP is NP-complete. We have to prove that -

(i) TSP $\in$ NP

(ii) Taking an NP-complete problem (Hamiltonian cycle), (given)

we will show that (hamiltonian cycle) $\leq_P$ (TSP)

So, we have to reduce hamiltonian cycle problem to TSP. This will prove TSP is NP-hard.

~~Proof~~

~~(i)~~

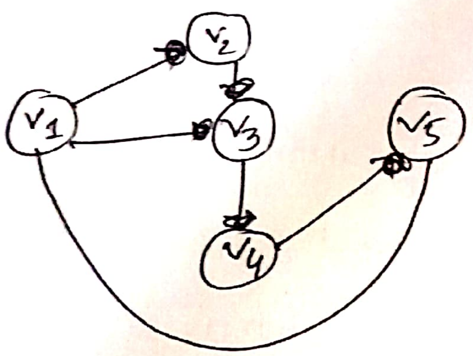
<u>Decision version of TSP:</u> Given a set of cities and is there any tour of length at most k where every city is visited exactly once and returns to start point.

<u>proof:</u>

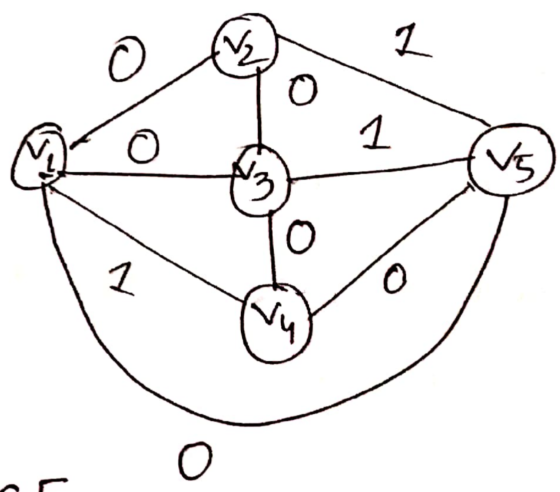
(i) If we are given a graph G and we have to verify the 'Yes' answer where we are given a threshold length k of tour. We can easily verify 'Yes' answer in polynomial time by traversing the graph. So, as we can verify 'Yes' answer with proper certificate in polynomial time, tsp belongs to NP.

(11) To. reduce hamiltonian cycle to tsp. we first formulate the graph. For hamiltonian cycle, we are given an ~~g~~ undirected unweighted graph G. We will construct G' where G' will be an ~~gra~~ undirected weighted graph. We will take the edges those belong to G and give them the weight 0. and the missing edges of G in G' with the weight 1.

G



G'



$\boxed{G = (V, E)}$

$\boxed{G' = (V, E')}$

$$\text{for } E',\ d(u, v) = \begin{cases} 0 & ; \text{ if } (u, v) \in E \\ 1 & ; (u, v) \notin E \end{cases}$$

Now, we will prove that

(a) if hamiltonian cycle exists in G, then G' has a tsp ~~path~~ tour having a length at most 0.

(b) if ~~tsp~~ 'G' has a tsp tour of length at most 0, then hamiltonian cycle exists.

(a) hamiltonian cycle exists in G. Now, this cycle will only have 0 edges in G′ as the edges E in G has 0 weight in E′ in G′. So, the tour has a length of at most 0. Second thing, as hamiltonian cycle traverses every ~~edge~~ vertex exactly once (except the start), then tsp tour in G′ also traverses each vertex and returns to start. Thus, the proof completes.

(b) Now, we have tsp tour of length at most 0 in G′. As, we have taken only 0 weighted edges in G′, then in G, these edges ~~belongs to~~ will definitely belong to as we ~~have~~ give 0 weight to those edges only which belong to G. ~~So,~~ thus, ~~the~~ we conclude that, if tsp tour of length 0 exists in G′, then G must have hamiltonian cycle.

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