

Proving Trigonometric Formulas using Euler's Formula: $e^{ix} = \cos(x) + i \sin(x)$

$$\sin^2(x) + \cos^2(x) = 1$$

$$e^{ix} = \cos x + i \sin x$$

$$e^{-ix} = e^{i(-x)} = \cos(-x) + i \sin(-x) = \cos(x) - i \sin(x)$$

$$e^{ix} \cdot e^{-ix} = (\cos x + i \sin x)(\cos x - i \sin x)$$

$$1 = \cos^2 x - i \cos x \sin x + i \sin x \cos x - i^2 \sin^2 x$$

$$1 = \cos^2(x) + \sin^2(x)$$

We are going to prove some formulas from trigonometry, directly from Euler's formula, $e^{ix} = \cos(x) + i \sin(x)$.

First, we are going to prove that $\sin^2(x) + \cos^2(x) = 1$.

This can be proven from the Pythagorean theorem and the definitions of the trigonometric functions, but we are going to do it using Euler's formula.

First, use the fact that e^{ix} , we know that's $\cos(x) + i \sin(x)$.

We also use the fact that, we've got $e^{i(-x)}$ is $e^{i(-x)}$ that's the same thing as $\cos(-x) + i \sin(-x)$.

Furthermore, since \cos is an even function, $\cos(-x)$ is $\cos(x)$, and since \sin is an odd function, $\sin(-x)$ is negative $\sin(x)$.

We write the negative out in front of the i .

Next, we are going to multiply together these two equations right here.

We have $e^{ix} \cdot e^{-ix}$ is equal to $(\cos(x) + i \sin(x))$ times the quantity $(\cos(x) - i \sin(x))$.

Baseline

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$$e^{ix} \cdot e^{-ix} = (\cos x + i \sin x)(\cos x - i \sin x)$$

$$1 = \cos^2 x - i \cos x \sin x + i \sin x \cos x - i^2 \sin^2 x$$

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We are going to prove some formulas from trigonometry, directly from Euler's formula, $e^{ix} = \cos(x) + i \sin(x)$.

$$\sin^2(x) + \cos^2(x) = 1$$

This can be proven from the Pythagorean theorem and the definitions of the trigonometric functions, but we are going to do it using Euler's formula.

$$e^{ix} = \cos x + i \sin x$$

$$e^{-ix} = e^{i(-x)} = \cos(-x) + i \sin(-x) = \cos(x) - i \sin(x)$$

Next, we are going to multiply together these two equations right here.

$$e^{ix} \cdot e^{-ix} = (\cos x + i \sin x)(\cos x - i \sin x)$$

Now, on the left hand side, we have the same base. These exponents we will add them together, get zero for an exponent, and e^0 is 1.

$$1 = \cos^2 x - i \cos x \sin x + i \sin x \cos x - i^2 \sin^2 x$$

Visual Transcript

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$$e^{ix} = \cos x + i \sin x$$

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$$e^{ix} \cdot e^{-ix} = (\cos x + i \sin x)(\cos x - i \sin x)$$

$$1 = \cos^2 x - i \cos x \sin x + i \sin x \cos x - i^2 \sin^2 x$$

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Angle sum formula

$$e^{ia} = \cos a + i \sin a$$

$$e^{ib} = \cos b + i \sin b$$

$$e^{ia} e^{ib} = (\cos a + i \sin a)(\cos b + i \sin b)$$

$$e^{i(a+b)} = \frac{\cos a \cos b - \sin a \sin b}{(\cos a \sin b + \sin a \cos b) i}$$

$$e^{i(a+b)} = \cos(a+b) + i \sin(a+b)$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a+b) = \cos a \sin b + \sin a \cos b$$

Let $a=x$
 $b=x$: $\cos(2x) = \cos^2(x) - \sin^2(x)$
 $\sin(2x) = 2 \sin(x) \cos(x)$

NoteVideo

