Indefinite integrals of x raised to a power

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$$\frac{d}{dx}\left[\frac{x^{n+1}}{n+1}+C\right]$$

$$n \neq -1$$

And we're going to assume here, because we want this expression to be defined, we're going to assume that n does not equal negative 1. If it equaled negative 1, we'd be dividing by 0, and we haven't defined what that means. So let's take the derivative here.

$$\frac{d}{dx}\left[\frac{x^{n+1}}{n+1}+C\right] = \frac{(n+1)}{n+1}x^{n} + 0 = x^{n} \quad *$$

So the derivative of this thing-- and this is a very general terms-- is equal to x to the n. So given that, what is the antiderivative-- let me switch colors here. What is the antiderivative of x to the n?

$$\boxed{\underbrace{\int x^{n}\,dx}_{\text{indef. integral}} = \frac{x^{n+1}}{n+1}+C} \qquad n \neq -1$$

n does not equal negative 1. Once again, this thing would be undefined if n were equal to negative 1. So let's do a couple of examples just to apply this-- you could call it the reverse power rule if you want, or the anti-power rule.

$$\int x^{5}\,dx$$

What is the antiderivative of x to the fifth? Well, all we have to say is, well, look, the 5 is equal to the n.

$$\boxed{\underbrace{\int x^{n}\,dx}_{\text{indef. integral}} = \frac{x^{n+1}}{n+1}+C} \qquad n \neq -1$$

We just have to increment the exponent by 1.

$$\int x^{5}\,dx = \frac{x^{5+1}}{5+1}+C$$

$$= \frac{x^{6}}{6}+C$$

And you can verify. Take the derivative of this using the power rule, you indeed get x to the fifth. Let's try another one. Let's try-- now we'll do it in blue.

$$\int 5x^{-2}\,dx = 5\int x^{-2}\,dx$$

$$= 5\left(\frac{x^{-2+1}}{-2+1}+C\right)$$

And now we can just use, I guess we could call it this anti-power rule, so this is going to be equal to 5 times x to the negative 2 power plus 1 over the negative 2 power plus 1 plus some constant right over here.

$$= 5\left(\frac{x^{-1}}{-1}+C\right)$$

$$= 5\left(-x^{-1}+C\right)$$

And then if we want, we can distribute the 5.

$$= -5x^{-1} + \because$$

Now, we could write plus 5 times some constant, but this is just an arbitrary constant. So this is still just an arbitrary constant. So maybe we could [INAUDIBLE] this.

$$= 5\left(\frac{x^{-2+1}}{-2+1}+C_1\right)$$

$$= 5\left(\frac{x^{-1}}{-1}+C_1\right)$$

$$= 5\left(-x^{-1}+C_1\right)$$

If you want it to show that it's a different constant, you could say this is c1, c1, c1. You multiply 5 times c1, you get another constant.

$$= \boxed{-5x^{-1}+C}$$

$$\int 5x^{-2}\,dx = 5\int x^{-2}\,dx$$

And once again, all of these, try to evaluate the derivative, and you will see that you get this business, right over there.

Fundamental theorem of calculus

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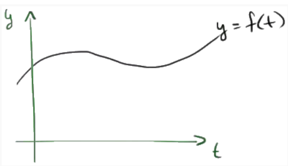
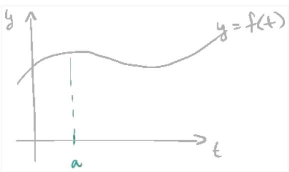
f continuous on $[a, b]$

And I have these brackets here, so it also includes a and b in the interval. So let me graph this just so we get sense of what I'm talking about.



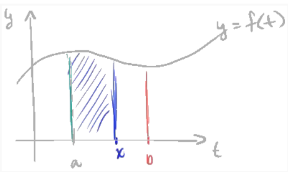
f continuous on $[a, b]$



Now our lower endpoint is a, so that's a right over there.

f continuous on $[a, b]$

Our upper boundary is b. Let me make that clear.



Well, how do we denote the area under the curve between two endpoints? Well, we just use our definite integral. That's our Riemann integral. It's really that right now before we come up with the conclusion of this video, it really just represents the area under the curve between two endpoints. So this right over here, we can say is the definite integral from a to x of f(t)dt.

$$F(x) = \int_a^x f(t)\,dt, \text{ where } x \text{ in } [a,b]$$

So all fair and good. Uppercase F(x) is a function. If you give me an x value that's between a and b, it'll tell you the area under lowercase f(t) between a and x. Now the cool part, the fundamental theorem of calculus. The fundamental theorem of calculus tells us-- let me write this down because this is a big deal.

Fund. theorem of calculus

$$\boxed{\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)}$$

Well, it tells us that for any continuous function f, if I define a function, that is, the area under the curve between a and x right over here, that the derivative of that function is going to be f. So let me make it clear.

- Every cont. f has an antiderivative F(x)
- Connection between derivatives / integration

Now we see it has a connection to derivatives. Well, how would you actually use the fundamental theorem of calculus? Well, maybe in the context of a calculus class. And we'll do the intuition for why this happens or why this is true and maybe a proof in later videos. But how would you actually apply this right over here? Well, let's say someone told you that they want to find the derivative. Let me do this in a new color just to show this is an example.

$$\frac{d}{dx}\left(\int_{\underset{a}{\pi}}^{x} \overbrace{\frac{\cos^2 t}{\ln(t-\sqrt{t})}}^{f(t)}\,dt\right) = \overbrace{\frac{\cos^2 x}{\ln(x-\sqrt{x})}}^{f(x)} \checkmark$$

And notice, it doesn't matter what the lower boundary of a actually is. You don't have anything on the right hand side that is in some way dependent on a. Anyway, hope you enjoyed that. And in the next few videos, we'll think about the intuition and do more examples making use of the fundamental theorem of calculus.