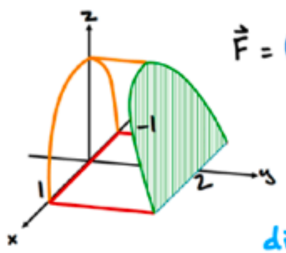
Divergence Theorem – Example



$0 \le y \le 2-z$

$0 \le z \le 1-x^2$

$-1 \le x \le 1$

$$\vec{F} = \left(\tfrac{1}{2}x^2 + e^{\cos z}y\right)i + \left(yx + \ln|z|\right)j + \tan(xy)k$$

$$\iint_S \vec{F}\, ds = ? \quad = \iiint_R \mathrm{div}\vec{F}\, dV \qquad \text{(a)}$$

$$\mathrm{div}\vec{F} = x + x + 0 = 2x \qquad \text{(b)}$$

$$= \int_{-1}^{1}\int_0^{1-x^2}\int_0^{2-z} 2x\, dy\, dz\, dx \qquad \text{(c)}$$

$$= \int_{-1}^{1}\int_0^{1-x^2} 2x(2-z)\, dz\, dx \qquad \text{(d)}$$

(a) $$\iint_S F\, ds = ? \quad = \iiint_R \mathrm{div}\vec{F}\, dV$$

(b) $$\mathrm{div}\vec{F} = \frac{d}{dx}\left(\frac{1}{2}x^2 + e^{\cos z}y\right) + \frac{d}{dy}\left(yx + \ln|z|\right) + \frac{d}{dz}\tan(xy)$$

$$= x + x + 0 = 2x$$

(c) $$= \int_{-1}^{1}\int_0^{1-x^2}\int_0^{2-z} 2x\, dy\, dz\, dx$$

(d) $$= \int_{-1}^{1}\int_0^{1-x^2} 2x(2-z)\, dz\, dx$$

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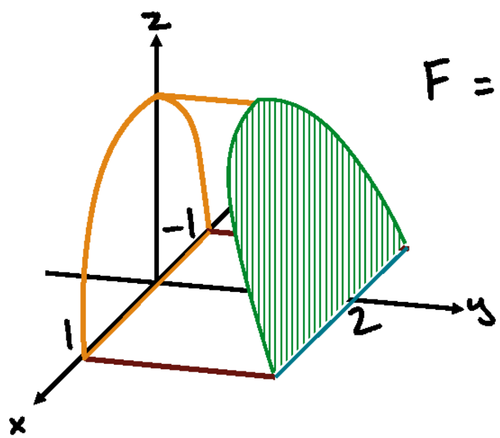

$$F = \left(\tfrac{1}{2}x^2 + e^{\cos zy}\right)i + \left(yx + \ln|z|\right)j + \tan(xy)\,k$$

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$$0 \le y \le 2 - z$$

$$0 \le z \le 1 - x^2$$

$$-1 \le x \le 1$$

$$= \int_{-1}^{1}\int_{0}^{1-x^2}\int_{0}^{2-z} 2x \; dy\; dz\; dx$$

point out range of x,y,z in graph

$$= \int_{-1}^{1}\int_{0}^{1-x^2} 2x(2-z)\; dz\; dx$$

explain surface integral volume integral

$$= \int_{-1}^{1} \left(3x - 2x^3 - x^5\right) dx \; = 0$$

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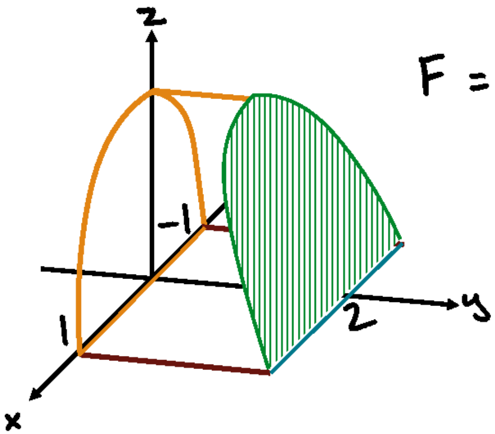

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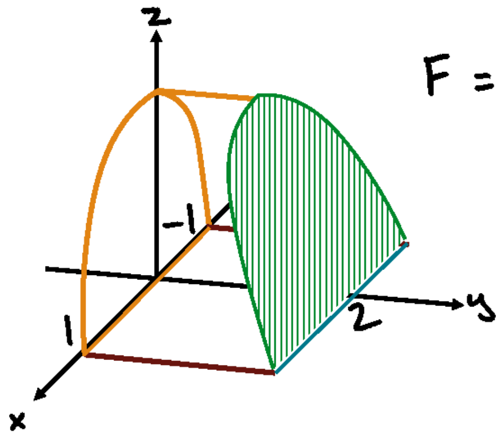

$$F = \left(\frac{1}{2}x^2 + e^{\cos zy}\right)i + \left(yx + \ln|z|\right)j + \tan(xy)k$$

$$\iint_S \vec{F}\, ds =$$

$$\int_{-1}^{1}\int_{0}^{1-x^2}\int_{0}^{2-z} 2x \, dy \, dz \, dx$$

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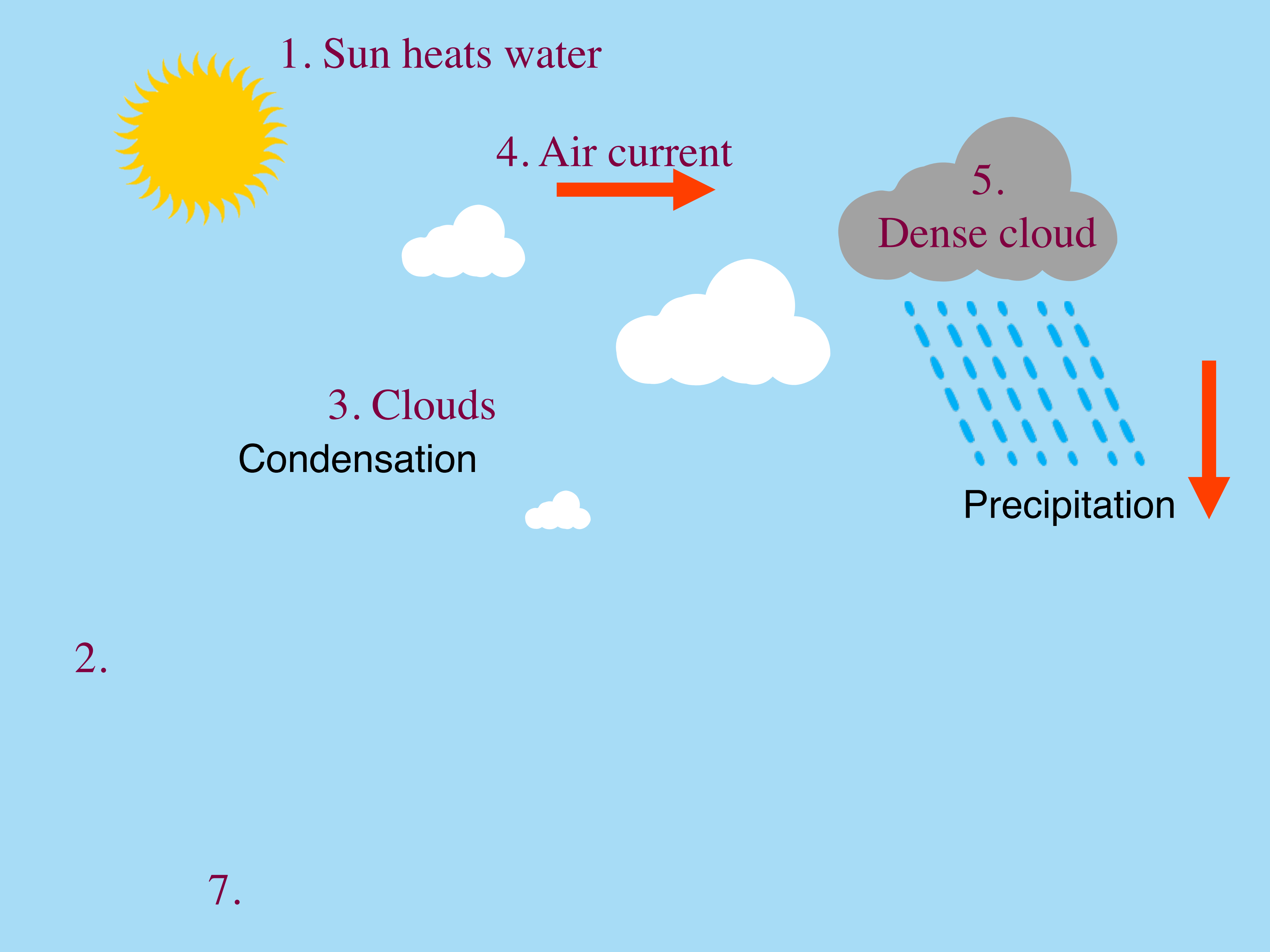
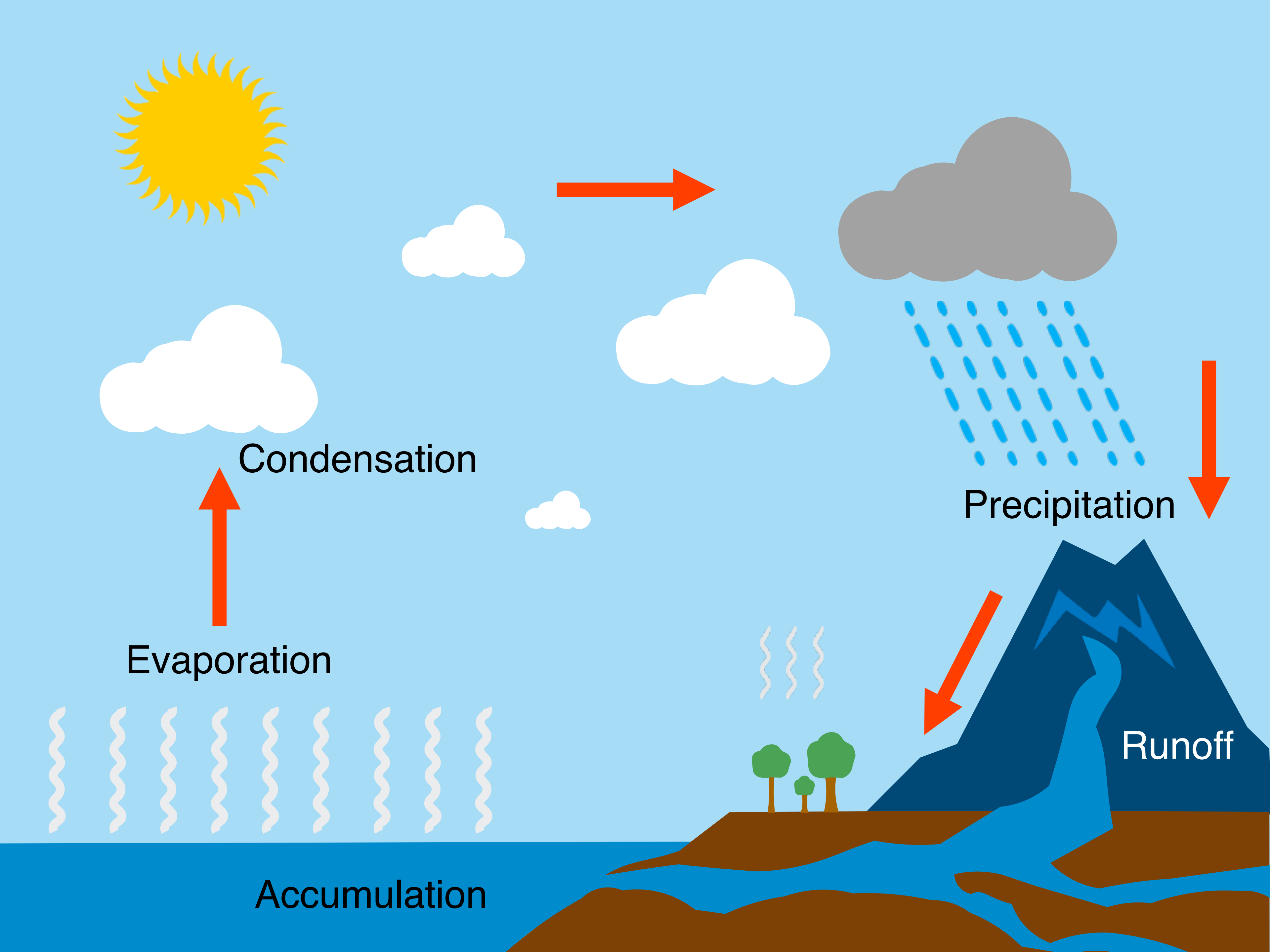
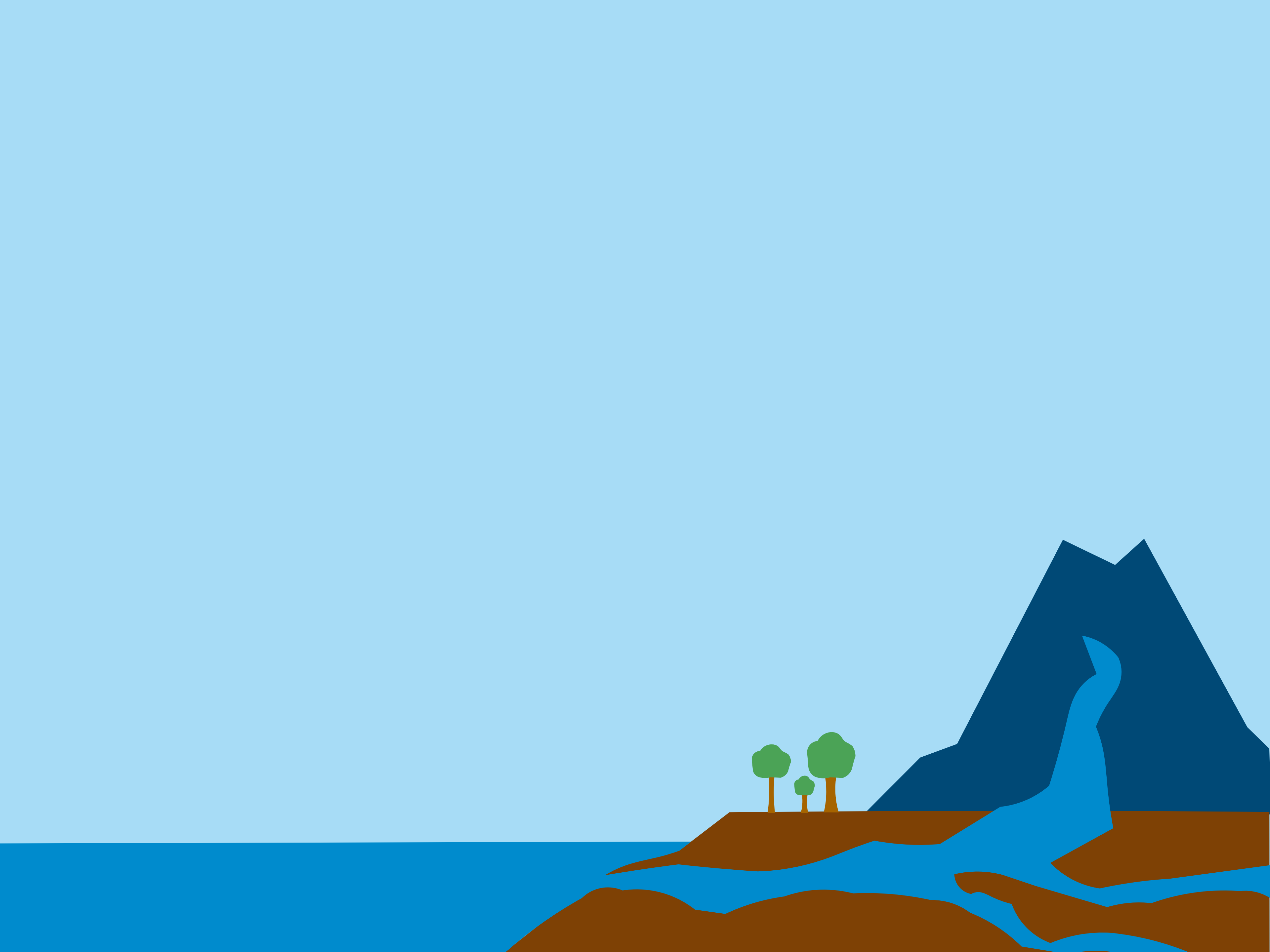
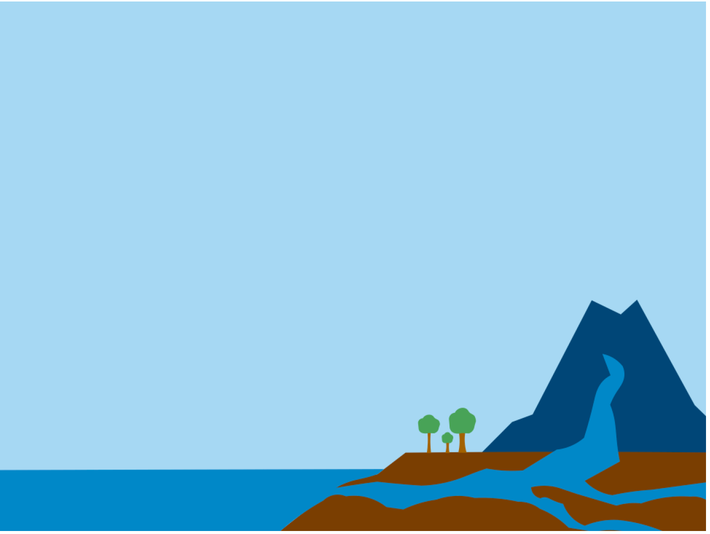
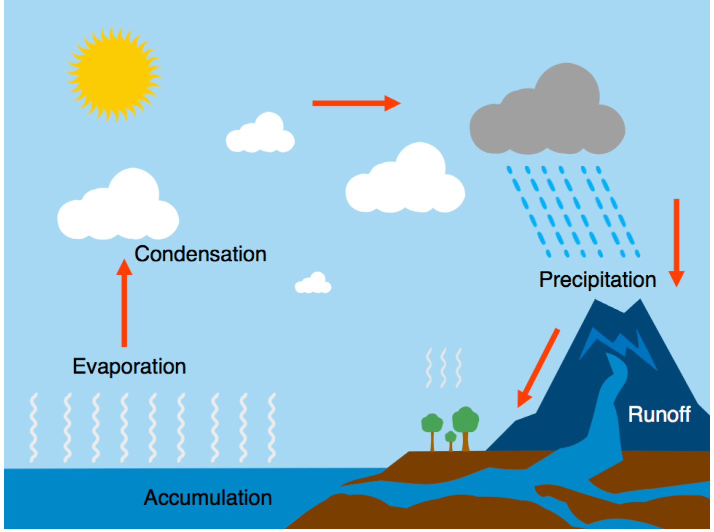
$$= x + x + 0 = 2x$$


1. Sun heats water
4. Air current
5.
Dense cloud
3. Clouds
Condensation
Precipitation
2.
7.


Condensation
Precipitation
Evaporation
Runoff
Accumulation



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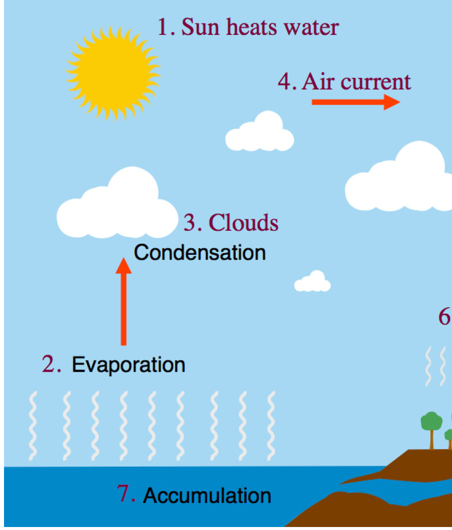
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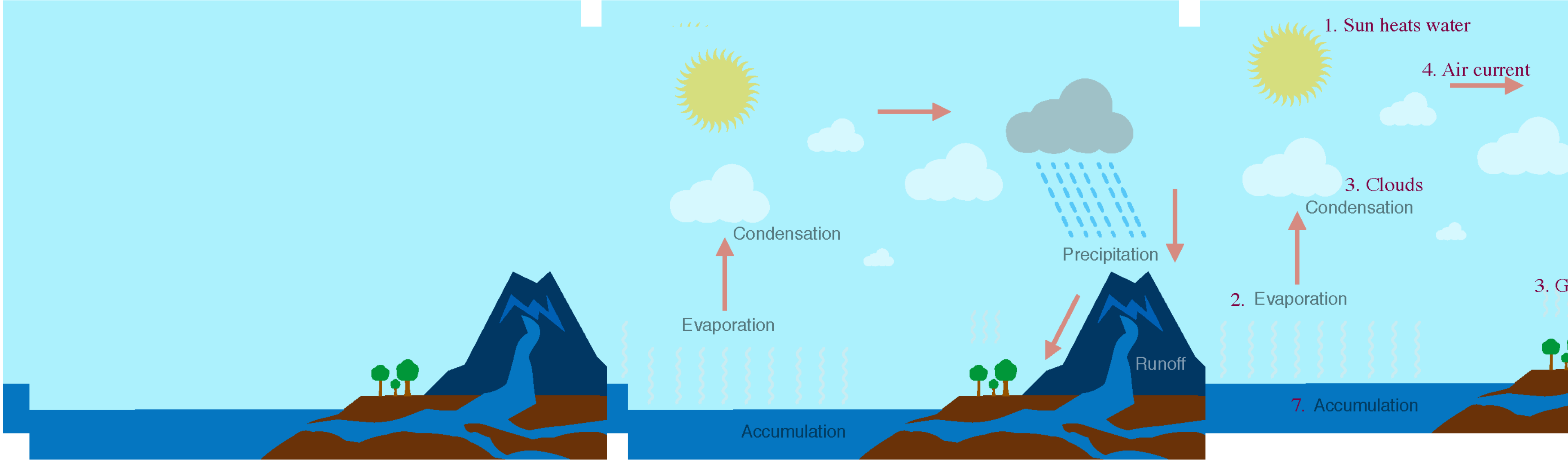
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