Indefinite integrals of x raised to a power – Khan Academy

$$\frac{d}{dx}\left[\frac{x^{n+1}}{n+1}+c\right]$$

$$n \neq -1$$

And we're going to assume here, because we want this expression to be defined, we're going to assume that n does not equal negative 1. If it equaled negative 1, we'd be dividing by 0, and we haven't defined what that means. So let's take the derivative here.

$$\frac{d}{dx}\left[\frac{x^{n+1}}{n+1}+c\right] = \frac{(n+1)x^{n}}{n+1} + 0 = x^{n} \quad *$$

So the derivative of this thing-- and this is a very general terms-- is equal to x to the n. So given that, what is the antiderivative-- let me switch colors here. What is the antiderivative of x to the n?

$$\boxed{\underbrace{\int x^n\,dx}_{\text{in def. integral}} = \frac{x^{n+1}}{n+1} + c} \qquad n \neq -1$$

n does not equal negative 1. Once again, this thing would be undefined if n were equal to negative 1. So let's do a couple of examples just to apply this-- you could call it the reverse power rule if you want, or the anti-power rule.

$$\int x^5\,dx$$

What is the antiderivative of x to the fifth? Well, all we have to say is, well, look, the 5 is equal to the n.

$$\boxed{\underbrace{\int x^{n}\,dx}_{\text{in def. integral}} = \frac{x^{n+1}}{n+1} + c} \qquad n \neq -1$$

We just have to increment the exponent by 1.

$$\int x^5\,dx = \frac{x^{5+1}}{5+1} + C$$

$$= \frac{x^6}{6} + C$$

And you can verify. Take the derivative of this using the power rule, you indeed get x to the fifth. Let's try another one. Let's try-- now we'll do it in blue.

$$\int 5x^{-2}\,dx = 5\int x^{-2}\,dx$$

$$= 5\left(\frac{x^{-}}{-2+1} + c\right)$$

And now we can just use, I guess we could call it this anti-power rule, so this is going to be equal to 5 times x to the negative 2 power plus 1 over the negative 2 power plus 1 plus some constant right over here.

$$= 5\left(\frac{x^{-1}}{-1} + c\right)$$

$$= 5\left(-x^{-1} + c\right)$$

And then if we want, we can distribute the 5.

$$= -5x^{-1} + \vdots$$

Now, we could write plus 5 times some constant, but this is just an arbitrary constant. So this is still just an arbitrary constant. So maybe we could [INAUDIBLE] this.

$$= 5\left(\frac{x^{-2+1}}{-2+1} + c_1\right)$$

$$= 5\left(\frac{x^{-1}}{-1} + c_1\right)$$

$$= 5\left(-x^{-1} + c_1\right)$$

If you want it to show that it's a different constant, you could say this is c1, c1, c1. You multiply 5 times c1, you get another constant.

$$= \boxed{-5x^{-1} + C}$$

$$\int 5x^{-2}\,dx = 5\int x^{-2}\,dx$$

And once again, all of these, try to evaluate the derivative, and you will see that you get this business, right over there.