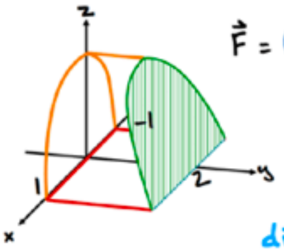
Divergence Theorem – Example



$$\vec{F} = \left(\tfrac{1}{2}x^2 + e^{\cos z}y\right)i + \left(yx + \ln|z|\right)j + \tan(xy)k$$

$0 \le y \le 2-z$

$0 \le z \le 1-x^2$

$-1 \le x \le 1$

$$\iint_S \vec{F}\, ds = ? \quad = \iiint_R \operatorname{div}\vec{F}\, dV \qquad \text{a}$$

$$\operatorname{div}\vec{F} = x + x + 0 = 2x \qquad \text{b}$$

$$= \int_{-1}^{1}\int_{0}^{1-x^2}\int_{0}^{2-z} 2x \, dy\, dz\, dx \qquad \text{c}$$

$$= \int_{-1}^{1}\int_{0}^{1-x^2} 2x(2-z)\, dz\, dx \qquad \text{d}$$

a

$$\iint_S F\, ds = ? \quad = \iiint_R \operatorname{div}\vec{F}\, dV$$

b

$$\operatorname{div}\vec{F} = \frac{d}{dx}\left(\tfrac{1}{2}x^2 + e^{\cos z}y\right) + \frac{d}{dy}\left(yx + \ln|z|\right) + \frac{d}{dz}\tan(xy)$$

$$= x + x + 0 = 2x$$

c

$$= \int_{-1}^{1}\int_{0}^{1-x^2}\int_{0}^{2-z} 2x\, dy\, dz\, dx$$

d

$$= \int_{-1}^{1}\int_{0}^{1-x^2} 2x(2-z)\, dz\, dx$$