

Divergence Theorem - Example

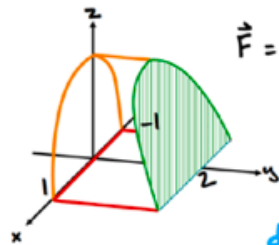
$$\vec{F} = \left(\frac{1}{2}x^2 + e^{\cos z}y\right)\mathbf{i} + (yx + \ln|z|)\mathbf{j} + \tan(xy)\mathbf{k}$$

$$\iint_S \vec{F} \cdot d\vec{s} = ? = \iiint_R \text{div} \vec{F} \, dV \quad \text{a}$$

$$\text{div} \vec{F} = x + x + 0 = 2x \quad \text{b}$$

$$= \int_{-1}^1 \int_0^{1-x^2} \int_0^{2-z} 2x \, dy \, dz \, dx \quad \text{c}$$

$$= \int_{-1}^1 \int_0^{1-x^2} 2x(2-z) \, dz \, dx \quad \text{d}$$



$$0 \leq y \leq 2 - x^2$$

$$0 \leq z \leq 2 - x^2$$

$$-1 \leq x \leq 1$$

$$\text{a} \quad \iint_S \vec{F} \cdot d\vec{s} = ? = \iiint_R \text{div} \vec{F} \, dV$$

$$\text{b} \quad \text{div} \vec{F} = \frac{d}{dx} \left(\frac{1}{2}x^2 + e^{\cos z}y \right) + \frac{d}{dy} (yx + \ln|z|) + \frac{d}{dz} \tan(xy) \\ = x + x + 0 = 2x$$

$$\text{c} \quad = \int_{-1}^1 \int_0^{1-x^2} \int_0^{2-z} 2x \, dy \, dz \, dx$$

$$\text{d} \quad = \int_{-1}^1 \int_0^{1-x^2} 2x(2-z) \, dz \, dx$$