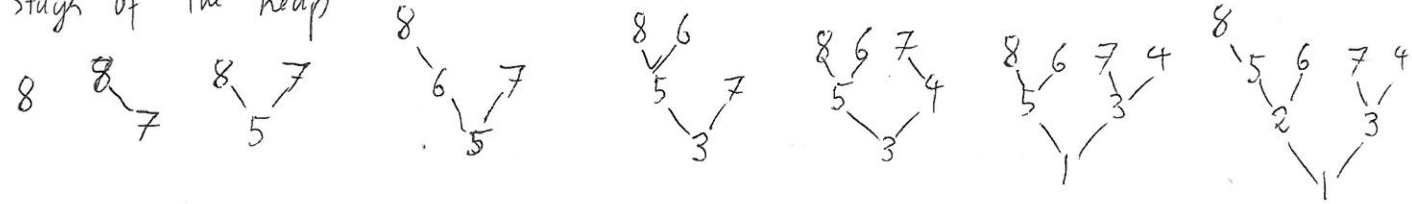
CSc 220 Algorithms Midterm Exam Sample Solution

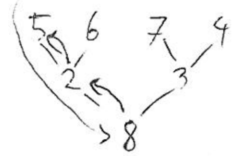
10/11/2017

(1) $f(n) = n!$ does not satisfy $f(n+1) \le C_1 f(n)$ for any C_1, so $f(n+1) \ne O(f(n))$

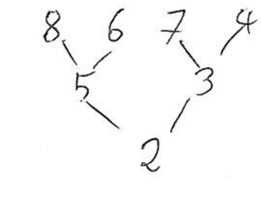
(2) Stages of the heap



deletemin



result



(3) Have: $f(n) \le \sum_{i=1}^{n-1} \frac{1}{i} f(i) + C_1$ for some given C_1

Want: $f(n) \le A \cdot n$ for some A (we choose)

Inductive Assumption: $f(k) \le Ak$ for $k < n$, so $f(i) \le A \cdot i$ for $i = 1 \cdots n-1$

so $f(n) \le \sum_{i=1}^{n-1} \frac{1}{i} f(i) + C_1 \le \sum_{i=1}^{n-1} \frac{1}{i} A i + C_1 = (n-1) \cdot A + C_1 \le n \cdot A$ for $A \geq C_1$ ✓

(4) Have: $f(n) \le \frac{1}{2} n^2 f(\frac{1}{2} n)$

Want: $f(n) \le C_1 \cdot 2^{(\log n)^2}$ for some C_1

Inductive Assumption: $f(k) \le C_1 \cdot 2^{(\log k)^2}$ for all $k < n$,

so $f(\frac{1}{2} n) \le C_1 \cdot 2^{(\log \frac{1}{2} n)^2} = C_1 2^{(\log n - 1)^2} = C_1 2^{(\log n)^2 - 2(\log n) + 1} = C_1 2^{(\log n)^2} \cdot \frac{1}{n^2} \cdot 2$

then $f(n) \le \frac{1}{2} n^2 f(\frac{1}{2} n) \le \frac{1}{2} \cdot n^2 \cdot C_1 \cdot 2^{(\log n)^2} \cdot \frac{1}{n^2} \cdot 2 = C_1 2^{(\log n)^2}$ ✓

(5) list of comparisons (1,2), (5,2), (5,3), (5,4), (5,6), (7,6), (7,10), (8,10), (9,10), (12,10)

(6)

0	1	2	3	4	5	6	7	8	9
90	91	42	13	44	15		07	98	89
00	51	92	73				37		09
10	01		23						29
	21								

sequence after first round 90 00 10 91 51 01 21 42 92 13 73 23 44 15 07 37 98 89 09 29

0	1	2	3	4	5	6	7	8	9
00	10	21	37	42	51		73	89	90
01	13	23		44					91
07	15	29							92
09									98

sequence after second round 00 01 07 09 10 13 15 21 23 29 37 42 44 51 73 89 90 91 92 98

(7)

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int lowerdigit (int i, int n) { return (i % n); }
int upperdigit (int i, int n) { return (i / n); }
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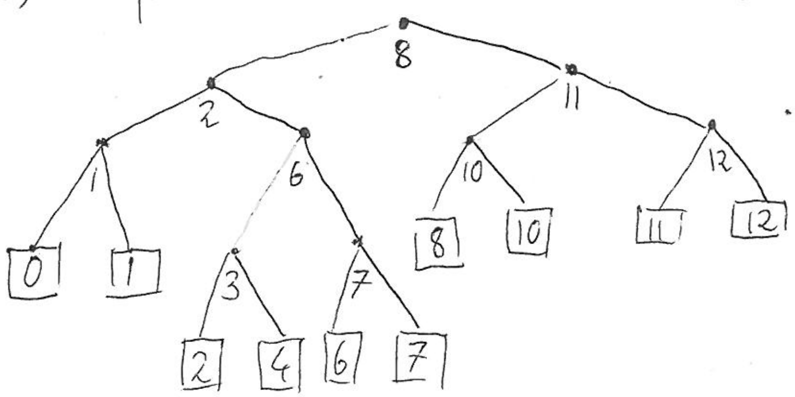
(8) after insert 10 and rebalance (needs double rotation)



after delete 1 and rebalance (needs single rotation)