Final Exam Algorithms 220 Fall 2017 Sample Solution

12/11/2017

1. Example of function f with $f(n+1) - f(n) = \Theta(n^2)$ is $f(n) = n^3$

$$(n+1)^3 - n^3 = n^3 + 3n^2 + 3n + 1 - n^3 = 3n^2 + 3n + 1 = \Theta(n^2)$$

2. Have $f(n) \leq (\log n) * f(\frac{1}{2}n)$

Want $f(n) \leq C' * (\log n)^{(\log n)}$

Inductive Assumption $f(k) \leq C * (\log k)^{(\log k)}$, for $k < n$; so $f(\frac{1}{2}n) \leq C' (\log(\frac{1}{2}n))^{(\log(\frac{1}{2}n))} = C' (\log(\frac{1}{2}n))^{(\log n - 1)}$

so $f(n) \leq (\log n) * C' * (\log(\frac{1}{2}n))^{(\log n - 1)} < (\log n) \cdot C' \cdot (\log n)^{(\log n - 1)} = C' (\log n)^{(\log n)}$

3.
```
int largestStep(int *a, int n)
{
    int i, result;
    result = a[1] - a[0];
    for (i = 2; i < n; i++)
        if ((a[i] - a[i-1]) > result)
            result = a[i] - a[i-1];
    return (result);
}
```

4.

item	1	2	3	4	5	6
weight	4	3	2	1	2	3
value	3	3	2	3	3	4

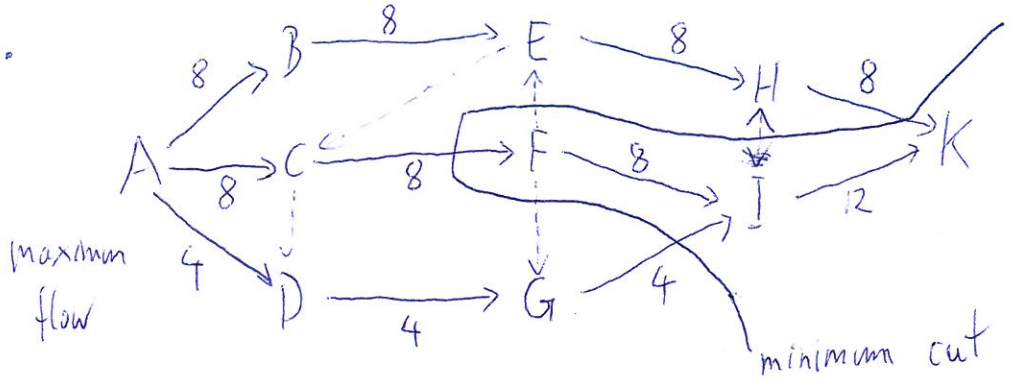
weight limit 6

weight limit \ item	1	2	3	4	5	6
1	0	0	0	3	3	3
2	0	0	2	3	3	3
3	0	3	3	5	6	6
4	3	3	3	6	6	7
5	3	3	5	6	8	8
6	3	3	5	8	9	10

optimum solution items 4, 5, 6

5. Kruskal's Algorithm for Minimum Spanning Tree
chooses edges BC, DE, HI, GH, CE, AB, DG

6.



7.

hash table

0	1	2	3	4	5
3	6	1		5	
				2	

distribution for $a=2$

3	2	5	1	
	6			

distribution for $a=4$

8. a) Given a graph G of n vertices, and a number k, and as additional information a set of k vertices, we can test in $O(k^2) \leq O(n^2)$ whether the set is an independent set: we check for any two vertices in the given set of k vertices that they are not connected by an edge. This is the required checking algorithm for class NP

b)

many other solutions    etc