EM314 - NUMERICAL METHODS ASSIGNMENT - 3

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Theory.

1. $f(x) = \ln(x)$. on interval $[1, 4]$

third order polynomial approximation.

$\therefore$ 4 points.

$x = 1, 2, 3, 4.$

$x = 1 \Rightarrow f(1) = \ln(1) = 0$

$x = 2 \Rightarrow f(2) = \ln(2) = 0.69$

$x = 3 \Rightarrow f(3) = \ln(3) = 1.10$

$x = 4 \Rightarrow f(4) = \ln(4) = 1.39$

lagrange interpolation basis function,

$$l_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$$l_0(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} = \frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)}$$

$$= \frac{(x-2)(x-3)(x-4)}{-6}$$

$$l_1(x) = \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)}$$

$$= \frac{(x-1)(x-3)(x-4)}{2}$$

$$l_2(x) = \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)}$$

$$= \frac{(x-1)(x-2)(x-4)}{-2}$$

$$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$= \frac{(x-1)(x-2)(x-3)}{6}$$

$\therefore$ polynomial $P(x)$,

$$P(x) = \sum_{i=0}^{n} f(x_i) \cdot l_i(x)$$

$$P(x) \approx \ln(x) \approx \ln(1) \cdot l_0(x) + \ln(2) \cdot l_1(x) + \ln(3) \cdot l_2(x) + \ln(4) \cdot l_3 x$$

$$\approx 0 + \frac{\ln(2) \cdot (x-1)(x-3)(x-4)}{2} + \frac{\ln(3)(x-1)(x-2)(x-4)}{-2} + \frac{\ln(4)(x-1)(x-2)(x-3)}{6}$$

$$\approx 0.0264x^3 - 0.3143x^2 + 1.4373x - 1.1514$$

Q2. Consider function $f(x) = 1$.

when $f(x)$ is approximated with lagrange interpolating polynomial it can be expressed as,

$$P(x) = \sum_{i=0}^{n} f(x_i) \cdot l_i(x)$$

since $f(x_i) = 1$ for x.

$$P(x) = \sum_{i=0}^{n} 1 \cdot l_i(x)$$

so $p(x) = f(x) = 1$ polynomial perfectly interpolates any $x_1, x_2, \ldots x_n$ by reason of the interpolating polynomial is unique.

$$\therefore \sum_{i=0}^{n} l_i(x) = 1 \quad \text{for all } x.$$

Implementation

3. (a)

<u>OCTAVE code (LagrangeInterpolant.m)</u>

```
pkg load symbolic;
function LagrangeInterpolant( X , Y )
 syms x
 Px=0;
 len = length(X);
 for i=1:len
      l= x^0;
      for j=1:len
            if i!=j
                  l = simplify(( l*( x-X(j) )/( X(i)-X(j) ) ));
            endif
      endfor
      Px = Px + l*Y(i);
 endfor
 Px = simplify(Px)
 hold on;
 ezplot(Px)
 plot(X,Y,'ob')
endfunction
```

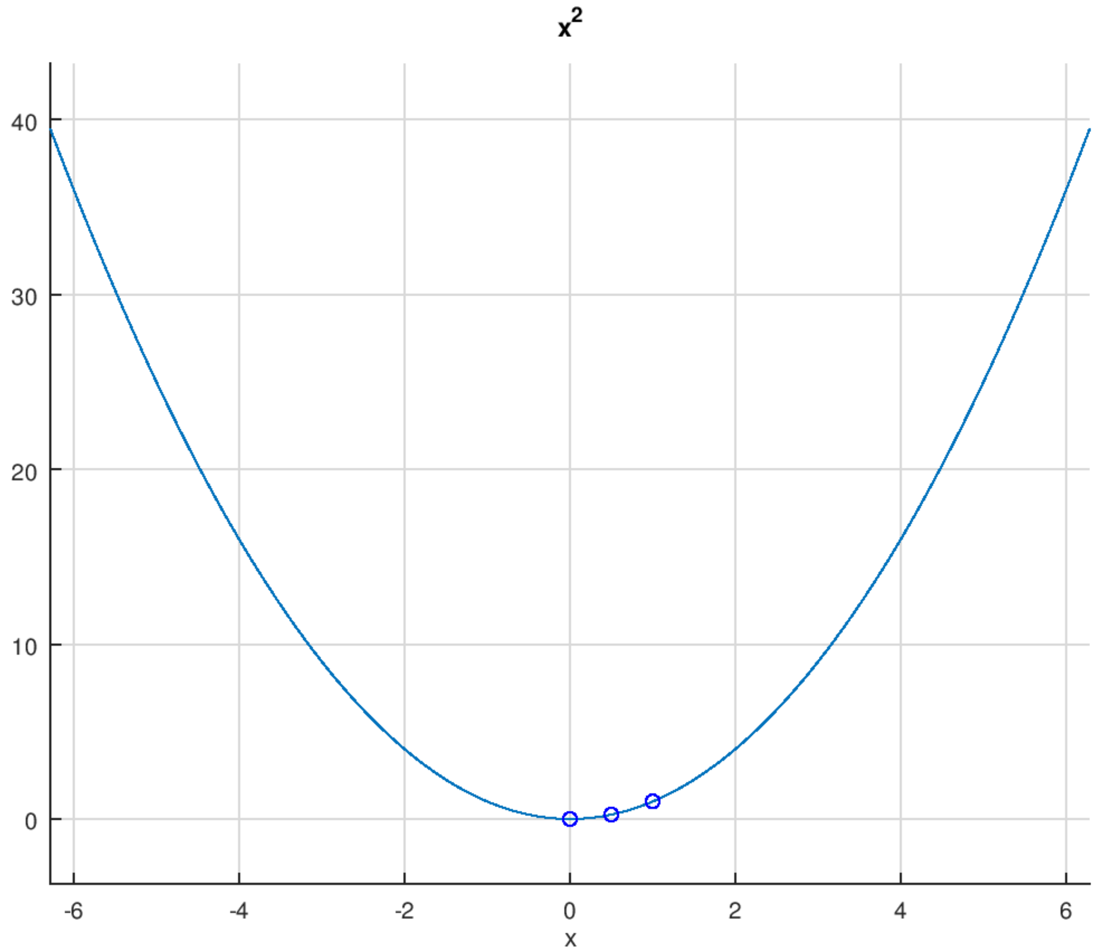
(b) **OCTAVE code (Q3.m)**

```
X = [0 1/2 1];
Y = [0 1/4 1];
LagrangeInterpolant(X, Y);
```

Output of the code

>> Q3

Px = (sym) x^2



Expected answer which $P(x) = x^2$ is obtained.

Q3.
(b)

x	0	$1/2$	1
y	0	$1/4$	1

lagrange interpolating polynomial,

$$l_0(x) = \frac{(x-1/2)(x-1)}{(-1/2)\cdot(-1)} = (2x-1)(x-1)$$

$$l_1(x) = \frac{(x-0)(x-1)}{1/2\cdot(1/2-1)} = -4x(x-1)$$

$$l_2(x) = \frac{(x-0)(x-1/2)}{1\cdot(1-1/2)} = x(2x-1)$$

$$\therefore P(x) = 0\cdot l_0(x) + 1/4\cdot l_1(x) + 1\cdot l_2(x)$$

$$= 0 + \frac{1}{4}(-4x)(x-1) + x(2x-1)$$

$$= -x^2 + x + 2x^2 - x$$

$$= x^2$$

$\therefore$ Expected answer $P(x) = x^2$

4. (b) OCTAVE code (Q4.m)

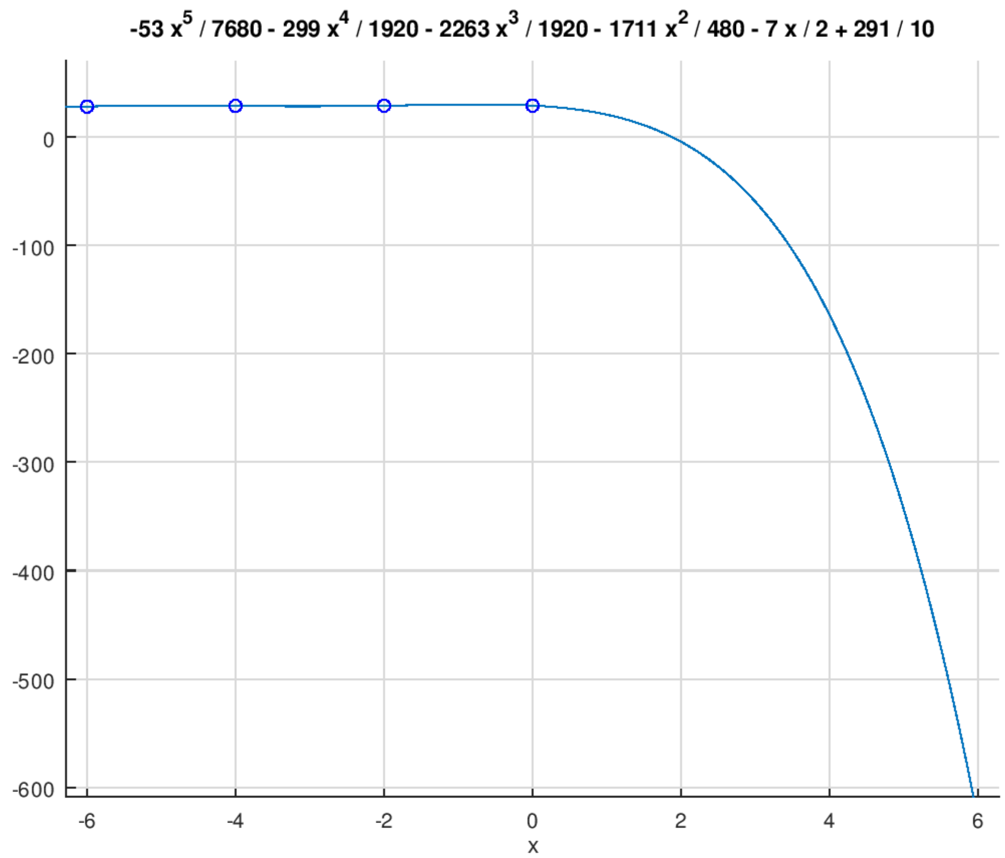
```
pkg load symbolic;
syms x
X = -10:2:0;
Y = [19.1 20.7 28.2 28.7 29 29.1];
Px=0;
len = length(X);

for i=1:len
 l= x^0;
 for j=1:len
  if i!=j
   l = simplify(( l*( x-X(j) )/( X(i)-X(j) ) ));
   endif
  endfor
  Px = Px + l*Y(i);
endfor

Px = simplify(Px)
hold on;
grid on;
ezplot(Px)
plot(X,Y,'ob')

Temp = double(subs(Px,x,-7 ))

p1 = simplify(diff(Px,x));
p2 = simplify(diff(p1,x))

points = real( double(solve(p2==0,x) ) )
x0 = -9:0.2:-1;
y0 = double(subs(p1,x,x0));
y1 = double(subs(p1,x,points));
plot(x0,y0)
plot(points,y1,'ob')
```

Output of the code

>> Q4

Px = (sym)

$-(53/7680)*x^5 - (299/1920)*x^4 - (2263/1920)*x^3 - (1711/480)*x^2 - (7/2)*x^1 - (291/10)$



Temp = 25.291

p1 = (sym) $-(53/1536)*x^4 - (299/480)*x^3 - (2263/640)*x^2 - (1711/240)*x - (7/2)$

p2 = (sym) $-(53/384)*x^3 - (299/160)*x^2 - (2263/320)*x - (1711/240)$

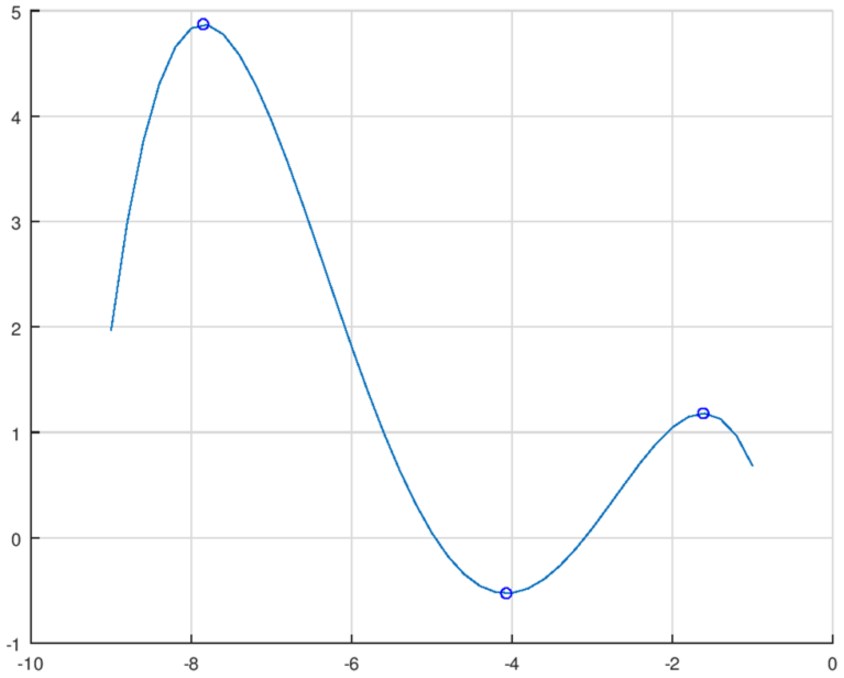
points =

-4.0723

-1.6154

-7.8519

Graph of $\frac{dT}{dz}$



(a) Lagrange interpolating polynomial P(x),

P(x) = - (53/7680)*x^5 - (299/1920)*x^4 - (2263/1920)*x^3 - (1711/480)*x^2 - (7/2)*x^1 (291/10)

(b) Temperature at depth 7m of the lake is 25.291 C.

When comparing with the values in the table it is a valid answer.

(c) The thermocline locates at the maximizer of $\frac{dT}{dz}$,

Critical points of $\frac{dT}{dz}$ are given by $\frac{dT^2}{d^2z} = 0$

$$\frac{dT}{dz} = -(53/1536) * x^4 - (299/480) * x^3 - (2263/640) * x^2 - (1711/240) * x - (7/2)$$

$$\frac{dT^2}{d^2z} = -(53/384) * x^3 - (299/160) * x^2 - (2263/320) * x - (1711/240)$$

Therefore, critical points are (-1.6154), (-4.0723), (-7.8519).

But by the graph $\frac{dT}{dz}$ maximum occurs at x= (-7.8519)

Therefore Thermocline locates at -7.8519m.