EM 314 - Assignment 3

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December 4, 2018

This assignment covers "interpolation".

Theory

1. Suppose you wish to find a polynomial approximation to $f(x) = \ln x$ on the interval [1,4]. Use Lagrange interpolation to find a third order interpolant $p(x) \approx \ln x$. Choose equally spaced points on the interval $[1, 4]$.

2. Let $l_i(x), i = 0, 1, \ldots, n$ be the Lagrange basis functions defined by

$$l_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}.$$

   Prove that $\sum_{i=0}^{n} l_i(x) = 1$ for all x.

Implementation

3. (a) Write a MATLAB/Octave function `LagrangeInterpolant()` that **finds** and **plots** the Lagrange interpolant $p(x)$ for a given data set $\mathbf{x}, \mathbf{y}$. Use "symbolic toolbox" in MATLAB (for Octave: load symbolic package) and print $p(x)$ in a symbolic format. Moreover, on the plot of $p(x)$, denote the given data points by small circles.
   Hint: `syms`, `ezplot()`, `simplify()`.

   (b) Test your code with the following:

$$\mathbf{x} = [0 \quad 1/2 \quad 1], \ \mathbf{y} = [0 \quad 1/4 \quad 1]$$

   Do you obtain the expected answer?

Remark: You may use your code here to solve the next question.

Application

4. (*Note: You may use MATLAB/Octave as necessary to solve this problem.*)
A thermocline in a lake is a thin, distinct layer in which the temperature changes more rapidly with depth, than it does in the layers above or below.
Suppose you wish to find the thermocline in a lake. To this end, assume you have measured the temperature T at different depths z of the lake, as tabulated below:

z (m)	T (℃)
0	29.1
-2	29
-4	28.7
-6	28.2
-8	20.7
-10	19.1

(a) Use Lagrange interpolation to model the relationship $T(z)$.

(b) Predict the temperature at depth $7\,m$ of the lake using part (a). By observing the table, determine the validity of your answer.

(c) The thermocline locates at the maximizer of $T'(z)$. That is

$$\max_z \frac{dT}{dz}$$

Using the approximation in part (a), find the depth z at which the thermocline exists.

Some Remarks

- Recall that the critical points of $\frac{dT}{dz}$ are given by

$$\frac{d^2T}{dz^2} = 0.$$

As $T(z)$ is a 5th order polynomial, $\frac{d^2T}{dz^2}$ is a third order polynomial which might have 3 real roots. If this is the case, select the root at which $\frac{dT}{dz}$ is a maximum.

- To find derivatives, find roots and do simplifications, you may use MATLAB/Octave. For Octave, the associated functions are `diff()`, `fzero()` and `simplify()`. For MATLAB, search the documentation to find the corresponding functions.
- Plotting graphs of $\frac{dT}{dz}$, $\frac{d^2T}{dz^2}$ may help you in solving/validating your results.

Submission Guidelines:

- Produce a document that includes solutions, code, output and figures appropriately. Use a suitable cover page that includes the course title and the course code, assignment number, your name and the registration number.
- Submit your document in **PDF** format, online via **FEeLS**.
- **Due date**: December 17 (Monday), 2018, 11:55 pm. *Please note that marks will be deducted for late submissions.*