

Q2. any $x \in \mathbb{F}$ can be represented as,

$$x = \pm (d_0, \dots, d_{t-1}) \cdot \beta^E$$

the sign bit can only assume 2 values.

d_0 cannot assume 0. therefore it can only assume $\beta-1$ values. but each of digits d_1, d_2, \dots, d_{t-1} can assume β different values. therefore mantissa assumes $(\beta-1) \times \beta^{t-1}$ values.

Range of exponent is $[L, U]$ therefore exponent can assume $(U-L+1)$ values.

∴ number of elements, $= 2 \times (\beta-1) \beta^{t-1} \times (U-L+1)$

∴ set \mathbb{F} contains $2(\beta-1)\beta^{t-1}(U-L+1)$ elements.

Q3. Taylor series.

$$f(x) = \sum_{k=0}^N \frac{1}{k!} f^{(k)}(x_0)(x-x_0)^k + E_{N+1}(f, \xi)$$

$$\text{where } E_{N+1}(f, \xi) = \frac{1}{(N+1)!} f^{(N+1)}(\xi)(x-x_0)^{N+1}$$

$$\therefore f(x+h) = f(x) + h f'(x) + \frac{h^2}{2} f''(\xi) \quad \text{where } x < \xi < x+h$$

$$f'(x) = \frac{f(x+h) - f(x) - \frac{h^2}{2} f''(\xi)}{h}$$

$$= \frac{f(x+h) - f(x)}{h} - \frac{h f''(\xi)}{2} = f'_h(x) - \frac{h f''(\xi)}{2}$$

$$\therefore E_h(x) = |f'(x) - f'_h(x)| = \left| h \frac{f''(\xi)}{2} \right|$$

$$\lim_{h \rightarrow 0} \frac{|E_h(x)|}{|h|} = \lim_{h \rightarrow 0} \left| \frac{f''(\xi)}{2} \right| = \left| \frac{f''(\xi)}{2} \right| = \text{constant.}$$

$$\therefore E_h = O(h)$$