

Theory

Q1. use Taylor series $\sin x$,

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \dots$$

$$f(x) = \sin x.$$

$$a = 0$$

$$\therefore \sin x = 0 + x \cdot \cos(0) + \frac{x^2}{2!} \cdot 0 + \frac{x^3}{3!}(-\cos 0) + \dots$$

$$\sin x \approx 0 + x + 0 + E_3(f, \xi)$$

$$\begin{aligned} |\sin x - x| &= |E_3(f, \xi)| \\ &= \left| \frac{\cos(\xi) \cdot x^3}{3!} \right| \end{aligned}$$

if approximation has to give correct result rounded to six decimal place then.

$$\text{error} < 10^{-6}$$

$$\left| \frac{\cos(\xi) \cdot x^3}{3!} \right| < 10^{-6}$$

$$|\cos \xi \cdot x^3| < 10^{-6} \times 6$$

$$|x^3| < 6 \times 10^{-6}$$

$$\therefore |\cos \xi| < 1$$

$$x < 1.8171 \times 10^{-2}$$

$$x < 0.018171$$