Theory.

1. $f(x) = \ln(x)$. on interval $[1,4]$

third order polynomial approximation.

$\therefore$ 4 points.

$x = 1, 2, 3, 4.$

$x = 1 \Rightarrow f(1) = \ln(1) = 0$

$x = 2 \Rightarrow f(2) = \ln(2) = 0.69$

$x = 3 \Rightarrow f(3) = \ln(3) = 1.10$

$x = 4 \Rightarrow f(4) = \ln(4) = 1.39$

lagrange interpolation basis function,

$$l_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$$l_0(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} = \frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)}$$

$$= \frac{(x-2)(x-3)(x-4)}{-6}$$

$$l_1(x) = \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)}$$

$$= \frac{(x-1)(x-3)(x-4)}{2}$$

$$l_2(x) = \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)}$$

$$= \frac{(x-1)(x-2)(x-4)}{-2}$$