

Notes on "Topoi" by Goldblatt

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Contents

1 Chapter 3: Arrows instead of epsilons	1
1.1 Exercise Solutions	1
1.1.1 3.1.	1

1 Chapter 3: Arrows instead of epsilons

1.1 Exercise Solutions

1.1.1 3.1.

In any category

1. $g \circ f$ is monic if both f and g are monic.
2. If $g \circ f$ is monic, then so is f .

Solutions

(1)

If $(g \circ f) \circ h_1 = (g \circ f) \circ h_2$, then

$$\begin{aligned}(g \circ f) \circ h_1 &= (g \circ f) \circ h_2 \\ g \circ (f \circ h_1) &= g \circ (f \circ h_2) \\ f \circ h_1 &= f \circ h_2 \quad (\because g \text{ is monic}) \\ h_1 &= h_2 \quad (\because f \text{ is monic})\end{aligned}$$

Therefore, $(g \circ f)$ is monic.

(2)

If $f \circ h_1 = f \circ h_2$, then

$$\begin{aligned}g \circ (f \circ h_1) &= g \circ (f \circ h_2) \\ (g \circ f) \circ h_1 &= (g \circ f) \circ h_2 \\ h_1 &= h_2 \quad (\because f \circ g \text{ is monic})\end{aligned}$$

Hence, f is monic.