Notes on "Combinatorics" by Bollobas

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Contents

1 Chapter 2 Representing Sets

Theorem 1.
Let $\mathcal{F} = \{A_1, A_2, \ldots, A_n\}$ be a set system on $X = [n]$. Then there is an element $x \in X$ such that $A_1 - \{x\}, A_2 - \{x\}, \ldots, A_n - \{x\}$ are all distinct. The set system $\mathcal{F} = \{\emptyset, \{1\}, \{2\}, \ldots, \{n\}\}$ shows that such en x need not exist if $|\mathcal{F}| = n + 1$.

Proof1.

Set $\mathcal{D} = \{D \subset X : |\mathcal{F}_{\mathcal{D}}| \geq |D| + 1\}$, where $\mathcal{F}_{\mathcal{D}} = \{D \cap A_i : i \in \mathcal{F}\}$. If $A_1, A_2 \in \mathcal{F}$ and $d \in A_1 \triangle A_2$ then $\{d\} \in \mathcal{D}$ so $\mathcal{D} \neq \emptyset$. Let D be a maximal set in $\mathcal{D}$. Then $|D| \leq n - 2$(??) and $|\mathcal{F}_{\mathcal{D}}| \leq n - 1$ (so, in fact $|\mathcal{F}_{\mathcal{D}}| = n - 1$)

$\boxed{\text{What about the case } D = \{1, 2, \ldots, n-1\}\ \mathcal{F} = \{\{1\}, \{2\}, \ldots, \{n-1\}, \{n\}\}, \text{ where we have } |D| = n-1 \text{ and } |\mathcal{F}_{\mathcal{D}}| = n?}$

When $D = \{1, 2, \ldots, n\}$ $\mathcal{F} = \{\{1\}, \{2\}, \ldots, \{n-1\}, \{n\}\}$,