Pointless Topology 勉強ノート

中村仁宣

2024 年 6 月 13 日˜

1 Preliminary

1.1 Topology トポロジー

Let $\mathcal{P}(X)$ denote the power set of X.

定義 1.1 (Topology トポロジー)**.** *A **topological space** is an ordered pair* (X, τ), $\tau \subseteq \mathcal{P}(X)$ *which satisfies the following properties*

1. $\varnothing \in \tau$ *and* $X \in \tau$.
2. *if* $U, V \in \tau$, *then* $U \cap V \in \tau$.
3. *if* $\forall I, U_i \in \tau$ *forall* $i \in I$, *then* $\bigcup_{i \in I} U_i$.

τ *is called the **topology** of* X. *The members of the topology* $U \in \tau$ *is said to be **open** and* $V \subseteq X$ *is said to be closed if* $\exists U$ *open such that* $V = U^c$.

定義 1.2 (Separation Axioms 分離公理)**.** *A space* (X, τ) *is called* T_i *,if respectively satisfies the following conditions,*

1. T_0*:* $\forall x, y \in X$ $\exists$ *an open set* $U \in \tau$ *such that* U *contains one of* x, y *and not the other.*
2. T_1*:* $\forall x, y \in X$ $\exists$ *a nhood of each not containing the other.*

1.2 Posets, Lattices 半順序集合、束

定義 1.3 (Posets)**.** *A **partial order (**半順序**)** on a set* X *is a binary relation* $R \subseteq X \times X$ *satisfying,*

1. $\forall a, aRa$ *(reflexivity,* 反射律*),*
2. $\forall a, b, c,\ aRb \ \&\ bRc \Rightarrow aRc$ *(transitivity,* 推移律*),*
3. $\forall a, b,\ aRb \ \&\ bRa \Rightarrow a = b$ *(antisymmetry,* 反対称律*).*

if moreover

4. $\forall a, b$ *either* aRb *or* bRa *holds,*

*it is said to be a **linear** or **total** order.*
*A **poset** or **partially ordered set**,* $(X, \leq)$ *is a set with a partial order. If the order of a poset is linear (or total), it is called a **linearly ordered set**, **totally ordered set** or **chain**. A relation that satisfies only (1) and (2) is called **preorder**.*

2 Spaces and Lattices of Open Sets

We will suppose that all topological spaces that appear here will be T_0.

参考文献

[1] Jorge Picado, Aleš Putlr, Frames and Locales:Topology without points, Birkhäuser.