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Designing Wideband RF Impedance Transformers

This simplified design approach can lead to low-cost, low-insertion-loss 1:4 impedance transformers capable of operating over bandwidths as wide as three octaves.

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Impedance matching devices are often used in high-frequency circuits, typically to match the impedance of a device or component to the characteristic impedance of a circuit or system. In some circuits, it is desirable for the impedance matching to achieve multiple-octave frequency coverage accompanied by low insertion loss. To help designers working with impedance transformations, this article explores the design of unbalanced-to-unbalanced (unun) wideband impedance transformers with an impedance ratio of 1:4. Such transformers are useful in radio communications systems, typically in hybrid circuits, signal combin-ers and dividers, and for interstage coupling of amplifier chains.

Such wideband unun impedance transformers are also useful for test circuits, optical receiver systems,[1] microwave circuits with wideband impedance matching,[2] and antenna coupling.[3] Modern computational programs usable for high-frequency circuit design and simulation include this device in their tool boxes.[4] A wideband unun impedance transformer consists of a toroidal ferrite core wrapped with a twisted bifilar transmission line, with the wires isolated by means of an enamel film. By combining design elements of conventional and transmission-line impedance transformers, it is possible to create a true wideband component. The design offers high efficiency for an impedance trans-
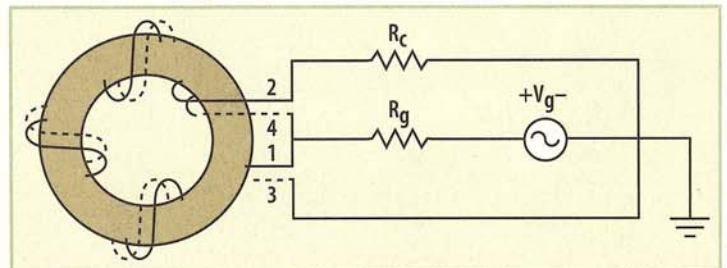
1. This electrical diagram shows a unbalanced-to-unbalanced (unun) transformer with 1:4 impedance transformation ratio.
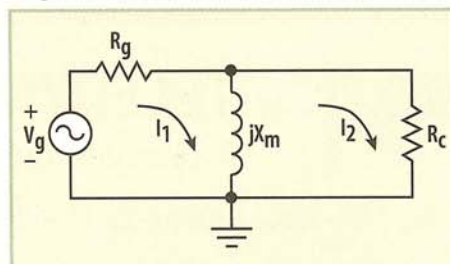

2. Conventional transformers can be analyzed with this low-frequency model.

formation ratio of 1:4 (*Fig. 1*).[5]

In a conventional impedance transformer, the energy transference between the primary and secondary windings occurs mainly through magnetic coupling which also accounts for the transformer's capability of providing good low-frequency response. Assuming a lossless ferrite core and purely resistive load and source impedances, and considering only the influence of its magnetization inductance, the simplified low-frequency model for the transformer can be represented by the structure in *Fig. 2*.[6,7] The low-frequency model response under maximum power transference condition is determined by the device's insertion loss:

$$A_{gc} = \frac{P_g}{P_c} = \frac{R_g^2 + 4X_m^2}{4X_m^2} \quad (1)$$

where:

P_g = the maximum available power of the source,

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P_c = the power delivered to the load, R_g = the source impedance, and X_m = the magnetization reactance. This last parameter is determined by the operating frequency, f, and the core magnetization inductance, L_m, through:

$$X_m = 2\pi f L_m \qquad (2)$$

The value of L_m depends on the number of turns for the primary winding and a core inductance factor, A_l. Usually, this factor is specified by ferrite core manufacturers in nanoHenries/turns squared (nH/turns^2). Therefore, the magnetization inductance in nH is:
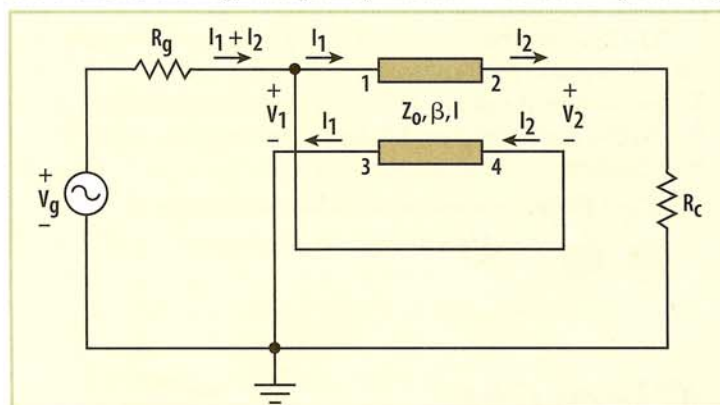
$$L_m = N_p^2 A_l \qquad (3)$$

By applying this parameter within the corresponding reactance formula, and substituting the result of that calculation into the insertion loss equation, the lower cutoff frequency of the transformer can be determined by putting the relationship in Eq. 2. Therefore:

$$f_i = \frac{10^9 \, R_g}{4\pi \, N_p^2 \, A_l} \qquad (4)$$

This value decreases as the number of turns for the primary winding increases. This formula can also be applied for a given desired cutoff frequency in order to calculate the proper number of turns for the primary winding. The factor of 10^9 is used so that the inductance specification ca be presented in nH.

The electrical coupling between the primary and secondary windings of a transmission-line transformer improves the transfer of high-frequency energy. *Figure 3* shows the high-frequency model

$$A_{gc} = \frac{P_g}{P_c} = \frac{Z_o^2 \left\{ 2R_g \left[1 + \cos(\beta\ell)\right] + R_c \, \cos(\beta\ell) \right\}^2 + \left(R_g R_c + Z_o^2\right)^2 \sin^2(\beta\ell)}{4 R_g R_c Z_o^2 \left[1 + \cos(\beta\ell)\right]^2} \qquad (5)$$
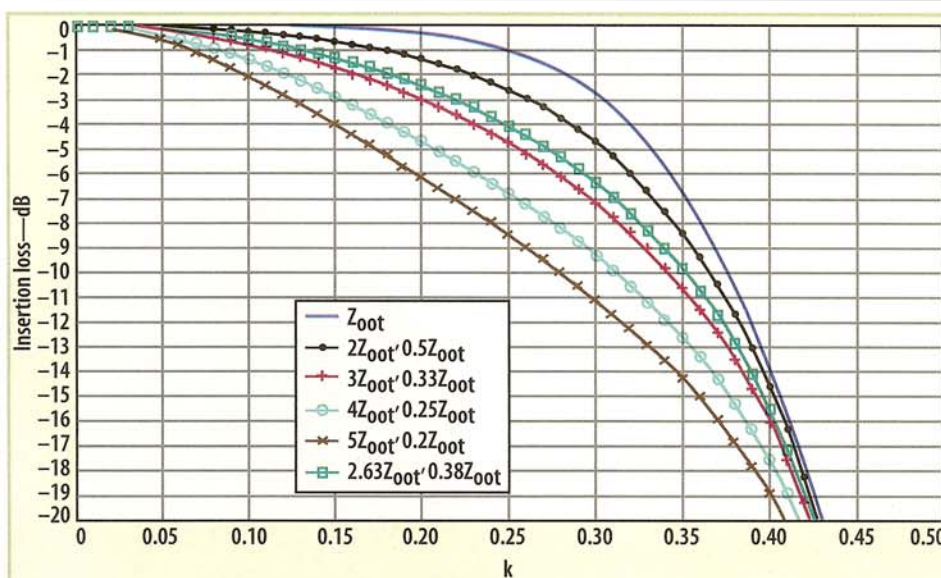
4. This plot shows insertion loss in function of k for variations in the optimum value of Z_o.

for a transmission-line 1:4 unun transformer, considered without losses because of its short length.[5] In this idealized model, the source and load impedances are assumed to be pure resistances. The high-frequency model response is quantified also by its insertion loss. Again, the ratio between the available source power and the secondary load power is:

SEE EQ. 5 IN BOX ABOVE

where:

R_g = the source impedance,
R_c = the load impedance,
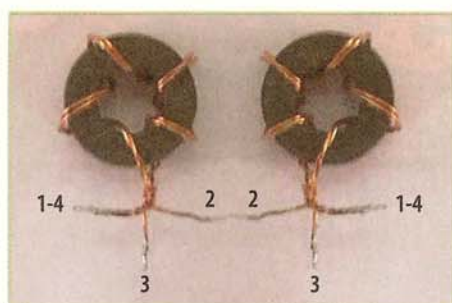Z_o = the transmission-line characteristic impedance,

$\beta = 2\pi/\lambda$ = the phase factor, and
$\ell = k\lambda$ = the length of the transmission line with k a fraction of wavelength λ.

Equation 5 shows the importance of an optimum Z_o value to achieve good wideband high-frequency response. The power transference is nullified for a half-wavelength ($\lambda/2$) line length, and will be 1 dB less than the maximum value for a quarter-wavelength ($\lambda/4$) line length. From this is can be seen that shorter lengths of line result in broader-bandwidth high-frequency responses. The optimum line characteristic impedance and the load impedance for maximum power transmission are:

3. This model represents a transmission-line unun transformer with 1:4 impedance transformation ratio.
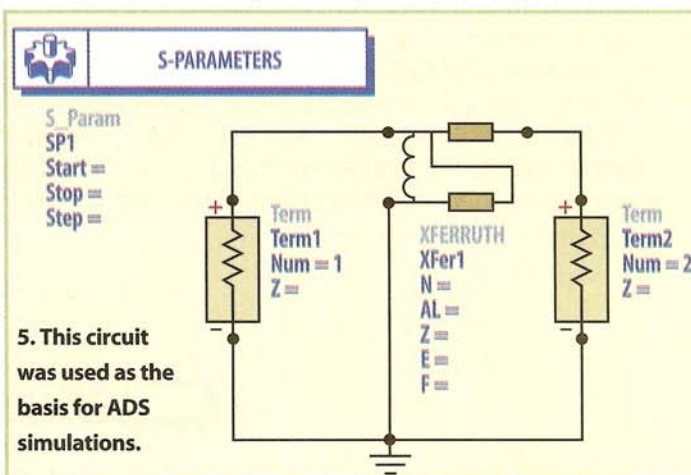
5. This circuit was used as the basis for ADS simulations.

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6. Two transformers were needed in order to connect the device under test (DUT) to the 50-ohm VNA-based measurement setup.

$$Z_{oot} = \sqrt{R_g R_c} \quad (6)$$

$$R_c = 4 R_g \quad (7)$$

A 1:4 transformation is necessary between the source and load impedances for impedance matching. With this result, a relationship between line characteristic impedance and the source and load impedances can be established:

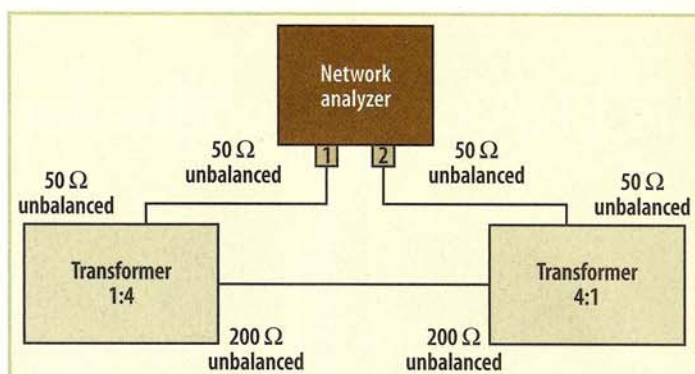$$Z_o = 2 R_g = \frac{R_c}{2} \quad (8)$$

The use of twisted transmission line in a transformer makes it possible to set the characteristic impedance almost at an optimum value for a desired passband by varying the number of twists per unit length of transmission line. An increase in the number of twists per unit length results in a decrease in the characteristic impedance. *Figure 4* plots insertion loss behavior as a function of k for optimized and non-optimized values of characteristic impedance. For a case with non-optimized characteristic impedance, the insertion loss increases and bandwidth decreases relative to a case using optimized characteristic impedance. Thus, the use of twisted transmission line is readily justified for obtaining optimum values of characteristic impedance.[8,9]

As a comparison, simulated performance was predicted using the Advanced Design System (ADS®) computer-aided-engineering (CAE) software suite from Agilent Technologies (www.agilent.com) while design prototypes were measured with a commercial microwave vector network analyzer (VNA). The analysis indicated the relationship between the load and source power.
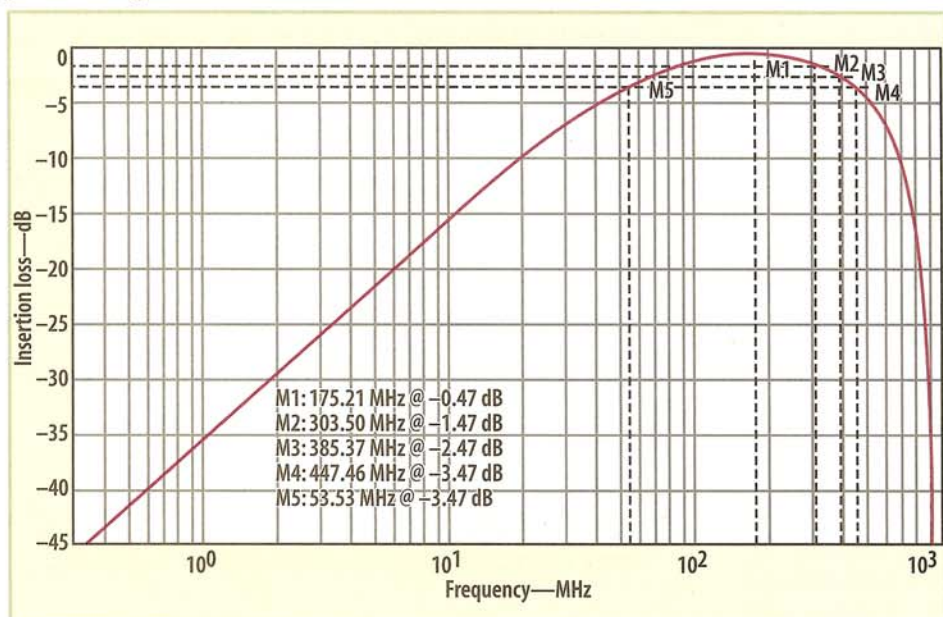
In order to determine a transformer's low-frequency response, the characteristics of the ferrite core must be known since the inductance factor A_l is determined relative to a specific frequency. Knowing this as well as the source's internal impedance (R_g), a designer can establish the low-frequency cutoff frequency (f_i) and, using Eq. 4, can calculate the required number of turns (N_p) for the primary winding. To determine the high-frequency response, information about the transmission line is required, such as its characteristic impedance (Z_o), the propagation velocity (v_p), and the phase factor (β), all at the desired operating frequency. Along with the values of the source impedance (R_g) and the load impedance (R_c), the optimum theoretical value of the characteristic impedance (Z_{opt}) can be determined by applying Eq. 6. Knowing the characteristics of the transmission line, the high-frequency cutoff frequency (f_s), and the true characteristic impedance of the transmission line, Z_o, it is possible to calculate the propagation velocity (v_p) and the phase factor (β). With the value of the true characteristic impedance, Z_o, the difference between it and Z_{opt} can be verified and the definitive insertion loss for f_s be specified. Figure 4 shows the determination of values of k as a function of true characteristic impedance (Z_o) and insertion loss. With values for k, v_p, and f_s, the line length (ℓ) required for achieving the previous specifications can be calculated through:


7. This connection scheme was used to make VNA measurements of the 1:4 unun impedance transformer.

M1: 175.21 MHz @ −0.47 dB
M2: 303.50 MHz @ −1.47 dB
M3: 385.37 MHz @ −2.47 dB
M4: 447.46 MHz @ −3.47 dB
M5: 53.53 MHz @ −3.47 dB

8. These analytical responses were generated by using MATLAB software to study a transformer with four turns and 9-cm line length.

$$\ell = k\lambda = k \frac{v_p}{f_s} \quad (9)$$

MATLAB mathematical analysis software from The MathWorks

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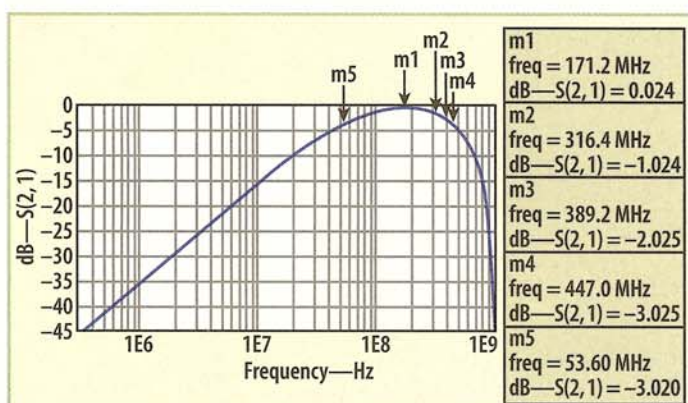

9. These numerical response were generated with the ADS software for a transformer with four turns and 9-cm line length.

m1
freq = 171.2 MHz
dB—S(2, 1) = 0.024
m2
freq = 316.4 MHz
dB—S(2, 1) = −1.024
m3
freq = 389.2 MHz
dB—S(2, 1) = −2.025
m4
freq = 447.0 MHz
dB—S(2, 1) = −3.025
m5
freq = 53.60 MHz
dB—S(2, 1) = −3.020

10. These experimental responses were acquired through VNA measurements for a transformer with four turns and 9-cm line length.

(www.mathworks.com) was used to analyze the response of the transformer device model.[10] In this analysis, the insertion-loss responses for the individual low-frequency (Eq. 1) responses and high-frequency (Eq. 5) responses were combined. The desired target values were substituted into the MATLAB equations to obtain the final response of the wideband transformer. To achieve an electrical simulation of the MATLAB model's numerical response, the ADS modeling software was used. The software features a useful internal model for the source, called XFERRUTH, with variable parameters that include the number of turns (N), the inductance factor (AL), the characteristic impedance of the line (Z), the electrical length of the

transmission line (E), and the reference frequency (F) for the calculation of the transmission-line length.

In order for ADS to performance a device simulation on the transformer with the responses as scattering parameters (S-parameters), it employs its S_Param modeler, adjusting the initial (start) and the final (stop) sweep frequencies according to a specified step size. and the graduation step size. The source and load impedances are represented by a specific termination called Term, with an impedance value of Z. *Figure 5* shows the circuit that was used in the ADS simulation.

Measurements were performed on a
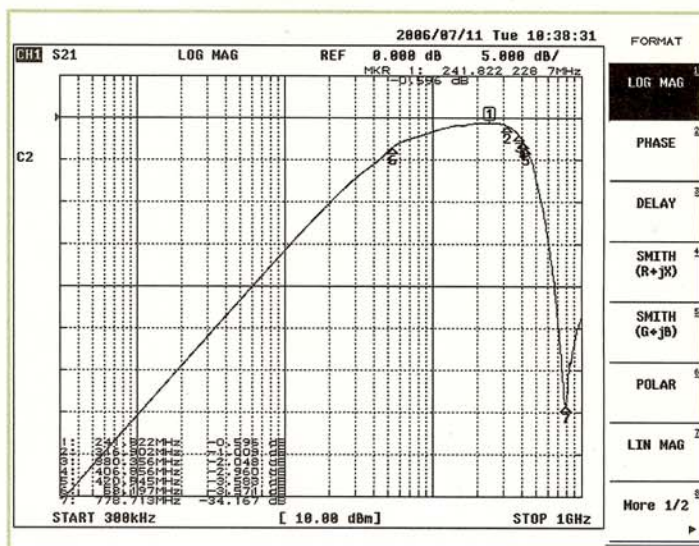
commercial VNA from Advantest (www.advantest.com), a 300-kHz-to-3.8-GHz model R3765CG. The analyzer features unbalanced test ports with terminal impedance of 50 ohms. Since the wideband unun impedance transformer has unbalanced terminals and a transformation ratio of 1:4, another device with 4:1 transformation ratio was needed to perform the impedance conversion needed to adapt the device to the measurement equipment. *Figures 6 and 7* show all the terminals connections. Both test terminals and all cables used with the VNA were calibrated in order to minimize their error contributions. The insertion loss and the passband response were analyzed by means of transmission coefficient S_{21} measurements presented in log-magnitude form.

The comparative results among the analytical (MATLAB), numerical (ADS), and the experimental models were obtained under several measurement conditions. A model E1003C5 toroidal ferrite core from Sontag Componentes Eletronicos (www.sontag.com.br) was used in the experiments. Its geometric and electromagnetic data include an external diameter of 10 mm, an internal diameter of 5 mm, a width of 3 mm, relative permeability (μ_r) of 11, and an inductance factor (A_l) of 4.2 nH/turns^2.[11] It is specified for use in the frequency range from 500 kHz and 50
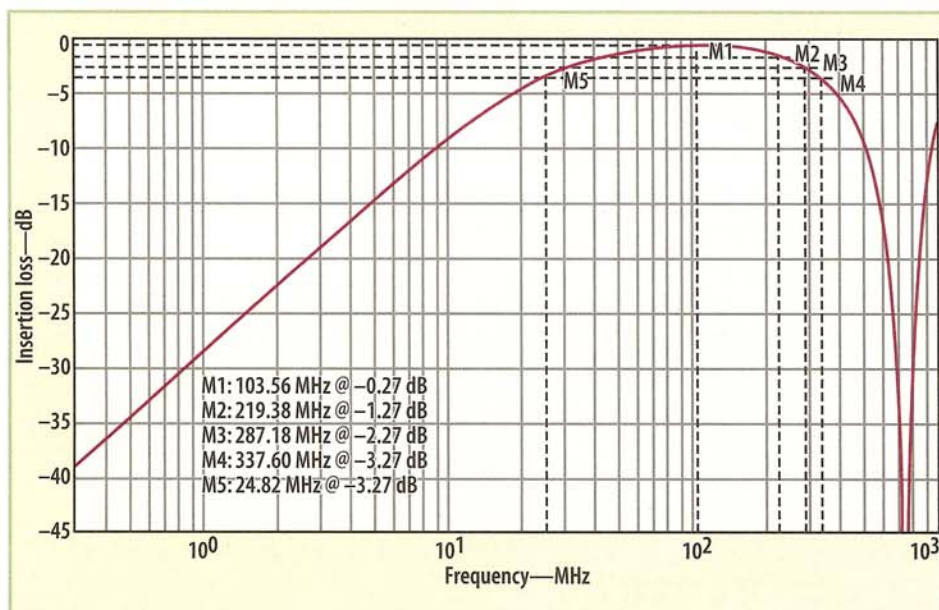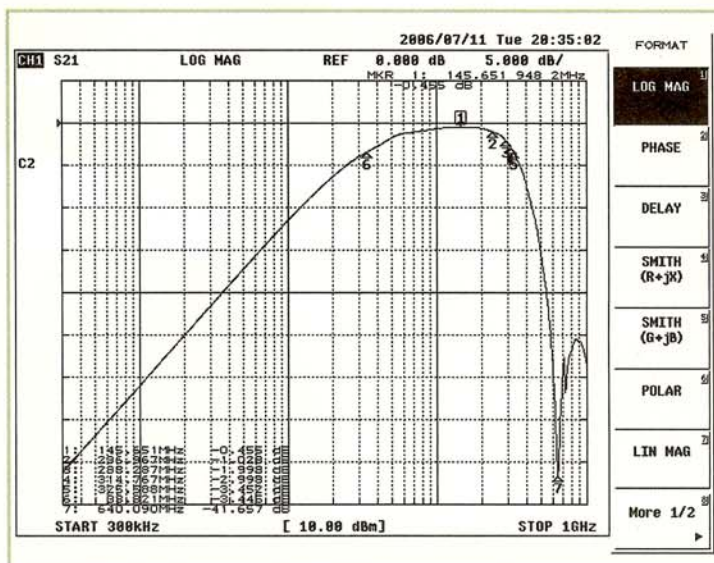

11. These analytical responses were generated by using MATLAB software to study a transformer with six turns and 11-cm line length.

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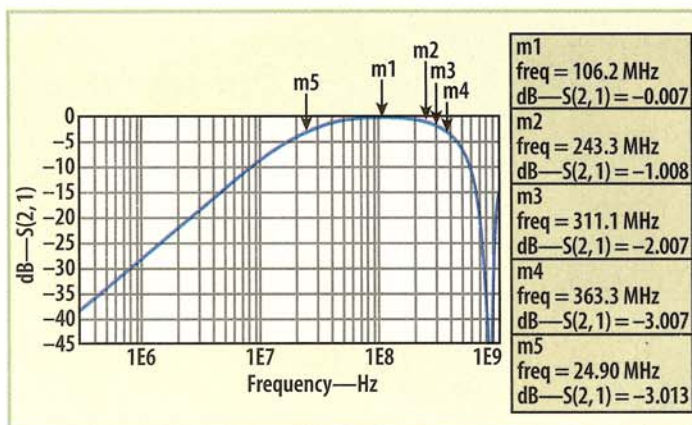
12. These numerical response were generated with the ADS software for a transformer with six turns and 11-cm line length.

13. These experimental responses were acquired through VNA measurements for a transformer with six turns and 11-cm line length.

MHz. A five twists per centimeter line with 30AWG-conductor transmission line was used. At 130 MHz, this line has a 38-ohm characteristic impedance, phase factor (β) of 4.5501 rad/m, and propagation velocity (v_p) of 1.7952×10^8 m/s. The optimum value of the characteristic impedance for a 50-ohm source impedance must be 100 ohms, given in Eq. 8, meaning a 0.38 times relationship. The k value for this difference and a insertion loss of 3 dB is 0.2207.

The first device was fabricated with four turns, resulting in a 9-cm line length. *Figures 8, 9, and 10* show the frequency insertion loss behaviors for analytical, numerical, and experimental cases. The *table* summarizes the main values, including insertion-loss results at maximum amplitude, at –3 dB frequencies (f_{max}, f_{i-3dB}, and f_{s-3dB}), the proper bandwidth (BW), and the percent frequency differences compared to model values. The differences among the analytical, numerical, and experimental results are very small, except at the maximum signal frequency. This can be attributed to limitations in the test system caused by noise and other parasitic effects in the measurement setup. Across the test frequency band with essentially constant amplitude, variations in signal level are imper-

ceptible, perhaps accounting for differences in reporting the maximum signal amplitude frequency.

A second device was constructed with six turns and 11-cm line length. With the increase in the number of turns, the lower cutoff frequency decreases and the higher cutoff frequency is also lowered as a result of the increased line
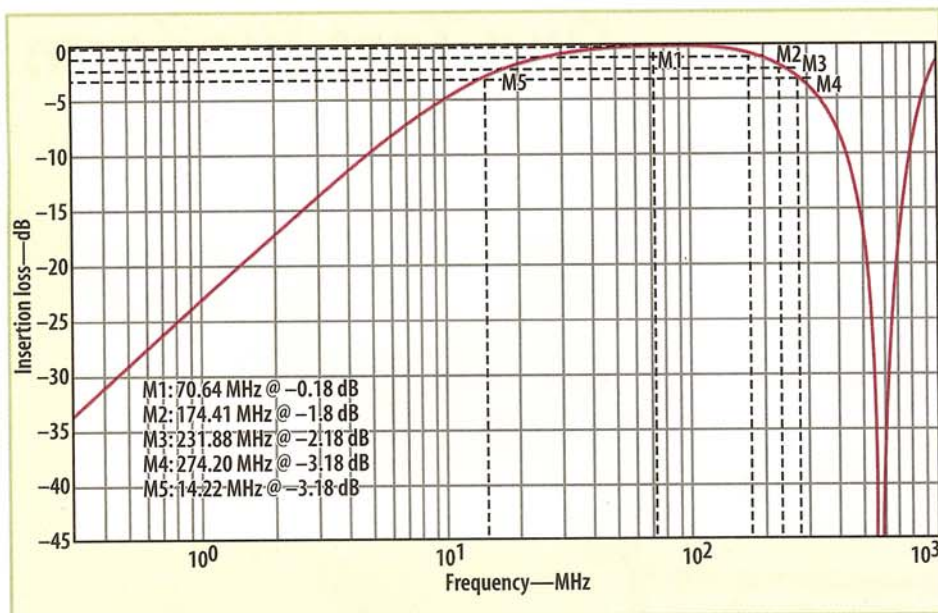
length. For the lower cutoff frequency, the values of the analytical and numerical results were as expected. However, the experimental responses do not agree exactly with the theoretical model. The values of the high-frequency responses are as expected, with good agreement among the three sets of values. *Figures 11, 12, and 13* show insertion loss as a function of frequency for the analytical, numerical, and experimental cases (shown also in the table). There is a small difference between the analytical and numerical results, due to the inherent limitations of the model. On the other hand, the experimental results prove the validity of the model, except at the low-frequency limit, where the largest error occurred. This is due to the fact that the theoretical model does not account for all of the parasitic elements of the components used in the constructed transformers.

For a further comparison, a transformer with eight turns and 14-cm line length was constructed. The results of the analytical, numerical, and experimental cases are summarized in the table and shown in *Figs. 14, 15, and 16*. At the

Comparing analytical, numerical, and experimental results for the devices

	ANALYTICAL	NUMERICAL	EXPERIMENTAL
	$N_P = 4$ e $\ell = 9$ cm		
f_{max}	175.21 MHz −0.470 dB	172.20 MHz / 2.3% −0.024 dB	241.82 MHz / 38% −0.596 dB
f_{i-3dB}	447.46 MHz −3.470 dB	447.00 MHz / 0.1% −3.024 dB	420.95 MHz / 5.9% −3.583 dB
f_{s-3dB}	53.53 MHz −3.470 dB	53.60 MHz / 0.1% −3.024dB	53.20 MHz / 0.6% −3.571 dB
BW	393.93 MHz	393.40 MHz / 0.1%	367.75 MHz / 6.7%
	$N_P = 6$ e $\ell = 11$ cm		
f_{max}	103.56 MHz −0.270 dB	106.20 MHz / 2.5% −0.007 dB	145.65 MHz / 40.6% −0.455 dB
f_{i-3dB}	337.60 MHz −3.270 dB	363.30 MHz / 7.6% −3.007 dB	325.58 MHz / 3.6% −3.452 dB
f_{s-3dB}	24.82 MHz −3.270 dB	24.90 MHz / 0.3% −3.013 dB	33.82 MHz / 36.3% −3.445 dB
BW	312.78 MHz	338.40 MHz / 8.2%	291.76 MHz / 6.7%
	$N_P = 8$ e $\ell = 14$ cm		
f_{max}	70.64 MHz −0.180 dB	71.50 MHz / 1.2% −0.004 dB	108.18 MHz / 53.1% –dB
f_{i-3dB}	274.20MHz −3.180 dB	284.80 MHz / 3.9% −3.004 dB	267.63 MHz / 2.4% −3.347 dB
f_{s-3dB}	14.22 MHz −3.180 dB	14.30 MHz / 0.6% −2.986 dB	21.51MHz / 51.3% −3.356 dB
BW	259.98 MHz	270.50 MHz / 4%	246.12 MHz / 5.3%

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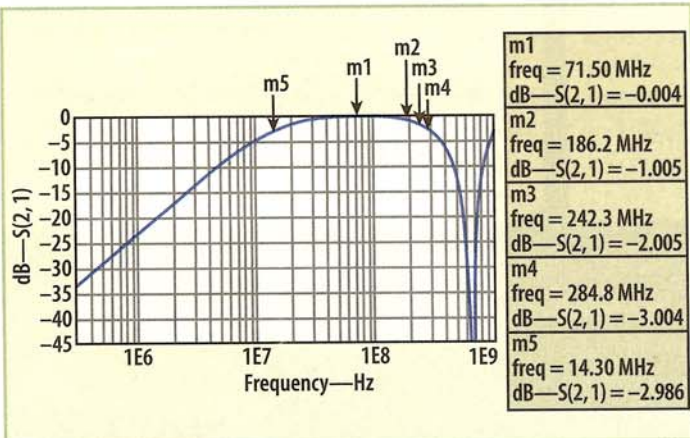**14. These analytical responses were generated by using MATLAB software to study a transformer with six turns and 14-cm line length.**

of inter-winding capacitance, which decreases the self-resonant frequency of the inductor. Still, these simplified design equations can provide reasonable results, replacing the more involved empirical processes usually involved in the design of a 1:4 impedance transformer. As these simplified equations show, they can be used to design transformers over a wide frequency range (three octaves) with low insertion loss and at low cost. MRi

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lower cutoff frequency, the analytical and numerical cases agree but the experimental results do not agree with the theoretical model. However, the higher cutoff frequency values are close to either other and to the expected results. As the number of turns was increased, the lower cutoff frequency was reduced; similarly, as the line length increased, the upper cutoff frequency was also reduced.

Although the three sets of results were determined by different methods, they appear satisfactory and in good agreement with each other. The analytical (MATLAB) and numerical (ADS) model responses and the experimental responses (VNA measurements) compared closely. The analytical and numerical

cases yielded approximately the same values, with some differences compared to the experimental results. This is best explained by the fact that the theoretical models do not fully account for the complex characteristics of the components used in the fabrication of the transformers, which were modeled as more or less "ideal" components.

These model equations represent a simplified equivalent-circuit model for a coil transformer. Recent studies have shown the need to use a more elaborate model that includes resistive and reactive effects as a function of increasing frequency and number of turns.[12] These advanced models also include the effects
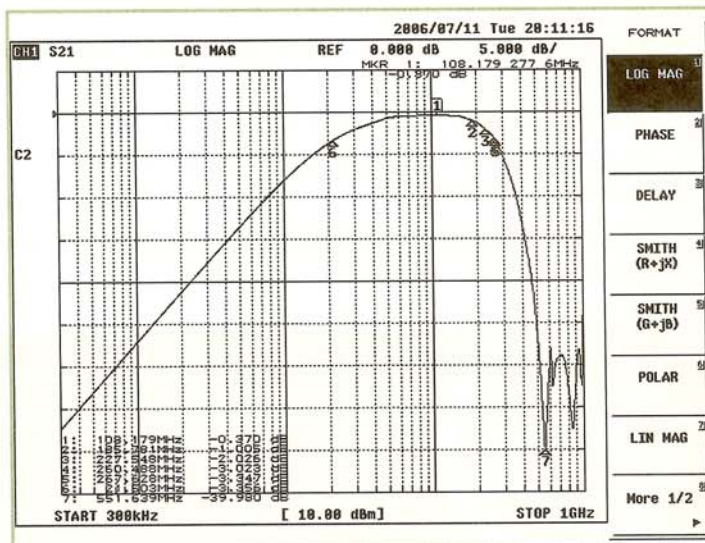

15. These numerical response were generated with the ADS software for a transformer with eight turns and 14-cm line length.

16. These experimental responses were acquired through VNA measurements for a transformer with eight turns and 14-cm line length.