4.4 最佳线性滤波器

$$Z = \sqrt{G_s^2 + G_c^2} \qquad \mathbf{2}$$

$$G_s = \int_0^T S_s(t)x(t)dt \quad (\quad S_s(t) = A\sin\omega_c t \quad) \qquad G_c = \int_o^T S_c(t)x(t)dt$$

$$S_c(t) = A\cos\omega_c t$$

$$\frac{S(\omega)}{N(\omega)}$$

$H(\omega)$ $h(t)$

$H(\omega)$ $h(t)$

$$H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt \qquad \textbf{4-1}$$

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{j\omega t} d\omega \qquad \textbf{4-2}$$

$$z(t) = s(t) + n(t) \qquad \textbf{4-3}$$

s(t) **n(t)**

$$z_0(t) = s_o(t) + n_0(t) \qquad \textbf{4-4}$$

s t **s(t)**

$$S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-j\omega t} dt$$ **4-5**

$$s_0(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) S(\omega) e^{j\omega t} d\omega$$ **4-6**

$t=t_0$

$$s_0(t_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) S(\omega) e^{j\omega t_0} d\omega$$ **4-7**

n(t) **n_0(t)**

$$s_{n_0}(\omega)$$

$$P_{n_0}(\omega) = |H(\omega)|^2 P_n(\omega) \qquad \textbf{4-8}$$

$P_n(\omega)$ **n(t)** $P_{n_0}(\omega)$

$n_0(t)$

$$E\left[n_0^2(t)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} P_{n_0}(\omega)d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 P_n(\omega)d\omega \qquad \textbf{4-9}$$

$$SNR_0=\frac{\text{输出信号峰值功率}}{\text{输出噪声平均功率}}=\frac{s_0^2(t_0)}{E\left[n_0^2(t)\right]}$$

$$=\frac{\left(\frac{1}{2\pi}\int_{-\infty}^{\infty}H(\omega)S(\omega)e^{j\omega t_0}d\omega\right)^2}{\frac{1}{2\pi}\int\left|H(\omega)\right|^2P_n(\omega)d\omega} \qquad 4\text{-}10$$

Schwarz

$$\left|\frac{1}{2\pi}\int_{-\infty}^{\infty} F^*(x)\theta(x)dx\right|^2$$

$$\leq \frac{1}{2\pi}\int_{-\infty}^{\infty} F^*(x)F(x)dx \cdot \frac{1}{2\pi}\int_{-\infty}^{\infty} \theta^*(x)\theta(x)dx \qquad \textbf{4-10}$$

F x x *

$$\theta(x) = aF(x)$$

$$F^*(x) = \frac{S(\omega)e^{j\omega\ t_0}}{\sqrt{P_n(\omega)}} \qquad 4\text{-}11$$

$$\theta(x) = \sqrt{P_n(\omega)}H(\omega) \qquad 4\text{-}12$$

Parseval

$$\varepsilon = \int_{-\infty}^{\infty} \left| s(t) \right|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| S(\omega) \right|^2 d\omega$$

4-13

4-10

$$SNR_0 = \frac{\left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) \sqrt{P_n(\omega)} \cdot \frac{1}{\sqrt{S_n(\omega)}} S(\omega) e^{j\omega \ t_0} d\omega \right\}^2}{\frac{1}{2\pi} \int_{-\infty}^{\infty} \left| H(\omega) \right|^2 P_n(\omega) d\omega}$$

$$\le \frac{\frac{1}{2\pi} \int_{-\infty}^{\infty} \left| H(\omega) \right|^2 P_n(\omega) d\omega \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\left| S(\omega) \right|^2}{P_n(\omega)} d\omega}{\frac{1}{2\pi} \int_{-\infty}^{\infty} \left| H(\omega) \right|^2 P_n(\omega) d\omega}$$

$$SNR_0 \le \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\left| S(\omega) \right|^2}{P_n(\omega)} d\omega \qquad \text{4-14}$$

$$\theta(x) = aF(x)$$ **4-11**

4-12

$$H(\omega) = \frac{\alpha S^*(\omega)}{2\pi P_n(\omega)} e^{-j\omega t_0}$$ **4-15**

4-14

4-15

$$\frac{N_0}{2}$$ 4-14

$$SNR_0 \le \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\left|S(\omega)\right|^2}{N_0/2}d\omega$$ 4-16

4-13

$$SNR_0 \le \frac{2\varepsilon}{N_0}$$ 4-17

SNR₀ $\frac{2\varepsilon}{N_0}$

4-15

$$H(\omega) = KS^*(\omega)e^{-j\omega t_0}$$ 4-18

$$K = \frac{\alpha}{\pi N_0}$$

4-18

$$Ke^{-j\omega t_0}$$

S*

t_0

S **H**

$$S(\omega) = |S(\omega)| e^{j \arg[s(\omega)]}$$

$$H(\omega) = |H(\omega)| e^{j \arg[H(\quad)]}$$ **4-20** **4-18**

$$|H(\omega)| = K |S(\omega)|$$ **4-21**

$$\arg H(\omega) = -\arg[S(\omega)] - \omega t_0$$

0 **t_0**

t_0

K

K=1

4-21 4-22

t_0

h(t) 4-18 H

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} KS^*(\omega)e^{-j\omega t_0}e^{j\omega t}d\omega$$

$$= K\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)e^{j\omega(t-t_0)}d\omega\right\}$$

$$= Ks^*(t_0 - t) \qquad \textbf{(4-23)}$$

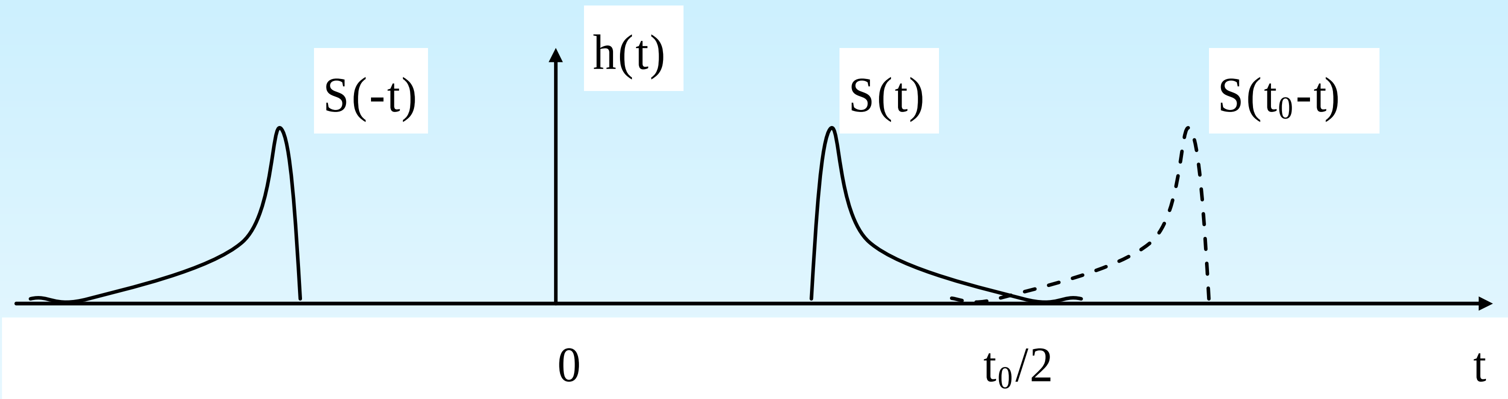
s(t)

$$h(t) = Ks(t_0 - t) \qquad \textbf{4-24}$$

t_0 K

4-24

h(t) **s(t)** $\frac{t_0}{2}$ **4.8**



4.8

$$h(t) = 0, \qquad t < 0$$

4-24

$$h(t) = \begin{cases} 0, & t < 0 \\ Ks(t_0 - t) & t \geq 0 \end{cases}$$ **4-25**

s(t) **s_0(t)**

$$s(t) = 0, \qquad t > t_0$$ **4-26**

t=t_0 **s(t)**

4.4.2

1

s(t) $H(\omega) = KS^*(\omega)e^{-j\omega t_0}$

s(t) **A**

$s_1(t) = As(t-\tau)$ **s_1(t)**

$S_1(\omega) = AS(\omega)e^{-j\omega\tau}$ **s_1(t)** **(K=1)**

$$H_1(\omega) = AS_1^*(\omega)e^{-j\omega t_0'} = AS^*(\omega)e^{-j\omega(t_0'-\tau)}$$

$$= AH(\omega)e^{-j\omega\left[t_0'-(t_0+\tau)\right]}$$

t_0 $H(\omega)$ t_0'

$H_1(\omega)$

$s_1(t)$ **s(t)**

t_0' **t_0** $t_0' = t_0 + \tau$

$H_1(\omega) = AH(\omega)$

A

$H(\omega)$

$$s_1(t) = As(t - \tau)$$

$$S_2(\omega) = S(\omega + \upsilon)$$

$$H_2(\omega) = S^*(\omega + \nu)e^{-j\omega t_0}$$

$$H(\omega)$$

$$s_0(t) = \int_{-\infty}^{\infty} s(t-\mu)h(\mu)d\mu$$

$$h(t) = Ks(t_0 - t)$$

$$s_0 \;\; t \;\; = \int_{-\infty}^{\infty} s(t-\mu)Ks(t_0-\mu)d\mu$$

$$\tau = t_0 - \mu$$

$$s_0(t) = K\int_{-\infty}^{\infty} s(\tau)s\left[\tau - (t_0 - t)\right]d\tau = KR_s(t - t_0)$$

$$R_s(t - t_0) = \int_{-\infty}^{\infty} s(t - \mu)s(t_0 - \mu)d\mu$$

$$s_0(t) = KR_s(t - 0) = KR_s(t)$$

$$s_0(t) = KR_s(0) = K\int_{-\infty}^{\infty} s^2(t)dt = K\varepsilon$$

$$t = t_0 + \varphi$$

$$S_0(t_0 + \varphi) = KR_s(t_0 + \varphi - t_0) = KR_s(\varphi) = KR_s(-\varphi) = S_0(t_0 - \varphi)$$

$$n_0(t) = \int_{-\infty}^{\infty} n(t-\mu)h(\mu)d\mu$$

$$= \int_{-\infty}^{\infty} n(t-\mu)Ks(t_0-\mu)d\mu$$

$$= KR_{ns}(t - t_0)$$

$$R_{ns}(t - t_0)$$

$$s\left[\tau - (t_0 - t)\right]$$

$$x(\tau) = s(\tau) + n(\tau)$$

$$s(\tau)$$

$S(\omega)$

$S_0(\omega)$

$$S_0(\omega) = H(\omega)S(\omega) = S*(\omega)e^{-j\omega t_0}S(\omega) = \left|S(\omega)\right|^2 e^{-j\omega t_0}$$

4.1

s(t)

A

$$s(t)=\begin{cases} A, & t \le \dfrac{\tau}{2} \\ 0, & |t| > \dfrac{\tau}{2} \end{cases}$$

$$S(\omega)=\int_{-\infty}^{\infty} s(t)e^{-j\omega t}dt=\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} Ae^{-j\omega t}dt = A\tau\frac{\sin\dfrac{\omega\tau}{2}}{\dfrac{\omega\tau}{2}}$$

s(t)

$$H(\omega) = KA\tau \frac{\sin \frac{\omega\tau}{2}}{\frac{\omega\tau}{2}} e^{-j\omega t_0}$$

t_0 $\frac{\tau}{2}$

$$h(t) = Ks(t_0 - t) = \begin{cases} KA, & 0 \le t \le \tau \\ 0, & t < 0, t > \tau \end{cases}$$

$$s_0(t) = KR_s(t - t_0) = K\int_{-\infty}^{\infty} s(t-\alpha)s(t_0-\alpha)d\alpha$$

$$R_s(t)=\int_{-\infty}^{\infty}s(t+\alpha)s(\alpha)d\alpha$$

$$R_s(t)=\int_{-\frac{\tau}{2}+t}^{\frac{\tau}{2}}A^2 d\alpha = A^2(\tau-t)$$

$$-\tau<t<0$$

$$R_s(t)=\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}+t}A^2 d\alpha = A^2(\tau+t)$$

$$R_s(t)=\begin{cases} A^2(\tau-t), & 0<t<\tau \\ A^2(\tau+t), & -\tau<t<0 \end{cases}$$

$$S_0(t) = KR_s(t-t_0) = \begin{cases} KA^2(\tau - t + t_0), & t_0 < t < \tau + t_0 \\ KA^2(\tau + t - t_0), & -\tau + t_0 < t < t_0 \end{cases}$$

4-8 b **t=t$_0$** $t_0 = \dfrac{\tau}{2}$

$$s_0(t) = KR_s(t-t_0) = \begin{cases} KA^2(\dfrac{3\tau}{2} - t), & \dfrac{\tau}{2} < t < \dfrac{3\tau}{2} \\ KA^2(\dfrac{\tau}{2} + t), & -\dfrac{\tau}{2} < t < \dfrac{\tau}{2} \end{cases}$$

$t = t_0 = \dfrac{\tau}{2}$ **s(t)**

$$s(\frac{\tau}{2}) = KA^2\tau = K\varepsilon$$

$$s_0(t) = \int_{-\infty}^{\infty} s(t')h(t-t')dt'$$

4.2

A

$$s(t) = Arect(\frac{t}{\tau})\cos\omega_0 t$$

rect

$$rect(x) = \begin{cases} 1, & |\mathrm{x}| \le \frac{1}{2} \\ 0, & |\mathrm{x}| > \frac{1}{2} \end{cases}$$

s(t)

$$S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-j\omega t} dt = A \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \cos \omega_0 t e^{-j\omega t} dt$$

$$= \frac{A}{2} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \left[e^{-j(\omega-\omega_0)} + e^{-j(\omega+\omega_0)t} \right] dt$$

$$= \frac{A\tau}{2} \left[\frac{\sin(\omega-\omega_0)\frac{\tau}{2}}{(\omega-\omega_0)\frac{\tau}{2}} + \frac{\sin(\omega+\omega_0)\frac{\tau}{2}}{(\omega+\omega_0)\frac{\tau}{2}} \right]$$

$\omega = \pm\omega_0$ $\frac{\sin x}{x}$ **sinc**

4dB $\frac{1}{\tau}$

$f_0 \tau >> 1$ **f=0**

$$H(\omega) = KS^*(\omega)e^{-j\omega t_0}$$

$$= \frac{KA\tau}{2}\left[\frac{\sin(\omega-\omega_0)\frac{\tau}{2}}{(\omega-\omega_0)\frac{\tau}{2}} + \frac{\sin(\omega+\omega_0)\frac{\tau}{2}}{(\omega+\omega_0)\frac{\tau}{2}}\right]e^{-j\omega t_0}$$

$$h(t) = Ks(t_0 - t) = A\cos[\omega(t_0 - t)]$$

$$\varepsilon = \int_{-\infty}^{\infty} s^2(t)dt = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A^2 \cos^2 \omega_0 t dt$$

$$= \frac{A^2}{2} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} (1 + \cos 2\omega_0 t)dt = \frac{A^2 \tau}{2}$$

$$SNR_0 = \frac{2\varepsilon}{N_0} = \frac{A^2 \tau}{N_0}$$

$$s(t) = Arect(\frac{t}{\tau})\cos(\omega_0 t + \frac{\mu t^2}{2})$$

$$\omega = \frac{d\varphi}{dt} = \omega_0 + \mu t$$

$$\omega_0 - \frac{\mu\tau}{2} \qquad \omega_0 + \frac{\mu\tau}{2}$$

$$B = \frac{\mu\tau}{2\pi}$$

$$S(\omega)=\int_{-\infty}^{\infty} s(t)e^{-j\omega t}dt$$

$$=A\int_{-\infty}^{\infty} rect(\frac{t}{\tau})\cos(\omega_0 t+\frac{\mu\tau^2}{2})e^{-j\omega t}dt$$

$$=\frac{A}{2}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\exp\left\{+\left[j(\omega_0-\omega)t+j\frac{\mu t^2}{2}\right]\right\}dt$$

$$+\frac{A}{2}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\exp\left\{-\left[j(\omega_0-\omega)t+j\frac{\mu t^2}{2}\right]\right\}dt$$

$$\pm\omega_0$$

ω_0

$$S_+(\omega) = \frac{A}{2}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \exp\left\{+\ j\left[(\omega_0 - \omega)t + \frac{\mu}{2}t^2\right]\right\}dt$$

$$(\omega_0 - \omega)t + \frac{\mu}{2}t^2 = \frac{\mu}{2}\left[t^2 + \frac{2}{\mu}(\omega_0 - \omega)t + \left(\frac{\omega_0 - \omega}{\mu}\right)^2\right] - \frac{\mu}{2}\left(\frac{\omega_0 - \omega}{\mu}\right)^2$$

$$= \frac{\mu}{2}\left(t + \frac{\omega_0 - \omega}{\mu}\right)^2 - \frac{1}{2\mu}(\omega_0 - \omega)^2$$

$$S_+(\omega)=\frac{A}{2}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\exp\left[j\frac{\mu}{2}\left(t+\frac{\omega_0-\omega}{\mu}\right)^2-j\frac{1}{2\mu}(\omega_0-\omega)^2\right]dt$$

$$=\frac{A}{2}\exp\left[-j\frac{(\omega_0-\omega)^2}{2\mu}\right]\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\exp\left[j\frac{\mu}{2}\left(t+\frac{\omega_0-\omega}{\mu}\right)^2\right]dt$$

$$\sqrt{\mu}(t+\frac{\omega_0-\omega}{\mu})=\sqrt{\pi}x$$

$$dt=\sqrt{\frac{\pi}{\mu}}\cdot dx$$

$$S_{+}(\omega)=\frac{A}{2}\sqrt{\frac{\pi}{\mu}}\exp\left[-j\frac{(\omega-\omega_0)^2}{2\mu}\right]\int_{-X_1}^{X_2}\exp\left[j\frac{\pi x^2}{2}\right]dx$$

$$X_1=\frac{-\frac{\mu\tau}{2}+(\omega-\omega_0)}{\sqrt{\pi\mu}}$$

$$X2=\frac{\frac{\mu\tau}{2}+(\omega-\omega_0)}{\sqrt{\pi\mu}}$$

$$S_+(\omega) = \frac{A}{2}\sqrt{\frac{\pi}{\mu}}\left\{\exp\left[-j\frac{(\omega-\omega_0)^2}{2\mu}\right]\right\}$$

$$\bullet\left[C(X_1) + jS(X_1) + C(X_2) + jS(X_2)\right]$$

$$C(X) = \int_0^X \cos\frac{\pi y^2}{2}dy$$

$$S(X) = \int_0^X \sin\frac{\pi y^2}{2}dy$$

Fresnel

$$C(-X) = -C(X)$$

$$S(-X) = -S(X)$$

$$S_{-}(\omega)=\frac{A}{2}\sqrt{\frac{\pi}{\mu}}\left\{\exp\left[j\frac{(\omega+\omega_0)^2}{2\mu}\right]\right\}$$

$$\bullet\left[C(X_3)+jS(X_3)+C(X_4)-jS(X_4)\right]$$

$$\left|S_{+}(\omega)\right|=\frac{A}{2}\sqrt{\frac{\pi}{\mu}}\left\{\left[C(X_1)+S(X_2)\right]^2+\left[S(X_1)+S(X_2)\right]^2\right\}^{1/2}$$

$$\arg S_{+}(\omega)=tg-1\left[\frac{S(X_1)+S(X_2)}{C(X_1)+C(X_2)}\right]-\frac{(\omega-\omega_0)^2}{2\mu}$$

$$H(\omega)=KS*(\omega)\exp(-j\omega t_0)$$

$$=K\left[S_{+}^{*}(\omega)+S_{-}^{*}(\omega)\right]\exp(-j\omega t_0)$$

4-11

$$B = \mu\tau / 2\pi$$

$$\beta(\omega) = \frac{(\omega - \omega_0)^2}{2\mu} - \mu t_0$$

$$T(\omega) = -\frac{d\beta(\omega)}{d\omega} = \frac{\omega - \omega_0}{\mu} + t_0$$

t_0

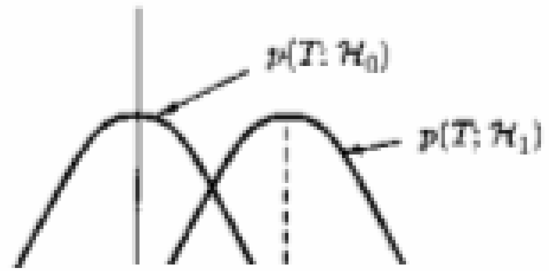
4.4.3

$$T(\mathbf{x}) = \sum_{n=0}^{N-1} x[n]s[n] > \gamma'$$

$$\begin{aligned}
E(T;\mathcal{H}_0) &= E\left(\sum_{n=0}^{N-1} w[n]s[n]\right) = 0 \\
E(T;\mathcal{H}_1) &= E\left(\sum_{n=0}^{N-1} (s[n] + w[n])s[n]\right) = \mathcal{E} \\
\operatorname{var}(T;\mathcal{H}_0) &= \operatorname{var}\left(\sum_{n=0}^{N-1} w[n]s[n]\right) = \sum_{n=0}^{N-1} \operatorname{var}(w[n])s^2[n] \\
&= \sigma^2 \sum_{n=0}^{N-1} s^2[n] = \sigma^2 \mathcal{E}
\end{aligned}$$

$$\mathrm{var}(T;\mathcal{H}_1)=\sigma^2\mathcal{E}$$

$$T\sim\begin{cases}\mathcal{N}(0,\sigma^2\mathcal{E}) & \text{在 } \mathcal{H}_0 \text{ 条件下}\\ \mathcal{N}(\mathcal{E},\sigma^2\mathcal{E}) & \text{在 } \mathcal{H}_1 \text{ 条件下}\end{cases}$$



$$T' = T/\sqrt{\sigma^2 \mathcal{E}}$$

$$T' \sim \begin{cases} \mathcal{N}(0,1) & \text{在}\mathcal{H}_0\text{条件下} \\ \mathcal{N}(\sqrt{\mathcal{E}/\sigma^2},1) & \text{在}\mathcal{H}_1\text{条件下} \end{cases}$$

$$\sqrt{\mathcal{E}/\sigma^2}$$

$$P_{FA} = \Pr\{T > \gamma'; \mathcal{H}_0\} = Q\left(\frac{\gamma'}{\sqrt{\sigma^2 \mathcal{E}}}\right)$$

$$P_D = \Pr\{T > \gamma'; \mathcal{H}_1\} = Q\left(\frac{\gamma' - \mathcal{E}}{\sqrt{\sigma^2 \mathcal{E}}}\right)$$

其中

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}t^2\right) dt = 1 - \Phi(x)$$

$$\gamma = \sqrt{\sigma^2 \mathcal{E}}\, Q^{-1}(P_{FA})$$

$$P_D = Q\left(\frac{\sqrt{\sigma^2 \mathcal{E}}\, Q^{-1}(P_{FA})}{\sqrt{\sigma^2 \mathcal{E}}} - \sqrt{\frac{\mathcal{E}}{\sigma^2}}\right) = Q\left(Q^{-1}(P_{FA}) - \sqrt{\frac{\mathcal{E}}{\sigma^2}}\right)$$

$$\sqrt{\mathcal{E}/\sigma^2}$$

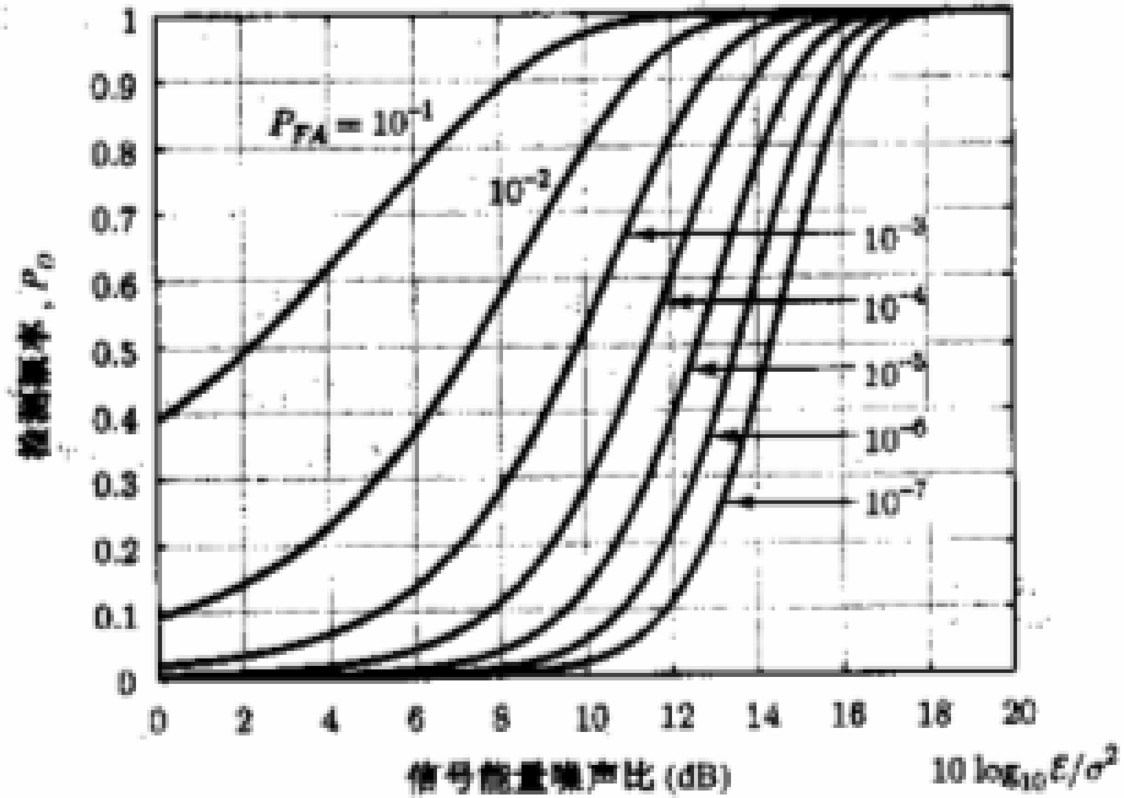


图 4.5 匹配滤波器的检测性能

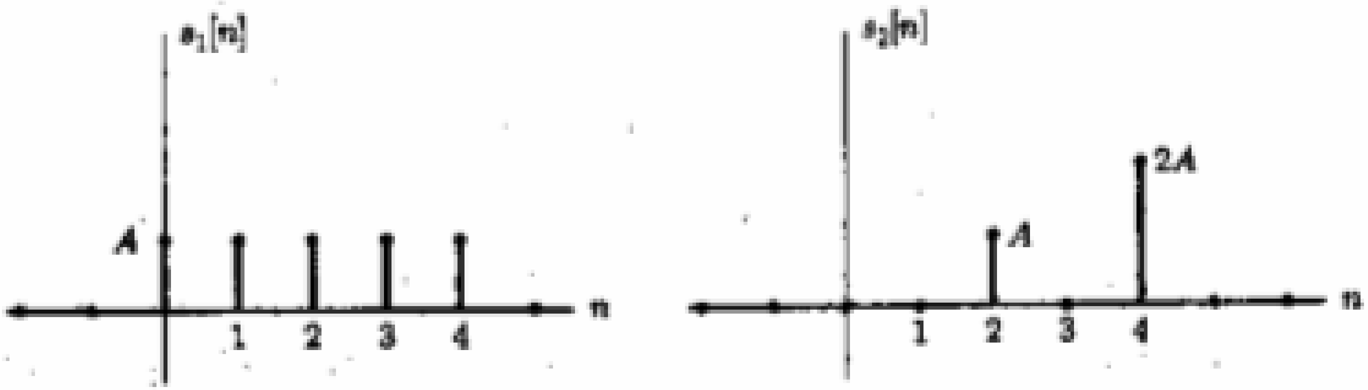


图 4.6 产生相同检测性能的信号

$$\eta_{in} = \frac{\mathcal{E}}{\sigma^2} = \frac{A^2}{\sigma^2}$$

$$\eta_{out} = \frac{\mathcal{E}}{\sigma^2} = \frac{NA^2}{\sigma^2}$$

$$\mathrm{PG} = 10\log_{10}\frac{\eta_{out}}{\eta_{in}} = 10\log_{10} N \quad \mathrm{dB}$$

$$[\mathbf{C}]_{mn} = \operatorname{cov}(w[m], w[n]) = E(w[m]w[n]) = r_{ww}[m-n]$$

$$p(\mathbf{x};\mathcal{H}_1) = \frac{1}{(2\pi)^{\frac{N}{2}}\det^{\frac{1}{2}}(\mathbf{C})}\exp\left[-\frac{1}{2}(\mathbf{x}-\mathbf{s})^T\mathbf{C}^{-1}(\mathbf{x}-\mathbf{s})\right]$$

$$p(\mathbf{x};\mathcal{H}_0) = \frac{1}{(2\pi)^{\frac{N}{2}}\det^{\frac{1}{2}}(\mathbf{C})}\exp\left[-\frac{1}{2}\mathbf{x}^T\mathbf{C}^{-1}\mathbf{x}\right]$$

σ

$$l(\mathbf{x}) = \ln \frac{p(\mathbf{x}; \mathcal{H}_1)}{p(\mathbf{x}; \mathcal{H}_0)} > \ln \gamma$$

$$\begin{aligned} l(\mathbf{x}) &= -\frac{1}{2}\left[(\mathbf{x}-\mathbf{s})^T \mathbf{C}^{-1}(\mathbf{x}-\mathbf{s}) - \mathbf{x}^T \mathbf{C}^{-1}\mathbf{x}\right] \\ &= -\frac{1}{2}\left[\mathbf{x}^T \mathbf{C}^{-1}\mathbf{x} - 2\mathbf{x}^T \mathbf{C}^{-1}\mathbf{s} + \mathbf{s}^T \mathbf{C}^{-1}\mathbf{s} - \mathbf{x}^T \mathbf{C}^{-} \right. \end{aligned}$$

$$= \mathbf{x}^T \mathbf{C}^{-1}\mathbf{s} - \frac{1}{2}\mathbf{s}^T \mathbf{C}^{-1}\mathbf{s}$$

$$T(\mathbf{x}) = \mathbf{x}^T \mathbf{C}^{-1} \mathbf{s} > \gamma'$$

σ

$$\frac{\mathbf{x}^T \mathbf{s}}{\sigma^2} > \gamma'$$

$$\mathbf{x}^T \mathbf{s} = \sum_{n=0}^{N-1} x[n] s[n] > \gamma''$$

$$T(\mathbf{x}) = \mathbf{x}^T \mathbf{C}^{-1} \mathbf{s} > \gamma'$$

$$\mathbf{s}' = \mathbf{C}^{-1} \mathbf{s}.$$

$$T(\mathbf{x}) = \mathbf{x}^T \mathbf{C}^{-1} \mathbf{s} = \mathbf{x}^T \mathbf{s}'$$

$$w[n] \sim \mathcal{N}(0, \sigma_n^2)$$

$$\mathbf{C} = \mathrm{diag}(\sigma_0^2, \sigma_1^2, \ldots, \sigma_{N-1}^2)$$

$$T(\mathbf{x}) = \sum_{n=0}^{N-1} \frac{x[n]s[n]}{\sigma_n^2} > \gamma'$$

$$T(\mathbf{x}) = \sum_{n=0}^{N-1} \frac{w[n] + s[n]}{\sigma_n} \frac{s[n]}{\sigma_n} = \sum_{n=0}^{N-1} \left(w'[n] + \frac{s[n]}{\sigma_n} \right) \frac{s[n]}{\sigma_n}$$

$$\mathbf{C}_{s'} = \mathbf{I}$$

$$s'[n] = s[n]/\sigma_n$$

$$T(\mathbf{x}') = \sum_{n=0}^{N-1} x'[n]s'[n]$$

$$x'[n] = x[n]/\sigma_n$$

,

$$\mathbf{C}^{-1} = \mathbf{D}^T\mathbf{D}$$

$$(1/\sigma_0, 1/\sigma_1, \ldots, 1/\sigma_{N-1})$$

$$T(\mathbf{x}) = \mathbf{x}^T\mathbf{C}^{-1}\mathbf{s} = \mathbf{x}^T\mathbf{D}^T\mathbf{D}\mathbf{s} = \mathbf{x}'^T\mathbf{s}'$$

$$\mathbf{x}' = \mathbf{D}\mathbf{x}, \mathbf{s}' = \mathbf{D}\mathbf{s}$$

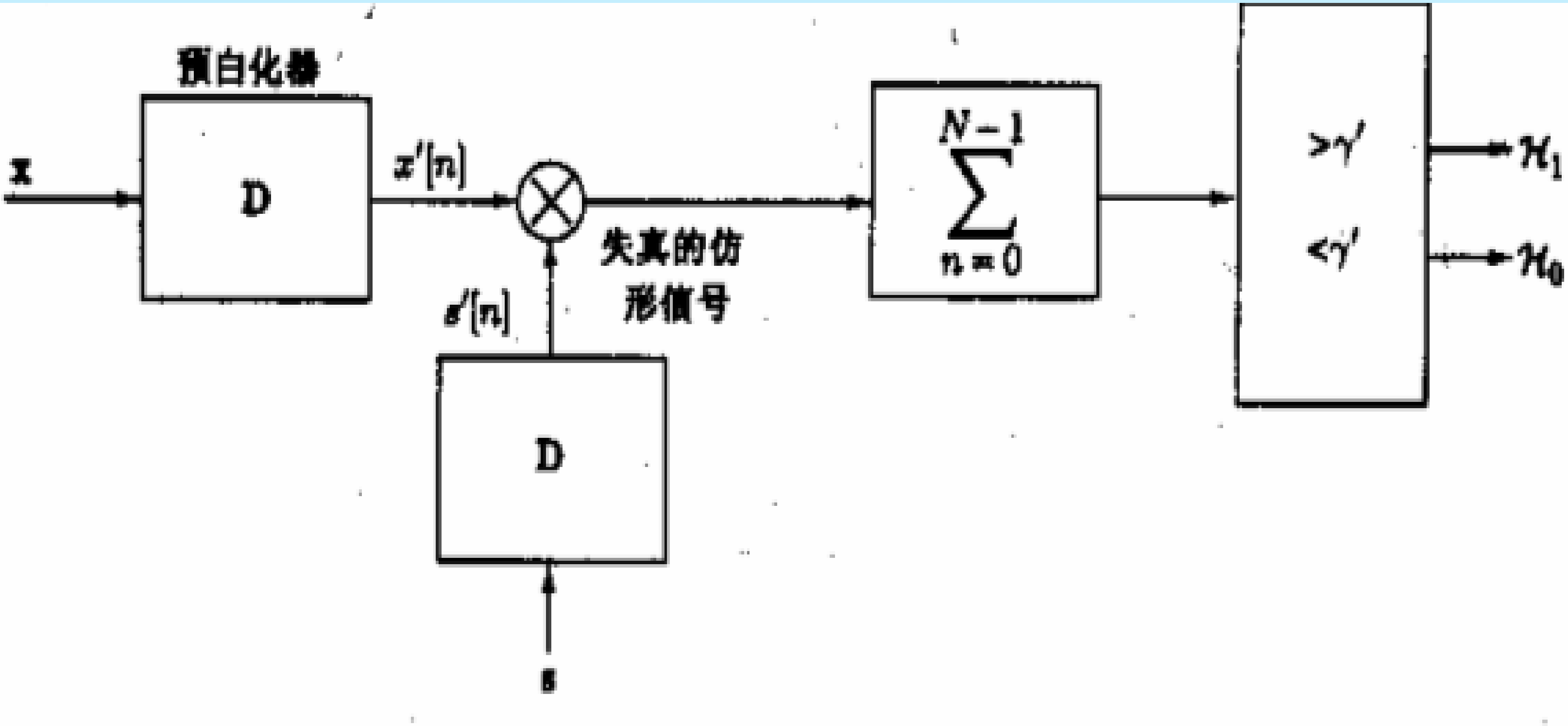


图 4.7　预白化器加仿形－相关器(匹配滤波器)的广义匹配滤波器

$$T(\mathbf{x}) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{X(f)S^*(f)}{P_{ww}(f)} df$$