. è ɚ ⁄|

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ɚ ⁄| ″ ᾱ

ɔ ɚ ⁄| ʌ
E ɚ ⁄| Ռ .
″ ᾱ ḣ ‡ ᾱ ¥ ˩ ʌ ῆ ‡

ɚ ⁄| ɚ ⁄| Ռ ɚ ⁄| Ч

4.1 ɋ /ꟸ

y í

ɋ /ꟸ ɋ /ꟸ ɶ

Ո̂ ᛌ ɋ /ꟸ

¥ ɋ ╟

. è

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$N_0/2$ ᛌ ɋ /ꟸ

y ɘ ≠

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γ ʌ

ט У э ὠ

$x(t)$ ɔ ɜ ч э Ǐ H_0 H_1 è

У э $x(t)$ ɜ ˧

ɷ $x(t)$ ɜ $p(x/o)$ $p(x/s)$

ט ̋l $x(t)$ $x(t)$ N э

˩ ˩ ̏Ạ N ŕ Ŝ

$$N(\omega)=\begin{cases}\dfrac{N_0}{2} & |\omega|\le\omega_c\\ 0 & |\omega|>\omega_c\end{cases}$$

$$R(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}N(\omega)\exp(j\omega\tau)d\omega=\frac{N_0\omega_c}{2\pi}\cdot\frac{\sin\omega_c\tau}{\omega_c\tau}$$

σ_n^2

$$6_N^2=R(0)\rightarrow\sigma_n^2=\frac{N_0\omega_c}{2\pi}=\frac{N_0}{2}\cdot\frac{1}{\Delta t}$$

$$x(t) = n(t)$$

$$p(x/0)$$

$$p(x, x_2, ..., x_N / 0) = \frac{1}{(2\pi\sigma_n^2)^{N/2}} \exp\{-\frac{1}{2\sigma_n^2}\sum_{k=1}^{N} x_k^2\}$$

$\mathbf{x_k}$ $\mathbf{x(t)}$

$$x(t) = s(t) + n(t)$$

$x(t)$ $p(x/s)$

$s(t)$

$$p(x_1, x_2, \cdots, x_N / s) = \frac{1}{(2\pi\sigma_n^2)^{N/2}} \exp\{-\frac{1}{2\sigma_n^2} \sum_{K=1}^{N} (x_k - s_k)^2\}$$

s_k $s(t)$

נ

$$\lambda(x_1, x_2, \cdots, x_N) = \frac{p(x_1, x_2, \cdots, x_N / s)}{p(x_1, x_2, \cdots, x_N / 0)}$$

$$= \exp\left\{ -\frac{1}{2\sigma_n^2} \left[\sum_{k=1}^{N} (x_k - s_k)^2 - \sum_{k=1}^{N} x_k^2 \right] \right\}$$

$$= \exp\left\{ \frac{\Delta t}{N_0} \left[2\sum_{k=1}^{N} x_k s_k - \sum_{k=1}^{N} s_k^2 \right] \right\}$$

ῆ ‡ ḣ

$$\lambda(x_1, x_2, \cdots, x_N) = \exp\left\{\frac{\Delta t}{N_0}\left[2\sum_{k=1}^{N} x_k s_k - \sum_{k=1}^{N} s_k^2\right]\right\} > \lambda_0$$

‡ ḯ **H_1** ″ ᾱ ə ʌ ī ‡ ″ ə ʌ

ɏ ´ γ ɺ

$$\Delta t \frac{2}{N_0}\sum_{k=1}^{N} x_k s_k > \ln \lambda_0 + \frac{\Delta t}{N_0}\sum_{k=1}^{N} s_k^2$$

y

$$\Delta t \sum_{k=1}^{N} x_k s_k > \frac{N_0}{2}\ln \lambda_0 + \frac{1}{2}\Delta t \sum_{k=1}^{N} s_k^2$$

¥ . ω_c G Δt ¥ 0 ɻ N ¥

ą Ռ ꜛ $T = N\Delta t$

O Í T ᾳ̈ ῆ

$$\int_0^T x(t)s(t)dt > \frac{N_0}{2}\ln\lambda_0 + \frac{1}{2}\int_0^T s^2(t)dt$$

$$\int_0^T x(t)s(t)dt = G$$

. **G** ὠ **x(t)** ὠ **x(t)** ə /ǃ **s(t)**

ǫ / ↑ ↑ $\int_0^T s^2(t)dt = E$ ə /ǃ

$$\frac{N_0}{2}\ln\lambda_0 + \frac{1}{2}E = G_0$$

″ ᾱ ‡ ɬ $G > G_0$ ″ ᾱ ə /ǃ **H_1**

Í ῗ ‡ ə /ǃ

x(t) ⸸ **G** **x(t)** ɔ

ʄ $G > G_0$ $\lambda > \lambda_0$ ʌ

ὠ **x(t)** ə˞ /‖ **s(t)** ⸸ **G** ɺ

ɣ **G₀** ″ ᾱ ə˞ /‖

ḣ э ɔ è

$x_k - s_k$ ῆ ǂ **x(t)** ∩ **s(t)**

ῆ **G** **x(t)** ∩ **s(t)** ḣ **R**

0 ɣ . ‾ ḣ ″ ᾱ ə˞ /‖

⸸ ʄ ɮ ʌ

ɮ ∩ ɮ ɮ ʌ è

ɏ У ə ʎ γ ĩ

$$H_0 : s_0(t)$$

$$H_1 : s_1(t)$$

ч э ɻ ǂ

$$s(t) = s_1(t) - s_0(t)$$ ʎ

ɔ

ɔ . **Pd**

Ռ ɚ . У ɿ

. ʇ ʎ ⁸ ת

G ǜ . † У э ″ ᾱ ̤ ˧

“ ” “ ” ɚ ʎ ɑ̢ ɏ **G** **x(t)**

ʎ **G** У э ɿ **G**

ḣ Ռ ч э **p_1** **G**

Ռ **p_0** **G** Ռ

Í **H_0** $x(t)=n(t)$

$$E[x(t)] = E[n(t)] = 0$$

G ḣ

$$E[G] = E[\int_0^T x(t)\cdot s(t)dt] = \int_0^T E[x(t)]\cdot s(t)dt = 0$$

Í **H_1** $x(t) = n(t) + s(t)$

$$E[G] = E\{\int_0^T [n(t)+s(t)]s(t)dt\} = \int_0^T s^2(t)dt = E$$

s(t) H_0 H_1 G

H_0

$$D[G]=E\{[G-E[G]]^2\}$$

$$=E\{[\int_0^T x(t)s(t)dt]^2\}$$

$$=E[\int_0^T\int_0^T n(t_1)n(t_2)s(t_1)s(t_2)dt_1dt_2]$$

$$=\int_0^T\int_0^T E[n(t_1)n(t_2)]s(t_1)s(t_2)dt_1dt_2]$$

$$=\frac{N_0}{2}\int_0^T\int_0^T \delta(t_1-t_2)s(t_1)s(t_2)dt_1dt_2]$$

$$=\frac{N_0}{2}\int_0^T s^2(t)dt=\frac{N_0E}{2}$$

¥ ῃ $p_0(G) \cap p_1(G)$

$$p_0(G) = \frac{1}{\sqrt{\pi N_o E}} \exp\left[- \frac{G^2}{N_o E} \right]$$

$$_1(\quad) \quad \frac{1}{\sqrt{\pi \quad _o}} \exp \quad \frac{(\qquad)^2}{_0}$$

‟ Ὠ

$$\alpha = \int_{G_0}^{\infty} P_o(G)dG = \int_{G_0}^{\infty} \frac{1}{\sqrt{\pi N_o E}} \exp\left[-\frac{G^{2}}{N_o E}\right] dG$$

$$u = \frac{G}{\sqrt{\frac{N_o E}{2}}}$$ ‡ ↑ **G₀** $$\frac{G_o}{\sqrt{\frac{N_o E}{2}}} = u_0$$

‡ $$\alpha = \int_{u_0}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du = 1 - \Phi(u_0)$$

è Ü **(x)** ↑

P_d

$$P_d = \int_{G_o}^{\infty} p_1(G)dG = \int_{G_o}^{\infty} \frac{1}{\sqrt{\pi N_o E}} \exp\left[-\frac{(G-E)^2}{N_o E}\right] dG$$

$$v = \frac{G-E}{\sqrt{\frac{N_o E}{2}}} \qquad u_0 = \frac{G_0}{\sqrt{\frac{N_o E}{2}}}$$ ‡ **G_0**

$$\frac{G_o}{\sqrt{\frac{N_o E}{2}}} - \sqrt{\frac{2E}{N_o}} = u_0 - \sqrt{d}$$

$$P_d = \int_{u_0-\sqrt{d}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{v^2}{2}\right) dv = 1 - \Phi(u_0 - \sqrt{d})$$

è $v = u - \sqrt{d}$ **d** ə˞ /ℓ ´ α̤̈ . ǯ

. ┤ ɷ ə˞ . à **P_d** ᾱ ɏ ə˞ /ℓ .

∩˫

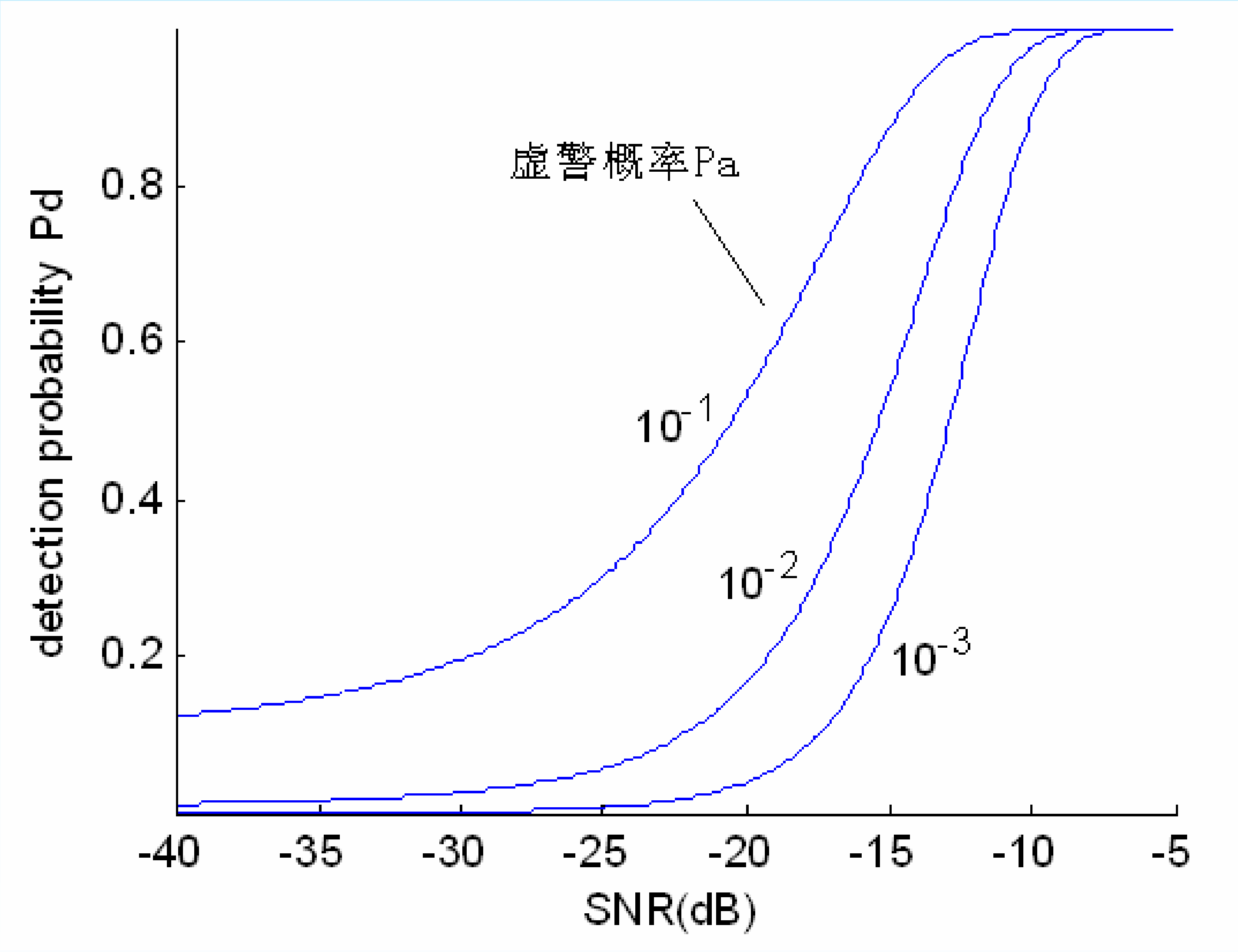
E **N_o** ∩˫ **G_o** э ⍏ ⅃ à **P_d** ḣ

ɔ ɔ ɷ à ∩˫

P_d ┤ ə˞ . **d** **ROC** ΰ

ȳ y à ┤ **d** ⍏

P_d **4.3**


虚警概率Pa
10^{-1}
10^{-2}
10^{-3}
detection probability Pd
0.8
0.6
0.4
0.2
-40
-35
-30
-25
-20
-15
-10
-5
SNR(dB)

y

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u_o d ⋂ u_o P_d

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ą̈ у ᛋ

ə /ˀl ᛋ ə /ˀl ə ╟ ə

/ˀl ⫯ Ռ ᛋ è У э ű э

4.2.1

$s(t, \varphi_s)$

φ_s φ_s

0 2 $p(\varphi_s) = \frac{1}{2\pi}$

$\frac{N_0}{2}$

ɣ ʌ

ɔ ɺ

ɚ /ˀ| ↑ x(t) ɺ . X(t)=n(t)

p(x/o) ᾱ ɏ .

ɚ /ˀ| ↑ $x(t) = n(t) + s(t, \varphi_s)$

φ_s ∩ n(t) x(t) ʎ

p(x, φ_s /s)

ɔ ê (**x**,φ_s) φ_s ɷ ¥ φ_s

ɔ . φ_s

p φ_s φ_s ٩

ə̨ ᚨ ᛅ ɔ

$$p(x,\varphi_s/s)=p_s(x/\varphi_s)p(\varphi_s)$$

$$p(x/s)=\int_0^{2\pi}p(x/\varphi_s)p(\varphi_s)d\varphi_s$$

è $p(x/\varphi_s)$ ¥ φ_s

ᛏ **x(t)**

$$\lambda(x) = \frac{p(x/s)}{p(x/o)}$$

$$= \frac{\int_0^{2\pi} p(x/\varphi_s)p(\varphi_s)d\varphi_s}{p(x/0)}$$

$$= \int_0^{2\pi} \lambda(x/\varphi_s)p(\varphi_s)d\varphi_s$$

è $\lambda(x/\varphi_s) = \dfrac{p(x/\varphi_s)}{p(x/0)}$ נ ≠ ٦

φ_s נ נ ﻟ $\lambda(x)$

נ

$\lambda(x/\varphi_s)$ φ_s

$\Delta t \rightarrow o$

$\lambda(x/\varphi_s)$

$$\lambda(x/\varphi_s) = \exp\left\{\frac{2}{N_0}\int_O^T x(t)S(t,\varphi_s)dt - \frac{1}{N_0}\int_o^T S^2(t,\varphi_s)dt\right\}$$

$$= \exp\left\{\frac{2}{N_o}G(\varphi_s) - \frac{E(\varphi_s)}{N_o}\right\}$$

è $E(\varphi_s) = \int_o^T S^2(t,\varphi_s)dt$ əʅ /ᛚ

$G(\varphi_s) = \int_o^T x(t)s(t,\varphi_s)dt$ φ_s ↑ ȕ φ_s

↑ ȕ əʅ /ᛚ

$G(\varphi_s)$ ɔ ″ ᾱ əʅ /ᛚ

ə̢ /\|

$$S(t,\varphi_s) = A\sin(\omega_c t + \varphi_s) \qquad 0 \le t \le T$$

–

Ὠ ↑

$$S(t,\varphi_s) = A\sin\omega_c t \cdot \cos\varphi_s + A\cos\omega_c t \cdot \sin\varphi_s$$

$$S_s(t) = A\sin\omega_c t$$

$$S_c(t) = A\cos\omega_c t$$

‡ $S(t,\varphi_s) = S_s(t)\cos\varphi_s + S_c(t)\sin\varphi_s$

ώ $G(\varphi_s)$

$$G(\varphi_s) = \int_o^T S(t, \varphi_s) x(t) dt$$

$$= \int_o^T \left[S_s(t) \cos\varphi_s + S_c(t) \sin\varphi_s \right] x(t) dt$$

$$= G_s \cos\varphi_s + G_c \sin\varphi_s$$

è

$$G_s = \int_0^T S_s(t) x(t) dt$$

$$G_c = \int_o^T S_c(t) x(t) dt$$

Z Ո ç

$$Z = \sqrt{G_s^2 + G_c^2}$$

$$\theta = tg^{-1} \frac{G_c}{G_s}$$

‡ $G_s = Z \cos\theta$

$$G_c = Z \sin\theta$$

¥

$$G(\varphi_s) = Z\cos\theta\cos\varphi_s + Z\sin\theta\sin\varphi_s = Z\cos(\varphi_s - \theta)$$

ə /ꟾ

$$E(\varphi_s) = \int_0^T S^2(t,\varphi_s)dt = \int_0^T A^2 \sin^2(\omega_c t + \varphi_s)dt$$

$$= \frac{A^2}{2}T = E \qquad \textbf{4-36}$$

ə /ꟾ ˥ φ_s ǘ ψ **4-35** Ռ **4-36** ώ

נ

$$\lambda(x/\varphi_s) = \exp\left\{\frac{2}{N_0}Z\cdot\cos(\varphi_s - \theta) - \frac{E}{N_0})\right\}$$

ώ נ נ $\lambda(x)$

$$\lambda(x) = \int_0^{2\pi} \lambda(x/\varphi_s)\, p(\varphi_s)\, d\varphi_s$$

$$= \exp(-\frac{E}{N_0})\frac{1}{2\pi}\int_0^{2\pi} \exp\left\{\frac{2}{N_0} Z \cos(\varphi_s - \theta)\right\} d\varphi_s$$

$$= \exp(-\frac{E}{N_0}) I_0(\frac{2Z}{N_0}) \qquad \textbf{4-37}$$

è $I_0(u) = \frac{1}{2\pi}\int_0^{2\pi} \exp\{u \cdot \cos(\varphi - \theta)\} d\varphi$ g

″ ᾱ ‡

$$\lambda(x)=\exp(-\frac{E}{N_0})I_0(\frac{2Z}{N_0})>\lambda_0 \qquad \textbf{(4-38)}$$

″ ᾱ ə /ˀ **H_1** Í

ч **H_1** Í ″ ᾱ

$$\ln I_0(\frac{2Z}{N_0})>\ln\lambda_0+\frac{E}{N_0}$$

∧ ‡ ¥ Z

´ $I_0(u)$ ***u*** ´ ≉ ´

ʎ Z ″ ᾱ ″ ᾱ ‡

$$Z > \frac{N_0}{2} arcI_0[\lambda_0 \exp(\frac{E}{N_0})] \qquad \textbf{4-39}$$

Z₀

$$Z_0 = \frac{N_0}{2} arcI_0[\lambda_0 \exp(\frac{E}{N_0})] \qquad \textbf{4-40}$$

$Z > Z_0$ $\lambda(x) > \lambda_0$ ∧ ῶ

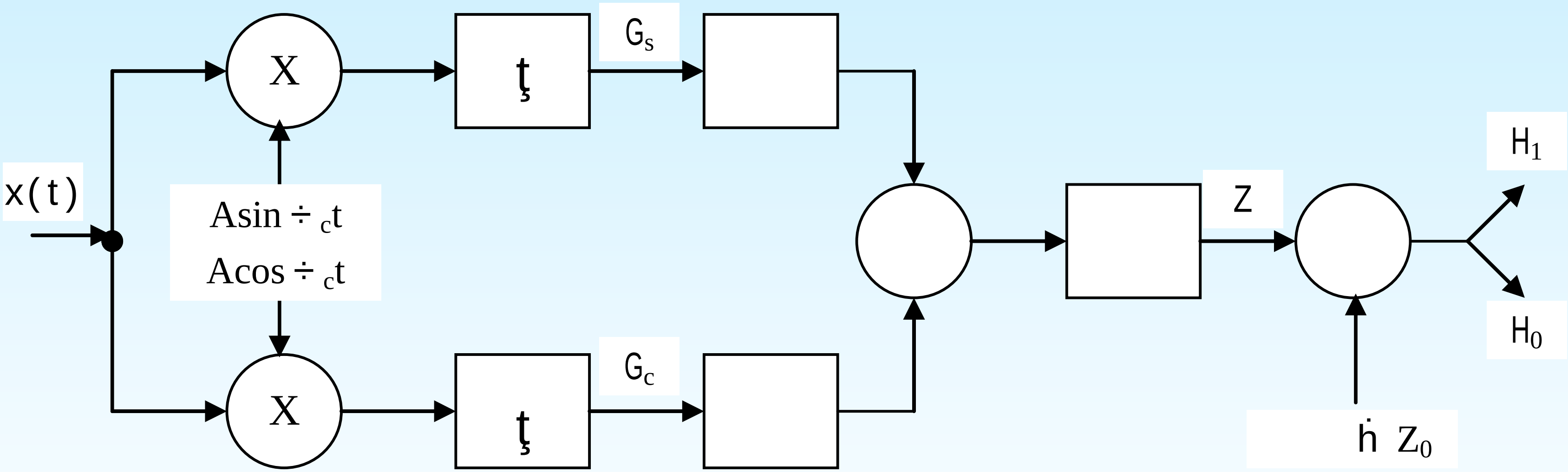
$x(t)$ **Z** ʎ Z_0 ɔ ″ ᾱ

∧ **4.4**

$Z > Z_0$ $\lambda(x) > \lambda_0$ ʌ ώ $x(t)$

Z ɹ Z_0 ɔ ″ ᾱ ʌ

4.4



4.4 əɻ ʎ ʌ

è ”l ᛩ ə̡ /ᛧ $x(t)$ᛨ ч ᛨ ⁞

$A\sin\omega_c t$ $A\cos\omega_c t$ ϕ ᛨ ɺ Ռ

Z $x(t)$ Z Z Ꭹ

ə̡ /ᛧ У ɻ У э Ꭹ G ɏ

э G ↑ ǖ ə̡ /ᛧ˥ φ_s ‡ G

ə̡ /ᛧ ″ ᾱ ə̡ /ᛧ ч э

Ʉ ə̡ /ᛧ ǯ Ռ ч э Ʉ ч э Ꭹᛨ

⁞ G_S Ռ G_C G_S Ռ G_C ə̡ /ᛧ ˥ φ_s ת

Ռ Z^2 φ_s ɷ ɷ φ_s ″ ᾱ

ĺ ə̡ /ᛧ ´ ə̡ /ᛧ \ᛧ ɏ ə̡ /ᛧ ʌ

ɵ Ꭹ ɏ ɏ ə̡ /ᛧ ʌ ‡ z

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ɔ

Z ȕ . † У э Z

″ ᾱ ᵜ ч

ɔ Z $p(Z/0)$ ∩

$p(Z/s)$ α ∩ $\mathbf{P_d}$

$$Z=\sqrt{G_S^2+G_C^2}$$ Z

″l $\mathbf{G_S}$ ∩ $\mathbf{G_C}$ ώ .

$\mathbf{G_S}$ $\mathbf{G_C}$ $\boldsymbol{x(t)}$ ≠ ᛇ ∩

ḣ ῆ

ĺ **H_1** ɏ У э ḣ **G_S** **G_C** ḣ

$$E[G_S/\varphi_S] = E\cos\varphi_S$$

Ռ $$E[G_C/\varphi_S] = E\sin\varphi_S$$

è $$E = \frac{A^2}{2}T$$

ə̡ ʎl ɹl ɹl

$$\sigma_{G_S}^2 = \sigma_{G_C}^2 = \sigma_{G_O}^2 = \frac{N_0 E}{2}$$

¥ G_S ∩ G_C

$$P_1(G_S, G_C / \varphi_S) = P_1(G_S / \varphi_S) P_1(G_C / \varphi_S)$$

$$= \frac{1}{2\pi\sigma_{G_0}^2} \exp\left[-\frac{(G_S - E\cos\varphi_S)^2 + (G_C - E\sin\varphi_S)^2}{2\sigma_{G_0}^2} \right]$$

$G_s = Z\cos\theta$ $G_c = Z\sin\theta$

$$P_1(Z,\theta/\varphi_S) = P_1(G_s, G_C/\varphi_S)\cdot|J|$$

$$J = \frac{\partial(G_S, G_C)}{\partial(Z,\theta)} = \begin{vmatrix} \cos\theta & \sin\theta \\ -Z\sin\theta & Z\cos\theta \end{vmatrix} = Z$$

$$p_1(Z,\theta/\varphi_S) = \frac{Z}{2\pi\sigma_{G_0}^2}\exp\left[-\frac{Z^2+E^2-2ZE\cos(\theta-\varphi_S)}{2\sigma_{G_0}^2}\right]$$

θ 0 2π ,

$$p_1(Z/\varphi_S) = \frac{Z}{\sigma_{G_0}^2} I_0\left(\frac{ZE}{\sigma_{G_0}^2}\right)\exp\left(-\frac{Z^2+E^2}{2\sigma_{G_0}^2}\right)$$

φ_s , φ_s ,

$$p_1(Z) = \int_0^{2\pi} p_1(Z/\varphi_S)\frac{d\varphi_s}{2\pi}$$

$$= \frac{Z}{\sigma_{G_0}^2} I_0\left(\frac{ZE}{\sigma_{G_0}^2}\right)\exp\left(-\frac{Z^2+E^2}{2\sigma_{G_0}^2}\right)$$

Z

ɏ Ỉ $H_0, E = 0, I_0(0) = 1$

$$p_0(Z) = \frac{Z}{\sigma_{G_0}^2} \exp\left(-\frac{Z^2}{2\sigma_{G_0}^2}\right)$$ **4-4**

ə ᴧ **Z** ‾ ↑

Z ℨ ᴧ α Ռ

Pd

$$\alpha = \int_{Z_0}^{\infty} \frac{Z}{\sigma_{G_0}^2} \exp(-\frac{Z^2}{2\sigma_{G_0}^2}) dz$$

$$= \exp(-\frac{Z_0^2}{2\sigma_{G_0}^2}) = \exp(-\frac{z_0^2}{2}) \qquad 4\text{-}43$$

è $$z_0 = \frac{Z_0}{\sigma_{G_0}}$$

$$P_d = \int_{Z_0}^{\infty} \frac{Z}{\sigma_{G_0}^2} \exp(-\frac{Z^2 + E^2}{2\sigma_{G_0}^2}) I_0 (\frac{ZE}{\sigma_{G_0}^2}) dZ$$

$$= \int_{z_0}^{\infty} z \exp(-\frac{z^2 + d^2}{2}) I_0 (d \cdot z) dz \qquad \text{4-44}$$

è $\quad d = \frac{E}{\sigma_{G_0}}, z = \frac{Z}{\sigma_{G_0}}$

э $\quad$ † $\qquad$ **Q**

ɣ d z_0 ḣ ⍏ $\mathbf{P_d}$ Ռ α

ɔ **ROC** **4.5** ɷ è ≠

ɷ ə˛ /᛫ ʌ **ROC** è

⍏ ɺ Ռ ə˛ /᛫

ə˛ /᛫ ə˛ . **1dB** ɺ ɣ

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4.2.2 ɚ /ꞁ

ʏ ɩ́

╟ ˥ ɚ /ꞁ ˧ ɚ /ꞁ

◗ $$\Delta f_d = f_c \cdot \frac{2v_r}{c}$$ è v_r ╟ ɹ

c ɏ v_r ◗ Δf_d ≢

˥ ɚ /ꞁ ɔ

$$S(t,\omega_s,\varphi_s) = A\sin(\omega_s t + \varphi_s) \qquad 0 \le t \le T$$

A φ_s **0** 2π

ω_s (ω_L, ω_M) $p(\omega_S)$

$$\frac{N_0}{2}$$

ɣ ʌ

ə ʎ ɔ ᛏ ω_s ω_s

ʏ ḣ ɔ

$$\lambda(x/\omega_s) = \exp(-\frac{E}{N_0}) I_0(\frac{2Z}{N_0}) \qquad \textbf{4-45}$$

è

$$Z = \{[\int_0^T x(t) A \sin \omega_s t dt]^2 + [\int_0^T x(t) A \cos \omega_s t dt]^2\}^{\frac{1}{2}}$$

ɔ $\lambda(x)$

$$\lambda(x) = \int_{\omega_L}^{\omega_M} \lambda(x/\omega_s) p(\omega_s) d\omega_s \qquad \textbf{4-46}$$

ω_L Í ω_M , **M** э ҿ ą̈ $P(\omega_s)$

$$\begin{cases} \omega_i = \omega_L + i(\Delta\omega) \\ M = \dfrac{\omega_M - \omega_L}{\Delta\omega} \end{cases} \qquad i = 1,2,\cdots,M$$

$\lambda(x)$ ῆ

$$\lambda(x) = \sum_{i=1}^{M} \lambda(x/\omega_i)P(\omega_i) \qquad \textbf{4-47}$$

è $P(\omega_i) = p(\omega_i)\Delta\omega$

M

ω_i **Hi**

H

$$
\begin{aligned}
H_0 &: \quad x(t) = n(t) \\
H_1 &: \quad x(t) = n(t) + A\sin(\omega_1 t + \varphi_s) \\
&\vdots \\
H_M &: \quad x(t) = n(t) + A\sin(\omega_M t + \varphi_s)
\end{aligned}
$$

$P(\omega_i)$ H_i **H_0**

$$\lambda_i = \frac{p(x/\omega_i)}{p(x/o)} = \exp(-\frac{E}{N_0}) I_o(\frac{2Z_i}{N_0}) \qquad \textbf{4-48}$$

У э λ_i ḣ λ_0 ‡ ″ **H_0** Ĩ

√ ǯ ‡ ″ ɏ λ_i Ĩ **H_i** у

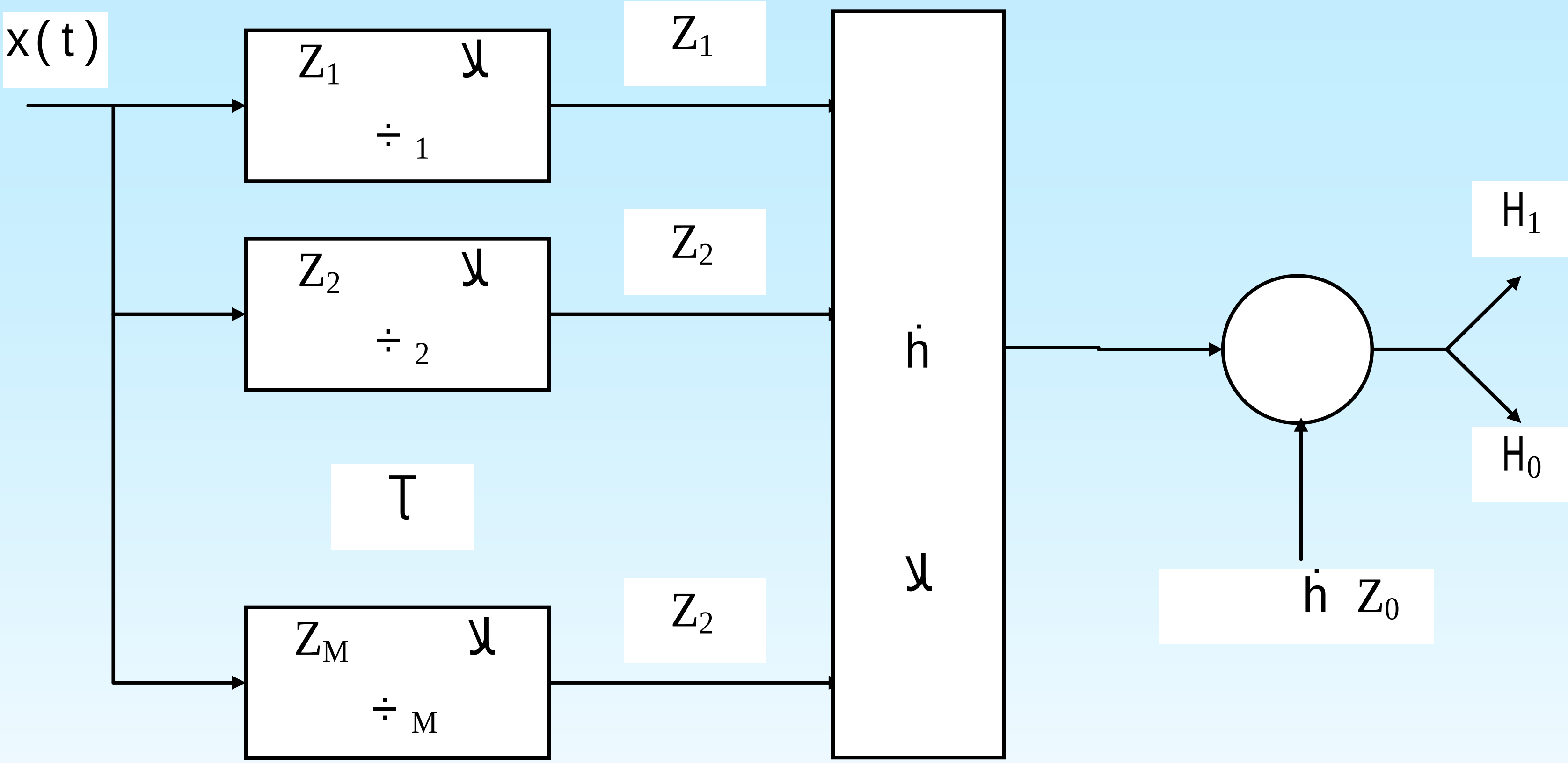
Z_i **Z_i** è

$$Z_0 = \frac{N_0}{2} arcI_0[\lambda_0 \exp(\frac{E}{N_0})]$$

Gs

‡ ″ **H_i** Ĩ √ ǯ У э **Z_i** ḣ ‡

H_0 Ĩ ʌ **4.5**



4.5

¥ ə ʎ ʮ ÷ i,

ʎ M è è ḣ Z_0 ″ ā

4.3 . ə ʎ

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ḯ ώ ə ʎ ∩ .

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э ɷ ɔ ʊ 2TW

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ǯ ʎ

ˮl

x(t) У ¥

-2πW,2πW ɔ **0≤ t≤T** **T**

  ɔ ʌ

$$x(t) = \sum_{n=-TW}^{TW} X(n)\exp(j\omega_n t) \qquad \textbf{(4-49)}$$

$$X(n) = \frac{1}{T}\int_0^T x(t)\exp(-j\omega_n t)dt \qquad \textbf{(4-50)}$$

è $\omega_n = n\frac{2\pi}{T}$ $X(n) = X(\omega_n)$ **n** ḣ

(-TW~TW)

X(n)=a(n)+jb(n) ‡ x(t)

$$x(t) = a_0 + \sum_{n=1}^{TW} [2a(n)\cos\omega_n t - 2b(n)\sin\omega_n t] \qquad \textbf{(4-51)}$$

$$a(n) = \frac{1}{T}\int_0^T x(t)\cos(\omega_n t)dt \qquad \textbf{(4-52)}$$

$$b(n) = -\frac{1}{T}\int_0^T x(t)\sin(\omega_n t)dt \qquad \textbf{(4-53)}$$

x(t) X(n) 0 T

X(n) a(n) b(n)

X(n) a(n) b(n) X(n) a(n) b(n)

x(t) E[x(t)]=0

X(n) a(n) b(n)

$$E[X(n)] = E[a(n)] = E[b(n)] = 0$$

ʎ $a(n)$ $a(m)$ $b(n)$ $b(m)$ ˧ $a(n)$ $b(m)$

˜ ḣ γ T

$$E[a(n)a(m)]=\begin{cases}\dfrac{S_x(\omega_n)}{2T} & n=m\\ 0 & n\neq m\end{cases}$$

$$E[b(n)b(m)]=\begin{cases}\dfrac{S_x(\omega_n)}{2T} & n=m\\ 0 & n\neq m\end{cases}$$

$E[a(n)b(m)]=0$ ˙ n ∩ m

è $S_x(\omega_n)$ $x(t)$ ω_n

ḣ

Ą **X(n)** **X(m)** ˜

$$E[X(n)X^*(m)] = E\left\{\left[a(n)+jb(n)\right]\left[a(m)-jb(m)\right]\right\}$$

$$= E[a(n)a(m)] + E[b(n)b(m)] + jE[a(m)b(n)] - jE[a(n)b(m)]$$

$$= \begin{cases} \dfrac{S_x(\omega_n)}{T} & n = m \\ 0 & n \neq m \end{cases}$$

ę **T** ɺ

ʄ . G ɺ $a(n)$ ∩ $b(m)$ ≠

$a(n)$ $b(n)$ $x(t)$

$X(n)$

$X(n)$ $a(n)$

$b(n)$

$a(n)$ $b(n)$.

$$p\left[a\left(n\right),b(n)\right]=\frac{1}{2\pi\cdot\frac{S_x(\omega_n)}{2T}}\exp\left\{-\frac{1}{2}\frac{2T}{S_x(\omega_n)}\left[a^2(n)+b^2(n)\right]\right\}$$

$$a^2(n) + b^2(n) = |X(n)|^2$$

$$\sigma_{X_n}^2 = \frac{S_x(\omega_n)}{T}$$

ὡ ↑ X(n)

$$p[X(n)] = \frac{1}{\pi\sigma_{X_n}^2}\exp\left[-\frac{|X(n)|^2}{\sigma_{X_n}^2}\right] \qquad \textbf{4-55}$$

$p[a(n), b(n)]$ Ռ $p[X(n)]$ У ܪ У э

$X(n)$ ↑ ܪ $p[a(n), b(n)]$ $X(n)$ Ռ ч э

$a(n), b(n)$

$\{X(n)\}$ ὤ ɷ \ɬ ὼ $x(t)$ ʊ ˧

$\bar{x}(t)=0$, $X(0)=0$ ˧

Ὤ **X(-n)=X*(n)** ʮ

˧ TW э : $X(1), X(2), \cdots x(TW)$. **TW** э У э

TW ɻ **X** $X=\left[X(1), X(2), \cdots X(TW)\right]$

x(t) X У э ɹ X

ҩ λ ɹ

ϯ $X(1), X(2), \cdots X(TW)$ ц . ц

ɹ X "

$$p(X)=\prod_{n=1}^{TW}\frac{1}{\pi\sigma_{X_n}^2}\exp\left[-\frac{\left|X(n)\right|^2}{\sigma_{X_n}^2}\right]$$

$$=[\prod_{n=1}^{TW}\frac{1}{\pi\sigma_{X_n}^2}]\cdot\exp\left[-\sum_{n=1}^{TW}\frac{\left|X(n)\right|^2}{\sigma_{X_n}^2}\right] \qquad \textbf{4-56}$$

H₀ ą . ˥ . $x(t) = n(t)$.

$N(\omega)$ ‡

$$\sigma^2_{X_n}\Big|_{H_0} = \frac{S_x(\omega_n)}{T}\Big|_{H_0} = \frac{N(\omega_n)}{T}$$

H₀ ą **X**

$$p_0(\mathbf{X}) = A\exp\left\{-\sum_{n=1}^{TW} T\frac{|X(n)|^2}{N(\omega_n)}\right\}$$

è **A** $X(n)$

H₁ ą . əˌ /ǀ . $x(t) = s(t) + n(t)$ əˌ /ǀ

$S(\omega)$ əˌ /ǀ ∩ . ц ‡

$$\sigma^2_{X_n}\Big|_{H_1} = \frac{N(\omega_n)}{T} + \frac{S(\omega_n)}{T}$$

H_1 ą **X**

$$p_1(\mathbf{X}) = B\exp\left\{-\sum_{n=1}^{TW} T \cdot \frac{\left|X(n)\right|^2}{N(\omega_n) + S(\omega_n)}\right\}$$

è **B** $X(n)$

ɔ ч ɔ ʓ̌ .

$$\lambda(\mathbf{X}) = \frac{p_1(\mathbf{X})}{P_0(\mathbf{X})}$$

$$= \frac{B}{A}\exp\left\{\sum_{n=1}^{TW} T\left|X(n)\right|^2 \left[\frac{1}{N(\omega n)} - \frac{1}{N(\omega_n) + S(\omega_n)}\right]\right\}$$

ɔ ʎ ɔ $\lambda(x)$ ʌ ῆ ‡

ḣ λ_0 ɷ ɔ ″ ᾱ ‡

$$\frac{B}{A}\exp\left\{\sum_{n=1}^{TW} T\left|X(n)\right|^2 \left[\frac{1}{N(\omega_n)} - \frac{1}{N(\omega_n) + S(\omega_n)}\right]\right\} > \lambda_0$$

$\mathbf{H_1}$ ΐ

¥ ´ ḣ Ո´ ч ɺ

ɺ

$$G=\sum_{n=1}^{TW} T\left|X(n)\right|^2\left[\frac{1}{N(\omega_n)}-\frac{1}{N(\omega_n)+S(\omega_n)}\right] \quad \textbf{4-57}$$

$$>\ln\left[\frac{A}{B}\lambda_0\right]=G_0$$

G **G_0** ḣ $G>G_0$

$\lambda(x)>\lambda_0$ ч

У Ɫ · $H(\omega)$

$$\left|H(\omega)\right| = \left[\frac{1}{N(\omega)} - \frac{1}{N(\omega)+S(\omega)}\right]^{\frac{1}{2}}$$

$$= \left[\frac{S(\omega)}{N(\omega)\left[N(\omega)+S(\omega)\right]}\right]^{\frac{1}{2}} \qquad \textbf{4-58}$$

3 $Y(n) = X(n)H(\omega_n), Y(n)$ $x(t)$ $X(n)$ ·

$H(\omega)$ Ɫ ‡ נ ῆ

$$G = T\sum_{n=1}^{TW}\left|X(n)H(\omega_n)\right|^2 = T\sum_{n=1}^{TW}\left|Y(n)\right|^2 > G_0$$

$$\int_o^T y^2(t)dt = 2T\sum_{n=1}^{TW}\left|Y(n)\right|^2$$

è **y(t)** **Y(n)** ω_n ¥

נ ″ ᾱ ‡ ῇ

$$G = \frac{1}{2}\int_o^T y^2(t)dt > G_0 \qquad \textbf{4-59}$$

H_1 ἷ

у

ʌ **0 T** ᾀ **x(t)**

У э **4-58** ʎ ʌ

ᵼ У э **Go** \ נ

″ ᾱ ‡ è $\frac{1}{2}$ **4.6**

∧ Ѫ

ᴛ ∧ η̃ † H_0 Ռ H_1

H_0 Ռ H_1 ώ ḣ ḣ

Ռ ώ̋ ḣ H_0 Ռ

H_1 ώ ⁞ ϴ ɏ ⅃ ↑

Ռ

⅃ ą

1

$S(\omega)$ $N(\omega)$

$(-2\pi W, 2\pi W)$

$\mathbf{H_0}$ $\mathbf{H_1}$

$H(\omega)$

$$\int_0^T [x(t)]^2 dt$$

T .

2 ˙ ą

ẇ . ‡ ‾ əʅ ϴ

3 əʅ ‾ E

Ɉ· $H(\omega)$

$$\left|H(\omega)\right| = \left\{ \frac{S(\omega)}{N(\omega)\left[N(\omega)+S(\omega)\right]} \right\}^{\frac{1}{2}}$$

ẇ əʅ . ą

$$\left|H(\omega)\right| \approx \frac{S^{\frac{1}{2}}(\omega)}{N(\omega)} \qquad \textbf{4-60}$$

(Eckart)

$|H(\omega)|$

$$|H(\omega)| = \frac{1}{N^{\frac{1}{2}}(\omega)} \cdot \frac{S^{\frac{1}{2}}(\omega)}{N^{\frac{1}{2}}(\omega)}$$

1 $\frac{1}{N^{\frac{1}{2}}(\omega)}$

1 . “ ”

$$\frac{S(\omega)}{N(\omega)}$$

2

$$\frac{S^{\frac{1}{2}}(\omega)}{N^{\frac{1}{2}}(\omega)}$$

$$\frac{S(\omega)}{N(\omega)}$$