ASSIGNMENT-2
LP - Model
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1. Solution: Given from the Questions,

2 types of backpacks : collegiate and Mini

Resources :

- ✓ 5000 Square feet of Nylon
- ✓ 35 workers, each working 40 hours per week

Constraints :

- ✓ At most 1000 Collegiates and 1200 Minis Can be Sold / week.
- ✓ Collegiate requires 3 square feet of Nylon, 45 min of Labor.
- ✓ Mini requires 2 Sq.ft of nylon, 40 min of Labor

Objective : Maximize profit where collegiate provides \$ 32 per unit
Mini provides \$ 24 per unit

a. Define Decision Variables

(z, a_1, a_2) z - objective function

a_1 - Number of collegiate backpacks produced Per week

a_2 - Number of Mini backpacks produced Per week

b. Define objective function

Maximize $z = 32a_1 + 24a_2$

Because 32 and 24 is the generated unit of profit and a_1 and a_2 are non-negative.

c. Constraints :

1. Material Constraint (5000 Sq.ft of Nylon)

Each collegiate backpack requires 3 sq.ft, and each

Mini backpack requires 2 Sq.ft. Therefore, total fabric used cannot exceed 5000 Sq.ft.

$$3a_1 + 2a_2 \leq 5000$$

2. Labor constraint (35 Laborers, 40 hours each, (or) 35×40 = 1400 hours per week)

Each collegiate backpack requires 45 minutes (0.75 hours) of labor, each Mini requires 40 minutes (or 2/3 hours) of labor. Thus, total labor used cannot exceed 1400 hours.

$$0.75\, a_1 + \frac{2}{3} a_2 \leq 1400$$

3. Sales constraint (at most 1000 can be sold per week)

$$a_1 \leq 1000$$

4. Sales constraint for Mini (at most 1200 can be sold/week)

$$a_2 \leq 1200$$

5. Non-negativity constraint: Number of backpacks produced must be non-negative

$$a_1 \geq 0, \quad a_2 \geq 0$$

d. full Mathematical formula Subject to

$$\text{Maximize } Z = 32a_1 + 24a_2$$

$$3a_1 + 2a_2 \leq 5000$$

$$0.75 a_1 + \frac{2}{3} a_2 \leq 1400$$

$$0 \leq a_1 \leq 1000 \text{ and } 0 \leq a_2 \leq 1200$$

Question 2
Solution

Given from the Question,

- Three product Sizes : Large, Medium, Small
- Profit per unit : \$420 for Large, \$360 for Medium, \$300 for Small
- Production capacities : plant 1 can produce 750 units/day, plant 2 can produce 900 units/day, plant 3 can produce 450 units/day.
- Space limitations : plant 1 has 13,000 sq ft, plant 2 has 12,000 Sq.ft and plant 3 has 5000 Sq.ft.
- Space requirement per unit : Large (20 Sq.ft), Medium (15 Sq.ft), Small (12 Sq.ft)
- Sales forecasts : Maximum Sales for large (900 units), Medium (1200 units), Small (750 units)
- Equal percentage use of production capacity across plants.

(a) Define the Decision Variables :

Let

- a_{1L}, a_{1M}, a_{1S} be the number of large, Medium and Small Products produced in plant 1, respectively.
- a_{2L}, a_{2M}, a_{2S} be the number of large, Medium and Small Products Produced in plant 2, respectively.
- a_{3L}, a_{3M}, a_{3S} be the number of large, Medium and Small products produced in plant 3, respectively.

b. Objective function

The goal is to maximize profit. The net profit for large, Medium and Small products is \$ 420, \$ 360, \$300 respectively. Thus, objective function is :

$$\text{Maximize } Z = 420(a_{1L} + a_{2L} + a_{3L}) + 360(a_{1M} + a_{2M} + a_{3M}) + 300(a_{1S} + a_{2S} + a_{3S})$$

c. Constraints :

1. Capacity constraint for plant 1 (750 units total capacity)

$$a_{1L} + a_{1M} + a_{1S} \leq 750$$

2. Capacity constraint for plant 2 (900 units total capacity)

$$a_{2L} + a_{2M} + a_{2S} \leq 900$$

3. Capacity constraint for plant 3 (450 units total capacity)

$$a_{3L} + a_{3M} + a_{3S} \leq 450$$

4. Storage Space constraint for plant 1 (13,000 Sq.ft):
Each large product requires 20 sq.ft, Medium requires 15 sq.ft and Small requires 12 sq.ft:

$$20a_{1L} + 15a_{1M} + 12a_{1S} \leq 13000$$

5. Storage Space constraint for plant 2 (12,000 sq.ft)

$$20a_{2L} + 15a_{2M} + 12a_{2S} \leq 12{,}000$$

6. Storage Space constraint for plant 3 (5000 sq.ft)

$$20a_{3L} + 15a_{3M} + 12a_{3S} \leq 5000$$

7. Sales forecast constraint,

For large products (900 can be sold)

$$a_{1L} + a_{2L} + a_{3L} \leq 900$$

8. For Medium products (1200 sold)

$$a_{1M} + a_{2M} + a_{3M} \leq 1200$$

9. For Small products (750 Sold)

$$a_{1S} + a_{2S} + a_{3S} \leq 750$$

10. Capacity utilization constraint (Same across all plants)

$$\frac{a_{1L} + a_{1M} + a_{1S}}{750} = \frac{a_{2L} + a_{2M} + a_{2S}}{900} = \frac{a_{3L} + a_{3M} + a_{3S}}{450}$$

11. Non-Negativity constraint: Number of Products ~~products~~ Produced must be non-negative:

$$a_{iL}, a_{iM}, a_{iS} \geq 0 \text{ for all } i = 1,2,3$$

✓ Full mathematical formulation:

$$\text{Maximize } Z = 420(a_{1L} + a_{2L} + a_{3L}) + 360(a_{1M} + a_{2M} + a_{3M}) + 300(a_{1S} + a_{2S} + a_{3S})$$

Subject to,

$$a_{1L} + a_{1M} + a_{1S} \leq 750$$

$$a_{2L} + a_{2M} + a_{2S} \leq 900$$

$$a_{3L} + a_{3M} + a_{3S} \leq 450$$

$$20a_{1L} + 15a_{1M} + 12a_{1S} \leq 13000$$

$$20a_{2L} + 15a_{2M} + 12a_{2S} \leq 12000$$

$$20a_{3L} + 15a_{3M} + 12a_{3S} \leq 5000$$

$$a_{1L} + a_{2L} + a_{3L} \leq 900$$

$$a_{1M} + a_{2M} + a_{3M} \leq 1200$$

$$a_{1S} + a_{2S} + a_{3S} \leq 750$$

$$\frac{a_{1L} + a_{1M} + a_{1S}}{750} = \frac{a_{2L} + a_{2M} + a_{2S}}{900} = \frac{a_{3L} + a_{3M} + a_{3S}}{450}$$

$$x_{iL}, x_{iM}, x_{iS} \geq 0 \text{ for all } i = 1, 2, 3$$