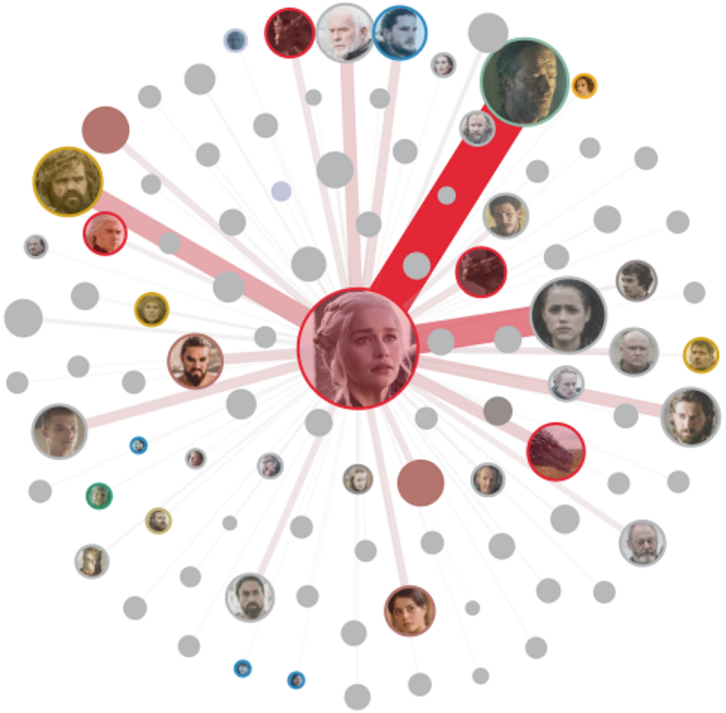
Social Network Analysis Review

Let’s get started from the basics!

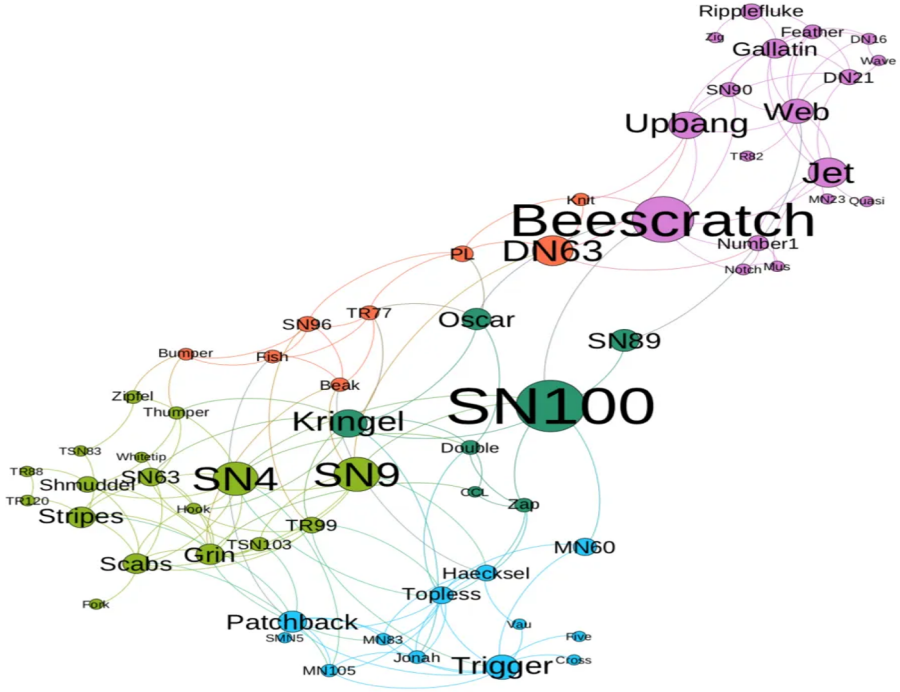
- Node/Vertex
- Tie/Edge
  - Tie direction
  - Tie weights
- Network Size

Image source https://news.northeastern.edu/2019/04/11/the-game-of-thrones-social-network/

Ego Network

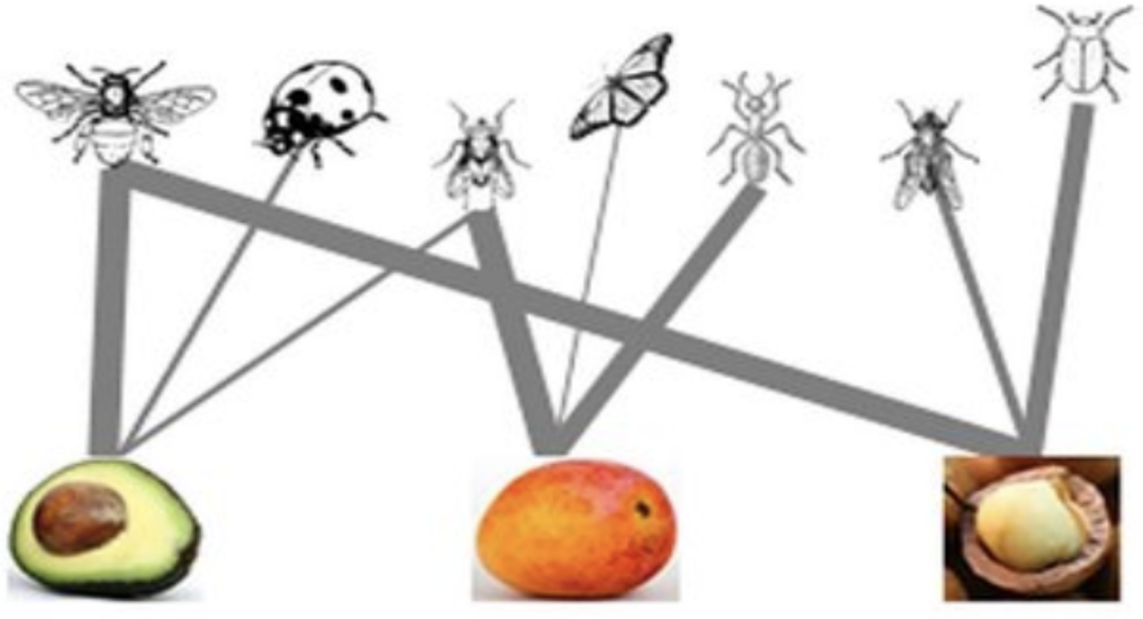


One mode networks



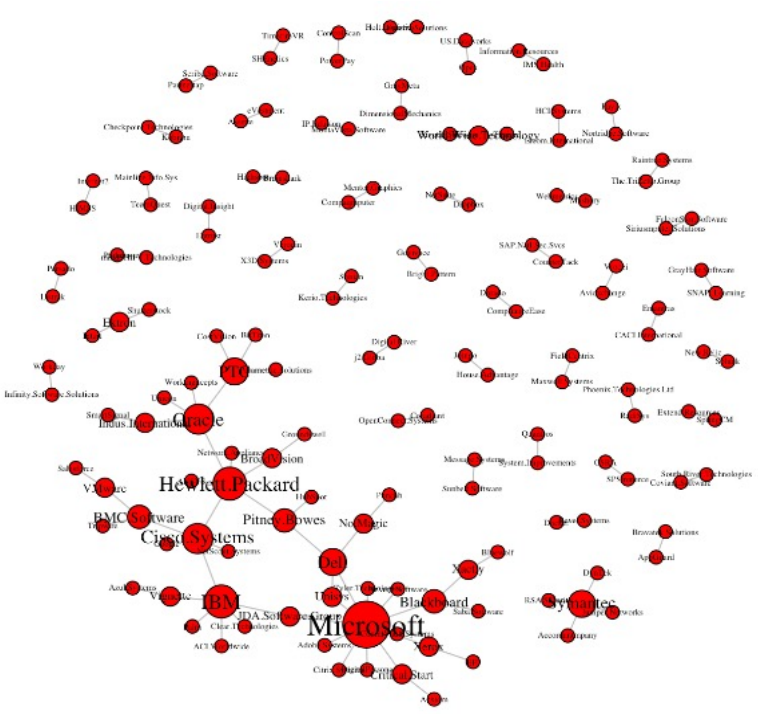
Dolphins swimming network From:
https://studentwork.prattsi.org/infovis/labs/mapping-the-social-structure-of-dolphins-using-gephi/

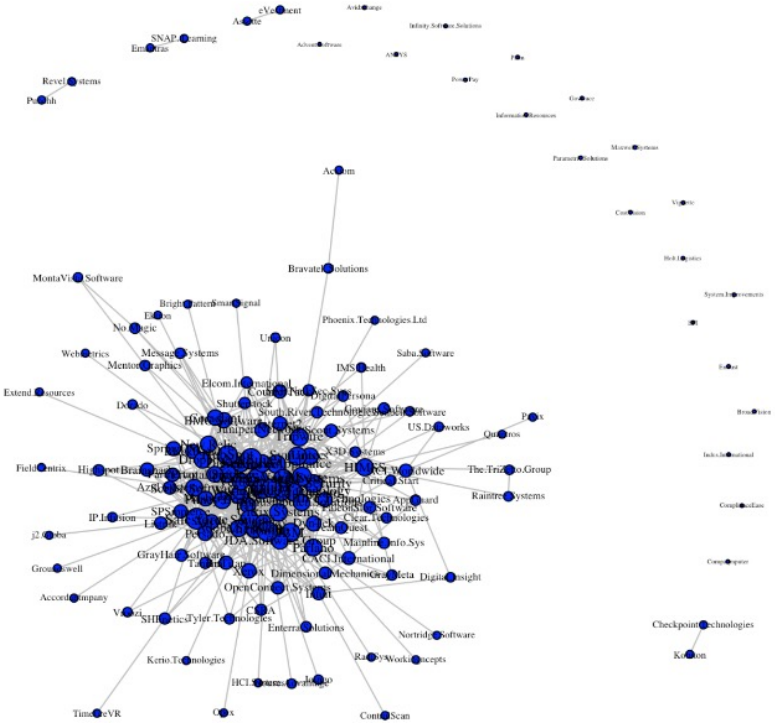
Two mode networks



Pollination network
Willcox et al. 2019

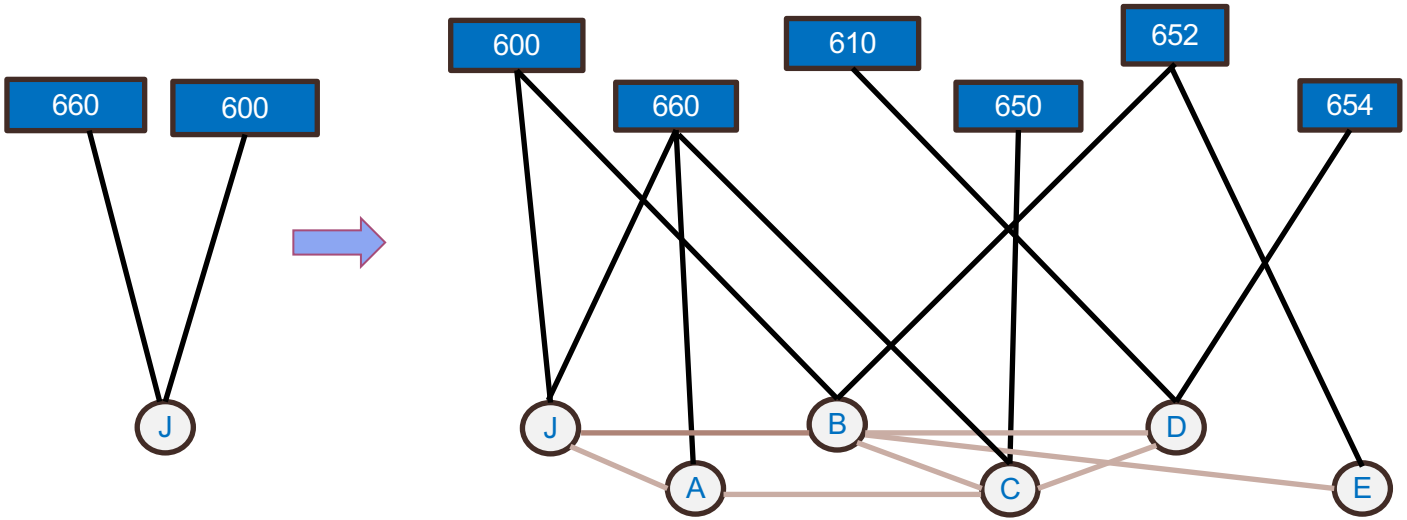
Multiplex networks





Multilevel networks

- Students and classes



How to represent networks?

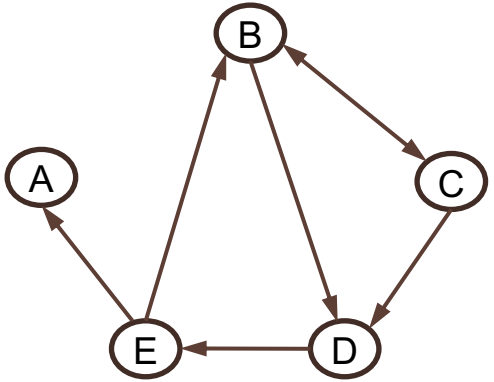
- Adjacency matrix
- Edgelist

Adjacency matrices

- Represent edges (who is adjacent to whom) as a matrix
  - A_{ij} = 1 if node *i* has an edge to node *j*; A_{ij} = 0 is node *i* doesn't have an edge to node *j*.

  - A_{ii} = 0 unless the network has self loops.
  - $A_{ij} = A_{ji}$ if the network is undirected or *i* and *j* share reciprocated edges.

Adjacency matrix example



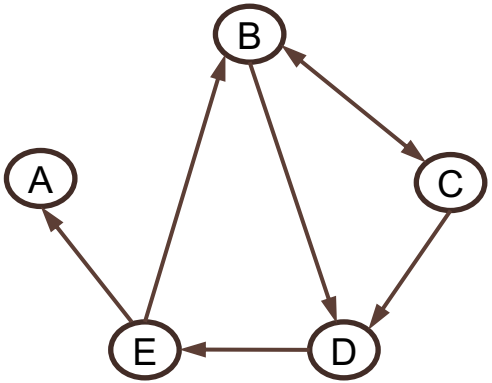
	A	B	C	D	E
A	0	0	0	0	0
B	0	0	1	1	0
C	0	1	0	1	0
D	0	0	0	0	1
E	1	1	0	0	0

How to calculate density?

- Network Density:
  - The number of observed ties/Maximum number of possible ties

	A	B	C	D	E
A	0	0	0	0	0
B	0	0	1	1	0
C	0	1	0	1	0
D	0	0	0	0	1
E	1	1	0	0	0

Node degrees



The outdegree of a node i is the number of nodes that are adjacent from i

Outdegree A = $\{a_{ij}\}$ =

	A	B	C	D	E
A	0	0	0	0	0
B	0	0	1	1	0
C	0	1	0	1	0
D	0	0	0	0	1
E	1	1	0	0	0

$a_{i\ +} = \sum_{j=1}^{n} a_{ij}$

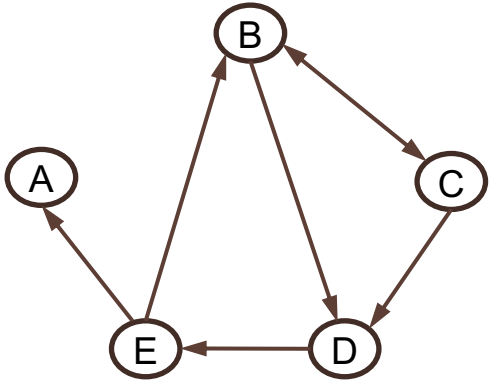
The indegree of a node i is the number of nodes that are adjacent to i

Indegree A = $\{a_{ij}\}$ =

	A	B	C	D	E
A	0	0	0	0	0
B	0	0	1	1	0
C	0	1	0	1	0
D	0	0	0	0	1
E	1	1	0	0	0

$a_{+\ j} = \sum_{i=1}^{n} a_{ij}$

Edge-list example



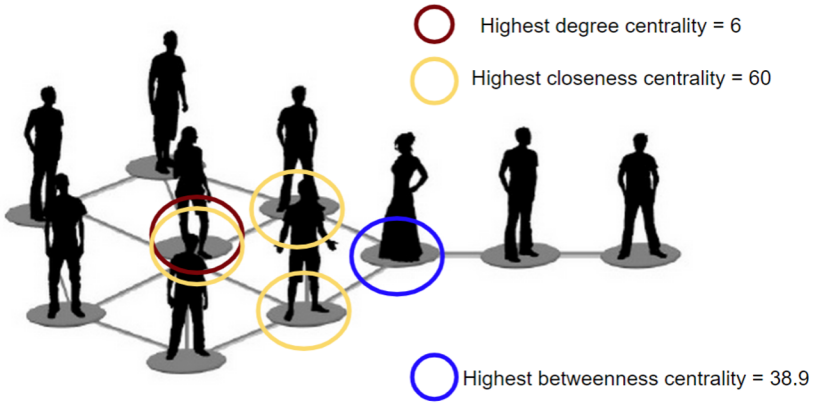
- B, C
- B, D
- C, B
- C, D
- D, E
- E, A
- E, B

Local measures

Centrality

Several types of centrality measurements:

- Degree centrality
- Closeness centrality
- Betweenness centrality
- Eigenvector centrality



Krackhardt kite graph. Image source: Slides by Hagen, 2018

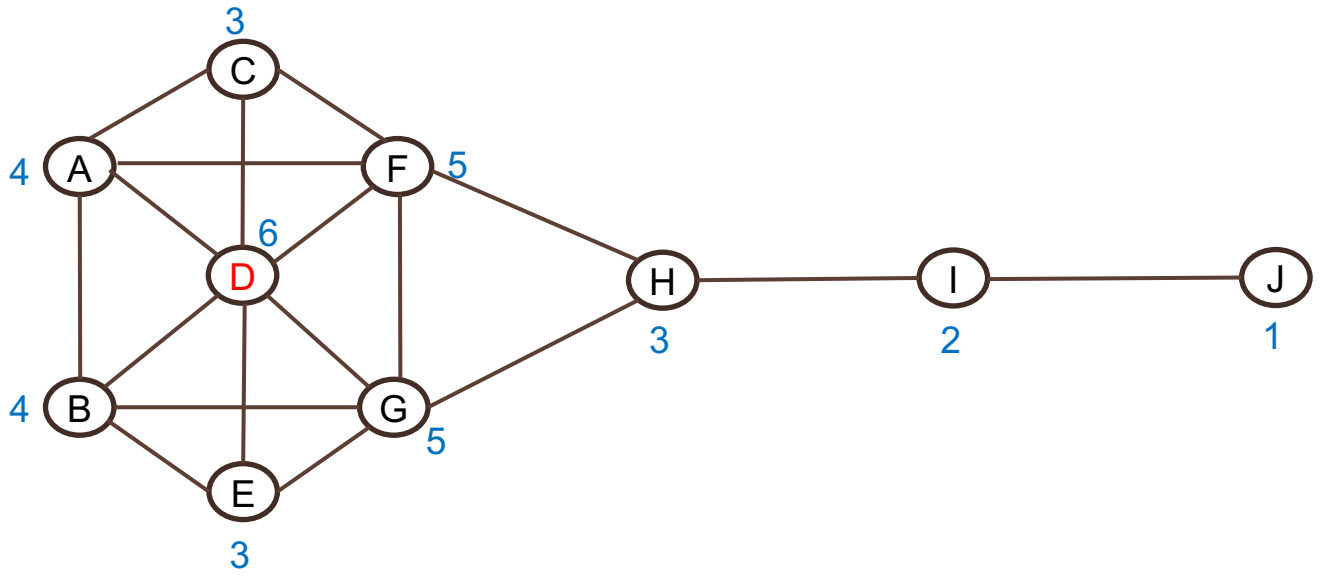
Degree centrality

"Being central means having ties to many others."

- Degree centrality is the count of all ties an actor has with all other actors:

$$c_i = \sum_{\substack{j=1, \\ j \neq i}}^{n} x_{ij} \qquad \text{or} \qquad c_i = x_{i+}$$

Degree centrality



Degree centrality

- Represents an index of *exposure* to what flows through the network;
- It indicates the opportunity to *influence others*
- but also *be influenced* by others.
- *Centrality and…*
    - *Creativity*
    - *Leadership*
    - *Job satisfaction*
    - *Task performance*

Closeness centrality

"Being well-connected means being able to reach all others quickly / be reached by others quickly."

- E.g., *gossip* – central actors hear any rumour quickly!
- Various definitions(!)
    - E.g., inverse of the sum of social distances to everybody else.
    - (occasionally computed as eigenvector centrality based on distance matrix)
- ***Work here with "sum of distances to all other nodes"*** (which is a negative measure of closeness, actually: "farness").

Closeness centrality

- “Central” nodes in a closeness are important, as they can reach (and be reached by) the whole network more quickly than non-central nodes
- Importance measured by how close a node is to other nodes

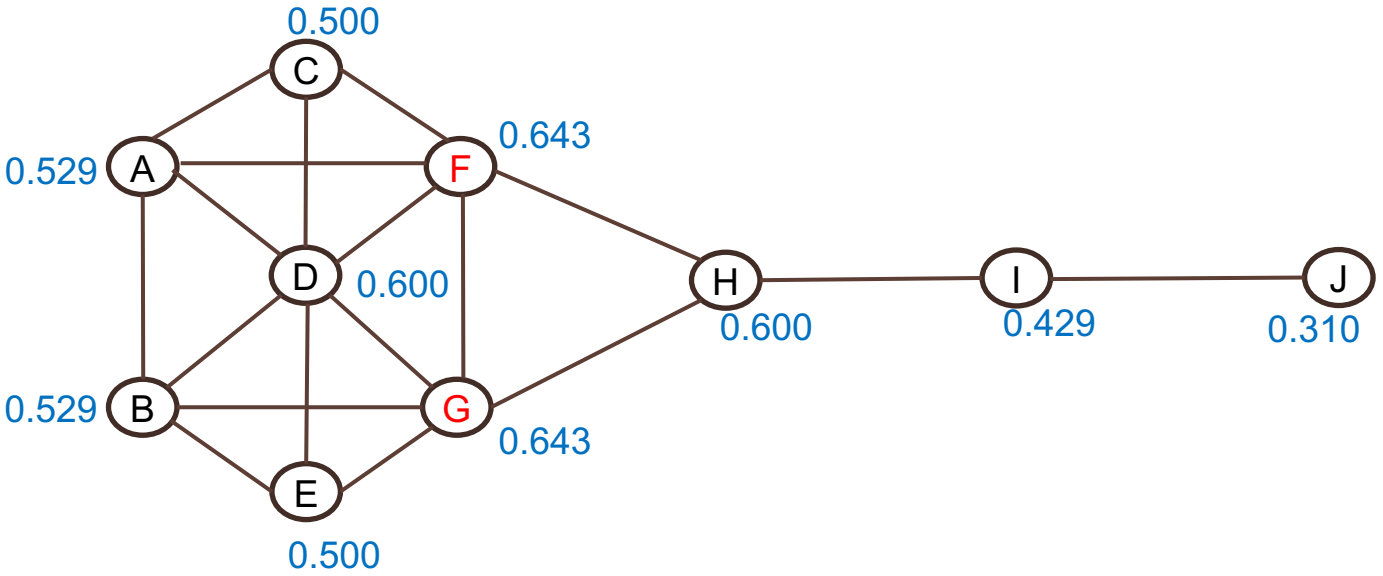
- Average Distance:

$$D_{avg}(v_i) = \frac{1}{n-1} \sum_{j \neq i}^{n} g(v_i, v_j)$$

- Closeness Centrality

$$C_C(v_i) = \left[\frac{1}{n-1} \sum_{j \neq i}^{n} g(v_i, v_j) \right]^{-1} = \frac{n-1}{\sum_{j \neq i}^{n} g(v_i, v_j)}$$

Closeness centrality



Betweenness centrality

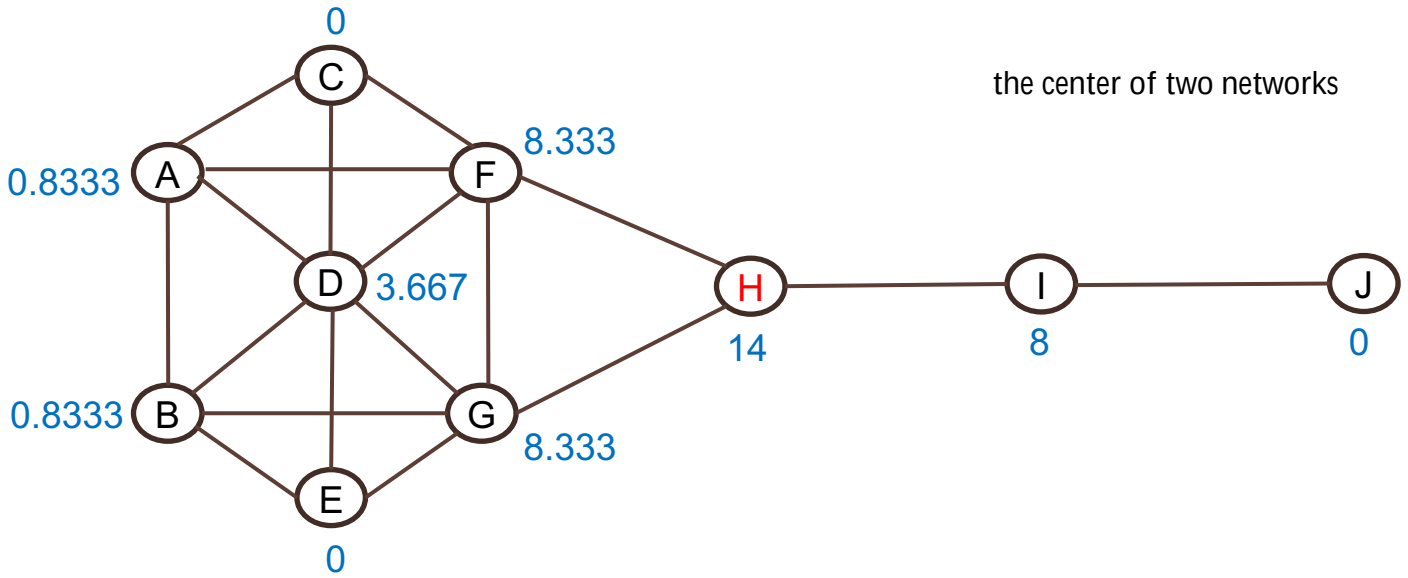
"Being well-connected means being able to hinder (or at least complicate) communication between two others."

- Sum of fractions of shortest paths between any two nodes that pass through a given node;
- Formula:

$$c_i = \sum_{\substack{j=1,\\ j\neq i}}^{n} \sum_{\substack{k=1,\\ k\neq i,j}}^{n} \frac{g_{jk\sim i}}{\tilde{g}_{jk}}$$

Here $g_{jk\sim i}$ is the number of shortest paths from j to k passing through i, while g_{jk} is the total number of shortest paths from j to k (others might not pass through i).

Betweenness centrality



Betweenness centrality

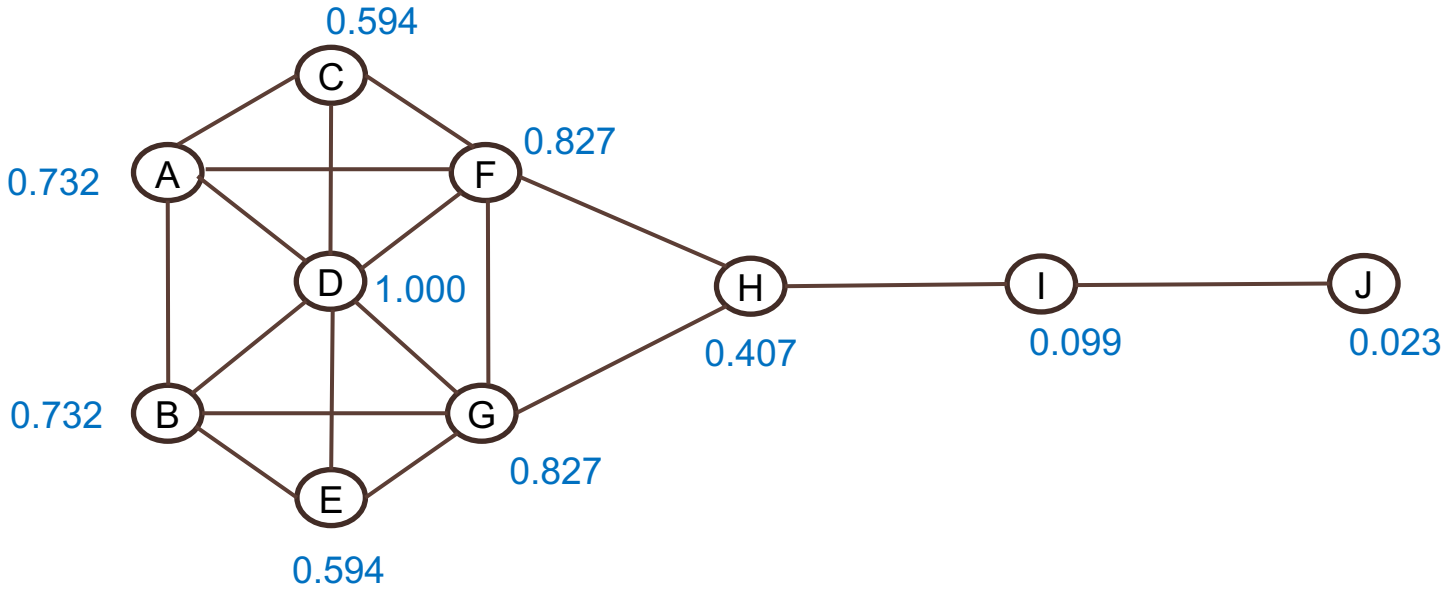
- The betweenness of a node counts the number of shortest paths that go through that node
- **Intuition**: how many pairs of individuals would have to go through you in order to reach one another in the minimum number of hops?
- Global network cohesion and connectivity are dependent on nodes connecting different parts (components) of the network
- Betweenness is typically associated with “influence,” “control” and the possibility of having access to information that is not generally shared

Eigenvector centrality

"Being well-connected means having ties to well-connected others." others will interpret as value.

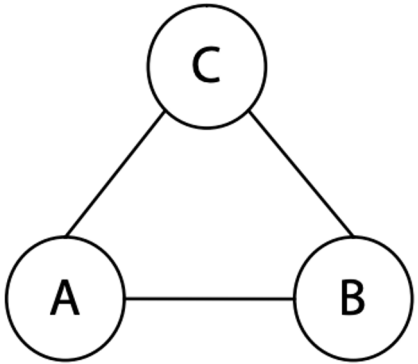
- Self-referencing definition:
  - Your centrality is proportional to the sum of your network neighbours' centrality.
  - Formula: $$c_i = \lambda \sum_{\substack{j=1, \\ j \neq i}}^{n} x_{ij} c_j$$
  - Here λ is the proportionality factor.
  - Mathematically, λ is the first eigenvalue of the adjacency matrix (x_{ij}) and the vector of centralities is the corresponding eigenvector (hence the name).

Eigenvector centrality

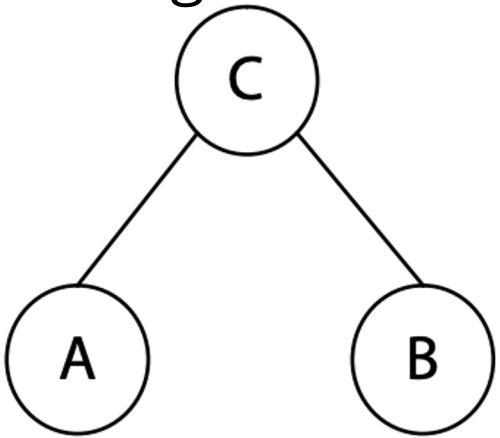


Social Capital and Networks

Closure



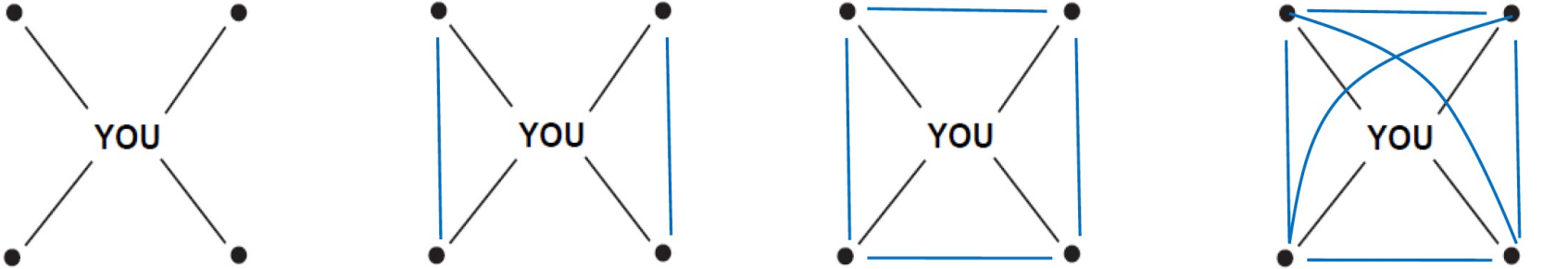
Brokerage



Borgatti, Jones, & Everett, 1998

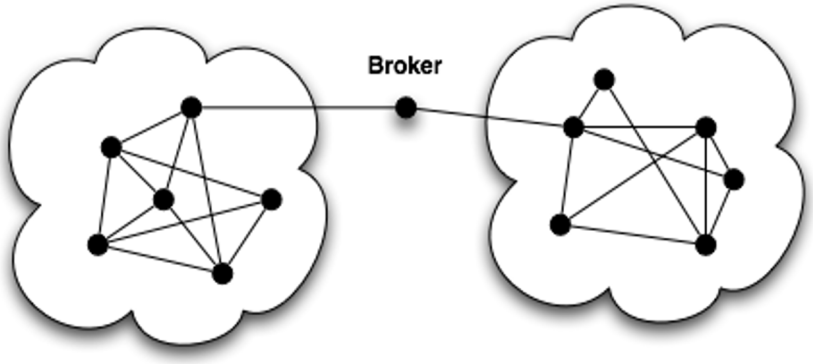
Ties among alters constrain ego

Increasing : constraint, redundancy
Decreasing: autonomy, opportunities



Network brokerage leads to good ideas

Network brokerage offers opportunity to exploit information advantage and access to novel information.



Artists' communication network and novelty

How structural holes can help entrepreneurs?

Animal + Ramen =

Threadless, Design by Vincent Trinidad Art

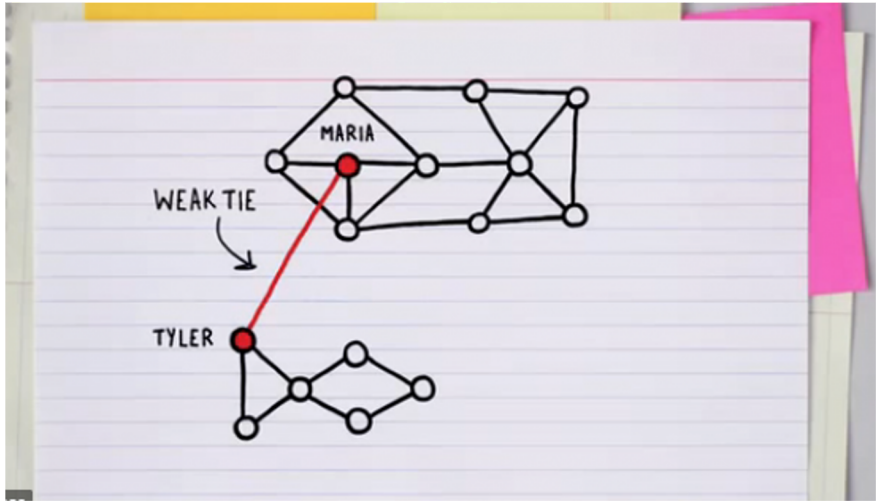
Weak Ties and Finding a Job (Granovetter, 1973)

A strong tie is someone you know well and a weak tie is someone you know but the interaction between you two is minimum.

No strong tie is a bridge.

Is it still true in the digital context?

Garg & Telang (2018) found weak ties from SNS are not effective leading to substantial job seeking outcomes.



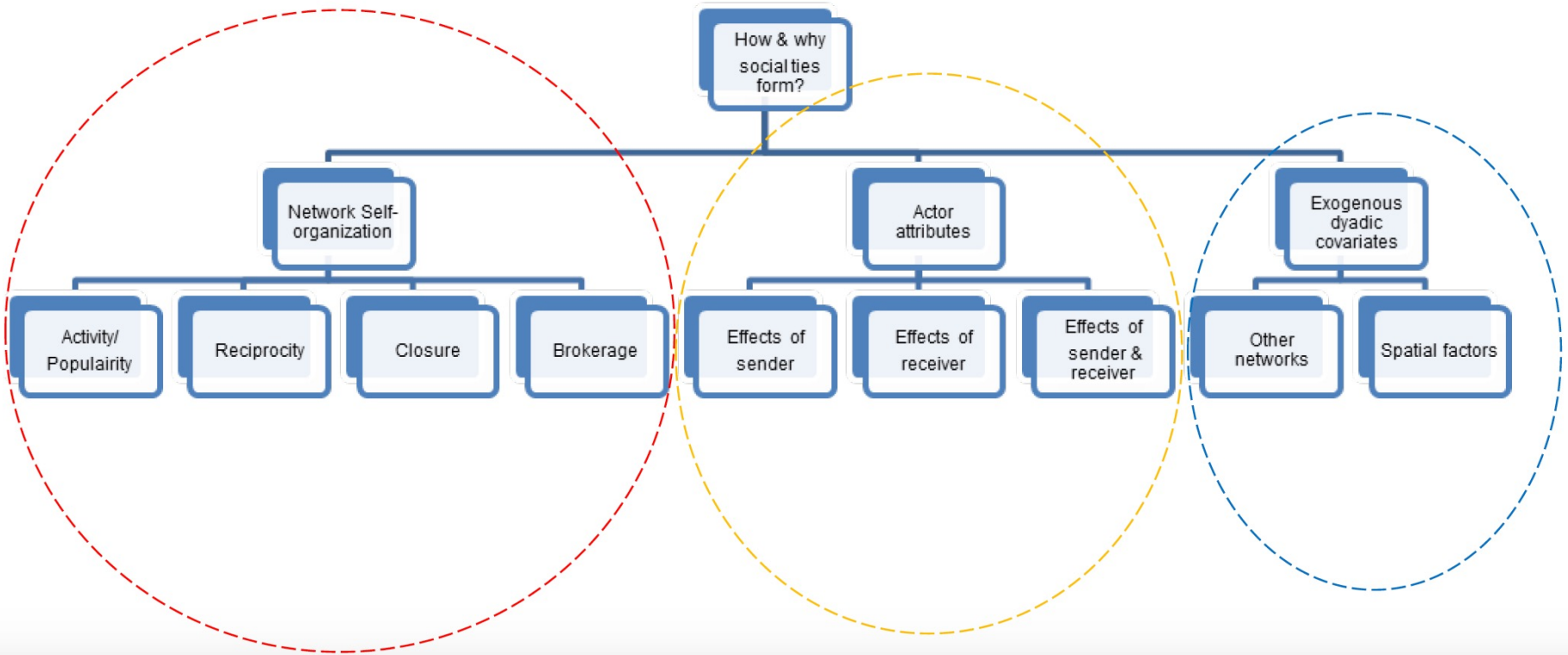
Network closure leads to trust

Network closure is conducive to trust, support and enforcement of social norm.

Eg1: Network closure and trust.

Eg2: Network closure and monitoring.

Where do networks come from?

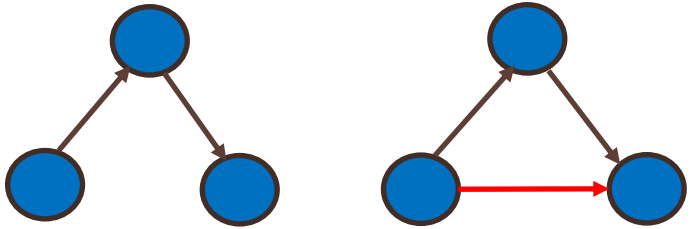

How & why social ties form?
Network Self-organization
Actor attributes
Exogenous dyadic covariates
Activity/ Populairity
Reciprocity
Closure
Brokerage
Effects of sender
Effects of receiver
Effects of sender & receiver
Other networks
Spatial factors

Network self-organization: dependence mechanisms

- Reciprocation

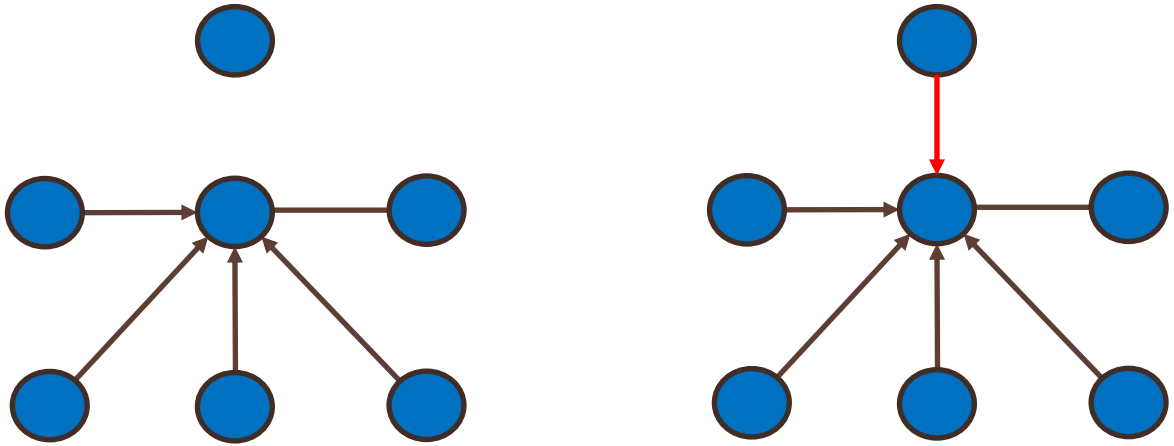
Network self-organization: dependence mechanisms

- Closure



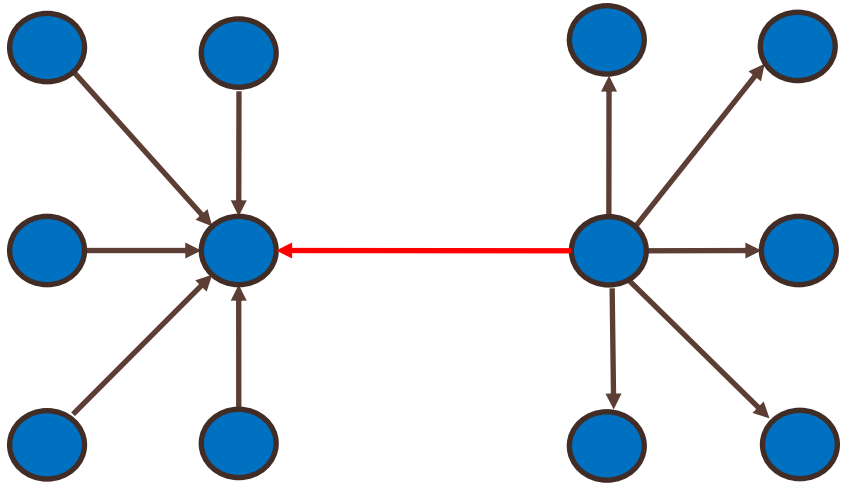
Network self-organization: dependence mechanisms

- Preferential attachment



Network self-organization: dependence mechanisms

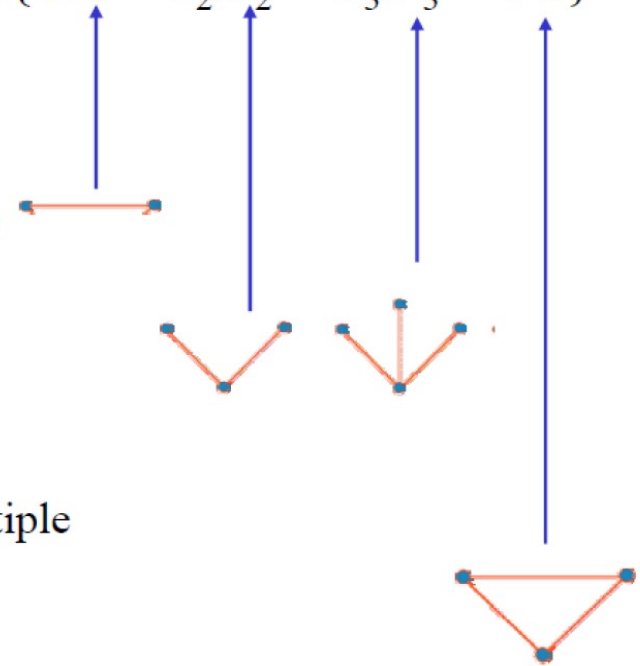
- Assortativity



A Markov random graph model:

Nondirected networks

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) = (1/\kappa)\exp\{\theta L + \sigma_2 S_2 + \sigma_3 S_3 + \tau T\}$$



- *Edge parameter (θ)*
  - L … number of edges
- *Star parameters (σ_k)*
  - Propensities for individuals to have connections with multiple network partners
- *Triangle parameter (τ)*
  - represents clustering

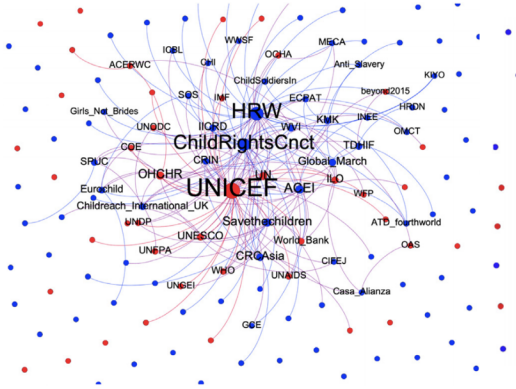
If θ is the only nonzero parameter, this is a Bernoulli random graph model.

Actor attributes: Birds of a Feather?

- “Similarity breeds connection.” – McPherson et al., 2001
- Homophily is a powerful organizing principle.
  - Race, gender, age, religion, education, geography, network positions, attitudes, etc.
- Beyond homophily
  - Heterophily: complementary skills
  - Aspiration: attracted to high values in capability (skills, expertise, information, social status, etc)

Networks on social media

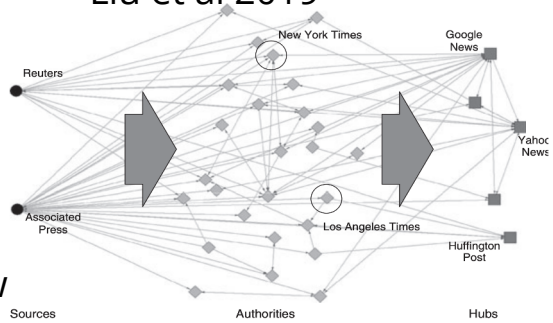
- What are examples of inter-organizational networks on line?
  - Follower-followee
  - Retweet
  - Mention
  - Co-share
  - hyperlinks etc.



Hyperlink Network

Twitter Network

NGO networks
Liu et al 2019



Hyperlink network of news flow
Weber & Monge 2011

Figure 5 Visualization of overall network.

Misinformation diffusion actor-URL network